Analytical solutions for geosynthetic tube resting on rigid foundation

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Abstract. Geosynthetic tubes inflated with water, clay slurry or sand have been widely used for large dike construction in land reclamation projects. In this paper, analytical solutions for geosynthetic tube resting on rigid foundation is presented by adopting an approach similar to that presented by Leshchinsky *et al*. (1996). The proposed method allows a quick preliminary design to be made for using a closed-form solution. To simplify the analysis, relationships between geometrical parameters and pumping pressure are established using numerical method. The analytical solutions were compared with several existing solutions and good agreements were achieved.

Keywords: geosynthetic tube; geomembrane tube; geotextile tube

1. Introduction

Geosynthetic tubes inflated with water, clay slurry, sand or waste sludge have been used to form dikes or breakwaters. Using this method, a dike can be constructed using either single or multiple layers of geosynthetic tubes (Kazimirowicz 1994, Miki *et al*. 1996, Leshchinsky *et al*. 1996, Oh and Shin 2006, Yan and Chu 2010). Geosynthetic tubes have also been adopted for other applications such as, increasing the height of existing dams or spillways (Perry 1993), diverting water for irrigation (Tam 1997), breakwaters in beach (Alvarez *et al*. 2007), flood control (Biggar and Masala 1998, Fowler 1997, Plaut and Suherman 1998), coastal erosion prevention (Shin and Oh 2007), groundwater recharging and control, sediment and industrial sludge dewatering (Worley *et al*. 2008, Yee *et al*. 2012) and optimization of the water level while maintaining a minimum flow over the weir at all times (Sehgal 1996).

Geosynthetic tubes filled with water or slurry have been analyzed by a number of researchers (Silvester 1986, Leshchinsky *et al*. 1996, Kazimirowicz 1994, Plaut and Suherman 1998, Malík 2009, Ghavanloo and Daneshmand 2009, Cantré and Saathoff 2010, Chu *et al*. 2011, Guo *et al*. 2011, 2013). However, most of the above solutions require the running of a computer program.

This is not convenient for preliminary design where some trial and error processes or parametric studies are involved in the selection of the dimensions of the geosynthetic tubes and types of geotextiles to be used.

In this paper, a numerical method, closed-form solution and coefficient method are proposed to analyze the geometry parameters of geosynthetic tube by assuming the tensile forces along their cross-sectional circumference were constant. Therefore, the proposed solutions are only applicable to the impermeable geosynthetic tubes filled with slurry/water. However, it may also be suitable to permeable geotextile tubes at the state after the filling and dewatering is completed. The proposed method allows a preliminary design to be carried out on geosynthetic tube before more sophisticated numerical analyses are carried out if necessary.

2. Basic assumptions

The following assumptions were made in deriving the closed-form solutions:
(1) The geosynthetic tube is sufficiently long to be assumed as a plane strain problem;
(2) The geosynthetic shell is thin and flexible so that its weight and extension can be neglected;
(3) Frictions between the geosynthetic tube and the fill material, or that between the geosynthetic tube and the rigid foundation are neglected;
(4) The tensile force along the geosynthetic sheet is constant;
(5) All the geosynthetic tubes are inflated with the same material and no external water pressure is applied.

The above assumptions simulates closely a rubber dam application where water or air is used to inflate an impervious tube or a permeable tube filled with sand after it has been completely consolidated under the pumping pressure. Some of the above assumptions were also adapted in the existing analytical solutions for geosynthetic tubes (Szyszkowski and Glockner 1987, Kazimierowicz 1994, Leshchinsky *et al*. 1996, Plaut and Suherman 1998, Cantré and Saathoff 2010, Malik and Sysala 2010, Cantré and Saathoff 2010).

3. Analytical solutions

As the problem is symmetric, only half of the simplified cross-section of a geosynthetic tube is needed as shown in Fig. 1. The geosynthetic tube is assumed to be inflated with only one type of liquid or slurry material with a unit weight of γ. The coordinates are set up with x in the vertical direction and y in the horizontal direction. The origin of the coordinates is taken as the top point of the cross-section. The width of the cross-section is written as B. The height of the geosynthetic tube is denoted as H. The contact width with ground surface is presented as b. The tensile force along the geosynthetic tube per unit length is denoted as T. The free body diagram of this half cross-section is plotted in Fig. 1. Due to symmetric of the cross-section, the of tensile forces direction on top and bottom surface are horizontal. The forces acting on horizontal direction only involve the hydraulic pressure and the tensile force as shown in Fig. 1. Similarly, the expression of tensile force can be derived as shown in Eq. (1). It can be seen that the tensile force along the geosynthetic tube is related to the height of the cross-section, the pumping pressure and the unit weight of filling slurry

$$T = \left(p_0 H + \frac{1}{2}\gamma H^2 \right) \Big/ 2 \tag{1}$$

An infinite small curve with a length of ds around an arbitrary point $S(x, y)$ can be treated as an arc with the center at random point C and radius of r as shown in Fig. 2. Then four geometrical equations relating the angle θ and the x and y coordinates are given in Eqs. (2) and (5).

$$\frac{y'}{\sqrt{1+y'^2}} = \sin\theta \tag{2}$$

$$y' = \frac{dy}{dx} = \tan\theta \tag{3}$$

$$\frac{dT}{ds} = 0 \tag{4}$$

$$\frac{d\theta}{ds} = \frac{1}{r} = \frac{1}{T}\left[P_0 + \gamma x \right] \tag{5}$$

A free body diagram for a section from point O to a point $S(x,y)$ on the cross-section is used for force equilibrium analysis as shown in Fig. 3. The angle between the tangent direction at point $S(x,y)$ and the x axis is denoted as θ. The hydraulic pressure acting internally on this point $S(x,y)$ is $p_0 + \gamma x$ where p_0 is the pumping pressure. For the free-body shown in Fig. 3, the forces equilibrium along the horizontal direction is written as Eq. (6). By deriving from Eq. (6), the expression of $\sin\theta$ can be obtained as shown in Eq. (7).

$$T - T\sin\theta = \int_0^S \left(p_0 + \gamma x \right)ds\sin\theta = \int_0^S \left(p_0 + \gamma x \right)dx = p_0 x + \frac{1}{2}\gamma x^2 \tag{6}$$
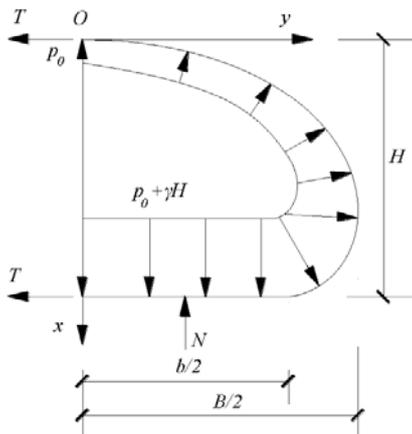
Fig. 1 Free body diagram of half cross-section   Fig. 2 The free body diagram of infinite unit
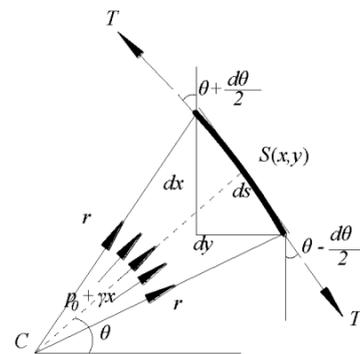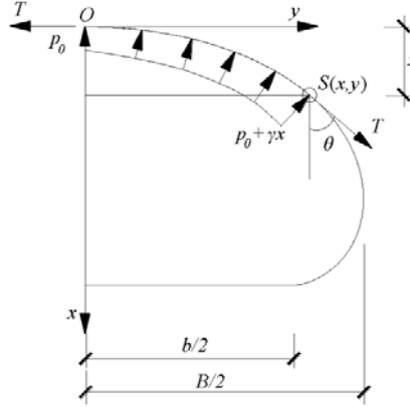
Fig. 3 Free body diagram of curve OS

$$\sin\theta = 1 - \left(p_0 x + \frac{1}{2}\gamma x^2 \right)\Big/T \tag{7}$$

Deriving from Eq. (7) obtains the solution of the *x*-coordinate shown as follows (as *x* > 0, the negative result is omitted)

$$x = \frac{1}{\gamma}\left[-p_0 + \sqrt{p_0^2 + 2\gamma T(1-\sin\theta)} \right] \tag{8}$$

$$\frac{dy}{d\theta} = -\frac{T\sin\theta}{\sqrt{p_0^2 + 2\gamma T(1-\sin\theta)}} \tag{9}$$

Integrating *y* in Eq. (9) with respect to θ using the boundary condition, $\theta = 0$, *y'* = 0, we have the *y*-coordinate of the geosynthetic tube.

$$y = -\sqrt{\frac{T}{2\gamma}}\int\left(\sqrt{Q-\sin\theta} - \frac{Q}{\sqrt{Q-\sin\theta}} \right)d\theta \tag{10}$$

and

$$Q = 1 + p_0^2 / 2\gamma T \tag{10a}$$

where *Q* is denoted as the factor of pumping pressure. It has no unit.

Knowing the unit weight of the filling slurry, γ, the pumping pressure, p_0, the height, *H*, or the perimeter, *L*, of cross-section, and combining with boundary conditions $x = 0$, $\theta = \pi/2$ and $x = H$, $\theta = -\pi/2$, the cross-section and tensile force can be calculated.

4. Calculation procedure

Eq. (10) contains the first and second elliptic integrals which have no closed-form solutions. A computer program was written using the adaptive Runge-Kutta-Merson method (RKM4) to solve

Eqs. (1), (7), (8) and (10). The standard programs for the RKM4 method had already been introduced by Press *et al*. (2007), Xu (1995), Christiansen (1970) and Lukehart (1963). The calculation was carried out using the following inputs parameters, the unit weight of the slurry, γ, the pumping pressure, p_0, the height, H, or the perimeter, L. The unknown parameters were searched by the trial-and-error method outlined as follows:

(1) Input the initial parameters: γ, p_0, H;
(2) Calculate the tensile force, T, using Eq. (1) and the factor of pumping pressure, Q, using Eq. (10a);
(3) Calculate $\sin\theta$ using Eq. (7);
(4) Solve Eqs. (8) and (10) using the RKM4 method;
(5) If the perimeter of the geosynthetic tube, L, is taken as an input rather than the height, H, the iteration can be done as follows: assume $H_t = L/\pi$ as the input height to calculate the trial length, L_t. If $L_t \neq L$, then modify H_t and repeat the calculation until the difference between L_t and L is less than 1E-6.

5. Closed-form solutions

As mentioned in above, the Eq. (10) contains the first and second elliptic intervals and thus has no closed-form solutions. The equation has to be solved by numerical method. However, by making an approximate, it is possible to derive an approximate closed-form solution in a closed-form format. In this paper, the following approximation is adopted to solve this problem.

$$\sqrt{Q-\sin\theta} \approx \sqrt{Q} - \left(\sqrt{Q} - \sqrt{Q-1}\right)\sin\theta \tag{11}$$

A comparison between the real value and the approximation is shown in Fig. 4. It can be seen that for $Q \geq 2$, the two curves are very close. Therefore, Eq. (11) is applicable when $Q \geqslant 2$.

By substituting Eq. (11) into Eq. (10) and integrating y with respect to θ using the boundary condition of $x = 0$, $y = 0$ and $\theta = \pi/2$, the final geometry equation of the cross-section is written as follows

$$y = \sqrt{\frac{T}{2\gamma}\left(\begin{array}{c}\sqrt{Q}\left(\theta - \dfrac{\pi}{2}\right) + \left(\sqrt{Q} - \sqrt{Q-1}\right)\cos\theta \\[2mm] + \dfrac{2Q}{\sqrt{1-Q+2\sqrt{Q(Q-1)}}}\tan^{-1}\left(\sqrt{\dfrac{2\sqrt{Q}-\sqrt{Q-1}}{\sqrt{(Q-1)}}}\tan\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)\right)\end{array}\right)} \tag{12}$$

The parameters related to geometry such as the width, B, the perimeter, L, and the area, A, of the cross-section can be calculated using Eq. (12). $B = 2y_{max}$. When $y = y_{max}$, $dy/d\theta = 0$ and $\theta = 0$. Thus B can be calculated from Eq. (12) for $\theta = 0$ which is re-written as Eq. (13). The contact width with ground b is calculated when $x = H$ or when $\sin\theta = -1$ or $\theta = -\pi/2$. Submitting these values into Eq. (12), b can be calculated as shown in Eq. (14). Similarly, L can be calculated by Eq. (15). Based on Fig. 3, the forces equilibrium on the contact edge between geosynthetic tube and the rigid foundation gives $\gamma A = N$ and $N = (p_0 + \gamma H) b$. Then the area of cross-section, A, is calculated as shown in Eq. (16). It should be point out that these equations were derived based on the approximation shown in Eq. (11). Thus Eqs. (13) to (16) are only applicable when $Q \geq 2.0$.
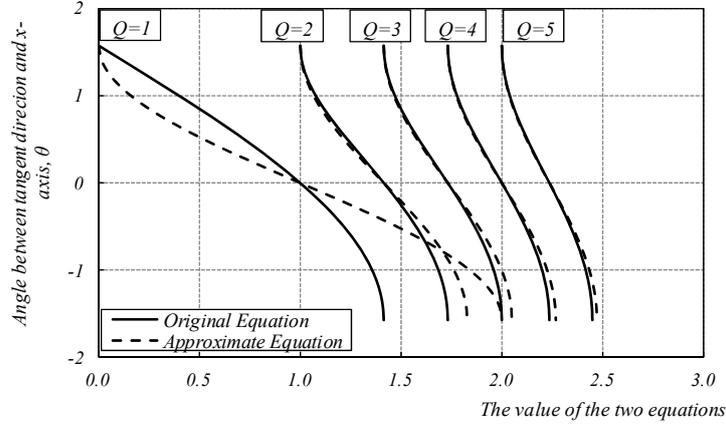
Fig. 4 Comparisons between the two equations

$$B = \sqrt{\frac{2T}{\gamma}}\left[\left(1-\frac{\pi}{2}\right)\sqrt{Q}-\sqrt{Q-1}+\frac{2Q}{\sqrt{1-Q+2\sqrt{Q(Q-1)}}}\tan^{-1}\left(\sqrt{\frac{2\sqrt{Q}-\sqrt{Q-1}}{\sqrt{(Q-1)}}}t\right)\right] \qquad (13)$$

$$b = \sqrt{\frac{2T}{\gamma}}\left(-\pi\sqrt{Q}+\frac{\pi(Q+1)}{\sqrt{1-Q+2\sqrt{Q(Q-1)}}}\right) \qquad (14)$$

$$L = 2\int_{\frac{\pi}{2}}^{-\frac{\pi}{2}}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta + b = \sqrt{\frac{2T}{\gamma}}\left(-\pi\sqrt{Q}+\frac{\pi(Q+1)}{\sqrt{1-Q+2\sqrt{Q(Q-1)}}}\right) \qquad (15)$$

$$A = \frac{P_0+\gamma H}{\gamma}b \qquad (16)$$

6. Coefficient methods

The closed-form equations given in Eqs. (13) to (16) are convenient. However, they are only applicable when $Q \geqslant 2.0$. To overcome this limitation, a coefficient method is also developed to establish some relationships similar to Eqs. (13) to (16) using numerical method. Eqs. (13) and (16) reveal that the geometric parameters, B, b, L and A are all related to a function of Q and $\sqrt{(2T/\gamma)}$. Thus Eqs. (13) to (16) can also be written as

$$B = f_1(Q)\sqrt{\frac{2T}{\gamma}} = C_B\sqrt{\frac{2T}{\gamma}} \qquad (17)$$

$$b = f_2(Q)\sqrt{\frac{2T}{\gamma}} = C_b\sqrt{\frac{2T}{\gamma}} \tag{18}$$

$$L = f_3(Q)\sqrt{\frac{2T}{\gamma}} = C_L\sqrt{\frac{2T}{\gamma}} \tag{19}$$

where C_B, C_b and C_L are factors related to Q.

If some relationships between the three geometry factors (B, b, L) and Q can be established, then Eqs. (17) to (19) can be used for preliminary design. For this purpose, some parametric studies were carried out using the computer program described in Section 3. Relationships between C_B, C_b and C_L and Q are established through the use of the computer program as shown in Fig. 5. In developing these charts, the factor of pumping pressure Q was selected in the range of 1.0 to 10.0 which will satisfy the general case of geosynthetic tube design. The values of the factor of pumping pressure (Q) and the three geometry parameters (B, b, L) were also presented in Fig. 6. It can be seen that the Q value were separated into 25 intervals. If the calculated Q just equal to the endpoint, the value of B, b, L can easily be calculated. However, if the required Q value locates between the endpoints, the linear interpolation method can be used by assuming their value are linear in the intervals. The calculation procedure can be further simplified using Microsoft Office
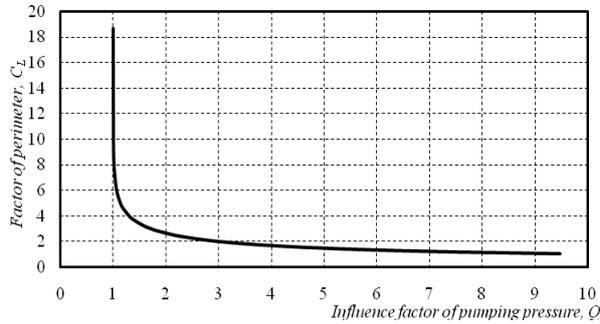
(a) Q versus C_L curve
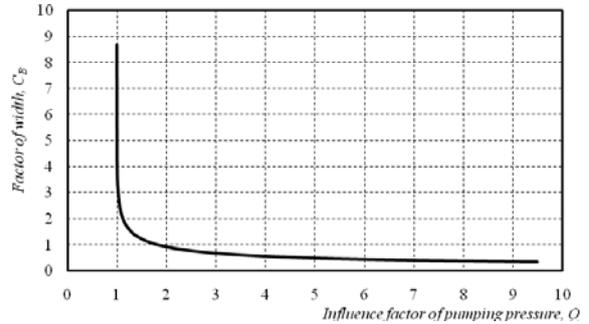
(b) Q versus C_B curve
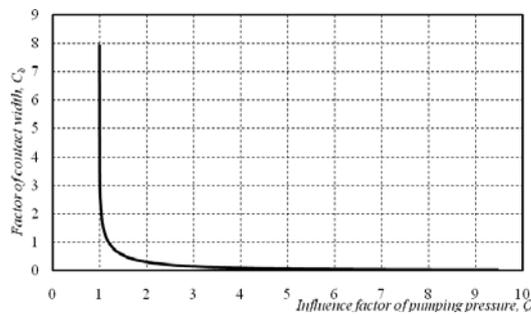
(c) Q versus C_b curve

Fig. 5 Designing charts method

linear interpolation method can be solved automatically by a subroutine, such as micro function Excel as shown in Fig. 6. The advantage of using Excel is that the calculation of Q value using *Qmatch*() shown in Fig. 6. Generally, there are two cases of calculation based on the inputs:

(1) When H, γ and p_0 are taken as inputs, the calculation can be carried out directly using the calculation procedure as shown in Fig. 6.
(2) When L, γ and p_0 are taken as inputs, the "Goal Seek" function in Microsoft Office Excel 2010 (Menu/Data/What-if Analysis/Goal Seek) can be used to search the value of L to the desired magnitude by changing H.

7. Comparisons with existing solutions

In order to verify the accuracy of the proposed methods, the numerical method (NM), the closed-form method (CFM) and the coefficient method (CM) are compared with the solutions given by Leshchinsky *et al.* (1996). Leshchinsky *et al.*'s method (1996) has been coded to a computer program (GeoCoPS) by Leshchinsky and Leshchinsky (1996) and calibrated against a case by using $L = 9$ m and $\gamma = 12$ kN/m³ as inputs. The same case is used for comparison and the results are given in Table 1. The percentage differences between each method proposed in this paper and that of Leshchinsky *et al.* (1996) are given in Table 1. It can be seen that for the NM and CM methods, the maximum difference is only 5.89%. The results of tensile force and height of cross-section calculated by CFM method have little difference with those from Leshchinsky's results. However, the areas of cross-sections and contact width with ground have large difference especially when pumping pressure becomes greater than 34.5 kPa.
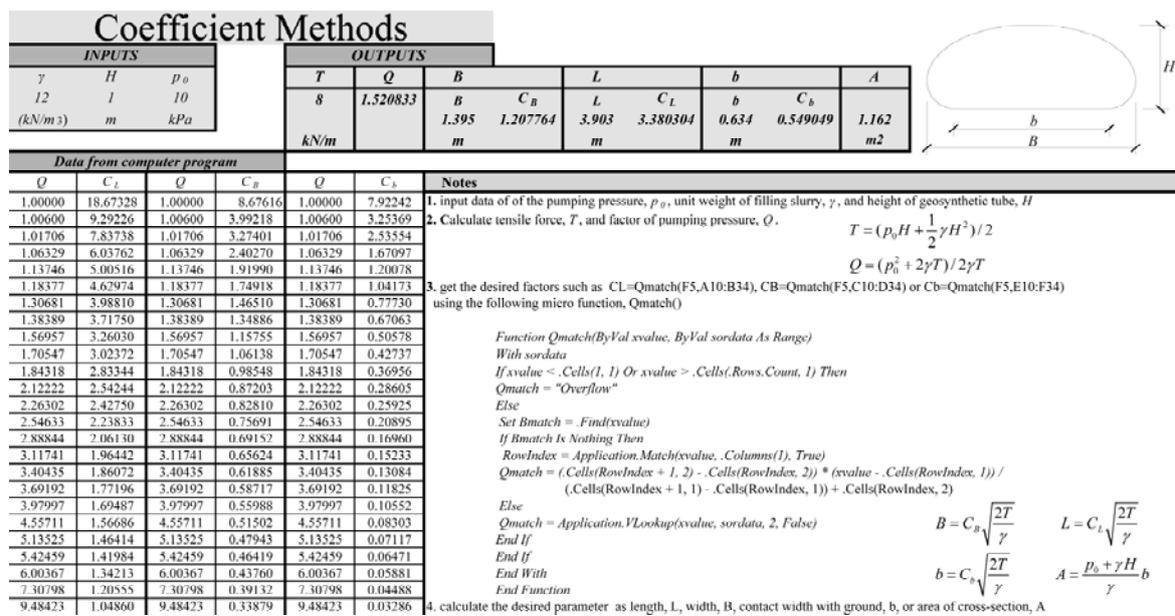

Fig. 6 Designing charts method (Guo 2012)

Table 1 Comparison of the proposed methods with that by Leshchinsky *et al.* (1996)
(For $L = 9$ m, $\gamma = 12$ kN/m^3)

p_0(kPa)	Source	H (m)		B (m)		A (m^2)		T (kN/m)	
		Values	Diff(%)	Values	Diff(%)	Values	Diff(%)	Values	Diff(%)
4.8	Leshchinsky	1.80		3.60		5.56		14.60	
	NM	1.79	0.56	3.57	0.83	5.35	3.78	13.90	4.79
	CFM	1.73	3.89	4.20	-16.67	4.98	10.43	13.15	9.93
	CM	1.78	1.11	3.57	0.83	5.37	3.42	13.74	5.89
6.9	Leshchinsky	2.00		3.64		5.76		18.10	
	NM	1.90	5.00	3.49	4.12	5.56	3.47	17.60	2.76
	CFM	1.90	5.00	4.10	-12.64	5.13	10.94	17.48	3.43
	CM	1.89	5.50	3.50	3.85	5.60	2.78	17.21	4.92
34.5	Leshchinsky	2.50		3.21		6.45		61.70	
	NM	2.40	4.00	3.13	2.49	6.25	3.10	59.80	3.08
	CFM	2.55	-2.00	3.49	-8.72	4.64	28.06	63.63	-3.13
	CM	2.43	2.80	3.13	2.49	6.29	2.48	59.76	3.14
52.4	Leshchinsky	2.60		3.13		6.51		87.50	
	NM	2.50	3.85	3.06	2.24	6.33	2.76	86.00	1.71
	CFM	2.65	-1.92	3.34	-6.71	4.33	33.49	90.6	-3.54
	CM	2.54	2.31	3.06	2.24	6.44	1.08	85.78	1.97
103.5	Leshchinsky	2.70		3.00		6.57		162.00	
	NM	2.70	0.00	2.98	0.67	6.40	2.59	159.80	1.36
	CFM	2.75	-1.85	3.50	-16.67	3.89	40.79	165.25	-2.01
	CM	2.68	0.74	2.98	0.67	6.63	-0.91	159.7	1.42
122.8	Leshchinsky	2.70		2.96		6.57		189.70	
	NM	2.70	0.00	2.96	0.00	6.41	2.44	187.60	1.11
	CFM	2.77	-2.59	3.11	-5.07	3.80	42.16	193.15	-1.82
	CM	2.69	0.37	2.96	0.00	6.55	0.30	187.20	1.32

The proposed methods are also compared with that presented by Kazimierowicz (1994) as shown in Table 2. The same input parameters as those used in Kazimierowicz (1994) were used in the calculation. The same conclusions can be made from this comparison, that is, the solutions by the NM and CM methods are close to those by the Kazimierowicz method (1994) and the solutions from the CFM method were quite different.

8. Parametric studies

Some parametric studies using the numerical method were carried out to investigate the influence of different design parameters. In order to use dimensionless parameters, the height and

Table 2 Comparison of present method with the results proposed by Kazimierowicz (1994)
      (For L = 3.6 m, γ = 14 kN/m³)

p_0 (kPa)	Source	H (m)		b (m)		T (kN/m)	
		Values	Diff. (%)	Values	Diff. (%)	Values	Diff. (%)
17.5	Kazimierowicz	1.00		0.46		11.80	
	NM	0.99	-1.00	0.46	0.00	11.90	0.85
	CFM	1.03	3.06	0.32	-30.22	12.74	7.93
	CM	0.98	-2.00	0.45	-2.17	11.92	1.02
10.4	Kazimierowicz	0.90		0.64		6.80	
	NM	0.90	0.00	0.65	1.56	6.77	-0.44
	CFM	0.97	7.78	0.46	-27.94	8.30	22.04
	CM	0.92	2.22	0.59	-7.81	7.71	13.38
4.6	Kazimierowicz	0.80		0.84		4.00	
	NM	0.81	1.25	0.82	-2.38	4.11	2.75
	CFM	0.83	3.75	0.71	-15.48	4.32	8.00
	CM	0.81	1.62	0.81	-3.57	4.18	4.50
3	Kazimierowicz	0.70		0.96		2.70	
	NM	0.76	8.57	0.89	-7.29	3.03	12.22
	CFM	0.75	7.14	0.85	-11.46	3.09	14.44
	CM	0.74	6.32	0.93	-3.12	3.05	12.96

width of geosynthetic tubes were normalized using the tube perimeter, L, the pumping pressure was normalized using $\gamma_w L$, the tensile force was normalized using $\gamma_w L^2$. The same normalization method had also been used by Plaut and Suherman (1998), Plaut and Klusman (1999), Plaut and Cotton (2005).

The normalized height versus the normalized pumping pressure curve is shown in Fig. 7. It can be seen that the larger the pumping pressure, the higher the cross-section of the tube. However, the height does not increase much when the normalized pumping pressure is greater than 0.3. The normalized cross-section area versus the normalized height curve is shown in Fig. 8. The normalized area increased rapidly with the normalized pumping pressure. However, when the normalized pumping pressure is greater than 0.3, it will have little influence on the area of cross-section anymore. The normalized tensile force versus normalized pumping pressure is shown in Fig. 9. The higher the pumping pressure, the higher the tensile force as expected. To achieve an economic design, the smaller the pumping pressure, the smaller the tensile strength required. On the other hand, the geotextile bag should be inflated as high as possible to reduce the number of bags used to reach the required design height for the dike. Based on Figs. 7-8, it appears that a normalized pumping pressure in the range of 0.2 to 0.3 would be desirable. It should be pointed out that the normalized tensile force will increase exponentially when the normalized height is higher than 0.25 as shown in Fig. 10. As such, the height to the length ratio for a geotextile tube should be controlled to be less than 0.25.
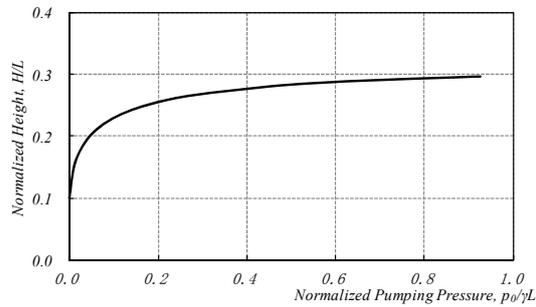
Fig. 7 Normalized pumping pressure vs height curve
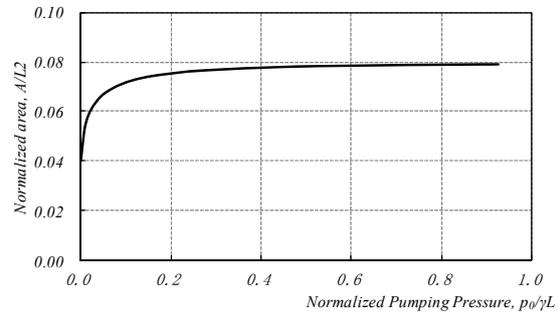


Fig. 8 Normalized pumping pressure vs area curve
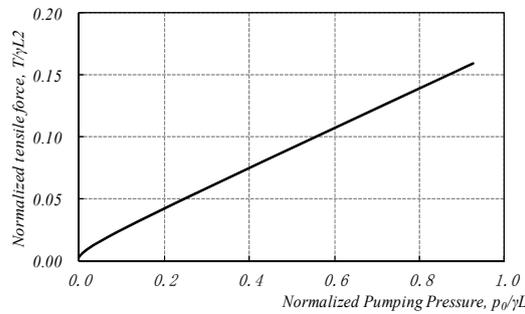
Fig. 9 Normalized pumping pressure vs tensile force curve
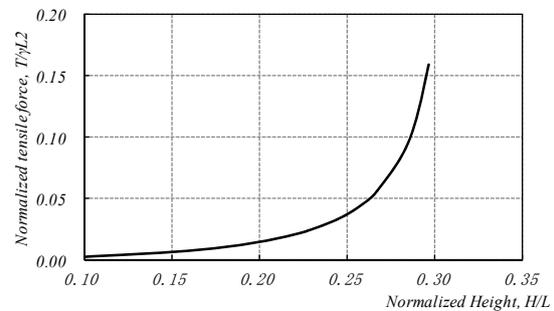
Fig. 10 Normalized height vs tensile force curve

8. Parametric studies

Some parametric studies using the numerical method were carried out to investigate the influence of different design parameters. In order to use dimensionless parameters, the height and width of geosynthetic tubes were normalized using the tube perimeter, L, the pumping pressure was normalized using $\gamma_w L$, the tensile force was normalized using $\gamma_w L^2$. The same normalization method had also been used by Plaut and Suherman (1998), Plaut and Klusman (1999), Plaut and Cotton (2005).

9. Conclusions

In this paper, an analytical method, a closed-form solution and the coefficient method were proposed to analyze the geometric parameters of geosynthetic tube by assuming the tensile forces along their cross-sectional circumference were constant. It should be pointed out that the proposed solutions are only applicable to impermeable geosynthetic tubes filled with slurry/water. However, it may also be suitable to permeable geotextile tubes at a state after the filling or dewatering is completed. The following conclusions can be drawn from this study.

- The proposed analytical solution resolved by numerical method is similar to that presented by Leshchinsky *et al*. (1996). The differences between the results obtained from the numerical method and those by Leshchinsky *et al*. (1996) are less than 6%.
- A closed-form solution was derived for calculation of geosynthetic tube resting on rigid foundation based on an approximation. The closed-form method was only valid when the factor of pumping pressure $Q = 1 + p_0^2/2\gamma T$ is greater than 2.0.
- By generating the relationships established by the closed-form solution using the data obtained from numerical simulations using the analytical solution, the coefficient method was proposed to calculate the geometry parameters of the geosynthetic tube. The coefficient method removes the condition for Q to be greater than 2.0. The results from the coefficient method were also compared with those from Leshchinsky *et al*. (1996) and the differences are less than 6%.
- A parametric study was also carried out. The study indicates that the normalized pumping pressure $p_0/(\gamma_w L)$ should be controlled within the range of 0.2 to 0.3 and the normalized height H_0/L to be controlled to be less than 0.25.

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