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Surface wave propagation in an initially stressed heterogeneous medium having a sandy layer and a point source

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Abstract. An attempt has been made here to study the propagation of SH-type surface waves in an elastic medium, which is initially stressed and heterogeneous and has a point source inside the medium. The upper portion of the composite medium is a sandy layer. It is situated on an initially stressed heterogeneous half-space, whose density, rigidity and internal friction are function of depth. The analysis has been carried out by using Fourier transform and Green's function approach. The phase velocity has been investigated for several particular situations. It has been shown that the results of the study agree with those the case of Love wave propagation in a homogeneous medium in the absence of the sandy layer, when the initial stress is absent. In order to illustrate the validity of the analysis presented here, the derived analytical expression has been computed numerically, by considering an illustrative example and the variances of the concerned physical variables have been presented graphically. It is observed that the velocity of shear wave is amply influenced by the initial stress and heterogeneity parameters and the presence of the sandy layer. The study has an important bearing on investigations of different problems in the earth's interior and also in seismological studies.

Keywords: surface wave; green function; dispersion equation; phase velocity; heterogeneity

1. Introduction

The theory of elastic waves propagating in heterogeneous media is of considerable importance to geophysicists and seismologists. Studies related to seismic waves help explore a variety of information about the internal structure of the earth. As we know, the structure of the earth is very complex. It contains different types of rocks and materials having amazing mechanical characteristics, such as heterogeneity and anisotropy. The constituent layers of the earth are in a state of pre-stressed condition. Some layers are sandy. Quite some information about various previous studies regarding plate tectonics of the earth is available in the classic works by Love (1911), Biot (1956), Ewing *et al.* (1957), Gubbins (1990) and others.

The existing scientific literatures have revealed that a large amount of elastic strain is stored in different layers of the earth. There are multiple factors for this, viz variation of temperature in the atmosphere, gravitating pull, slow creeping process and pressure exerted by the overburdened layer. When the elastic strain is released in the form of wave propagation, earthquakes are likely to occur. It is also known that in the interior of the earth, body forces may give rise to an impulsive loading in any arbitrary direction. Such a force may be represented mathematically through the use of the Dirac delta function, which is of particular importance in studies of phenomena arising out of a point action or an impulsive force.

The point source function is usually described by the delta function. Watanabe and Payton (2002) studied the propagation of SH-waves in a cylindrically monoclinic material using Greens function. Dispersion of shear waves in a pre-stressed heterogeneous orthotropic layer over a pre-stressed anisotropic porous half-space with self-weight was studied by Kakar and Kakar (2016). Shear wave propagation over the surface of a homogeneous medium or inhomogeneous elastic half-space is a prominent phenomenon in wave theory. Several researchers worked on the propagation of elastic (seismic) waves in a finite layer over half-space. Recently, Li and Tao (2015) studied the influence of initial stress on wave propagation and dynamic elastic coefficients. Structural damage detection through longitudinal wave propagation using the spectral finite element method was discussed by Kumar *et al.* (2017). In another recent paper, Tokhi *et al.* (2015) performed a wave equation analysis with the purpose of establishing a relation between ultimate capacity and net set blow. The authors claim that by using their study, it is possible to overcome

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some shortcomings of dynamic formulae and that by introducing a correction factor, the results can be improved considerably.

Consideration of heterogeneity, viscoelasticity and initial stresses are important in various studies relating to the earth. Theoretical analysis of torsional wave propagation in a heterogeneous aeolotropic stratum over a viscoelastic half-space of voigt type was studied by Manna *et al.* (2018). The influence of viscoelasticity was investigated by the same authors (Misra and Samanta 1982). The effect of initial stresses on wave propagation in arterial wall tissues was explored by Misra and Roychoudhury (1983). This study also explored the influence of heterogeneity and nonlinearity of the wall tissues of arteries. Misra and Chakravarty (1982) also performed a separate study on the free vibration of a human-sized skull and explored a variety of information. These studies reveal that the factors, heterogeneity and initial stress play important roles in vibration and wave propagation problems of both inanimate objects and living bodies. Theoretical/numerical analysis of the influence of initial stress gradient on wave propagations was performed by Tao *et al.* (2016). The rotational effect of Rayleigh, Love and Stoneley waves in non-homogeneous fiber-reinforced anisotropic general viscoelastic media of higher order was studied by Abo-Dahab *et al.* (2016). In a separate communication published in 2017, the same authors discussed the rotational effect on thermoelastic Stoneley, Love, and Rayleigh wave propagation in fibre-reinforced anisotropic viscoelastic media of higher order. Love wave dispersion in pre-stressed homogeneous medium over a porous half-space with irregular boundary surfaces was studied by Kundu *et al.* (2016). Generalized Rayleigh wave propagation in a covered half-space with liquid upper layer was discussed by Negin (2015).

It is known that the SH-wave is a horizontally polarized surface wave that can shift the earth during an earthquake. The disturbance energy generated by seismic waves, travel from the focus to the epicenter through the interior of the earth. Shear waves are transversely propagated surface waves, which can be easily felt during earthquakes. Information on surface wave propagation in a sandy/heterogeneous medium is of profuse interest. Chakraborty and Chandra (1984) discussed the nature of reflection and refraction of plane SH-wave at the boundary of the dry sandy layer/anisotropic elastic medium. SH-wave propagation at a corrugated interface between a dry sandy half-space and an anisotropic elastic half-space were studied by Tomar and Kaur (2007). Daros (2013) introduced Green's function for the study of propagation of SH-waves in inhomogeneous anisotropic elastic solid with power-function velocity variation. Ghorai *et al.* (2010) introduced the Love waves in a fluid-saturated porous layer under a rigid boundary and lying over an elastic half-space under gravity. Kundu *et al.* (2014) treated SH-type waves dispersion in an isotropic medium sandwiched between an initially stressed orthotropic medium and a heterogeneous semi-infinite medium.

Ghosh (1970) studied the effect of a point source on Love wave propagation in a layered medium, considering heterogeneity. Reflection of waves from a point source by an impedance boundary was discussed by Thomasson (1976). By using function theoretic methods, propagation of noise from a point source on a plane that consists of two half-planers were discussed by Nagesh and Hayek (1981), where each of the half-planes are covered by a different impedance. They made an observation that diffraction of ground waves is dependent on the impedance difference of the pair of half-planes. Manna *et al.* (2016) investigated the effect of reinforcement and heterogeneity on the propagation of Love waves. Chattopadhyay and Kar (1981) discussed the propagation of Love waves due to a point source in an initially stressed isotropic elastic solid, while Chattopadhyay and Pal (1983) studied analytically the dispersion curves during Love wave propagation in a heterogeneous layer containing a point source. Propagation of SH waves was discussed by Kochengin (1997) for a medium having a point source and a vertical boundary, in a situation where the velocity of wave propagation is a smoothly increasing function of depth. The effects of a point source and heterogeneity on SH wave propagation in a viscoelastic layer were studied by Chattopadhyay *et al.* (2012), where the layer is situated over a viscoelastic half-space. Kundu *et al.* (2016) studied the effect of the presence of a point source on the propagation of Love waves in a heterogeneous medium situated on a functionally graded heterogeneous half-space.

Recently, propagation of SH-wave was examined by Singh *et al.* (2017) in a piezoelectric layer containing a point source. Although different problems on wave propagation in solid structures were studied by various researchers, we observe that some problems, which are of particular importance to several parts of the Himalayan region have not received adequate attention. In the present communication, we have undertaken a study which is particularly relevant to north-western Himalayan region, Jammu and Kashmir situated on upper siwalik and lower siwalik in the earth's crust. It is known that the layers of the Siwalik regions are highly heterogeneous and that most parts of the upper and middle Siwalik regions are sandy. This has been taken care of, while considering the geometrical configuration of the medium for the present study. We have carried out an investigation that concerns the propagation of SH-waves in a composite medium that consists of a dry sandy layer lying over a heterogeneous half-space that contains a point source. Moreover, in view of the fact that the earth's crust is initially stressed, the effect of initial stresses has been paid due consideration in the wave propagation problem. The rigidity and density of the upper dry sandy layer are taken to vary uniformly and those for the half-space are supposed to vary with depth. The Green's function technique has been employed to determine the displacement in the sandy layer as well as in the half-space beneath the sandy layer. For the particular case of the propagation of Love waves, the dispersion equation has been derived. A computational study has been performed by using the theoretical analysis. The numerical results presented in graphicall form depict the variation of the phase velocity with the change in the values of the physical parameters. These graphs also reveal the effects of sandiness of the upper layer, as well as the effects of heterogeneity and initial stresses of the structure on the propagation of SH-waves.

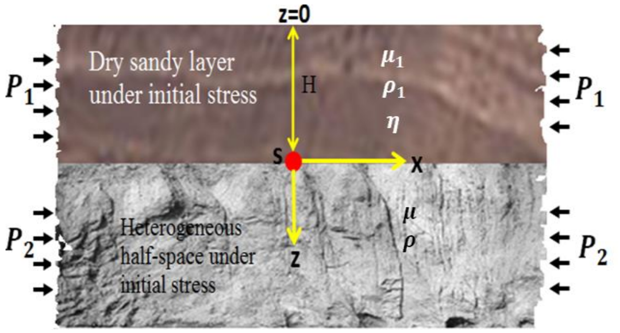

Fig. 1 Geometry of the problem

2. Formulation of the problem

We consider shear wave propagation in an initially stressed dry sandy layer of finite thickness H, lying over an initially stressed heterogeneous elastic half-space. The z-axis is taken along the vertically downward direction in the half-space, and the x-axis along a direction parallel to that of wave propagation. The origin is taken at a fixed point on the surface of the upper layer.

The variation of heterogeneity with rigidity and density in the lower semi-infinite medium has been taken as in Eq. (1) below.

$$\left.\begin{aligned} \mu &= \mu_2 + \varepsilon \sinh b(z-H) \\ \text{and } \rho &= \rho_2 + \varepsilon \cosh b(z-H), \end{aligned}\right\} \tag{1}$$

where ε is a small positive real constant such that $O(\varepsilon^2) \to 0$. The source of disturbance S is located at the point of intersection of the upper layer and lower half space. The rigidity and density of the dry sandy layer and heterogeneous half-space are denoted by (μ_1, ρ_1) and (μ, ρ) respectively. η is the sand parameter of the upper layer (Fig. 1).

3. Solution of the problem

Let (u_1, v_1, w_1) denote the displacement components along x, y and z directions for sandy layer and (u_2, v_2, w_2) those for lower half-space. The constitutive relations get modified the rigidity modulus of a medium due to slippage of granules. The modified constitutive relations (Pal and Mandal 2014) in an isotropic sandy medium are given by

$$\tau_{ij} = \lambda \delta_{ij} + 2\frac{\mu_1}{\eta} e_{ij}, \qquad \eta > 1 \tag{2}$$

where λ and μ_1 are Lame's constants and $\eta (> 1)$ is a measure of sandiness. δ_{ij} is the Kronecker delta function. In the above equation, $\eta > 1$ corresponds to sandy media, e_{ij} are strain components and $\eta = 1$ corresponds to an isotropic elastic solid.

In the presence of a point source, equation of motion can be written as

$$\tau_{ij,j} + F_i = \rho \ddot{u}_i \tag{3}$$

where $\tau_{ij}'s$ are the stress components, ρ is the density of the medium and F_i denotes the force at a point.

3.1 Dynamical behavior of the upper dry sandy layer

Here we use the conventional shear wave conditions $u_1 = 0 = w_1$ and consider v_1 as a function of the variables x, z and t.

Then, the equation of motion of the plane SH-wave propagating in a dry sandy elastic medium [cf. Kar *et al.* (1986)] assumes the form

$$\frac{\mu_1}{\eta}\left[\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2}\right] - \frac{P_1}{2}\frac{\partial^2 v_1}{\partial x^2} = \rho_1 \frac{\partial^2 v_1}{\partial t^2} + 4\pi\sigma_1(r,t), \tag{4}$$

where v_1 is the displacement along the y-direction and μ_1, ρ_1, η are respectively rigidity, density and sandiness parameter of the layered medium. $\sigma_1(r,t)$ represents the force density distribution in the upper sandy layer due to the point source. r denotes the distance from the origin, where the force is applied at a point of the medium, t is the time and P_1 is the initial stress at the upper layer. For any linearized wave problem, if the range of the spatial co-ordinate x is $(-\infty, \infty)$, all the coefficients are constants, and the force density distribution is independent of impulsive force and time t, then there will be a sinusoidal wave train of the form

$$v_1(x,z,t) = V_1(x,z)e^{i\omega t} \text{ and } \sigma_1(r,t) = \sigma_1(r)e^{i\omega t} \tag{5}$$

where $\omega = kc$ is the angular frequency, k is the wave number and c is the phase velocity. In order to study the physics of this particular type of waves, we can use the principle of superposition to construct more general solutions. Introducing (5) in (4), we obtain

$$\frac{\partial^2 V_1}{\partial z^2} + \left(1 - \frac{P_1}{2\mu_1}\eta\right)\frac{\partial^2 V_1}{\partial x^2} + \frac{\omega^2}{\beta_1^2}\eta V_1(x,z) = \frac{\eta}{\mu_1} 4\pi\sigma_1(r), \tag{6}$$

in which $\beta_1 = \sqrt{\frac{\mu_1}{\rho_1}}$ stands for the shear wave velocity in the upper layer. Due to the application of the impulsive force $\sigma_1(r)$ a disturbance is which may be described in terms of Dirac-delta function at the source point as $\sigma_1(r) = \delta(x)\delta(z-H)$.

The equation of motion for the upper dry sandy layer with an impulsive point source is then given by

$$\begin{aligned} \frac{\partial^2 V_1}{\partial z^2} + \left(1 - \frac{P_1}{2\mu_1}\eta\right)\frac{\partial^2 V_1}{\partial x^2} + \frac{\omega^2}{\beta_1^2}\eta V_1(x,z) \\ = \frac{\eta}{\mu_1} 4\pi\delta(x)\delta(z-H). \end{aligned} \tag{7}$$

We define the Fourier transform $\bar{V}_r(\xi, z)$ as

$$\bar{V}_r(\xi,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V_r(x,z)e^{i\xi x}dx. \tag{8}$$

and the inverse Fourier transform as

$$V_r(x,z) = \int_{-\infty}^{\infty} \bar{V}_r(\xi,z)e^{-i\xi x}d\xi. \tag{9}$$

Applying Fourier transformation on Eq. (7), we get

$$\frac{d^2\bar{V}_1}{dz^2} - \alpha^2\bar{V}_1 = 2\frac{\eta}{\mu_1}\delta(z-H), \tag{10}$$

with $\alpha^2 = \xi^2 - \eta k_1^2$ and $k_1^2 = \frac{\omega^2}{\beta_1^2} + \frac{P_1}{2\mu_1}\xi^2$.

3.2 Dynamics of the lower heterogeneous medium

In the absence of body forces, the equation of motion for lower the heterogeneous half-space due to shear wave propagation may be written as

$$\frac{\partial}{\partial x}\left(\mu\frac{\partial v_2}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial v_2}{\partial z}\right) - \frac{\partial}{\partial x}\left(\frac{P_2}{2}\frac{\partial v_2}{\partial x}\right) = \rho\frac{\partial^2 v_2}{\partial t^2}, \tag{11}$$

where μ and ρ are respectively rigidity and density as defined in relation (1) and P_2 is the initial stress parameter for the lower half-space. Using the relation (1), Eq. (11) takes the form

$$\begin{aligned}\{\mu_2 + \varepsilon\sinh b(z-H)\}\left(\frac{\partial^2 v_2}{\partial z^2} + \frac{\partial^2 v_2}{\partial x^2}\right) + \varepsilon b\cosh b(z-H)\frac{\partial v_2}{\partial z} \\ -\frac{P_2}{2}\frac{\partial^2 v_2}{\partial x^2} = \{\rho_2 + \varepsilon\cosh b(z-H)\}\frac{\partial^2 v_2}{\partial t^2},\end{aligned} \tag{12}$$

in which μ_2 and ρ_2 stand respectively for the rigidity modulus and density of the half-space at $z = H$.

Similarly, using Eq. (1), the equation of motion for the lower half-space in the Fourier transform space reads

$$\frac{d^2\bar{V}_2}{dz^2} - \beta^2\bar{V}_2 = 4\pi\sigma(z), \tag{13}$$

where $\beta^2 = \left(1 - \frac{P_2}{2\mu_2}\right)\xi^2 - \frac{\omega^2}{\beta_2^2}$, the shear wave velocity, β_2 for the half-space is $\sqrt{\frac{\mu_2}{\rho_2}}$ and

$$\begin{aligned}4\pi\sigma(z) = -\frac{\varepsilon}{\mu_2}\Big[\sinh b(z-H)\frac{d^2\bar{V}_2}{dz^2} + b\cosh b(z-H)\frac{d\bar{V}_2}{dz} \\ +\{\omega^2\cosh b(z-H) - \xi^2\sinh b(z-H)\}\bar{V}_2\Big].\end{aligned} \tag{14}$$

The displacement in the lower half-space can be determined from the Eq. (13) for the isotropic homogeneous lower half-space having $\sigma(z)$ as the source density disturbance.

Now the Eqs. (10) and (13) will be solved by using the Green's function technique under the prescribed boundary conditions, which in the Fourier transform domain read

$$\frac{d\bar{V}_1}{dz} = 0 \quad \text{at} \quad z = 0 \tag{15}$$

$$\bar{V}_1(z) = \bar{V}_2(z) \quad \text{at} \quad z = H \tag{16}$$

$$\frac{\mu_1}{\eta}\frac{d\bar{V}_1}{dz} = \mu_2\frac{d\bar{V}_2}{dz} \quad \text{at} \quad z = H \tag{17}$$

As in Kundu *et al.* (2016), using Green's function technique and boundary conditions (15)-(17) on equations (10), (13) and (14), we get

$$\begin{aligned}tan\left[kH\sqrt{\left(\frac{c^2}{\beta_1^2} + \frac{P_1}{2\mu_1}\right)\eta - 1}\right] = \frac{\mu_2}{\mu_1}\frac{\eta\sqrt{1 - \frac{P_2}{2\mu_2} - \frac{c^2}{\beta_2^2}}}{\sqrt{\left(\frac{c^2}{\beta_1^2} + \frac{P_1}{2\mu_1}\right)\eta - 1}} + \frac{\varepsilon\eta}{2\mu_1\sqrt{\left(\frac{c^2}{\beta_1^2} + \frac{P_1}{2\mu_1}\right)\eta - 1}} \\ \times\frac{\frac{b}{2k}\left(1 - \frac{P_2}{2\mu_2} - \frac{c^2}{\beta_2^2}\right) + \frac{b}{2k} - \frac{c^2}{\beta_1^2}\left(\frac{\mu_1}{\rho_1}\right)\sqrt{1 - \frac{P_2}{2\mu_2} - \frac{c^2}{\beta_2^2}}}{\left\{\left(1 - \frac{P_2}{2\mu_2} - \frac{c^2}{\beta_2^2}\right) - \left(\frac{b}{2k}\right)^2\right\}}\end{aligned} \tag{18}$$

in which $\beta_1 = \sqrt{\frac{\mu_1}{\rho_1}}$ and $\beta_2 = \sqrt{\frac{\mu_2}{\rho_2}}$.

This gives the dispersion equation of shear waves in dry sandy layer over an initially stressed heterogeneous half-space.

4. Particular cases

4.1 Case I

When the sand parameter η of the upper layer is unity, i.e, if we consider the upper layer as an isotropic elastic solid medium, the dispersion Eq. (18) reduces to

$$\begin{aligned}tan\left[kH\sqrt{\frac{c^2}{\beta_1^2} + \frac{P_1}{2\mu_1} - 1}\right] = \frac{\mu_2}{\mu_1}\frac{\sqrt{1 - \frac{P_2}{2\mu_2} - \frac{c^2}{\beta_2^2}}}{\sqrt{\frac{c^2}{\beta_1^2} + \frac{P_1}{2\mu_1} - 1}} + \frac{\varepsilon}{2\mu_1\sqrt{\frac{c^2}{\beta_1^2} + \frac{P_1}{2\mu_1} - 1}} \\ \times\frac{\frac{b}{2k}\left(1 - \frac{P_2}{2\mu_2} - \frac{c^2}{\beta_2^2}\right) + \frac{b}{2k} - \frac{c^2}{\beta_1^2}\left(\frac{\mu_1}{\rho_1}\right)\sqrt{1 - \frac{P_2}{2\mu_2} - \frac{c^2}{\beta_2^2}}}{\left\{\left(1 - \frac{P_2}{2\mu_2} - \frac{c^2}{\beta_2^2}\right) - \left(\frac{b}{2k}\right)^2\right\}},\end{aligned} \tag{19}$$

which is the dispersion equation for propagation of shear waves in an isotropic elastic medium placed over an initially stressed heterogeneous half-space.

4.2 Case II

If initial stress parameters for the upper layer as well as the lower half-space are negligibility small, putting $P_1 = 0$ and $P_2 = 0$ in case I, the dispersion equation (19) assumes the simplified form

$$\begin{aligned}tan\left[kH\sqrt{\frac{c^2}{\beta_1^2} - 1}\right] = \frac{\mu_2}{\mu_1}\frac{\sqrt{1 - \frac{c^2}{\beta_2^2}}}{\sqrt{\frac{c^2}{\beta_1^2} - 1}} + \frac{\varepsilon}{2\mu_1\sqrt{\frac{c^2}{\beta_1^2} - 1}} \\ \times\frac{\frac{b}{2k}\left(1 - \frac{c^2}{\beta_2^2}\right) + \frac{b}{2k} - \frac{c^2}{\beta_1^2}\left(\frac{\mu_1}{\rho_1}\right)\sqrt{1 - \frac{c^2}{\beta_2^2}}}{\left\{\left(1 - \frac{c^2}{\beta_2^2}\right) - \left(\frac{b}{2k}\right)^2\right\}}.\end{aligned} \tag{20}$$

This is the dispersion equation of shear wave propagation in an isotropic medium over a heterogeneous half-space.

4.3 Case III

If we set the parameter $\varepsilon = 0$ in case II, the dispersion Eq. (20) further reduces to

$$tan\left[kH\sqrt{\frac{c^2}{\beta_1^2} - 1}\right] = \frac{\mu_2}{\mu_1}\frac{\sqrt{1 - \frac{c^2}{\beta_2^2}}}{\sqrt{\frac{c^2}{\beta_1^2} - 1}}. \tag{21}$$

This is the standard dispersion equation of shear waves of an isotropic homogeneous layer, lying over a homogeneous half-space (Chattopadhyay and Pal 1984). This equation also describes the dispersion of Love waves.

5. Numerical results and discussion

Eq. (18) gives the phase velocity of shear wave

Table 1 Values of rigidity and density for the layer and the half-space

Medium	Rigidity $(\times 10^{10} N/m^2)$	Density (kg/m^3)
Sandy layer	$\mu_1 = 6.54$	$\rho_1 = 3409$
Inhomogeneous half-space	$\mu_2 = 11.77$	$\rho_2 = 4148$

Table 2 Values of various dimensionless parameters

Parameters	Fig. 2	Fig. 3	Fig. 4	Fig. 5	Fig. 6	Fig. 7	Fig. 8
$\frac{P_1}{2\mu_1}$	-	-	-	0.5	0.0	0.5	0.4
$\frac{P_2}{2\mu_2}$	-	-	-	0.5	0.0	0.5	0.4
$\frac{\varepsilon}{2\mu_1}$	0.45	0.45	0.0	-	-	0.45	0.45
$\frac{b}{2k}$	0.4	0.4	0.0	0.6	0.6	-	0.4
η	1.0	1.0	1.0	1.0	1.0	0.1	-

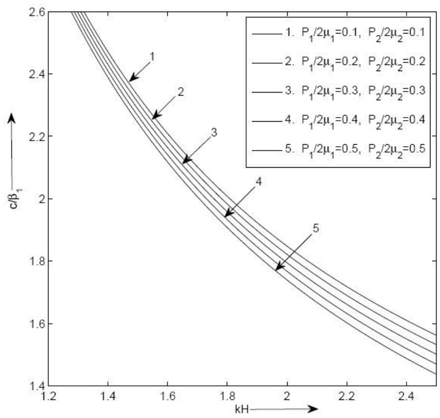

Fig. 2 Dimensionless phase velocity versus dimensionless wave number demonstrating the effect of tensile initial stresses

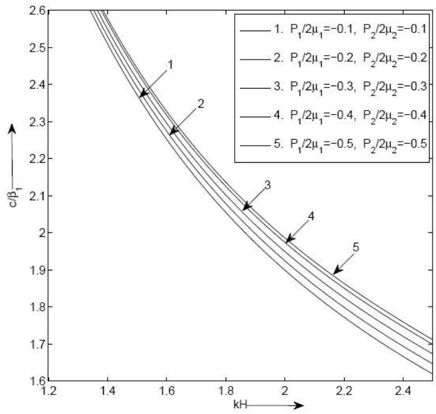

Fig. 3 Dimensionless phase velocity versus dimensionless wave number showing the influence of compressive initial stresses

propagation in an initially stressed dry sand layer over a

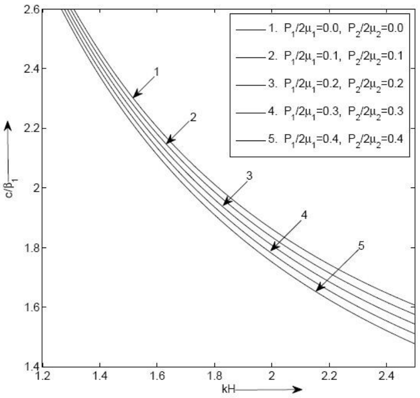

Fig. 4 Variation of phase velocity with wave number exhibiting the effect of tensile initial stresses in the absence of heterogeneity

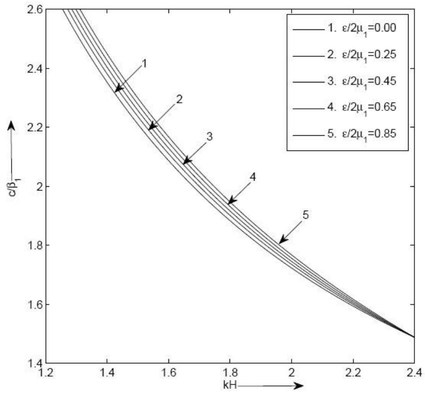

Fig. 5 Phase velocity versus wave number demonstrating the influence of heterogeneity $(\frac{\varepsilon}{2\mu_1})$ associated with rigidity of the initially stressed half-space

heterogeneous half-space under initial stress. This equation is very useful in the study of the effects of sandiness, heterogeneity, initial stress parameters, wave number and time period on the phase velocity of shear waves. Based on the dispersion equation (18), the graphs have been plotted by using the numerical data presented in Table 1 (Gubbins 1990).

Based upon our computational study, the effects of various parameters are presented in Figs. 2-8. All these figures give the variation of dimensionless phase velocity for different values of non-dimensional wave number.

Compression and tensile axes can be obtained by projecting polarity of wave along the direction of the observatory. Directions of compression and tension axes are sometimes regarded as those of the maximum and minimum principal stress, respectively. Compression and tension axes represent stress change due to fault slip and their directions need not always coincide with those of the initial stress. The directions of initial stress can be easily estimated from enormous fault data for earthquake although the magnitude of initial stress can not be estimated.

Fig. 2 shows the effect of compressive initial stresses $(\frac{P_1}{2\mu_1}, \frac{P_2}{2\mu_2} > 0)$ in the dry sand layer and heterogeneous half-space. The values of initial stress parameters $\frac{P_1}{2\mu_1}$ and

$\frac{P_2}{2\mu_2}$ considered for plots 1-5 in Fig. 2 have been taken equal to $0.1, 0.2, 0.3, 0.4, 0.5$ respectively. From this figure, it may be observed that the dimensionless phase velocity $(\frac{c}{\beta_1})$ decreases, as the tensile initial stresses are enhanced. The curves being a little far apart from each other at lower phase velocity, it is revealed that the tensile initial stress has much dominance for large values of non-dimensional wave number.

Fig. 3 gives the dispersion curves for shear wave propagation, when the initial stresses $(\frac{P_1}{2\mu_1}, \frac{P_2}{2\mu_2} < 0)$ are of compressive nature, have been taken for both dry sandy layer and half-space. Here, the values of initial stress parameters for the curves 1-5 have been taken to be $-0.1, -0.2, -0.3, -0.4, -0.5$, respectively. Values of other parameters used for this figure are presented in Table 2. Thus Fig. 3 reveals that the phase velocity increases with a reduction in the values of initial stress parameters.

In Fig. 4, an attempt has been made to study the effect of initial stress parameters for the propagation of shear waves in the absence of heterogeneity of the lower half-space. The different values of $\frac{P_1}{2\mu_1}$ and $\frac{P_2}{2\mu_2}$ for this figure have been taken to be $0.0, 0.1, 0.2, 03$ and 0.4, respectively. From this figure, one may observe that the speed of shear wave propagation decreases, as the values of initial stress parameters $\frac{P_1}{2\mu_1}, \frac{P_2}{2\mu_2}$ increase. Curve 1 of this figure represents the phase velocity in a dry sandy medium over a homogeneous half-space.

Fig. 5 illustrates the effect of heterogeneity parameter $\frac{\varepsilon}{2\mu_1}$ for the propagation of shear waves in the sand layer overlaying the heterogeneous half-space. Curves are plotted for different values of $\frac{\varepsilon}{2\mu_1}$ and fixed values of initial stress parameters (Table 2). The value of $\frac{\varepsilon}{2\mu_1}$ for curves 1-5 have been taken equal to $0.00, 0.25, 0.45, 0.65$ and 0.85, respectively. This figure shows that the phase velocity reduces, as the wave number increases and that they accumulate, when $kH = 2.4$, when the wave number is considerably large. It is also interesting to note from Fig. 5 that the phase velocity rises, as the value of the heterogeneity parameter $\frac{\varepsilon}{2\mu_1}$ gradually increases.

Fig. 6 presents the dispersion curves of shear waves for different values of the heterogeneity parameter $\frac{\varepsilon}{2\mu_1}$ in case the initial stress is absent. The different values of $\frac{\varepsilon}{2\mu_1}$ considered here are the same as those for Fig. 5. The plots of this figure show that the phase velocity increases with the increase in the value of $\frac{\varepsilon}{2\mu_1}$ up to a certain point, where $kH = 2.2$. When the value of KH exceeds 2.2, a reverse trend is noticed for the phase velocity.

Fig. 7 demonstrates the effect of the heterogeneity parameter $\frac{b}{2k}$ for the shear wave propagation. Curves are plotted here for different values of the said parameter and fixed values of initial stress parameter given in Table 2. The values of $\frac{b}{2k}$ for curves 1-4 are equal to $0.0, 0.3, 0.6$ and 0.9, respectively. From this figure, one may observed that

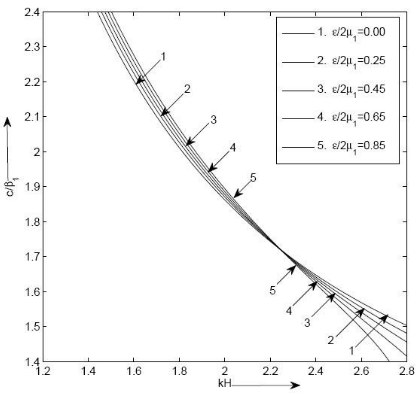

Fig. 6 Dimensionless phase velocity against dimensionless wave number illustrating the effect of parameter $\frac{\varepsilon}{2\mu_1}$ associated with rigidity in the absence of initial stress

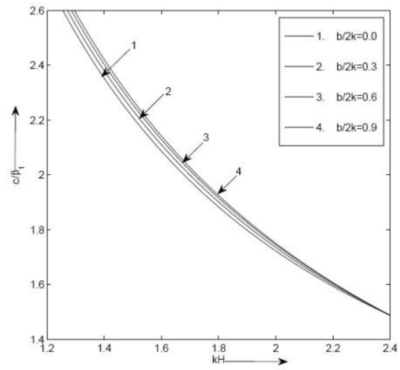

Fig. 7 Phase velocity versus wave number indicting the influence of heterogeneity parameter $\frac{b}{2k}$ of the half-space

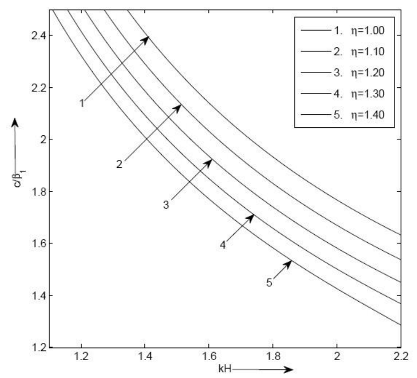

Fig. 8 Variation of the phase velocity with wave number demonstrating the effect of sand parameter

shear wave velocity increases at a slower rate, when the value of the heterogeneity parameter is on the rise and that the plots accumulate, when the wave number rises.

The influence of the sand parameter η on the propagation of shear waves is illustrated through Fig. 8. The values of η for curves 1-5 are taken equal to $1.0, 1.10, 1.20, 1.30$ and 1.40, respectively. It is important to note from this figure that in the presence of heterogeneity and initial stresses, an increase in the sand parameter η brings about a reduction of the phase velocity of the shear wave propagation. Graph 1 of this figure that corresponds to $\eta = 1$ represents the variation of the phase velocity for the sand layer, which is less isotropic and homogeneous, while the plots 2-5 give an idea of the phase velocity variation in the case of a sand layer situated over an initially stressed heterogeneous half-space.

5. Conclusions

The central objective of the present investigation has been to determine an estimate of the influence of the presence of a sand layer on SH-wave propagation in a heterogeneous initially stressed medium situated below the sandy layer, which is considered to be of finite thickness. The novel aspect of the study has been the propagation of shear waves in a layered medium described above, that has a point source within it. The geometry considered in the present study is compatible with the actual scenario of the earth's crust. It may be mentioned that in the north-west Himalayan region (e.g., Jammu & Kashmir and the adjacent areas), the earth's crust contains upper Siwalik and lower Siwalik. The layers of the crust in these regions are highly heterogeneous. Keeping in view the fact that many regions of the earth's crust, including the Siwalik region have predominant heterogeneous property and that most of the areas of upper and middle Siwalik regions are sandy, the present investigation has been made for a medium that consists of a dry sandy layer overlaying a heterogeneous half-space. While formulating the problem mathematically, appropriate boundary conditions have been taken into consideration. The Fourier transform method along with Green's function approach is used for the derivation of the dispersion equation. Using the MATLAB software the dispersion curves are plotted for increasing wave number for different values of the heterogeneity parameter, initial stress parameters and the sand parameter. When the layer and half-space are both homogeneous, our dispersion equation coincides with the classical dispersion equation of Love wave. The study reveals the following:

1. Phase velocity of shear waves is enhanced as the wave number diminishes.
2. Phase velocity reduces as the tensile initial stress increases.
3. With a reduction in the compressive initial stress, the phase velocity of shear wave increases.
4. For higher values of the heterogeneity parameters of the lower half-space the speed of shear wave propagation changes slowly.
5. The phase velocity is significantly influenced by the sand parameter.

Following table gives a comparison of the results of the present study with those of similar studies performed by other researchers in the past.

In all the three studies, the results shown above have been computed by considering $\mu_1 = 6.54 \times 10^{10} N/m^2$ and $\rho = 3400\ \mathrm{kg/m^3}$. Kundu *et al.* (2016) considered wave propagation of Love waves in a heterogeneous medium containing a point source. Chattopadhyay *et al.* (2010) considered the propagation of SH-waves in a heterogeneous medium. However, both these studies were carried out in the absence of any sandy layer. Thus the difference in the results presented in the first two columns denotes the effect of the layer of sand on the earth's crust, which is very prominent in some regions of the Himalayan belt, including Jammu and Kashmir. The difference may also be partly attributed to the variation in the type of waves. The difference in the results of the first and third columns may be attributed to only the consideration of the layer of sand in the present study. The comparative study depicted in Table 3 validates the results of the present study.

Table 3 Comparison of results for non-dimensional phase velocity when $kh = 2.0$

Results of the present study	Results of Kundu *et al.* (2016)	Results of Chattopadhyay *et al.* (2010)
1.75	1.25	1.12

From the study, we can conclude that in the case of wave propagation in the above-mentioned layered medium, there exists a curvi-linear relationship between the phase velocity and the wave number, when the heterogeneity and initial stresses of the medium are taken into account. Mogi (1962) made an important observation that with the increase in heterogeneity in a layered medium, the elastic strain reduced, but during earthquakes, the strain released is high. This implies that the phase velocity is enhanced, when the heterogeneity parameter assumes higher values. It is to be noted that during severe earthquakes, the elastic strain is very large. The study reveals that the nature of wave propagation in a layered medium containing a point source depends on multiple factors, including the heterogeneity of the medium, the density and the rigidity modulus and the thickness of the sandy layer that overlays the semi-space. The results of the study have an important bearing on the study of propagation of seismic waves generated by artificial explosions and also on investigations aimed at exploring the internal structure of the earth. The study will also find applications in exploration geophysics and civil engineering.

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References

Abo-Dahab, S.M., Abd-Alla, A.M. and Khan, A. (2016) "Rotational effect on Rayleigh, Love and Stoneley waves in non-homogeneous fibre-reinforced anisotropic general viscoelastic media of higher order", *Struct. Eng. Mech.*, **58**(1), 181-197.

Biot, M.A. (1956), *Mechanics of Incremental Deformation*, Wiley, New York, U.S.A.

Chakraborty, S.K. and Chandra, A. (1984), "Reflection and refraction of plane SH-waves at the boundary of dry sandy layer and anisotropic elastic medium", *Acta Geophys. Polonica*, **32**(1), 69-79.

Chattopadhyay, A. and Kar, B.K. (1981), "Love waves due to a point source in an isotropic elastic medium under initial stress", *J. Non-Linear Mech.*, **16**(3), 247-258.

Chattopadhyay, A. and Pal, A.K. (1984), "Dispersion curves of Love waves from a point source in heterogeneous medium", *Acta Geophys. Pol.*, **32**(3), 323-332.

Chattopadhyay, A., Gupta, S., Kumari, P. and Sharma, V.K. (2012), "Effect of point source and heterogeneity on the propagation of SH-waves in a viscoelastic layer over a viscoelastic half space", *Acta Geophys.*, **60**(1), 119-139.

Chattopadhyay, A., Gupta, S., Sharma, V.K. and Kumari, P. (2010), "Effect of point source and heterogeneity on the propagation of SH-waves", *J. Appl. Math Mech.*, **6**(9), 76-89.

Daros, C.H. (2013), "Green's function for SH-waves in inhomogeneous anisotropic elastic solid with power-function velocity variation", *Wave Motion*, **50**(2), 101-110.

Ewing, W.M., Jardetzky, W.S. and Press, F. (1957), *Elastic waves in Layered Media*, McGraw-Hill, New York, U.S.A.

Ghorai, A.P., Samal, S.K. and Mahanti, N.C. (2010), "Love waves in a fluid-saturated porous layer under a rigid boundary and lying over an elastic half-space under gravity", *Appl. Math. Model.*, **34**(7), 1873-1883.

Ghosh, M.L. (1970), "Love waves due to a point source in an inhomogeneous medium", *Gerl. Beit. Zur Geophys.*, **79**, 129-141.

Gubbins, D. (1990), *Seismology and Plate Tectonics*, Cambridge University Press, Cambridge, U.K.

Kakar, R. and Kakar, S. (2016), "Dispersion of shear wave in a pre-stressed heterogeneous orthotropic layer over a pre-stressed anisotropic porous half-space with self-weight", *Struct. Eng. Mech.*, **59**(6), 951-972.

Kar, B.K., Pal, A.K. and Kalyani, V.K. (1986), "Propagation of Love waves in an irregular dry sandy layer", *Acta Geophys. Pol.*, **34**(2), 157-170.

Kochengin, S.A. (1997), "A point source of Sh wave in a medium with vertical boundary in the case where the velocity of wave propagation is a smoothly increasing function of depth", *J. Math. Sci.*, **86**(3), 2685-2693.

Kumar, K.V., Saravanan, T.J., Sreekala, R., Gopalakrishnan, N. and Mini, K.M. (2017), "Structural damage detection through longitudinal wave propagation using spectral finite element method", *Geomech. Eng.*, **12**(1), 161-183.

Kundu, S., Gupta, S. and Manna, S. (2014), "SH-type waves dispersion in an isotropic medium sandwiched between an initially stressed orthotropic and heterogeneous semi-infinite media", *Meccanica*, **49**(3), 749-758.

Kundu, S., Gupta, S., Vaishnav, P.K. and Manna, S. (2016), "Propagation of Love waves in a heterogeneous medium over an inhomogeneous half-space under the effect of point source", *J. Vib. Control*, **22**(5), 1380-1391.

Kundu, S., Manna, S. and Gupta, S. (2014), "Love wave dispersion in pre-stressed homogeneous medium over a porous half-space with irregular boundary surfaces", *J. Solids Struct.*, **51**(21-22), 3689-3697.

Li, X. and Tao, M. (2015), "The influence of initial stress on wave propagation and dynamic elastic coefficients", *Geomech. Eng.*, **8**(3), 377-390.

Manna, S., Kundu, S. and Gupta, S. (2016), "Effect of reinforcement and inhomogeneity on the propagation of Love waves", *J. Geomech.*, **16**(2), 04105045.

Manna, S., Kundu, S. and Misra, J.C. (2018), "Theoretical analysis of torsional wave propagation in a heterogeneous aeolotropic Stratum over a voigt-type viscoelastic half-space", *J. Geomech.*, **18**(6), 0401850.

Misra, J.C. and Chakravarty, S. (1982), "A free-vibration analysis for the human cranial system", *J. Biomech.*, **15**(9), 635-645.

Misra, J.C. and Roychoudhury, K. (1983), "Effect of initial stresses on wave propagation in arteries", *J. Math. Biol.*, **18**(1), 53-67.

Misra, J.C. and Samanta, S.C. (1982), "Thermal shock in a viscoelastic half-space", *J. Therm. Stresses*, **5**(3-4), 365-375.

Mogi, K. (1962), "Magnitude-frequency relationship for elastic shocks accompanying fractures of various materials and some related problems in earthquakes", *Bull. Earthq. Res. Inst. Univ.*, **40**, 831-883.

Nagesh, M. and Hayek, S.I. (1981), "Diffraction of a point source by two impedance covered half-planes", *J. Acoust. Soc. Am.*, **69**(3), 629-637.

Negin, M. (2015), "Generalized Rayleigh wave propagation in a covered half-space with liquid upper layer", *Struct. Eng. Mech.*, **56**(3), 491-506.

Pal, P.C. and Mandal, D. (2014), "Generation of SH-type waves due to shearing stress discontinuity in a sandy layer overlying an isotropic and inhomogeneous elastic half-space", *Acta Geophys.*, **62**(1), 44-58.

Singh, A.K., Das, A., Mistri, K.C. and Chattopadhyay, A. (2017), "Green's function approach to study the propagation of SH-wave in piezoelectric layer influenced by a point source", *Math. Meth. Appl. Sci.*, **40**(13), 4771-4784.

Tao, M., Chen, Z., Li, X., Zhao, H. and Yin, T. (2016), "Theoretical and numerical analysis of the influence of initial stress gradient on wave propagations", *Geomech. Eng.*, **10**(3), 285-296.

Thomasson, S.I. (1976), "Reflection of waves from a point source by an impedance boundary", *J. Acoust. Soc. Am.*, **59**(4), 780-785.

Tokhi, H., Ren G. and Li, J. (2015), "An application of wave equation analysis program to pile dynamic formulae", *Geomech. Eng.*, **9**(3), 345-360.

Tomar, S.K. and Kaur, J. (2007), "SH-waves at a corrugated interface between a dry sandy half-space and an anisotropic elastic half-space", *Acta Mechanica*, **190**(1-4), 1-28.

Watanabe, K. and Payton, R.G. (2002), "Green's function for SH-waves in a cylindrically monoclinic material", *J. Mech. Phys. Solids*, **50**(11), 2425-2439.

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