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Technical Note

Consolidation of thick clay layer by radial flow – non-linear theory

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1. Introduction

Preloading with Prefabricated Vertical Drains (PVDs) is one of the most effective methods of soft ground improvement. PVDs shorten the drainage path considerably, promote radial flow and accelerate the rate of consolidation. Barron (1948) presented a comprehensive analytical solution for the problem of radial consolidation of soft soils based on the assumptions of a linear void ratio-effective stress relationship and constant coefficients of volume compressibility (m_v) and horizontal permeability (k_h). To overcome the limitations involved in the classical theory and to evolve new techniques for analysis of radial consolidation, several attempts have been made *e.g.*, by Basak and Madhav (1978), Hansbo (1981), Teh and Nie (2002), Indraratna *et al.* (2003, 2005a, 2005b), Conte and Troncone (2009) and Walker *et al.* (2009).

This paper presents a theory of non-linear consolidation for radial flow in thick clay deposits considering a linear void ratio-log effective stress relationship and the variation of initial in-situ stress with depth in a thick clay deposit assuming a constant coefficient of consolidation (c_r).

2. Vertical drains

The strip drain and the zone of influence of each drain are replaced by equivalent circular shapes and the flow pattern around the drain studied considering the flow to be axi-symmetric (Fig. 1). The equivalent diameter of the drain, $d_w = 2(a+b)/\pi$, where 'a' and 'b' are the width and thickness of the PVD respectively. The equivalent diameter of the influence zone, d_e = 1.13S & 1.05S for square and triangular patterns respectively, where S is the spacing of drains.

3. Formulation

The general equation of non-linear consolidation with radial flow in normally consolidated soils, can be derived following Davis and Raymond (1965) as

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$$\frac{\partial w}{\partial t} = c_r\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{1}$$

where $$w = \log_{10}\frac{\sigma'}{\sigma_f'} \quad \text{or} \quad w = \log_{10}\frac{(\sigma_f' - u)}{\sigma_f'} \tag{2}$$

σ' - the effective vertical stress, σ_f' - the final effective vertical stress, u - the excess pore pressure at a radial distance, r, from the centre of the drain and t - the time of consolidation. However, w varies with depth in thick deposits of clay as the initial effective in-situ stress, σ_o' and the final effective stress, σ_f' $(=\sigma_o' + q_0)$ vary with depth due to overburden effect, where q_o is the applied load intensity. Therefore, the thick clay layer of thickness, H is divided equally into m thin layers of thickness, $\Delta H = H/m$ (Fig. 2). The value of 'm' can be chosen depending upon the thickness of the deposit. In the present case, value of 'm' is 20. Even though the initial and final effective stresses are different in each layer, the flow in each layer is assumed to be purely radial and independent of the flows in the adjacent layers. Eq. (1) is identical in form to that of the conventional Barron's radial consolidation theory and can be solved in the same way with the boundary conditions that are the same in terms of u and w.

3.1. Initial and boundary conditions

$$\text{For } t = 0 \text{ and } r_w \le r \le r_e;\ u(r,0) = (\sigma_f' - \sigma_o') \text{ or } w(r,0) = w_o(r) = \log_{10}\frac{\sigma_o'}{\sigma_f'} \tag{3}$$

where, $\sigma_o' = \gamma'.z$; $\sigma_f' = \sigma_o' + \Delta\sigma'$; $\Delta\sigma' = q_o$; γ' -the submerged unit weight of soil; z - the depth of the soil layer from top; r_w and r_e - the equivalent radius of drain and influence zone respectively.

$$\text{For } t > 0 \text{ and } r = r_w;\ u(r = r_w, t) = 0 \text{ or } w(r = r_w, t) = 0 \tag{4}$$

$$\text{For } t > 0 \text{ and } r = r_e;\ \left.\frac{\partial u}{\partial r}\right|_{r=r_e} = 0 \text{ or } \left.\frac{\partial w}{\partial r}\right|_{r=r_e} = 0 \tag{5}$$

The unit cell is discretized and solved numerically by finite difference method for various d_e/d_w $(=n)$ and σ_f'/σ_o' of the corresponding layer. The numerical analysis is carried out for each layer independently for the corresponding boundary and initial conditions.

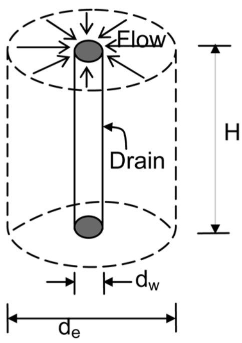

Fig. 1 Axisymmetric unit cell

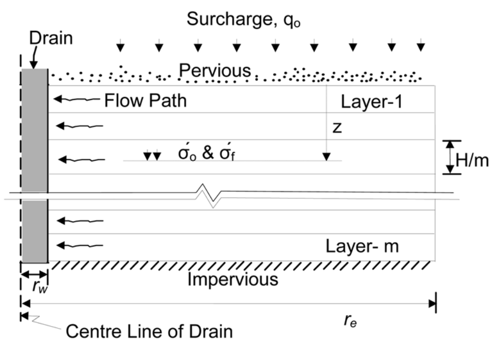

Fig. 2 Radial flow in clay layers

4. Results and discussion

The proposed non-linear consolidation is mainly influenced by the stress ratio, σ_f'/σ_o' which in turn depends on the non-dimensional loading parameter, $q^*_o = q_o/(\gamma'.H)$. Increases in q^*_o can be either due to increase of load intensity, q_o for a given thickness, H, or due to smaller thickness of clay deposit for a given load intensity. In either case, only the non-dimensional stress ratio, σ_f'/σ_o', influences the consolidation process and not the thickness of the deposit or the magnitude of loading individually.

The average excess pore pressure, $u_{avg}(z)$ is determined along the radial distance for $r = r_w$ to r_e at different layers for various n and q^*_o values and the normalized average excess pore pressures, $U^*_{avg}(z) = (u_{avg}(z)/u_o)$. 100 are obtained, where u_o is the initial excess pore pressure ($= \sigma_f' - \sigma_o'$). Average normalized excess pore pressures for the entire thickness or for all the layers, U^*_{avg} is obtained and the degree of dissipation of average excess pore pressure for the entire thickness, $U_p = (100 - U^*_{avg})$ is presented in Fig. 3 along with the degree of settlement, U_s. While the degree of settlement is independent of q^*_o, the degree of dissipation of pore pressure, U_p decreases with the increase of q^*_o values. Davis and Raymond (1965) have established that the degree of settlement is independent of stress ratio, σ_f'/σ_o', because of non-linear void ratio-effective stress relationship and the residual pore pressure is dependent on σ_f'/σ_o'. The increase of q^*_o is equivalent to increase of σ_f'/σ_o' for a given clay thickness. Fig. 4 shows that the degree of settlement is identical at all depths and the excess pore pressure, $U^*_{avg}(z)$ is dependent on depth. Similar observations are made for all other n values.

The remarkable phenomenon observed from Fig. 5 is that average pore pressure values from the non-linear radial consolidation theory vary with depth in contrast to the depth-independent U^*_{avg} values of linear theory. The difference between the pore pressures of non-linear and linear theory is relatively large at shallow depths due to large values of σ_f'/σ_o' compared to those at greater depths. This difference increases with increase of q^*_o. While the excess pore pressures in the linear theory are independent of, q^*_o, the pore pressures according to non-linear theory are dependent on q^*_o as the variation of q^*_o influences the ratio σ_f'/σ_o'. The residual average excess pore pressures thus are underestimated in the conventional linear theory.

Fig. 6 shows that at any depth, the degree of dissipation of average excess pore pressure, $U_p(z)$, is relatively smaller in the non-linear theory compared to that in the linear theory since the variation of permeability during consolidation is considered in the present non-linear theory. The dissipation of pore pressures is relatively very slow at shallow depths compared to that at greater depths in view of the large ratio of final to initial stress at shallow depths. Obviously, at the initial stage of

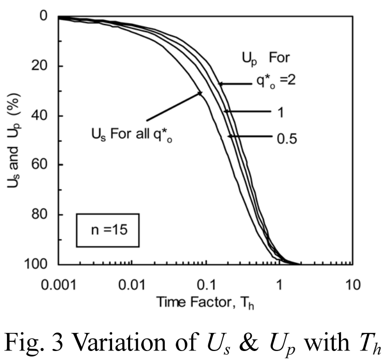

Fig. 3 Variation of U_s & U_p with T_h

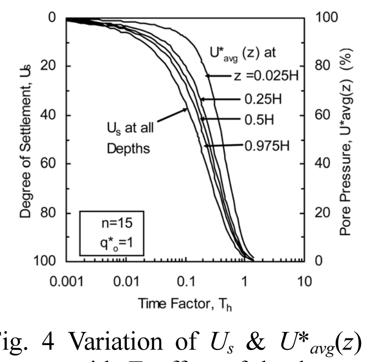

Fig. 4 Variation of U_s & $U^*_{avg}(z)$ with T_h-effect of depth

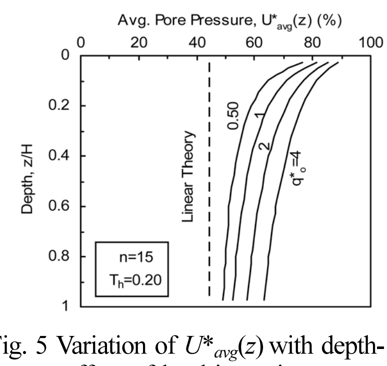

Fig. 5 Variation of $U^*_{avg}(z)$ with depth-effect of load intensity

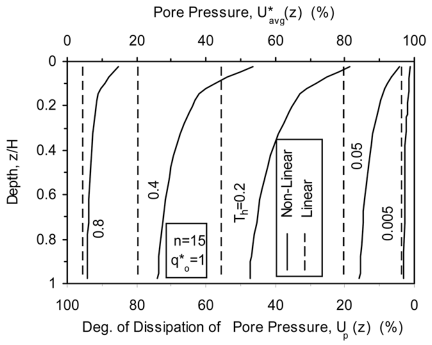

Fig. 6 Variation of $U^*_{avg}(z)$ with depth-effect of time

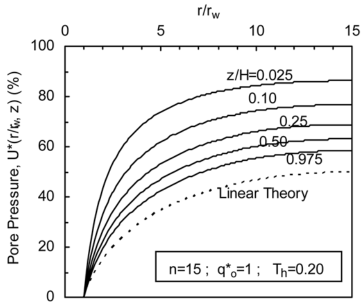

Fig. 7 Variation of pore pressures with radial distance-effect of depth

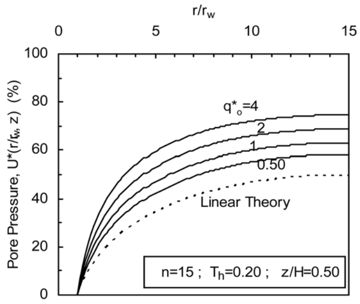

Fig. 8 Variation of pore pressures with radial distance-effect of load intensity

consolidation ($T_h = 0.005$) and towards the end ($T_h = 0.80$), the difference in the degrees of dissipation of average excess pore pressures of the two theories is relatively less compared to those at intermediate stages of consolidation.

The distribution of normalized excess pore pressures at any radial distance and depth, $U^*(r/r_w, z)$ depends on the depth and load intensity in the proposed theory in contrast to the conventional linear theory in which the normalized pore pressure distribution is independent of load and depth (Figs. 7 and 8). The effect of non-linear consolidation in a thick clay deposit is relatively more significant in the upper half of the deposit compared to that in the lower half.

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