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Shape factors of cylindrical permeameters

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Abstract. This paper presents an analytical solution for steady state flow into a close-ended cylindrical permeameter. The soil medium is considered to be uniform, isotropic, and of infinite thickness. Laplace equation is solved by considering rotational symmetry and by using curvilinear coordinates obtained from conformal mapping. The deduced shape factors, which are compared to approximate relationships obtained from both numerical and physical modelling, and idealizations involving ellipsoidal cavities, are proposed for use in field measurements. It is shown that some of the shape factors obtained are significantly different from published values and show a much higher dependence of the rate of flow on the aspect ratio, than deduced from approximate solutions.

Keywords: shape factors; cylindrical permeameters; hydraulic conductivity; analytical solution; infinite medium; constant-head test; comparisons.

1. Introduction

A shape factor is a geometric constant in the interpretation of in situ permeability tests. It depends on the geometry of the test cavity, the boundary conditions, and the anisotropy of the soil. It is independent of the hydraulic conductivity in an isotropic medium and is generally a characteristic of an axisymmetric flow net, since the porous probe is nearly always symmetric in shape (Brand and Premchitt 1980). For a spherical or an ellipsoidal cavity, Laplace equation can be integrated directly to obtain a closed-form solution for the shape factors. However, in view of the difficulty in obtaining an exact solution for cylindrical piezometers used in the field, numerous investigators employed various types of experimental methods and geometrical idealizations to determine shape factors applicable in practice. Some of these expressions are widely used in spite of approximations made in their derivation and inconsistencies in their results (Brand and Premchitt 1980). For close-ended piezometers, the most serious attempt to obtain a closed-form solution was made by Randolph and Booker (1982). These authors solved Laplace equation by assuming a particular form for the velocity distribution on the cylindrical intake area of the piezometer.

This paper presents an analytical solution for the steady flow into a close-ended cylindrical piezometer placed in an infinite, isotropic and incompressible medium. The deduced shape factors are compared with previously published results.

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2. Historical background

There are several methods which are available for field measurements of the hydraulic conductivity of soils. These include tests carried out at the bottom of boreholes, in piezometers installed in a section of a borehole isolated by means of plugs or packers, in piezometers pushed in the soil, and in special permeameters (Tavenas *et al.* 1990). Typically, a constant-head test is carried out and interpreted in the form of Hvorslev's expression (Hvorslev 1951)

$$Q = F\,k\,H \tag{1}$$

where Q is the rate of flow, F is the shape factor, k is the hydraulic conductivity, and H is the fixed excess head.

Numerous investigations have been carried out using various types of analytical, numerical or experimental methods to determine shape factors. These studies include: a) replacement of the cylindrical probe with a spherical or an ellipsoidal cavity (Hvorslev 1951, Schneebeli 1954, Kallstenius and Wallgren 1956, Wilkinson 1968, Mathias and Butler 2006); b) numerical analyses involving finite difference, finite element, and boundary element methods (Al-Dhahir and Morgenstern 1969, Brand and Premchitt 1980, Tavenas *et al.* 1990, Lafhaj and Shahrour 2000, Ratnam *et al.* 2001); and c) experimental measurements involving electric analogue models (Smiles and Youngs 1965, Al-Dhahir and Morgenstern 1969, Brand and Premchitt 1980, Youngs 1980, Lafhaj and Shahrour 2002).

The double-packer cylindrical permeameter considered in the present paper is shown in Fig. 1. The cylindrical intake area is of length l and diameter d, yielding the aspect ratio $N = l/d$. The surface of the casing above and below the intake area is impervious. As such the boundary conditions are of the mixed type: a Dirichlet condition over the pervious area, and a Neumann condition along the casing (Mathias and Butler 2007). The double-packer permeameter is placed in an infinite, isotropic, and incompressible medium. According to Hvorslev (1951), a soil medium

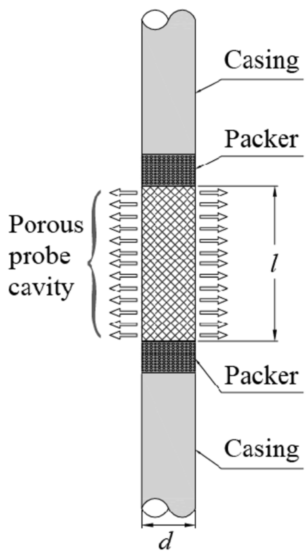

Fig. 1 Close-ended cylindrical piezometer

may be considered to be of infinite thickness and lateral extent when the inflow or outflow is so small that it does not cause any appreciable change in the ground-water level or pressure.

As the geometry of the cylindrical piezometer analyzed in this paper is that shown in Fig. 1, only the most pertinent results published in the literature are reviewed in the present section. These are the relationships of Hvorslev (1951), Schneebeli (1954), Youngs (1968, 1980), Al-Dhahir and Morgenstern (1969), Brand and Premchitt (1980), Randolph and Booker (1982), Tavenas *et al.* (1990), Lafhaj and and Shahrour (2000, 2002), and Mathias and Butler (2006). Among these, the geometrical conditions that resemble the most those shown in Fig. 1 are the ones found in the papers by Randolph and Booker (1982), and Ratnam *et al.* (2001).

The various shape factors obtained in these studies are compared in Figs. 2 and 3. Unless otherwise stated, the shape factor relationships, which appear in these two figures, were derived on the assumption that the soil stratum in which the piezometer is placed is of infinite extent.

On the basis of earlier studies, Hvorslev (1951) proposed the well-known expression (See Fig. 2)

$$F = \frac{2\pi l}{ln\left[\frac{l}{d} + \sqrt{1+\left(\frac{l}{d}\right)^2}\right]} \tag{2a}$$

from which, it follows that

$$\frac{F}{d} = \frac{2\pi N}{Ln[N+\sqrt{1+N^2}]} \tag{2b}$$

where again l and d are respectively the length and the diameter of a cylindrical piezometer, and $N = l/d$ is the aspect ratio. Eq. (2a) was derived on the basis of flow from a line source for which the equipotentials are ellipsoids. Thus, this equation also applies to a porous ellipsoidal cavity. In

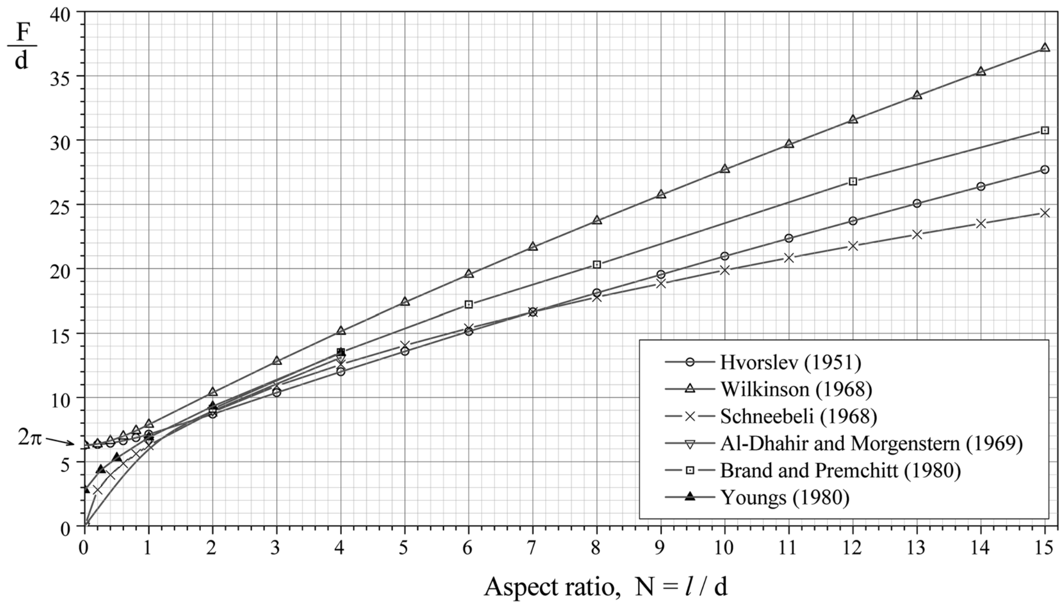

Fig. 2 Comparison of some *F/d* ratios

addition, it is better suited for a long cavity than for a short one since, when the length is zero, application of L'Hospital's rule shows that F reduces to $2\pi d$, that is, the value obtained theoretically for a spherical intake cavity. It is also evident that the flow around the ellipsoidal cavity cannot simulate adequately the flow regime in the vicinity of an open-ended cylindrical piezometer. In spite of such shortcomings, Eq. (2) is still currently used in practice, even for close-ended piezometers, particularly for large aspect ratios.

Schneebeli (1954), and Kallstenius and Wallgren (1956) suggested replacing the intake surface of the cylindrical piezometer with a sphere of equal surface area. Their relationship becomes

$$\frac{F}{d} = 2\pi\sqrt{N} \tag{3}$$

for close-ended piezometers Again, such relationship can only give approximate F values. It predicts F/d values much below those of Hvorslev's values for $N < 1$ and for $N > 15$, as shown in Fig. 2. However, for the usual range of $4 \le N \le 10$, Eq. (3) gives F/d values which are within $\pm$ 5% of those obtained from Eq. (2b).

Wilkinson (1968) proposed improving Hvorslev's relationship by replacing the cylindrical piezometer with a prolate spheroid with its major (vertical) axis adjusted so that its volume was equal to that of the piezometer. Essentially, this consists in replacing the length l in Eq. (2a) with 1.5 l, leading to

$$\frac{F}{d} = \frac{3\pi N}{ln[1.5N + \sqrt{1 + 2.25N^2}]} \tag{4}$$

As shown in Fig. 2, this equation predicts F/d values well above those of Hvorslev (1951). Also, the ratio F/d equals 2π for $N = 0$, as in the case of Hvorslev's expression. In addition, according to Wilkinson (1968), Eq. (4) gives F/d values similar to those obtained from a modified form of a solution reported by Schmid (1967), for steady flow to a cylindrical piezometer. However, close examination of Schmid's solution indicates that it is approximate.

Smiles and Youngs (1965), and Youngs (1968, 1980), on the basis of an electric analogue model, found shape factors differing considerably from those proposed by Hvorslev (1951). Since their analyses applies to cylindrical piezometers with pervious bottoms, these authors found a non-zero F value for $N = 0$. In addition, Youngs (1968) indicated that the outer boundaries of the medium ceased to affect the calculated values of the shape factor for a piezometer with $N = 8$ when the distances between the top of the piezometer and the ground-water table, and between the bottom of the piezometer and an underlying impervious layer, exceeded about 10 d.

Al-Dhahir and Morgenstern (1969) solved Laplace equation by a finite difference method and obtained shape factors similar to those of Smiles and Youngs (1965) for the range of $1 \le N \le 4$. The values of the shape factors given by Al-Dhahir and Morgenstern (1969) are meant to apply to a piezometer with a pervious bottom placed in an infinite soil mass. These authors varied the distances from the piezometers to outer boundaries until the calculated values of the shape factors remained essentially constant. In addition, Al-Dhahir and Morgenstern (1969) showed that the presence of the upper impervious casing was insignificant.

Brand and Premchitt (1980) used both a finite difference method and an electric analogue model to determine shape factors applicable to close-ended cylindrical piezometers. The calibration of the electrolytic tank by means of spherical piezometers verified that infinite values of shape factor (i.e.

shape factors corresponding to piezometers placed in infinite soil masses) would be determined for 3-mm diameter cylindrical piezometers located at mid-depth in the center of tank. The electrolytic tank measured 900 mm in diameter and 500 mm in depth. Thus, F/d values reported by Brand and Permchitt (1980) correspond to distances to outer boundaries that were sufficiently large not to affect the calculated shape factors for $2 \leq N \leq 15$. Moreover the piezometers analyzed by Brand and Premechitt (1980) were without any upper or lower casing. The results are in good agreement with those of Al-Dhahir and Morgenstern (1969) for the range of $2 \leq N \leq 4$. Brand and Premchitt (1980) also showed that their results could be approximated by a relationship obtained by replacing the length l in Hvorslev's expression (Eq. 2a) with $1.2\,l$, especially for $N \geq 1$. In addition, they found that the relationship

$$\frac{F}{d} = 7 + 1.65N \tag{5}$$

could be used to determine shape factors with negligible error for $N \geq 4$.

Randolph and Booker (1982) solved Laplace equation for the steady state flow in an infinite medium, by assuming the following form

$$f(z) = 1 + \lambda\left[1 + \left(\frac{2z}{l}\right)^2\right]^{-1/2}, \; |z| \leq \frac{l}{2} \tag{6}$$

for the distribution of the fluid velocity on the intake area of the piezometer. In the above, z is the axial coordinate and λ is a factor which depends on the aspect ratio. The relationship

$$\frac{F}{d} = 12.52\left[\left(\frac{N + 8.92}{8.92}\right)^2 - 1\right]^{-1/2} \tag{7}$$

gives F values which are within $\pm 5\%$ of those reported by Randolph and Booker (1982) for the range of $1 \leq N \leq 15$. Further, the F/d values obtained by these investigators, which are reported in Fig. 3, are quite similar to those of Brand and Premchitt (1980) for $N \geq 4$.

Chapuis (1989) indicates how to take into account the influence of the proximity of impervious or recharging boundaries on shape factors deduced from spherical or ellipsoidal cavity idealizations.

Tavenas *et al.* (1990) determined shape factors using a finite element method. The results compare extremely well to those of Randolph and Booker (1982), as shown in Fig. 3. To insure spherical flow at a great distance from the probe, a model with equal width and height was selected. A parametric analysis was carried out to determine the necessary dimensions of the model. The results showed that for probes with $N \leq 11$, the width and height of the model were taken equal to $125\,d$; a factor of 200 was used for longer probes. The open-ended cylindrical piezometers were placed at mid-depth in the center of the model. In addition, there was no casing in the model.

More recently, Lafhaj and Shahrour (2000, 2002) used both an electric analogue model and a boundary element method to determine shape factors. For close-ended piezometers, the results fall about a maximum of 6% below those of Tavenas *et al.* (1990), as evidenced in Fig. 3. For piezometers having $1 \leq N \leq 14$ the condition of infinite soil mass was reproduced with a radial boundary placed at $100\,d$ and a vertical distance of about $50\,d$ between the piezometers and both the ground-water table and an underlying impervious layer.

Ratnam *et al.* (2001) used a powerful commercial finite element code to analyze the geometry of Fig. 1. The soil domain measured 12 m in width and 7 m in depth. The piezometers were also placed at mid-depth in the center of the soil mass. The results which may be approximated with

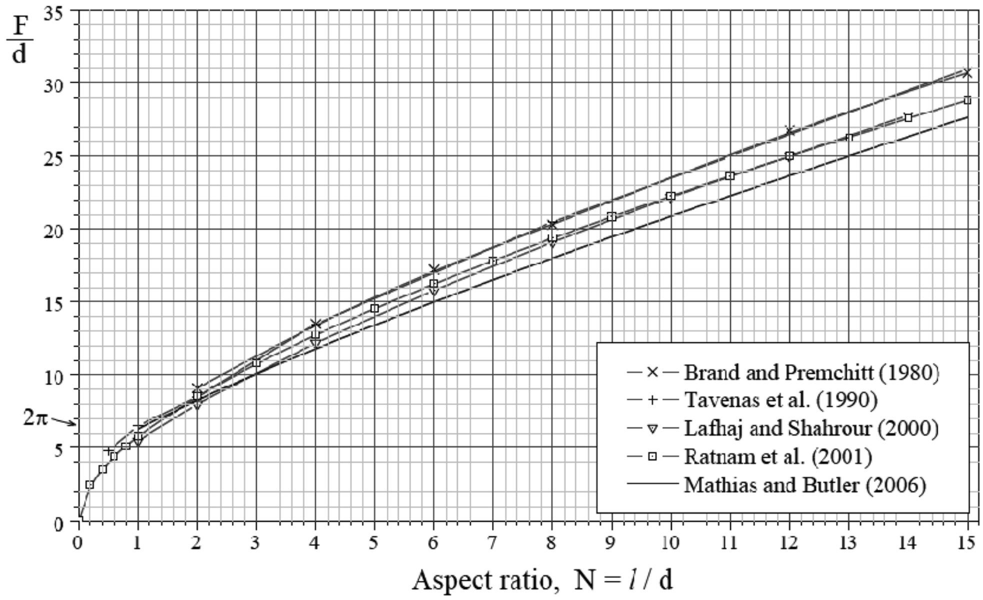

Fig. 3 Comparison of additional *F/d* ratios

negligible error by the expression (See Fig. 3).

$$\frac{F}{d} = 0.5691N + 5.2428N^2 \tag{8}$$

are very similar to those of Lafhaj and Shahrour (2000).

Quite recently, Mathias and Butler (2006) indicated that Hvorslev's relationship (i.e. Eq. (2)) was incorrect. These authors obtained the following expression

$$F/d = \frac{2\pi}{\sinh\left[\tanh^{-1}\left(\frac{1}{N}\right)\right]\ln\left\{\coth\left[\frac{1}{2}\tanh^{-1}\left(\frac{1}{N}\right)\right]\right\}} \tag{9}$$

which is meant to apply to an open-ended cylindrical piezometer placed in an isotropic, infinite, and incompressible soil medium. *F/d* values obtained from Eq. (9) are also reported in Fig. 3. Eq. (9) is based on the solution of Laplace equation for the distribution of the electrostatic charge on a prolate spheroid placed in an infinite field (Moon and Spencer 1961). This type of spheroid is generated by rotating an ellipse about its major vertical axis. Eq. (9) gives $F/d = 2\pi$ for $N = 1$, that is, the value corresponding to a spherical piezometer. However, Hvorslev's expression gives $F/d = 7.13$ for $N = 1$. In addition, the difference between *F/d* values given by Eqs. (2) and (9) decreases considerably as N increases. For example, the difference is less than 0.5 % for $N \geq 10$. Moreover, Eq. (9) is valid for $N \geq 1$, because the inverse hyperbolic tangent in the denominator is only defined for $N \geq 1$.

In 2007, Mathias and Butler obtained a semi-analytical solution for a constant-head double-packer permeameter placed in an infinite field, but for the case of a finite packer length. They showed that ignoring the length of the packer and assuming a fully cased well lead to an overestimate of hydraulic conductivity.

As a result of the comparisons reported in Figs. 2 and 3, it may be concluded that the shape factors, which are obtained by replacing the actual cylindrical piezometer by an ellipsoidal or a spherical cavity, are not entirely consistent. Concerning the analytical solution obtained by Randolph and Booker (1982), which compares well with the numerical solutions reported in Fig. 3, it is the most realistic.

3. Analysis

A close-ended cylindrical porous probe of length l and of diameter d is embedded in an infinitely long impervious casing, as shown in Fig. 4. As indicated above, the piezometer is placed in an isotropic, infinite, and incompressible soil medium. Because of the geometry of the porous element, the flow is axisymmetric. In order to solve Laplace equation, it is convenient to use orthogonal cylindrical coordinates u, v, θ.

In view of the axisymmetric flow, Laplace equation reduces to (Cassan 1980)

$$\frac{\partial}{\partial u}\left(\frac{e_v e_\theta}{e_u}\frac{\partial h}{\partial u}\right) = 0 \tag{10}$$

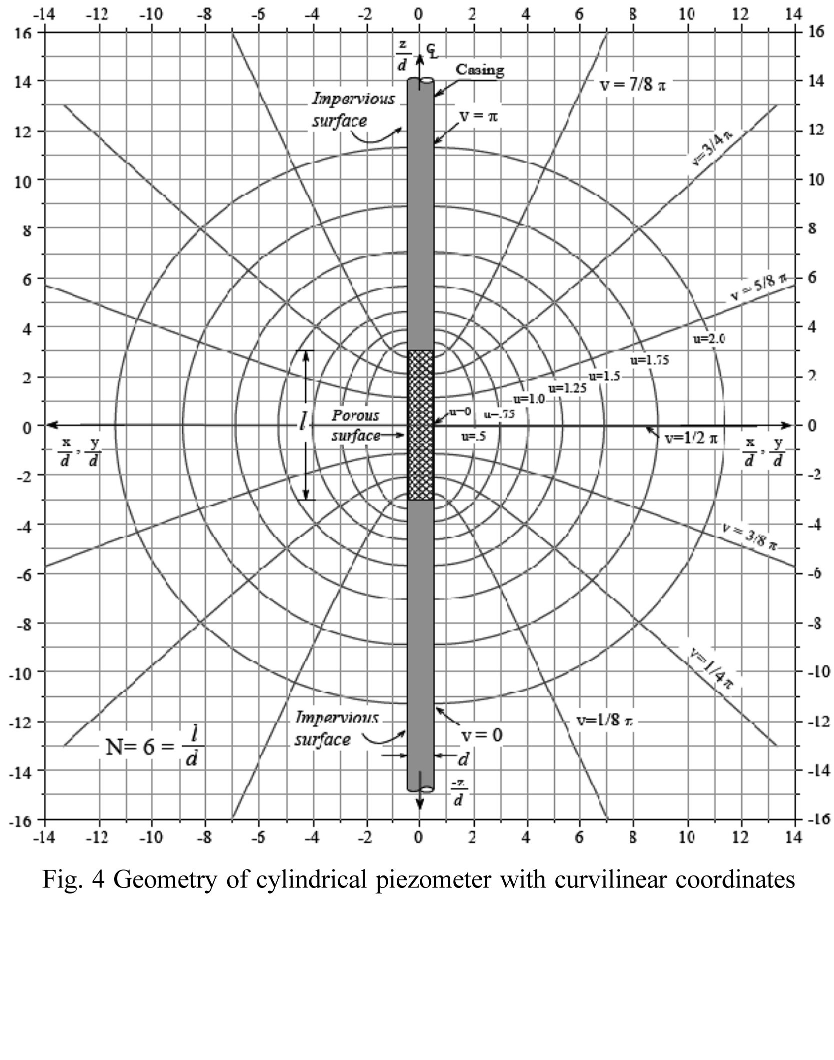

Fig. 4 Geometry of cylindrical piezometer with curvilinear coordinates

where h is the excess head, and e_u, e_v, e_θ are the metric coefficients or scale factors. The latter are defined as

$$e_u^2 = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2$$
$$e_v^2 = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 \quad (11)$$
$$e_\theta^2 = \left(\frac{\partial x}{\partial \theta}\right)^2 + \left(\frac{\partial y}{\partial \theta}\right)^2 + \left(\frac{\partial z}{\partial \theta}\right)^2$$

where x, y, z are the cartesian coordinates.

Integration of Eq. (10) yields

$$\frac{e_v e_\theta}{e_u}\frac{\partial h}{\partial u} = A(v,\ \theta) \quad (12)$$

The function $A(v, \theta)$ is given by (Cassan 1980)

$$A(v,\ \theta) = \frac{h_o - h_\infty}{\int_{u=\infty}^{u=u_o} \frac{e_u e_\theta}{e_v} du} \quad (13)$$

in which h_o is the fixed excess head on the intake surface of the piezometer, which corresponds to $u = u_o$, and h_∞ is the excess head at $u = \infty$.

The rate of flow Q is equal to

$$Q = -k \int_v \int_\theta \frac{e_v e_\theta}{e_u}\frac{\partial h}{\partial u} dv\ d\theta \quad (14a)$$

or

$$Q = -k \int_v \int_\theta A(v,\ \theta) dv\ d\theta \quad (14b)$$

on the basis of Eq. (12).

For the piezometer shown in Fig. 4, the orthogonal curvilinear coordinates are related to the cartesian coordinates by the following expressions

$$x = \frac{1}{2}(d + l \sinh u \sin v)\sin\theta$$
$$y = \frac{1}{2}(d + l \sinh u \sin v)\cos\theta \quad (15)$$
$$z = \frac{1}{2} l \cosh u \cos v$$

If the diameter d of the piezometer was equal to zero, Eq. (15) would represent the confocal families of: (1) prolate spheroids or ovary ellipsoids (u = constant) and (2) hyperboloids of revolution (v = constant) obtained by rotating the ellipses and hyperbolas around the z-axis (i.e. the major axis of the ellipses). In such a case, the solution of Laplace equation for the steady state flow in a constant-head test around a prolate spheroid would yield the shape factors given by Eq. (9)

(Mathias and Butler 2006).

For a cylindrical piezometer of diameter d, the curves u = constant are still ellipses in a meridional plane (i.e. a plane θ = constant). However, the ellipses are now rotated around the central axis of the piezometer, and not around the major (vertical) axis. Moreover, the flow becomes essentially spherical in nature for large distances from the piezometer.

It is seen in Fig. 4 that the intake surface of the piezometer corresponds to $u = u_o = 0$, the impervious surface of the casing to $v = 0$ and $v = \pi$, and the plane $z = 0$ to $v = \pi/2$. In addition, u = constant curves represent equipotentials, and those having v = constant correspond to flow lines in a meridional plane. Moreover, the intersection of u = constant and v = constant surfaces is a circle of radius $r = \sqrt{x^2 + y^2}$.

On basis of Eq. (14), the metric coefficients e_u, e_v, e_θ become

$$e_u = e_v = \frac{l}{2}(\sinh^2 u + \sin^2 v)^{1/2} = \frac{dN}{2}(\sinh^2 u + \sin^2 v)^{1/2} \tag{16}$$

and

$$e_\theta = \frac{1}{2}(d + l \sinh u \cdot \sin v) = \frac{d}{2}(d + N \sinh u \cdot \sin v)^{1/2} \tag{17}$$

Substitution of these expressions into Eq. (13) leads to

$$A(v,\ \theta) = \frac{Hd}{2\int_{u=\infty}^{u=u_o} \dfrac{du}{(1 + N \sinh u \cdot \sin v)}} \tag{18}$$

because $h_o = H$ at $u = u_o = 0$ and $h_\infty = 0$ at $u = \infty$.

Integration of Eq. (18) yields (Gradshteyn and Ryzhik 1980)

$$A(v,\ \theta) = -\frac{H\, d(1 + N^2 \sin^2 v)^{1/2}}{2\ ln\left(\dfrac{1 + N \sin v + \sqrt{1 + N^2 \sin^2 v}}{1 + N \sin v - \sqrt{1 + N^2 \sin^2 v}}\right)} \tag{19a}$$

or

$$A(v,\ \theta) = -\frac{H\, d(1 + N^2 \sin^2 v)^{1/2}}{4 \tanh^{-1}\left(\dfrac{\sqrt{1 + N^2 \sin^2 v}}{1 + N \sin v}\right)} \tag{19b}$$

Substitution of Eq. (19b) into Eq. (14) allows to find the rate of flow

$$Q = k \int_{v=0}^{v=\pi} \int_{\theta=0}^{\theta=2\pi} \frac{H\, d(1 + N^2 \sin^2 v)^{1/2}}{4 \tanh^{-1}\left(\dfrac{\sqrt{1 + N^2 \sin^2 v}}{1 + N \sin v}\right)} dv \tag{20}$$

However, since $A(v,\ \theta)$ is independent of θ, as shown in Eq. (19a), or Eq. (19b), and since the flow is symmetric about the plane $z = 0$, Eq. (20) reduces to

$$Q = \pi H d k \int_{v=0}^{v=\pi/2} \frac{(1+N^2 \sin^2 v)^{1/2}}{\tanh^{-1}\left(\frac{\sqrt{1+N^2 \sin^2 v}}{1+N \sin v}\right)} dv \quad (21)$$

By writing this equation in the form of $Q = F k H$, the ratio F/d becomes

$$\frac{F}{d} = \pi \int_{v=0}^{v=\pi/2} \frac{(1+N^2 \sin^2 v)^{1/2}}{\tanh^{-1}\left(\frac{\sqrt{1+N^2 \sin^2 v}}{1+N \sin v}\right)} dv \quad (22)$$

The integral in Eq. (22) was evaluated numerically for $1 \leq N \leq 15$ and the results are discussed in the next section.

4. Results and comparisons

The results found by means of Eq. (22) are compared in Figs. 5 and 6 with the values obtained from the relations proposed by Hvorslev (1951) and Wilkinson (1968), as well as with those determined by Randolph and Booker (1982), Tavenas *et al.* (1990), Ratnam *et al.* (2001), and Mathias and Butler (2006).

Comparison with the relationship given by Hvorslev (1951) indicates that the latter underpredicts F/d values for $N > 1.5$ and overpredicts them for $N \leq 1.5$. The divergence increases for increasing values of the absolute difference $|N-1.5|$. As for the modified relationship proposed by Wilkinson (1968), it overpredicts considerably F/d values for the whole range of N investigated.

Comparison with the results reported by Randolph and Booker (1982) indicates that the difference with the values obtained by means of Eq. (22) is always less than 5%. The same remarks apply to

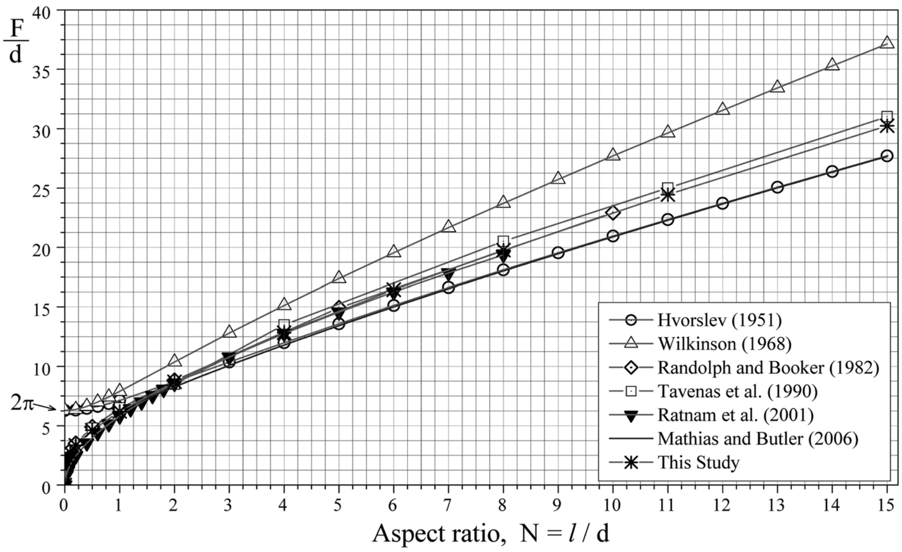

Fig. 5 Comparison of results obtained in this study and by others for $N \leq 15$

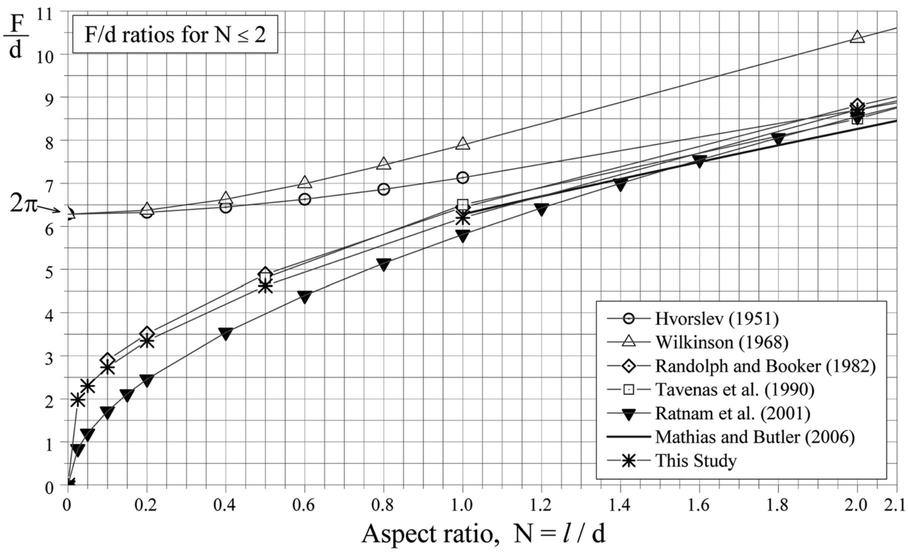

Fig. 6 Comparison of results obtained in this study and by others for $N \leq 2$

the results of Brand and Premchitt (1980) for $N \geq 4$, and of Tavenas *et al.* (1990) for ≤ 15, both of which are similar to those obtained by Randolph and Booker (1982).

Concerning the latter investigation, it is important to point out that the shape factors cannot be easily calculated from the expression reported by Randolph and Booker (1982), for the following reasons: a) the relationship given in their paper is very complex, and b) the factor λ is not given explicitly. Only three values of λ are mentioned in the paper; they are given for $N = 0.1$, 2, and 10.

As for the results obtained by Ratnam *et al.* (2001), they range on the average from 5 to 8% below those found in the present study, particularly for $N \leq 1$ and for $N \geq 6$. The same remarks apply to the results obtained by Lafhaj and Shahrour (2000), for $N \geq 4$.

Concerning the results obtained from the relationship proposed by Mathias and Butler (2006), they are quite similar to those determined from Hvorslev's expression, especially for $N \geq 4$, as shown by comparing Fig. 3 with Fig. 2. It should be again mentioned that Mathias and Butler's expression (i.e. Eq. (9)) represents an idealization of the flow regime that takes place around an open-ended cylindrical piezometer because it is based on the steady state flow around an ovary ellipsoid. As a consequence, the results obtained from such an approach can only be approximate when applied to cylindrical piezometers. However, it is believed that the accuracy of such idealization increases for increasing values of *N*, as also pointed out by Hvorslev (1951) regarding Eq. (2).

5. Conclusions

An analytical solution, albeit in the form of an integral, is presented for the determination of shape factors of close-ended cylindrical piezometers embedded in an infinite, incompressible and isotropic medium. It is shown that the results compare well with the analytical solution of Randolph and Booker (1982), and the numerical results obtained by Brand and Premchitt (1980), Tavenas *et al.* (2000), and Ratnam *et al.* (2001).

As for the various expressions deduced from idealizations involving spherical and ellipsoidal cavities, they should be abandoned in favour of the more realistic results mentioned above.

Because the shape factors found in this study apply to close-ended cylindrical piezometers, they can be used in practice when the geometry of the porous probes is similar to that shown in Fig. 1.

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