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# Vibration analysis of thick orthotropic plates using quasi 3D sinusoidal shear deformation theory

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**Abstract.** In this current work a quasi 3D “trigonometric shear deformation theory” is proposed and discussed for the dynamic of thick orthotropic plates. Contrary to the classical “higher order shear deformation theories” (HSDT) and the “first shear deformation theory” (FSDT), the constructed theory utilizes a new displacement field which includes “undetermined integral terms” and presents only three “variables”. In this model the axial displacement utilizes sinusoidal mathematical function in terms of z coordinate to introduce the shear strain impact. The cosine mathematical function in terms of z coordinate is employed in vertical displacement to introduce the impact of transverse “normal deformation”. The motion equations of the model are found via the concept of virtual work. Numerical results found for frequency of “flexural mode”, mode of shear and mode of thickness stretch impact of dynamic of simply supported “orthotropic” structures are compared and verified with those of other HSDTs and method of elasticity wherever considered.

**Keywords:** plate theory; vibration; orthotropic plate

## 1. Introduction

In the last two decades, the use of composite structures has increased steadily in many kinds of engineering application including automotive, civil engineering, aerospace, mechanical, underwater, ships and other industrial applications (Vel *et al.* 2005). The considerable advantages offered by composite structures over conventional structures are to present high strength-to-weight and stiffness-to-weight ratios, which make them ideally, suited for use in weight-sensitive structures (Sturzenbecher and Hofstetter, 2011, Ozturk, 2015). These structural elements are subjected to transverse forces, in-plane forces, and dynamic forces. Therefore, a number of studies have been performed to analyze the bending, buckling, and vibration behavior of composite structures due to the increased relevance of the composites structural components in the design of engineering structures. Since the early 1800s, various plates theories have been developed to predict more accurately their dynamic responses. The sources of plate theories can be classified into three main categories i.e., classical plate theory (CPT), first-order shear deformation plate theory (FSDT) and higher-order plate theory (HSDT) (Abualnour *et al.* 2018). The classical plate theory (Kirchhoff 1850a, b) ignores the transverse shear strain and is appropriate just to investigates thin plates. As a consequence it under predicts vertical displacements and over predicts natural frequencies. However, it is not appropriate for the moderately thick and thick plates, which require that the transverse and normal strain should be taken into account. The first-order shear deformation plate theory (Reissner 1945, Mindlin 1951) overcomes this problem by taking into account this effect. Many studies have been devoted for free vibration analysis of classical composite and orthotropic plates using first-order shear deformation plate theory (Chen and Liu 1990, Hashemi and Arsanjani 2005, Kshirsagar and Bhaskar 2009, Sadoune *et al.* 2014, Bellifa *et al.* 2016, Bouderba *et al.* 2016, Youcef *et al.* 2018). Since these models violate the equilibrium conditions at the top and bottom surfaces of the plate, and needs a shear correction factor which is hard to find as it depends on the geometries, material properties and boundary conditions of each problem (Ferreira *et al.* 2009, Thai and Kim 2012, Abualnour *et al.* 2018). To surmount the disadvantages of the first-order shear deformation plate theory, a number of higher-order shear deformation plate theories (HSDT), by applying a nonlinear variation of high order axial displacement in power series of the coordinate normal to the middle plane, have been proposed and it is not necessary to introduce the notion of shear correction factor. Many polynomial such as third-order shear deformation plate theory (Nelson and Lorch 1974, Krishna Murty 1977, Lo *et al.* 1977a, b, Levinson 1980, Kant 1982, Bhimaraddi and Stevens 1984, Reddy 1984, Hanna and Leissa 1994,

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Matsunaga 2008, Reddy 2011, Talha and Singh 2010, Bourada *et al.* 2012, Jha *et al.* 2013, Eltaher *et al.* 2013, Zidi *et al.* 2014, Ahmed 2014, Ait Amar Meziane *et al.* 2014, Ait Yahia *et al.* 2015, Zemri *et al.* 2015, Mahapatra *et al.* 2015, Kar *et al.* 2016, Bounouara *et al.* 2016, Becheri *et al.* 2016, Baseri *et al.* 2016, Mohammadimehr *et al.* 2016, Raminnea *et al.* 2016, Javed *et al.* 2016, Janghorban 2016, Rahmani *et al.* 2017, Fahsi *et al.* 2017, Yazid *et al.* 2018) and non polynomial such as sinusoidal, hyperbolic and exponential shear deformable plate theory (Levy 1977, Stein 1990, Touratier 1991, Soldatos 1992, Karama 2003, Zenkour *et al.* 2010a, 2011, Zenkour and Sobhy 2010, 2011, Ghugal and Sayyad 2011, Mantari 2012, Bouderba *et al.* 2013, Tounsi *et al.* 2013, Sobhy 2014, Ait Atmane *et al.* 2015, Sayyad and Ghugal 2015, Belkorissat *et al.* 2015, Al-Basyouni *et al.* 2015, Mahi *et al.* 2015, Akavci 2016, Bousahla *et al.* 2016, Laoufi *et al.* 2016, Beldjelili *et al.* 2016, Saidi *et al.* 2016, Ghorbanpour Arani *et al.* 2016, Wu and Chiu 2011, Ahouel *et al.* 2016, Boukhari *et al.* 2016, Hebali *et al.* 2016, Mehar and Panda 2017, Menasria *et al.* 2017, Zidi *et al.* 2017, Khetir *et al.* 2017, Klouche *et al.* 2017, Chikh *et al.* 2017, El-Haina *et al.* 2017, Mouffoki *et al.* 2017, Sekkal *et al.* 2017a, Bellifa *et al.* 2017a, Besseghier *et al.* 2017, Hirwani *et al.* 2017a, b, Hachemi *et al.* 2017, Abdelaziz *et al.* 2017, Mehar *et al.* 2017, Sahoo *et al.* 2017, Zine *et al.* 2018, Sobhy and Zenkour 2018) are developed for the bending and dynamic analysis of homogenous and laminated thick rectangular plates. A critical review of more works on the development of structures models can be found in (Noor and Burton 1989, Vasil'ev 1992, Kant 1993, Liew *et al.* 1995, Ghugal and Shimpi 2002, Wanji and Zhen 2008, Kreja 2011, Khandan *et al.* 2012). Recently, some studies are presented with considering shear and normal strains effects, in the open literature such as (Zenkour *et al.* 2010b, Fekrar *et al.* 2014, Belabed *et al.* 2014, Bousahla *et al.* 2014, Hebali *et al.* 2014, Bourada *et al.* 2015, Hamidi *et al.* 2015, Meradjah *et al.* 2015, Larbi Chaht *et al.* 2015, Sobhy and Radwan 2017, Zenkour and Sobhy 2015, Bennoun *et al.* 2016, Draiche *et al.* 2016, Sekkal *et al.* 2017b, Benahmed *et al.* 2017, Bouafia *et al.* 2017, Benchohra *et al.* 2018, Bouhadra *et al.* 2018, Karami *et al.* 2018a, b, Younsi *et al.* 2018).

This paper aims to improve the plate theory developed by Tounsi and his co-workers (Hebali *et al.* 2016) by including the so-called stretching effect. Using the proposed theory, both for vibration problem of orthotropic square and rectangular plates are investigated. The displacement model contains undetermined integral terms in addition to CPT terms. The numbers of variables are lower as that of conventional HSDTs. Equations of motion are derived from the Hamilton's principle. Analytical solutions are obtained for orthotropic plate, and accuracy is verified by comparing the obtained results with those reported in the literature.

## 2. Orthotropic plate under consideration

In this work, a rectangular plate of length $a$, width $b$, and a constant thickness $h$ is considered for investigation. The structure occupies (in O – $x$ – $y$ – $z$ right-handed Cartesian coordinate system) a region

$$0 \le x \le a\ ; \quad 0 \le y \le b\ ; \quad -h/2 \le y \le h/2 \qquad (1)$$

### *2.1 Assumptions made in theoretical formulation*

The displacement field of the proposed theory is constructed based on the following suppositions:

1. The displacement components $u$ and $v$ are the axial displacements in $x$ and $y$-directions respectively and $w$ is the vertical displacement in $z$-direction. These displacements are small in comparison with the plate thickness.

2. The axial displacement $u$ in $x$ -direction and $v$ in $y$-direction each consist of two parts:

a) Displacement component analogous to displacement in CPT of bending;

b) Displacement component due to shear deformation which is considered to be sinusoidal in nature with respect to thickness coordinate and is formulated by using the integral term.

3. The vertical displacement $w$ in z -direction is supposed to be a function of $x$, $y$ and $z$ coordinates.

The body forces are ignored in the investigation.

### *2.2 The displacement field*

Based upon the before mentioned assumptions and including the effect of transverse normal stress (thickness stretching effect), the displacement field of the proposed plate theory can be described as

$$u(x,y,z,t) = -z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y,t)\, dx \qquad (2a)$$

$$v(x,y,z,t) = -z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y,t)\, dy \qquad (2b)$$

$$w(x,y,z,t) = w_0(x,y,t) + g(z)\phi_\xi(x,y,t) \qquad (2c)$$

The coefficients $k_1$ and $k_2$ depends on the geometry. It can be observed that the kinematic in Eq. (2) uses only three unknowns ($w_0$, $\theta$ and $\phi_\xi$). In this work, the proposed HSDT is obtained by putting

$$f(z) = \frac{h}{\pi}\sin\left(\frac{\pi}{h}z\right) \quad \text{and} \quad g(z) = \frac{df}{dz} = \cos\left(\frac{\pi}{h}z\right) \qquad (3)$$

### *2.3 Strain displacement relationship*

Nonzero strains of the five variable plate model by employing displacement field expressed by Eq. (2) are expressed as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = +z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix},\quad \varepsilon_z = g'(z)\,\varepsilon_z^0 \qquad (4)$$

where

$$\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \dfrac{\partial}{\partial y}\int \theta\, dx + k_2 \dfrac{\partial}{\partial x}\int \theta\, dy \end{Bmatrix}, \tag{5}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta\, dy + \dfrac{\partial \phi_\xi}{\partial y} \\ k_1 \int \theta\, dx + \dfrac{\partial \phi_\xi}{\partial x} \end{Bmatrix}, \quad \varepsilon_z^0 = \phi_\xi$$

It can be observed from Eq. (4) that the transverse shear strains ($\gamma_{xz}$, $\gamma_{yz}$) are equal to zero at the upper ($z=h/2$) and lower ($z=-h/2$) surfaces of the plate. A shear correction coefficient is, hence, not required.

The integrals used in the above equations shall be resolved by a Navier type procedure and can be expressed as follows

$$\frac{\partial}{\partial y}\int \theta\, dx = A'\frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta\, dy = B'\frac{\partial^2 \theta}{\partial x \partial y}, \quad \int \theta\, dx = A'\frac{\partial \theta}{\partial x}, \quad \int \theta\, dy = B'\frac{\partial \theta}{\partial y} \tag{6}$$

where the coefficients $A'$ and $B'$ are considered according to the type of solution employed, in this case via Navier method. Therefore, $A'$, $B'$, $k_1$ and $k_2$ are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, k_1 = \alpha^2, k_2 = \beta^2 \tag{7}$$

where $\alpha$ and $\beta$ are defined in expression (20b).

### 2.4 Stress-strain relationship

For elastic and orthotropic materials, the constitutive relations can be written as (Karami *et al.* 2018c)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{8}$$

where $C_{ij}$ are the reduced stiffness coefficients as given by Jones (1975) are as follows

$$C_{22} = \frac{E_2(1-\nu_{13}\nu_{31})}{\Delta}, \quad C_{23} = \frac{E_2(\nu_{32}-\nu_{12}\nu_{31})}{\Delta}, \quad C_{33} = \frac{E_3(1-\nu_{12}\nu_{21})}{\Delta},$$
$$C_{66} = G_{12}, \quad C_{55} = G_{13}, \quad C_{44} = G_{23}, \tag{9}$$
$$\Delta = 1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{31}\nu_{13}-2\nu_{21}\nu_{32}\nu_{13}$$

### 2.5. Equations of motion

Hamilton's principle is utilized herein to obtain the equations of motion. The principle can be analytically written as (Taibi *et al.* 2015, Attia *et al.* 2015, 2018, Houari *et al.* 2016, Bellifa *et al.* 2017b, Benadouda *et al.* 2017, Bakhadda *et al.* 2018, Meksi *et al.* 2018, Kaci *et al.* 2018, Belabed *et al.* 2018, Fourn *et al.* 2018, Mokhtar *et al.* 2018)

$$0 = \int_0^T (\delta U - \delta K)\, dt \tag{10}$$

where $\delta U$ is the variation of strain energy; and $\delta K$ is the variation of kinetic energy. The variation of strain energy is written explicitly by

$$\begin{aligned} \delta U &= \int_V [\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_z \delta\varepsilon_z + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}]dV \\ &= \int_A [N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_z \delta\varepsilon_z^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \\ &\quad + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta\gamma_{yz}^s + S_{xz}^s \delta\gamma_{xz}^0 ]dA = 0 \end{aligned} \tag{11}$$

where $A$ is the area of top surface and the stress resultants $N$, $M$, and $S$ are expressed by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz, \quad (i = x, y, xy);$$
$$N_z = \int_{-h/2}^{h/2} g'(z)\sigma_z dz \quad \text{and} \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g(\tau_{xz}, \tau_{yz})dz \tag{12}$$

Substituting Eq. (4) into Eq. (8) and the subsequent results into Eq. (16), the stress resultants can be written in terms of generalized displacements ($w_0$, $\theta$ and $\varphi_z$) as

$$\begin{Bmatrix} M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \\ N_z \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 & Y_{13} \\ D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 & Y_{23} \\ 0 & 0 & D_{66} & 0 & 0 & D_{66}^s & 0 \\ D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 & Y_{13}^s \\ D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 & Y_{23}^s \\ 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s & 0 \\ Y_{13} & Y_{23} & 0 & Y_{13}^s & Y_{23}^s & 0 & Z_{33} \end{bmatrix} \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \\ k_x^s \\ k_y^s \\ k_{xy}^s \\ \phi_\xi \end{Bmatrix} \tag{13a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} k_2 B' \dfrac{\partial \theta}{\partial y} + \dfrac{\partial \varphi_z}{\partial y} \\ k_1 A' \dfrac{\partial \theta}{\partial x} + \dfrac{\partial \varphi_z}{\partial x} \end{Bmatrix} \tag{13b}$$

where

$$\left(D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2} C_{ij}\left(z^2, f(z), z\, f(z), f^2(z)\right) dz \tag{14a}$$

$$\left(X_{ij}, Y_{ij}, Y_{ij}^s, Z_{ij}\right) = \int_{-h/2}^{h/2} \left(1, z, f(z), g'(z)\right) g'(z) C_{ij} dz \tag{14b}$$

The variation of kinetic energy is given by

$$\begin{aligned} \delta K &= \int_V [\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w}]\ \rho\, dV \\ &= \int_A \Big\{ I_0 \dot{w}_0 \delta\dot{w}_0 + J_0\left(\dot{\phi}_\xi \delta\dot{w}_0 + \dot{w}_0 \delta\dot{\phi}_\xi\right) \\ &+ I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right) + K_2\left((k_1 A')^2\left(\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}\right) + (k_2 B')^2\left(\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}\right)\right) \\ &- J_2\left((k_1 A')\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right) + (k_2 B')\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\right) + K_0\left(\dot{\phi}_\xi \delta\dot{\phi}_\xi\right)\Big\} dA \end{aligned} \tag{15}$$

where dot-superscript convention indicates the differentiation with respect to the time variable $t$; $\rho$ is the mass density; and ($I_i$, $J_i$, $K_i$) are mass inertias expressed by

$$(I_0, I_2) = \int_{-h/2}^{h/2} (1, z^2) \rho dz \tag{16a}$$

$$(J_0, J_2) = \int_{-h/2}^{h/2} (g, z\, f) \rho dz \tag{16b}$$

$$(K_0, K_2) = \int_{-h/2}^{h/2} (g^2, f^2) \rho(z) dz \tag{16c}$$

The equations of motion can be obtained by substituting the equations for $\delta U$ and $\delta K$ from Eqs. (11) and (15) into Eq. (10), integrating by parts and collecting the coefficients of $\delta w_0$, $\delta\theta$, and $\delta\phi\xi$

$$\begin{aligned}
&\delta\, w_0:\ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = I_0 \ddot{w}_0 - I_2 \nabla^2 \ddot{w}_0 + J_2\left(k_1\, A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + k_2\, B' \frac{\partial^2 \ddot{\theta}}{\partial y^2}\right) + J_0\, \ddot{\phi}_\xi \\
&\delta\, \theta:\ -k_1 M_x^s - k_2 M_y^s - (k_1 A' + k_2 B') \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1\, A' \frac{\partial S_{xz}^s}{\partial x} + k_2\, B' \frac{\partial S_{yz}^s}{\partial y} = -K_2\left((k_1\, A')^2\, \frac{\partial^2 \ddot{\theta}}{\partial x^2} + (k_2\, B')^2\, \frac{\partial^2 \ddot{\theta}}{\partial y^2}\right) \\
&\qquad + J_2\left(k_1\, A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + k_2\, B' \frac{\partial^2 \ddot{w}_0}{\partial y^2}\right) \\
&\delta\, \phi_\xi:\ -N_z + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} = J_0\, \ddot{w}_0 + K_0\, \ddot{\phi}_\xi
\end{aligned} \tag{17}$$

Substituting Eq. (13) into Eq. (17), the equations of motion of the present quasi-3D sinusoidal shear deformation theory can be written in terms of displacements ($w_0$, $\theta$, $\phi_\xi$) as

$$\begin{aligned}
&Y_{13}\, d_{11}\phi_z + Y_{23}\, d_{22}\phi_z - D_{11} d_{1111} w_0 - 2(D_{12} + 2D_{66})\, d_{1122} w_0 - D_{22}\, d_{2222} w_0 + (D_{11}^s\, k_1 + D_{12}^s\, k_2)\, d_{11}\theta \\
&\quad + 2(D_{66}^s (k_1 A' + k_2 B'))\, d_{1122}\theta + (D_{12}^s k_1 + D_{22}^s k_2)\, d_{22}\theta = I_0 \ddot{w}_0 \\
&-I_2 (d_{11}\ddot{w}_0 + d_{22}\ddot{w}_0) + J_2 (k_1 A'\, d_{11}\ddot{\theta} + k_2\, B'\, d_{22}\ddot{\theta}) + J_0\, \ddot{\phi}_\xi
\end{aligned} \tag{18a}$$

$$\begin{aligned}
&-k_1 Y_{13}^s \phi_z - k_2 Y_{23}^s \phi_z + (D_{11}^s k_1 + D_{12}^s k_2) d_{11} w_0 + 2\,(D_{66}^s (k_1 A' + k_2 B')) d_{1122} w_0 + (D_{12}^s k_1 + D_{22}^s k_2) d_{22} w_0 \\
&-H_{11}^s k_1^2\, \theta - H_{22}^s\, k_2^2\, \theta - 2 H_{12}^s\, k_1\, k_2 \theta - ((k_1 A' + k_2 B')^2\, H_{66}^s) d_{1122}\theta + A_{44}^s (k_2 B')^2\, d_{22}\theta + A_{55}^s\, (k_1\, A')^2\, d_{11}\theta \\
&+ A_{44}^s (k_2 B')\, d_{22}\phi_\xi + A_{55}^s (k_1 A')\, d_{11}\phi_\xi = J_2 (k_1\, A' d_{11}\ddot{w}_0 + k_2\, B' d_{22}\ddot{w}_0) \\
&- K_2 ((k_1 A')^2\, d_{11}\ddot{\theta} + (k_2\, B')^2\, d_{22}\ddot{\theta})
\end{aligned} \tag{18b}$$

$$\begin{aligned}
&Z_{33}\phi_\xi + Y_{13}\, d_{11} w_0 + Y_{23}\, d_{22} w_0 \\
&\quad + (A_{44}^s - Y_{23}^s)(k_2 B') d_{22}\theta + (A_{55}^s - Y_{13}^s)\,(k_1\, A') d_{11}\theta + A_{44}^s\, d_{22}\phi_\xi + A_{55}^s\, d_{11}\phi_\xi = J_0\, \ddot{\phi}_\xi + K_0 \ddot{w}_0\,,
\end{aligned} \tag{18c}$$

where $d_{ij}$, $d_{ijl}$ and $d_{ijlm}$ are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j},\quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},\quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m},\quad d_i = \frac{\partial}{\partial x_i},\ (i, j, l, m = 1, 2). \tag{19}$$

## 3. Analytical solutions

In this part, a simply supported rectangular plate is considered with length $a$ and width $b$. Using the Navier solution procedure, the following expressions of displacements ($w_0$, $\theta$, $\phi\xi$) are taken

$$\begin{Bmatrix} w_0 \\ \theta \\ \phi_\xi \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} W_{mn} e^{i\omega\, t} \sin(\alpha\, x) \sin(\beta\, y) \\ X_{mn} e^{i\omega\, t} \sin(\alpha\, x) \sin(\beta\, y) \\ Y_{mn} e^{i\omega\, t} \sin(\alpha\, x) \sin(\beta\, y) \end{Bmatrix} \tag{20}$$

with

$$\alpha = m\pi / a \quad \text{and} \quad \beta = n\pi / b$$

where $i = \sqrt{-1}$, ($W_{mn}$, $X_{mn}$, $Y_{mn}$) are the unknown maximum amplitudes of displacement, and $\omega$ is the frequency of vibration.

Substituting Eq. (20) into Eq. (18), the analytical solutions can be determined by

$$\left( \begin{bmatrix} s_{33} & s_{34} & s_{35} \\ s_{34} & s_{44} & s_{45} \\ s_{35} & s_{45} & M_{55} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{33} & m_{34} & m_{35} \\ m_{34} & m_{44} & 0 \\ m_{35} & 0 & m_{55} \end{bmatrix} \right) \begin{Bmatrix} W_{mn} \\ X_{mn} \\ Y_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{21}$$

where

$$\begin{aligned}
s_{33} &= \alpha^4 D_{11} + \beta^4 D_{22} + 2\alpha^2 \beta^2 (D_{12} + 2D_{66}) \\
s_{34} &= \alpha^2 k_1 D_{11}^s + (k_2 \alpha^2 + k_1 \beta^2) D_{12}^s + \beta^2 k_2 D_{22}^s \\
&\quad - 2\alpha^2 \beta^2 (k_1 A' + k_2 B') D_{66}^s \\
s_{35} &= \alpha^2 Y_{13} + \beta^2 Y_{23}, \\
s_{44} &= k_1{}^2 H_{11}^s + k_2{}^2 H_{22}^s + 2 k_1 k_2 H_{12}^s \\
&\quad + \alpha^2 \beta^2 (k_1 A' + k_2 B')^2 H_{66}^s + \alpha^2 (k_1 A')^2 A_{55}^s \\
&\quad + \beta^2 (k_2 B')^2 A_{44}^s \\
s_{45} &= k_1 Y_{13}^s + k_2 Y_{23}^s + \alpha^2 k_1 A' A_{55}^s + \beta^2 k_2 B' A_{44}^s, \\
s_{55} &= \alpha^2 A_{55}^s + \beta^2 A_{44}^s + Z_{33} \\
m_{33} &= I_0 + I_2 (\alpha^2 + \beta^2), \\
m_{34} &= -J_2 (k_1 A' \alpha^2 + k_2 B' \beta^2), \\
m_{44} &= K_2 ((k_1 A')^2 \alpha^2 + (k_2 B')^2 \beta^2), \quad m_{35} = J_0, \\
m_{55} &= K_0
\end{aligned} \tag{22}$$

The orthotropic plate has following material properties as given by Srinivas *et al.* (1970)

$$\begin{aligned}
&C_{11} = 32.2 \times 10^6\ psi,\ C_{22} = 12.6 \times 10^6\ psi,\ C_{33} = 12.3 \times 10^6\ psi \\
&C_{12} = 5.41 \times 10^6\ psi,\ C_{13} = 0.25 \times 10^6\ psi,\ C_{23} = 2.28 \times 10^6\ psi \\
&C_{44} = 6.19 \times 10^6\ psi,\ C_{55} = 3.71 \times 10^6\ psi,\ C_{66} = 6.10 \times 10^6\ psi
\end{aligned} \tag{23}$$

The density (ρ) of material can be taken as any arbitrary value for calculation of frequencies.

## 4. Numerical results and discussion

In the present work dynamic investigation of simply supported orthotropic plate for thickness ratio 10 is examined. The results computed using the proposed quasi 3D trigonometric shear deformation are compared with exact results and those of other HSDT results existing in literature wherever applicable. Following non-dimensional form is employed for the purpose of presenting the results in this work.

$$\bar{\omega} = \omega_{mn} h \sqrt{\frac{\rho}{Q_{11}}} \tag{24}$$

Table 1 Comparison of natural frequencies of simply-supported orthotropic square plate ($b/a$=1, $h/a$=0.1)

| $(m,n)$ | Exact[(a)] | | | Present | | | Ref[(b)] | | | | Ref[(c)] | | | FSDT | | | CPT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{\omega}_w$ | $\bar{\omega}_\varphi$ | $\bar{\omega}_\psi$ | $\bar{\omega}_w$ | $\bar{\omega}_\theta$ | $\bar{\omega}_\xi$ | $\bar{\omega}_w$ | $\bar{\omega}_\varphi$ | $\bar{\omega}_\psi$ | $\bar{\omega}_\xi$ | $\bar{\omega}_w$ | $\bar{\omega}_\varphi$ | $\bar{\omega}_\psi$ | $\bar{\omega}_w$ | $\bar{\omega}_\varphi$ | $\bar{\omega}_\psi$ | $\bar{\omega}_w$ |
| (1,1) | 0.0474 | 1.3077 | 1.6530 | 0.0477 | 1.5116 | 5.2542 | 0.0474 | 1.2999 | 1.6448 | 5.2662 | 0.0474 | 1.3086 | 1.6550 | 0.0474 | 1.3159 | 1.6647 | 0.0497 |
| (1,2) | 0.1033 | 1.3331 | 1.7160 | 0.1041 | 1.6745 | 5.2529 | 0.1033 | 1.3290 | 1.7105 | 5.2269 | 0.1033 | 1.3339 | 1.7209 | 0.1032 | 1.3410 | 1.7307 | 0.1120 |
| (1,3) | 0.1888 | 1.3665 | 1.8115 | 0.1903 | 1.7987 | 5.2511 | 0.1888 | 1.3638 | 1.8052 | 5.1530 | 0.1888 | 1.3772 | 1.8210 | 0.1884 | 1.3841 | 1.8307 | 0.2154 |
| (1,4) | 0.2969 | 1.4372 | 1.9306 | 0.2990 | 1.9228 | 5.2491 | 0.2969 | 1.4281 | 1.9249 | 5.0416 | 0.2969 | 1.4379 | 1.9466 | 0.2959 | 1.4445 | 1.9562 | 0.3599 |
| (2,1) | 0.1188 | 1.4205 | 1.6805 | 0.1198 | 1.5133 | 5.2512 | 0.1190 | 1.4168 | 1.6728 | 5.3073 | 0.1189 | 1.4216 | 1.6827 | 0.1187 | 1.4285 | 1.6922 | 0.1354 |
| (2,2) | 0.1694 | 1.4316 | 1.7509 | 0.1723 | 1.6740 | 5.2498 | 0.1697 | 1.4277 | 1.7462 | 5.2692 | 0.1695 | 1.4323 | 1.7562 | 0.1692 | 1.4393 | 1.7657 | 0.1987 |
| (2,3) | 0.2475 | 1.4596 | 1.8523 | 0.2525 | 1.8227 | 5.2480 | 0.2480 | 1.4562 | 1.8418 | 5.1904 | 0.2477 | 1.4603 | 1.8622 | 0.2459 | 1.4671 | 1.8717 | 0.3029 |
| (2,4) | 0.3476 | 1.5068 | 1.9749 | 0.3545 | 1.9657 | 5.2461 | 0.3482 | 1.5039 | 1.9701 | 5.0764 | 0.3479 | 1.5076 | 1.9912 | 0.3463 | 1.5142 | 2.0004 | 0.4480 |
| (3,1) | 0.2180 | 1.5777 | 1.7334 | 0.2198 | 1.6442 | 5.2465 | 0.2191 | 1.5744 | 1.7274 | 5.3706 | 0.2184 | 1.5789 | 1.7361 | 0.2178 | 1.5857 | 1.7452 | 0.2779 |
| (3,2) | 0.2624 | 1.5651 | 1.8195 | 0.2678 | 1.7645 | 5.2451 | 0.2637 | 1.5612 | 1.8068 | 5.3367 | 0.2629 | 1.5658 | 1.8255 | 0.2619 | 1.5727 | 1.8343 | 0.3418 |
| (3,3) | 0.3320 | 1.5737 | 1.9289 | 0.3413 | 1.9033 | 5.2434 | 0.3337 | 1.5701 | 1.9203 | 5.2653 | 0.3326 | 1.5744 | 1.9395 | 0.3310 | 1.5812 | 1.9418 | 0.4470 |
| (4,1) | 0.3319 | 1.7179 | 1.8458 | 0.3343 | 1.8383 | 5.2404 | 0.3351 | 1.7119 | 1.8437 | 5.4552 | 0.3330 | 1.7189 | 1.8583 | 0.3311 | 1.7265 | 1.7267 | 0.4773 |
| (4,2) | 0.3707 | 1.6940 | 1.9447 | 0.3778 | 1.9256 | 5.2392 | 0.3743 | 1.6890 | 1.9351 | 5.4284 | 0.3720 | 1.6947 | 1.9514 | 0.3696 | 1.7022 | 1.9588 | 0.5415 |

Table 2 Comparison of natural frequencies of simply-supported orthotropic rectangular plate (b/a =$\sqrt{2}$, $h/a$=0.1)

| $(m,n)$ | Present | | | Ref[(b)] | | | | Ref[(c)] | | | FSDT | | | CPT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{\omega}_w$ | $\bar{\omega}_\theta$ | $\bar{\omega}_\xi$ | $\bar{\omega}_w$ | $\bar{\omega}_\varphi$ | $\bar{\omega}_\psi$ | $\bar{\omega}_\xi$ | $\bar{\omega}_w$ | $\bar{\omega}_\varphi$ | $\bar{\omega}_\psi$ | $\bar{\omega}_w$ | $\bar{\omega}_\varphi$ | $\bar{\omega}_\psi$ | $\bar{\omega}_w$ |
| (1,1) | 0.0377 | 1.4405 | 5.2544 | 0.0376 | 1.3036 | 1.6420 | 5.2701 | 0.0378 | 1.3045 | 1.6437 | 0.0377 | 1.3118 | 1.6533 | 0.0390 |
| (1,2) | 0.0670 | 1.5903 | 5.2537 | 0.0653 | 1.3162 | 1.6738 | 5.2577 | 0.0676 | 1.3169 | 1.6786 | 0.0669 | 1.3242 | 1.6882 | 0.0701 |
| (1,3) | 0.1131 | 1.6901 | 5.2227 | 0.1066 | 1.3376 | 1.7224 | 5.2208 | 0.1142 | 1.3382 | 1.7336 | 0.1132 | 1.3453 | 1.7433 | 0.1210 |
| (1,4) | 0.1737 | 1.7775 | 5.2515 | 0.1768 | 1.3680 | 1.7835 | 5.1582 | 0.1750 | 1.3686 | 1.8054 | 0.1739 | 1.3755 | 1.8151 | 0.1903 |
| (2,1) | 0.1105 | 1.4708 | 5.2514 | 0.1104 | 1.4194 | 1.6683 | 5.3106 | 0.1104 | 1.4206 | 1.6696 | 0.1100 | 1.4276 | 1.6790 | 0.1225 |
| (2,2) | 0.1378 | 1.5800 | 5.2507 | 0.1371 | 1.4235 | 1.7054 | 5.2989 | 0.1377 | 1.4243 | 1.7094 | 0.1362 | 1.4313 | 1.7188 | 0.1533 |
| (2,3) | 0.1807 | 1.6928 | 5.2496 | 0.1728 | 1.4344 | 1.7583 | 5.2614 | 0.1804 | 1.4348 | 1.7696 | 0.1779 | 1.4417 | 1.7790 | 0.2032 |
| (2,4) | 0.2371 | 1.7979 | 5.2483 | 0.2136 | 1.4540 | 1.8209 | 5.1955 | 0.2366 | 1.4543 | 1.8456 | 0.2333 | 1.4611 | 1.8550 | 0.2711 |
| (3,1) | 0.2114 | 1.6203 | 5.2467 | 0.2114 | 1.5831 | 1.7171 | 5.3729 | 0.2110 | 1.5852 | 1.7172 | 0.2102 | 1.5920 | 1.7262 | 0.2575 |
| (3,2) | 0.2362 | 1.6900 | 5.2460 | 0.2365 | 1.5697 | 1.7691 | 5.3634 | 0.2352 | 1.5713 | 1.7704 | 0.2329 | 1.5782 | 1.7792 | 0.2870 |
| (3,3) | 0.2754 | 1.7809 | 5.2449 | 0.2701 | 1.5650 | 1.8326 | 5.3277 | 0.2735 | 1.5658 | 1.8400 | 0.2695 | 1.5728 | 1.8488 | 0.3352 |
| (4,1) | 0.3267 | 1.8218 | 5.2407 | 0.3269 | 1.8376 | 1.7267 | 5.4560 | 0.3262 | 1.8370 | 1.7289 | 0.3246 | 1.8439 | 1.7371 | 0.4381 |
| (4,2) | 0.3491 | 1.8694 | 5.2400 | 0.3500 | 1.8967 | 1.7047 | 5.4500 | 0.3475 | 1.8947 | 1.7066 | 0.3442 | 1.9020 | 1.7143 | 0.4661 |

Results determined for frequencies of bending mode, thickness shear mode and thickness stretching mode are compared and discussed with the corresponding results of CPT, FSDT, Reddy's theory and exact theory (Srinivas 1970) and the trigonometric shear deformation theory (Ghugal and Sayyad 2011).

### 4.1 Bending frequency ($\bar{\omega}_w$)

Table 1 demonstrates comparison of bending frequencies for all modes of dynamic for square plate (b/a=1). It can be observed from Table 1 that the proposed theory provides excellent values of frequencies for all modes of vibration. The proposed theory, Reddy's theory (Reddy 1984), Ghugal and Sayyad's theory (2011) and Mindlin's theory (1951) predicts exact result of bending frequency for fundamental mode i.e. m = 1, n = 1.

Table 2 shows a comparison study of bending frequency for the rectangular plate (b/a =$\sqrt{2}$). From Table 2 it can be seen that the proposed theory with only three unknown shows an excellent agreement with Ghugal and Sayyad's theory (2011) containing a higher number of unknowns. In case of rectangular plate, CPT overestimates the results of bending frequency as compared to those of other HSDTs because of the neglect of transverse shear deformation and thickness stretching effects in the CPT.

### 4.2 Thickness shear mode frequency ($\bar{\omega}_\theta$)

From the Table 1 it can be seen that, for square plate (b/a = 1) the proposed theory provides a good results of thickness shear mode frequency for all modes of vibration as compared to that of exact theory and other HSDTs.

The comparison of thickness shear mode frequency for rectangular plate (b/a =$\sqrt{2}$) is demonstrated in Table 2. The frequencies determined by present model are in good agreement with those of other HSDTs.

### 4.3 Thickness stretch mode frequency ($\bar{\omega}_{\xi}$)

In Table 1 and 2 results of frequency of thickness stretching mode of vibration are provided for square and rectangular plates. The results of this frequency by other HSDTs are not available in the literature because of the neglect of thickness stretching effect in these models. The results compared with those obtained by Ghugal and Sayyad's theory (2011) confirm a good agreement between the two theories.

## 5. Conclusions

A new three variable trigonometric plate theory is developed in this study for dynamic analysis of orthotropic plates. Following conclusions are drawn from the present study:

1. The frequencies computed by the proposed theory for bending and thickness shear modes of vibration for all modes of vibration are in good agreement with the exact values of frequencies for the square plate (b/a = 1).

2. The frequencies of bending and thickness shear modes of vibration computed by the present model are in good agreement with those of HSDT for rectangular plate (b/a =$\sqrt{2}$).

3. The present theory with only three unknowns is capable to provide frequencies of thickness stretching mode of vibration and the obtained results are in excellent agreement with those of Ghugal and Sayyad's theory (2011) containing four unknowns.

4. The present theory is not only accurate but also simple in predicting the vibration analysis of orthotropic plates.

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