**Geomechanics and Engineering**, *Vol. 16, No. 1 (2018) 11-19*
DOI: https://doi.org/10.12989/gae.2018.16.1.011 

# Seismic surface waves in a pre-stressed imperfectly bonded covered half-space

Masoud Negin*

*Faculty of Engineering and Natural Sciences, Bahcesehir University, Besiktas, 34353, Istanbul, Turkey*

*(Received April 13, 2016, Revised December 6, 2017, Accepted March 7, 2018)*

**Abstract.** Propagation of the generalized Rayleigh waves in an elastic half-space covered by an elastic layer for different initial stress combinations and imperfect contact conditions is investigated. Three-dimensional linearized theory of elastic waves in initially stressed bodies in plane-strain state is employed, the corresponding dispersion equation is derived and an algorithm is developed for numerical solution to this equation. Numerical results on the influence of the initial stress patterns and on the influence of the contact conditions are presented and discussed. The case where the external forces are “follower forces” is considered as well. These investigations provide some theoretical foundations for the study of the near-surface waves propagating in layered mechanical systems and can be successfully used for estimation of the degree of the bonded defects between layers, fault characteristics and study of the behavior of seismic surface waves propagating under the bottom of the oceans.

**Keywords:** generalized Rayleigh waves; initial stresses; imperfect contact conditions; follower forces; wave dispersion

## 1. Introduction

Surface waves have become a widespread technique for characterization of Earth’s interior features during the last two decades, especially because of the development of modern electronic devices and increased numerical possibilities. Geophysical and earthquake engineering applications are site characterization, determination of the shear wave velocity profiles, damping ratio, evaluation of soil improvement by measuring surface wave velocities before and after ground modification, and study of the earthquakes, fault dynamics, earthquake site response, dynamic soil-structure interaction, and so on. Comprehensive literature reviews on the topic are given by Goldstein and Maugin (2005), Socco *et al.* (2010b) and Foti *et al.* (2011). Other relevant applications, for example, to study water saturated sediments, determination of a small strain stiffness profile, characterization of the dissipative behavior of soils and damping estimations can be found in papers by Strobbia and Cassiani (2007), van der Kruk *et al.* (2010), Malagnini *et al.* (1995), Rix *et al.* (2000), Lai *et al.* (2002), Xia *et al.* (2002), Foti (2004) and Badsar *et al.* (2010). Therefore the theoretical study of the influence of the reference characteristics of the layered medium, such as the effect of existence of different initial stress components in the constituents of the system and also the imperfectness of the contact conditions between Earth’s crustal layers on the dispersion of these waves has not only theoretical but also a practical significance.

In Earth’s crustal layers these initial stresses are present due to the temperature variations or they might occur under the action of geostatic and geodynamic forces. This affects the dynamical behavior of the system and several studies have been presented so far on the influence of the initial stresses in the elements of the constructions as well as in the layered medium on the dispersion of the propagated surface waves. A detailed consideration of the results of these type of investigations were given in the monographs by Biot (1965), Eringen and Suhubi (1975), Guz (2004) and Akbarov (2015) and reviews of the more recent investigations can be found in papers by Akbarov (2012), Akbarov and Ipek (2012), and Akbarov *et al.* (2011). Some of those numerous studies on the near-surface wave propagation in initially stresses layered half-spaces which were carried out in the last twenty years will be presented here.

Rogerson and Fu (1995) carried out an asymptotic analysis of dispersion relations for wave propagation in a pre-strained incompressible elastic plate and obtained an asymptotic expansion for the wave speed as a function of wave number and pre-stresses. The propagation of elastic interfacial waves along the plane boundary separating two pre-strained compressible half-space has been studied by Sotiropoulos (1998) assuming that the half-space was subjected to pure homogeneous finite strains. Generalized Rayleigh wave propagation in a pre-stressed stratified half-plane was investigated by Akbarov and Ozisik (2003). It was assumed that complete contact conditions between the layer and half-plane were satisfied. Moreover, it was assumed that the initial strains were small and the strains and stresses corresponding to the initial state were determined within the scope of the classical linear theory of elasticity, corresponding dispersion equation was obtained and the dispersion curves which were constructed from the solution to this equation were analyzed. Wijeyewickrema *et*

*Corresponding author, Ph.D.
E-mail: masoud.negin@eng.bau.edu.tr


http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

*al.* (2008) investigated the time-harmonic wave propagation in a pre-strained and constrained homogeneous compressible high-elastic layer and the influence of the degree of this constraint on the dispersion relations. Ogden and Singh (2011) in the presence of initial stresses derived the general constitutive equation for a transversely isotropic hyperelastic solid based on the theory of invariants to examine the propagation of both homogeneous plane waves and Rayleigh surface waves. Extensional and flexural Lamb waves propagating in a sandwich plate made from compressible highly elastic materials with finite initial strains was studied by Akbarov *et al.* (2011). They assumed that the initial strains were caused by the uniformly distributed normal compression forces acting on the face planes of the plate and the cases where the compression forces are dead and follower were considered. Gupta *et al.* (2012) studied the propagation of torsional surface wave in an initially stressed non-homogeneous layer over a non-homogeneous half-space and they showed that the inhomogeneity parameter and the initial stress play an important role for the propagation of torsional surface waves. Zhang and Yu (2013) based on the mechanics of incremental deformations investigated the guided wave propagation in unidirectional plates under gravity and initial stresses. Shams and Ogden (2014) by applying the theory of the superposition of infinitesimal deformations on finite deformations in a hyperelastic material studied the propagation of Rayleigh waves in an initially stressed incompressible half-space subjected to a pure homogeneous deformation. Zhang *et al.* (2014) using quasistatic approximation and linearity assumption, investigated the propagation of Rayleigh waves in a magneto-electroelastic half-space with initial stress and obtained the wave propagation velocity for four types of electromagnetic boundary.

The other important issue is the effect of the imperfect bonding of the layers to the dispersion of propagated waves. The investigations of this type of problems are motivated by very high sensitivity of the wave propagation characteristics to imperfect interfaces and weak-bonding between the constituents. In fact, in the crustal depths of the Earth, contacts between layers of rocks are not in perfect contact conditions sometime for several kilometers. Furthermore, different types of discontinuities such as fractures, joints and faults are dominant crustal features. Such imperfectness of the interface conditions certainly affects the elastic wave propagation and its geometric dispersion characteristics.

Two classical boundary conditions, i.e., perfect contact interfaces and full slipping interfaces idealize real physical contact between the covering layer and the half-space. In perfect contact condition all the stress and displacement components are continuous across the interface, whereas, in the case of full slipping conditions there is a discontinuity in the shear component of the displacement (Rokhlin and Wang (1991)). Several theoretical methods are available in the literature for studying the influence of interfaces on elastic wave propagation. Martin (1992) has provided a brief review of different imperfect interface models in the literature and formulated the problem mathematically. He applied a simple linear modification to the perfect interface continuity conditions to model various intermediate imperfect conditions. In a particular case, this model is performed as a shear-spring type imperfect interface model, according to which, only shear displacements have discontinuity across this interface and the jump of this discontinuity is connected linearly with the corresponding shear stress. The idea of spring type interface is an old technique and it goes back to Goland and Reissner (1944). Other authors have extended their formulation to different configurations later. An application and review of the related investigations on the influence of the shear-spring type imperfectness have been considered in many papers. To summarize some, Bigoni *et al.* (1997) formulated the bifurcation problem for hyperelastic, layered solids in plane strain state and studied the effect of interfacial compliance on the bifurcation of a layer bonded to a substrate. Bigoni and Gei (2001) investigated the bifurcations in velocities from a state of homogeneous axisymmetric deformation for a coated elastic cylinder subject to axial tension or compression. Leungvichcharoen and Wijeyewickrema (2003) have discussed the effect of an imperfect interface on harmonic extensional wave propagation in a pre-stressed, symmetric layered composite by employing shear-spring type resistance model to simulate the imperfect interface. Gei *et al.* (2004) proposed a new framework for thermoelastic analysis of wave propagation in multilaminated structures subjected to an arbitrary, homogeneous deformation and to a condition of uniform temperature. Melkumyan and Mai (2008) have studied the effects of imperfect bonding in piezoelectric/piezomagnetic composites and showed that imperfection of the interface bonding has significant impact on the existence of interface waves and on their velocities of propagation. Gei *et al.* (2004) proposed a new framework for thermoelastic analysis of wave propagation in multilaminated structures subjected to an arbitrary, homogeneous deformation and to a condition of uniform temperature. Bigoni *et al.* (2008, 2013) studied the anti-plane and plane-strain, time-harmonic, small-amplitude vibrations of an elastic layer on elastic half space subjected to a state of finite, uniform stress and strain. Note that the bifurcation problem has exactly the same mathematical structure of the wave propagation problem. Vishwakarma *et al.* (2014) have been considered different types of imperfect interfaces to study the propagation of a torsional surface wave in a homogeneous crustal layer over an initially stressed mantle with varying rigidities, density and initial stresses. Akbarov and Ipek (2010, 2012) have studied the influence of the imperfectness of the interface conditions on the dispersion of the axisymmetric longitudinal waves in the pre-strained compound cylinder under the shear-spring type model of the contact condition between layers. Kepceler (2010) has also carried out investigations of a similar type for the initially stressed bi-material compounded circular cylinder.

In the present paper, the study of Negin *et al.* (2014), Akbarov and Negin (2015) and Negin (2015) on propagation of the generalized Rayleigh waves in an initially stressed elastic half-space covered by an elastic layer is extended for the case of imperfect contact conditions. The three-dimensional linearized theory of elastic waves in initially stressed bodies (TLTEWISB) is utilized and the plane-strain state is considered. The dispersion equation for this system is derived and a computer algorithm is developed for numerical solution to this equation. Numerical results are presented and discussed on the influence of the initial stresses and on the influence of the character of the external compressional stresses. Different initial stress combinations are considered. In some cases it is assumed that the external forces are “dead” forces, therefore, in those cases the external forces only cause the initial stresses and do not constrain the wave propagation in the layered half-space. However, in other cases it is assumed that the external forces are “follower” forces; consequently, in those cases the external forces not only produce the initial stresses, but also constrain the wave propagation in the system. The external forces caused by the weight of the water columns on the system are modeled as uniformly distributed normal compressional forces acting on the ocean-earth crust interface or the face surface of the covering layer. These forces are usually referred to as “pressure loading” or “follower forces” and are counterbalanced with the corresponding compressional forces acting at infinity. Consequently, as a result of the water weight an initial homogeneous compressional normal stresses are present before the appearing of the near-surface waves in the system under consideration and the propagation direction of the waves is perpendicular to the direction of those initial normal stress.

## 2. Formulation of the problem

Consider an elastic half-space covered by an elastic layer with thickness $h$ as shown in Fig. 1. We determine the positions of the points by the Lagrange coordinates in the Cartesian system of coordinates $Ox_1x_2x_3$. A plane-strain state in the $Ox_1x_2$ plane is considered, thus the displacement components along $Ox_1$ and $Ox_2$ directions, $u_1$ and $u_2$ are non-zero while displacement component $u_3$ along $Ox_3$ direction is zero. We assume that the Rayleigh waves propagate in the positive direction of $Ox_1$ axis. Values related to the layer and half-space are denoted by upper indices (1) and (2), respectively. Furthermore, the values relating to the initial state are denoted by the additional upper index 0.

Two cases are considered. In case 1, it is assumed that the system is under initial normal compressive stress $\sigma_{22}^{(m),0}$ along $Ox_2$ direction and compressive or tensile initial stress $\sigma_{11}^{(m),0}$ along $Ox_1$ direction, respectively. The initial stresses $\sigma_{22}^{(m),0}$, can be caused by the uniformly distributed normal "dead" forces with intensity $\sigma_{22}^{(1),0} = \sigma_{22}^{(2),0} = const$ and acting in the direction of the $Ox_2$ axis. The mentioned "dead" forces can also be taken as a model of the weight of the bodies which are located on the stratified half-space under consideration. Thus, the initial stresses in the constituents are determined within the scope of the classical linear theory of elasticity as follows

$$\sigma_{11}^{(m),0} = const_m \neq 0, \quad \sigma_{22}^{(1),0} = \sigma_{22}^{(2),0} = const_m \neq 0, \quad \sigma_{12}^{(m),0} = 0\ , \quad m = 1,2. \tag{1}$$

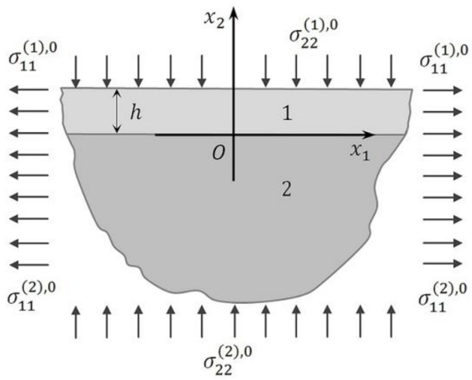



Fig. 1 Geometry of the considered mechanical system

The homogeneous initial stresses $\sigma_{11}^{(1),0} \neq \sigma_{11}^{(2),0}$ are caused by the external forces acting at $|x_1| \to \infty$ in the direction of the $Ox_1$ axis. It can be assumed that the layer and the half-space first have been stressed, i.e., have been stretched or compressed in the $Ox_1$ axis direction, and then have been bonded together. At the same time, it can be assumed that these initial stresses arose after bonding the constituents by the uniformly distributed normal forces acting at infinity in the direction of the $Ox_1$ axis.

In case 2, it is assumed that the considered system is compressed with the uniformly distributed normal forces with the intensity $P_0$ along its thickness. The uniformly distributed normal force $P_0$, as mentioned before, can be caused for instance, by the weight of the fluid on the stratified half-space if the system corresponds as a model for the soil of the bottom of the ocean or of the sea or it can also represents as a model of the weight of the bodies which are located on the stratified half-space under consideration.

According to Guz (2004), the equations of the three-dimensional linearized theory of elastic waves in initially stressed bodies are obtained from the corresponding geometrical non-linear equations of motion by their linearization with respect to the perturbations of the stresses, strains and displacements

$$\frac{\partial}{\partial x_i}\left[\sigma_{ij}^{m} + \sigma_{in}^{(m),0}\frac{\partial u_i^{(m)}}{\partial x_n}\right] = \rho^{(m)}\frac{\partial^2 u_i^{(m)}}{\partial t^2}, \tag{2}$$

Equations of motion for the case given by these equations can be written as follows

$$\begin{aligned}
&\frac{\partial \sigma_{11}^{(m)}}{\partial x_1} + \frac{\partial \sigma_{12}^{(m)}}{\partial x_2} + \sigma_{11}^{(m),0}\frac{\partial^2 u_1^{(m)}}{\partial x_1^2} + \sigma_{22}^{(m),0}\frac{\partial^2 u_1^{(m)}}{\partial x_2^2} = \rho^{(m)}\frac{\partial^2 u_1^{(m)}}{\partial t^2},\\
&\frac{\partial \sigma_{12}^{(m)}}{\partial x_1} + \frac{\partial \sigma_{22}^{(m)}}{\partial x_2} + \sigma_{11}^{(m),0}\frac{\partial^2 u_2^{(m)}}{\partial x_1^2} + \sigma_{22}^{(m),0}\frac{\partial^2 u_2^{(m)}}{\partial x_2^2} = \rho^{(m)}\frac{\partial^2 u_2^{(m)}}{\partial t^2}.
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
&\sigma_{ij}^{(m)} = \lambda^{(m)} e^{(m)} + 2\mu^{(m)}\varepsilon_{ij}^{(m)},\\
&e^{(m)} = \varepsilon_{11}^{(m)} + \varepsilon_{22}^{(m)} + \varepsilon_{33}^{(m)},\\
&2\varepsilon_{ij}^{(m)} = \frac{\partial u_i^{(m)}}{\partial x_j} + \frac{\partial u_j^{(m)}}{\partial x_i}.
\end{aligned} \tag{4}$$

Here $\lambda^{(m)}$ and $\mu^{(m)}$ are Lame’s constants.

As has been noted in the previous section we consider two cases with respect to the boundary conditions:

Case 1: we assume that the external compression stresses with intensity $\sigma_{22}^{(1),0} = \sigma_{22}^{(2),0} = const$ are "dead" forces, i.e., any magnitude in either direction does not change in the perturbation state. Therefore, in this case the boundary conditions are as follows

$$\sigma_{12}^{(1)}\Big|_{x_2=h} = 0, \qquad \sigma_{22}^{(1)}\Big|_{x_2=h} = 0. \tag{5}$$

Case 2: in this case we assume that the aforementioned forces are "follower" forces. The load is said to be "follower" if it is applied normally to the surface of a body and does not change its direction and magnitude during deformation, i.e., if it acts normally on the surface in the deformed state too. According to this definition of the "follower" forces, we have the boundary conditions written below

$$\begin{aligned} \sigma_{12}^{(1)}\Big|_{x_2=h} &= -P_0 \frac{\partial u_2^{(1)}}{\partial x_1}\Big|_{x_2=h}, \\ \sigma_{22}^{(1)}\Big|_{x_2=h} &= P_0 \frac{\partial u_1^{(1)}}{\partial x_1}\Big|_{x_2=h}. \end{aligned} \tag{6}$$

The expression for the calculation of the "follower forces" or "pressure loading" acting on the surface, which is also used for writing the conditions (6), is given in the monographs by Ogden (1984) and Guz (1999). Now we consider the formulation of the imperfect contact conditions on the interface plane between the covering layer and the half-space. It should be noted that, in general, the imperfectness of the contact conditions is identified by discontinuities of the displacements and forces across the mentioned interface. According to the discussion made in the previous subsection and according to Martin (1992) the mathematical formulation of the imperfectness of the contact conditions can be written as follows

$$\sigma_{i2}^{(1)}\,|_{x_2=0} = \sigma_{i2}^{(2)}\,|_{x_2=0} \quad, i=1,2, \tag{7}$$

$$\begin{aligned} u_1^{(1)}\,|_{x_2=0} - u_1^{(2)}\,|_{x_2=0} &= F_1 \frac{h}{\mu^{(2)}} \sigma_{12}^{(2)}\,|_{x_2=0}, \quad F_1 > 0 \\ u_2^{(1)}\,|_{x_2=0} - u_2^{(2)}\,|_{x_2=0} &= F_2 \frac{h}{\mu^{(2)}} \sigma_{22}^{(2)}\,|_{x_2=0}, \quad F_2 > 0 \end{aligned} \tag{8}$$

where $F_1$ and $F_2$ are the non-dimensional shear- and normal-spring parameters. This model is known as shear-spring type imperfect interface model in the literature, according to which, only shear displacements has discontinuity across the interface and the jump of this discontinuity is linearly connected to the corresponding shear stress. This is analogous to the interface conditions made in the paper Bigoni *et al.* (1997) where at the interface between the layers, the boundary conditions are such that the traction rate is continuous across the interface and this traction rate is related to the velocity jump across the interface by an elastic interfacial constitutive relation. Note that the case where $F_1 \to 0$ and $F_2 \to 0$, means that the displacement components across the interface are continuous and therefore the half-space and the covering layer are perfectly bonded together or to say that they are in welded contact condition. This limiting case coincides with the model considered by Bigoni *et al.* (2008) for perfect contact conditions between the layer and the substrate in the limiting case, imposing continuity of incremental displacements and nominal tractions. At the other extreme, $F_1 \to \infty$ and $F_2 \to \infty$ implies that the half-space and the covering layer are completely unbounded together and the full slipping condition is satisfied. Thus, any other finite positive values of $F_1$ and $F_2$ expresses different imperfect interface conditions in the problem.

Moreover, we assume that the following decay conditions also are satisfied

$$\sigma_{ij}^{(2)}\Big|_{x_2 \to -\infty} \to 0, \qquad u_i^{(2)}\Big|_{x_2 \to -\infty} \to 0, \qquad i = 1, 2. \tag{9}$$

Substituting Eq. (4) into Eq. (3) we get

$$\begin{aligned} &\lambda^{(m)}\left(\frac{\partial^2 u_1^{(m)}}{\partial x_1^2} + \frac{\partial^2 u_2^{(m)}}{\partial x_1 \partial x_2}\right) + 2\mu^{(m)} \frac{\partial^2 u_1^{(m)}}{\partial x_1^2} + \mu^{(m)}\left(\frac{\partial^2 u_1^{(m)}}{\partial x_2^2} + \frac{\partial^2 u_2^{(m)}}{\partial x_1 \partial x_2}\right) + \sigma_{11}^{(m),0} \frac{\partial^2 u_1^{(m)}}{\partial x_1^2} = \rho^{(m),0} \frac{\partial^2 u_1^{(m)}}{\partial t^2}, \\ &\lambda^{(m)}\left(\frac{\partial^2 u_2^{(m)}}{\partial x_2^2} + \frac{\partial^2 u_1^{(m)}}{\partial x_1 \partial x_2}\right) + 2\mu^{(m)} \frac{\partial^2 u_2^{(m)}}{\partial x_2^2} + \mu^{(m)}\left(\frac{\partial^2 u_2^{(m)}}{\partial x_1^2} + \frac{\partial^2 u_1^{(m)}}{\partial x_1 \partial x_2}\right) + \sigma_{11}^{(m),0} \frac{\partial^2 u_2^{(m)}}{\partial x_1^2} = \rho^{(m),0} \frac{\partial^2 u_2^{(m)}}{\partial t^2}. \end{aligned} \tag{10}$$

According to Helmholtz decomposition rule, these displacement fields can be described by the potentials for the longitudinal waves, $\phi$, and transverse waves, $\psi$, in the following form

$$u_1^{(m)} = \frac{\partial \phi^{(m)}}{\partial x_1} + \frac{\partial \psi^{(m)}}{\partial x_2}, \; u_2^{(m)} = \frac{\partial \phi^{(m)}}{\partial x_2} - \frac{\partial \psi^{(m)}}{\partial x_1}. \tag{11}$$

Substituting these relations into Eqs. (10) we obtain

$$\begin{aligned} &\left(\lambda^{(m)} + 2\mu^{(m)} + \sigma_{11}^{(m),0}\right)\left(\frac{\partial^3 \phi^{(m)}}{\partial x_1^3} + \frac{\partial^3 \psi^{(m)}}{\partial x_2 \partial x_1^2}\right) + \mu^{(m)}\left(\frac{\partial^3 \phi^{(m)}}{\partial x_1 \partial x_2^2} + \frac{\partial^3 \psi^{(m)}}{\partial x_2^3}\right) \\ &\qquad + \left(\lambda^{(m)} + \mu^{(m)}\right) + \left(\frac{\partial^3 \phi^{(m)}}{\partial x_2^2 \partial x_1} - \frac{\partial^3 \psi^{(m)}}{\partial x_1^2 \partial x_2}\right) + \rho^{(m),0} \frac{\partial^2}{\partial t^2}\left(\frac{\partial \phi^{(m)}}{\partial x_1} + \frac{\partial \psi^{(m)}}{\partial x_2}\right) = 0, \\ &\left(\lambda^{(m)} + 2\mu^{(m)}\right)\left(\frac{\partial^3 \phi^{(m)}}{\partial x_1^2 \partial x_2} + \frac{\partial^3 \psi^{(m)}}{\partial x_2^2 \partial x_1}\right) + \left(\mu^{(m)} + \sigma_{11}^{(m),0}\right)\left(\frac{\partial^3 \phi^{(m)}}{\partial x_2 \partial x_1^2} - \frac{\partial^3 \psi^{(m)}}{\partial x_1^3}\right) \\ &\qquad + \left(\lambda^{(m)} + 2\mu^{(m)}\right) + \left(\frac{\partial^3 \phi^{(m)}}{\partial x_2 \partial x_1^2} - \frac{\partial^3 \psi^{(m)}}{\partial x_1 \partial x_2^2}\right) + \rho^{(m),0} \frac{\partial^2}{\partial t^2}\left(\frac{\partial \phi^{(m)}}{\partial x_2} - \frac{\partial \psi^{(m)}}{\partial x_1}\right) = 0. \end{aligned} \tag{12}$$

Eq. (12) can also be written in the following form

$$\begin{aligned} &\frac{\partial}{\partial x_1}\left(\nabla \phi^{(m)} + \frac{\sigma_{11}^{(m),0}}{\lambda^{(m)} + 2\mu^{(m)}} \frac{\partial^2 \phi^{(m)}}{\partial x_1^2} - \frac{1}{\left(c_1^{(m)}\right)^2} \frac{\partial^2 \phi^{(m)}}{\partial t^2}\right) \\ &\qquad + \frac{\partial}{\partial x_2}\left(\nabla \psi^{(m)} + \frac{\sigma_{11}^{(m),0}}{\mu^{(m)}} \frac{\partial^2 \psi^{(m)}}{\partial x_1^2} - \frac{1}{\left(c_1^{(m)}\right)^2} \frac{\partial^2 \psi^{(m)}}{\partial t^2}\right) = 0, \\ &\frac{\partial}{\partial x_2}\left(\nabla \phi^{(m)} + \frac{\sigma_{11}^{(m),0}}{\lambda^{(m)} + 2\mu^{(m)}} \frac{\partial^2 \phi^{(m)}}{\partial x_1^2} - \frac{1}{\left(c_1^{(m)}\right)^2} \frac{\partial^2 \phi^{(m)}}{\partial t^2}\right) \\ &\qquad + \frac{\partial}{\partial x_1}\left(\nabla \psi^{(m)} + \frac{\sigma_{11}^{(m),0}}{\mu^{(m)}} \frac{\partial^2 \psi^{(m)}}{\partial x_1^2} - \frac{1}{\left(c_2^{(m)}\right)^2} \frac{\partial^2 \psi^{(m)}}{\partial t^2}\right) = 0. \end{aligned} \tag{13}$$

As can be seen from Eqs. (13), $\phi^{(m)}$ and $\psi^{(m)}$ potentials must satisfy the following wave equations

$$\begin{aligned} \nabla^2 \phi^{(m)} + \frac{\sigma_{11}^{(m),0}}{\lambda^{(m)} + 2\mu^{(m)}} \frac{\partial^2 \phi^{(m)}}{\partial x_1^2} &= \frac{1}{\left(c_1^{(m)}\right)^2} \frac{\partial^2 \phi^{(m)}}{\partial t^2}, \\ \nabla^2 \psi^{(m)} + \frac{\sigma_{11}^{(m),0}}{\mu^{(m)}} \frac{\partial^2 \psi^{(m)}}{\partial x_1^2} &= \frac{1}{\left(c_2^{(m)}\right)^2} \frac{\partial^2 \psi^{(m)}}{\partial t^2}. \end{aligned} \tag{14}$$

In Eqs. (14), $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$ is Laplace operator and

$$c_1^{(m)} = \sqrt{\frac{\lambda^{(m)} + 2\mu^{(m)}}{\rho^{(m)}}}, \qquad c_2^{(m)} = \sqrt{\frac{\mu^{(m)}}{\rho^{(m)}}}. \tag{15}$$

In the case where $\sigma_{11}^{(1),0} = \sigma_{11}^{(2),0} = \sigma_{22}^{(1),0} = \sigma_{22}^{(2),0} = 0$, this formulation transforms to the corresponding one made within the scope of the classical linear theory of elastodynamics. We assume that $\phi^{(m)}$ and $\psi^{(m)}$ functions are representing as follows

$$\begin{aligned} \phi^{(m)} &= \phi_0^{(m)}(x_2)\cos(kx_1 - \omega t), \\ \psi^{(m)} &= \psi_0^{(m)}(x_2)\sin(kx_1 - \omega t). \end{aligned} \tag{16}$$

Substituting relations (16) into Eq. (14) we obtain the following differential equations for the $\phi_0^{(m)}(x_2)$ and $\psi_0^{(m)}(x_2)$ as follows

$$\begin{aligned} \frac{d^2\phi_0^{(m)}}{dx_2^2} - \frac{k^2\sigma_{11}^{(m),0}}{\lambda^{(m)} + 2\mu^{(m)}}\phi_0^{(m)} &= -\left(\frac{c}{c_1^{(m)}}\right)^2 \phi_0^{(m)}, \\ \frac{d^2\psi_0^{(m)}}{dx_2^2} - \frac{k^2\sigma_{11}^{(m),0}}{\mu^{(m)}}\psi_0^{(m)} &= -\left(\frac{c}{c_2^{(m)}}\right)^2 \psi_0^{(m)}. \end{aligned} \tag{17}$$

We determine the solution to these differential equations as follows

$$\begin{aligned} \phi_0^{(1)} &= A_1^{(1)}\exp(ikp_1^{(1)}x_2) + A_2^{(1)}\exp(-ikp_1^{(1)}x_2), \\ \psi_0^{(1)} &= B_1^{(1)}\exp(ikp_2^{(1)}x_2) + B_2^{(1)}\exp(-ikp_2^{(1)}x_2), \\ \phi_0^{(2)} &= C_1^{(2)}\exp(kq_1^{(2)}x_2), \\ \psi_0^{(2)} &= D_1^{(2)}\exp(kq_2^{(2)}x_2), \end{aligned} \tag{18}$$

where

$$\begin{aligned} p_1^{(1)} &= \left(\frac{c^2}{\left(c_1^{(1)}\right)^2} - \left(1 + \frac{\sigma_{11}^{(1),0}}{\lambda^{(1)} + 2\mu^{(1)}}\right)\right)^{1/2}, \quad p_2^{(1)} = \left(\frac{c^2}{\left(c_2^{(1)}\right)^2} - \left(1 + \frac{\sigma_{11}^{(1),0}}{\mu^{(1)}}\right)\right)^{1/2}, \\ q_1^{(2)} &= \left(1 + \frac{\sigma_{11}^{(2),0}}{\lambda^{(2)} + 2\mu^{(2)}} - \frac{c^2}{\left(c_1^{(2)}\right)^2}\right)^{1/2}, \quad q_2^{(2)} = \left(1 + \frac{\sigma_{11}^{(2),0}}{\mu^{(2)}} - \frac{c^2}{\left(c_2^{(2)}\right)^2}\right)^{1/2}. \end{aligned} \tag{19}$$

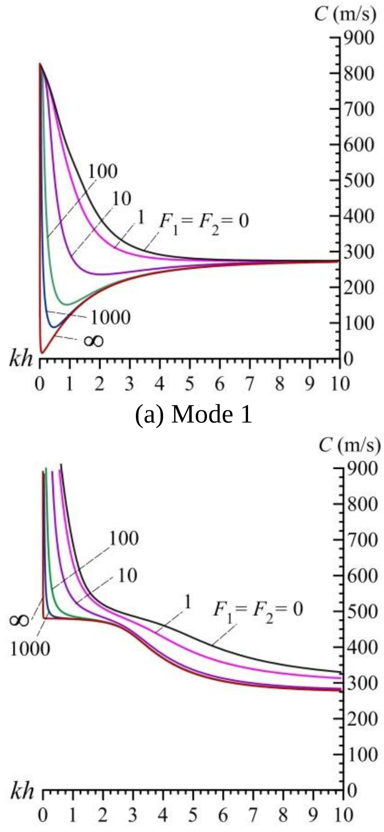
Finally, the dispersion equation after considering boundary conditions (7)-(8) can be expressed formally as follows

$$\det\left\| \alpha_{ij}\left(kh, c, F_1, F_2, \lambda^{(1)}, \lambda^{(2)}, \mu^{(1)}, \mu^{(2)}, \rho^{(1)}, \rho^{(2)}, \sigma_{11}^{(1),0}, \sigma_{11}^{(2),0}\right) \right\| = 0, \tag{20}$$

where $i;j$-1,2,...,6. This completes the solution method of the problem under consideration.

## 3. Numerical results and discussion

As a numerical example here we are looking at the soil model which was considered in a paper by Foti (2002), according to which the soil is modeled as a covering layer and a half-space. The thickness of the covering layer is $h$=10 m, the densities of the covering layer and half-space materials are equal to each other i.e., 1800 kg/m$^3$, shear and bulk waves velocities in the covering layer (half-plane) material are 300 m/s (900 m/s) and 500 m/s.



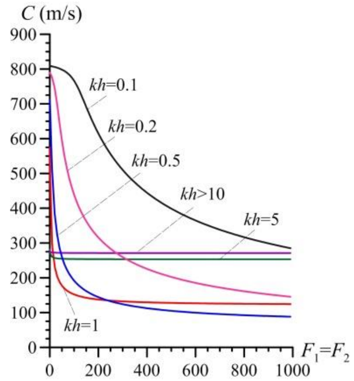

(a) Mode 1

(b) Mode 2

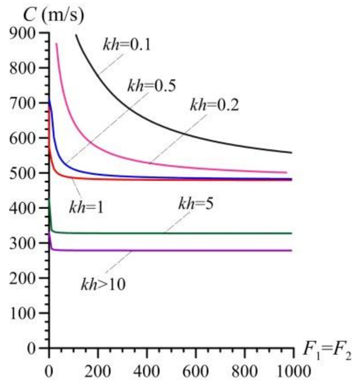
Fig. 2 Dispersion curves when imperfections exist both in transverse and normal directions. Numbers in the figures field show the values of the parameters $F_1$=$F_2$

First we consider the numerical results for the case where initial stresses in the constituents are absent. Note that the solution (18) corresponds to such a wave propagation in the layered half-space that the layer undergoes an oscillatory motion in the $Ox_2$ direction propagating in the $Ox_1$ direction with velocity $c$. The disturbances in the layer decay exponentially with depth in the half-space and therefore the wave can be considered as a generalized Rayleigh wave confined to the pre-stressed covered layer. The dispersion equation (20) has infinitely many modes unlike ordinary Rayleigh waves, which can propagate only in one mode. Moreover, the dispersion curves have two branches for each propagation mode which were denoted by $M_{1n}$ and $M_{2n}$ respectively for the mode. For the first $M_{1n}$ branch the displacement of the layer circumscribes the ellipse similar to the ordinary Rayleigh waves, but for the second $M_{2n}$ branch leads to an opposite type of motion. In addition, according to Tolstoy and Usdin (1953), it must be $c/c_2^{(2)}$<1 and $c/c_2^{(1)}$>1, i.e., the near-surface wave propagation in the system under consideration is subsonic in the half-space, but it is supersonic in the covering layer.

Fig. 2 shows the graphs related to the first and second branches of the first mode (presented as Mode 1 and Mode 2 in the figure, respectively) of the dispersion curves constructed for the above mentioned case where imperfections exist in both transverse and normal directions, i.e., $F_1$=$F_2$≠0. The numbers labeled on the curves




(a) Mode 1




(b) Mode 2

Fig. 3 Influence of the imperfection of the interface in transverse and normal directions on wave propagation velocity for different wavenumbers

correspond to the dimensionless shear- and normal-spring parameters $F_1$ and $F_2$ of the related curves. Note that, namely the graphs constructed in the case where $F_1$=$F_2$=0 and shown in Fig. 2 were used in the paper by Foti (2002) for validation of the experimentally constructed dispersion curves.

It follows from Fig. 2 that, first of all, the imperfectness of the contact conditions decreases the wave propagation velocity of both modes. In addition, the dimensionless wavenumber *kh* has cut off values for the second mode. However, the low and high wavenumber limit values of the wave propagation velocities as $kh\to 0$ and $kh\to\infty$, respectively, do not depend on the imperfectness of the interface, i.e., on the parameters $F_1$ and $F_2$. This result can also be seen more clearly from Fig. 3, which shows how imperfectness of the contact conditions affects the wave propagation velocity for both modes. It follows from these graphs that wave propagation velocity is more sensitive to the imperfectness of the contact conditions for small values of wavenumber *kh*, say *kh*<1 for this case. In other words, for wavenumbers *kh*<1, any small changes in interface conditions highly affect the wave propagation velocity. However, for high wavenumbers, say *kh*>5, the wave propagation velocity does not depend on the imperfectness of the contact conditions at all.

Now we consider the numerical results related to the influence of the initial stresses in the constituents of the system on the wave propagation velocity. For estimation of the magnitude of the initial stresses we introduce the following parameters

$$\psi^{(1)}=\sigma_{11}^{(1),0}/\mu^{(1)},\ \psi^{(2)}=\sigma_{11}^{(2),0}/\mu^{(2)},\ \psi^{(3)}=\sigma_{22}^{(1),0}/\mu^{(1)},\ \psi^{(4)}=P_0/\mu^{(2)}. \quad (21)$$

Here we will present the results for the following four initial stress combinations

$$\begin{aligned}&\text{Case 1. } \psi^{(1)}=\psi^{(2)}<0,\ \psi^{(3)}=0,\ \psi^{(4)}=0;\\ &\text{Case 2. } \psi^{(1)}=\psi^{(2)}=0,\ \psi^{(3)}<0,\ \psi^{(4)}=0;\\ &\text{Case 3. } \psi^{(1)}=\psi^{(2)}>0,\ \psi^{(3)}<0,\ \psi^{(4)}=0;\\ &\text{Case 4. } \psi^{(1)}=\psi^{(2)}=0,\ \psi^{(3)}=0,\ \psi^{(4)}<0.\end{aligned} \quad (22)$$

Moreover, we introduce the notation

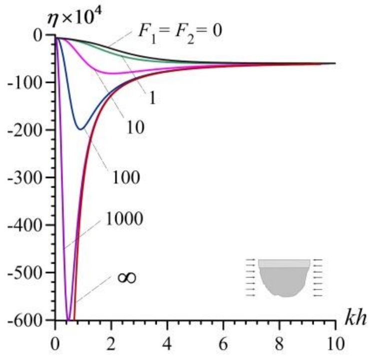
$$\eta=\frac{c\big|_{\psi\neq 0}-c\big|_{\psi=0}}{c\big|_{\psi=0}}, \quad (23)$$

for estimation of the influence of the initial stresses in the constituents, i.e., the influence of the parameters $\psi$ on the wave propagation velocity. Thus, through the graphs of the dependencies between $\eta$ (23) and *kh* we study the effect of different initial stress patterns in the constituents on the wave propagation velocity. We also assumed that the magnitude of the initial stresses in each layer is equal 1% of the elastic moduli of that layer.

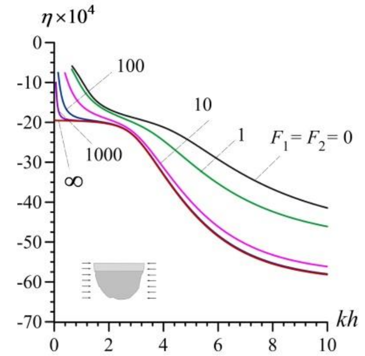
Fig. 4 shows the dispersion curves for initial stress combinations in case 1 (i.e., the case where constant compressive initial stresses act in horizontal direction both in covering layer and the half-space) for different values of the imperfect interface dimensionless parameters, i.e., for different values of $F_1=F_2\neq 0$. Note that, for clarity of the illustration, the graphs of the first and second modes of propagation are given separately in Fig. 4(a) and 4(b), respectively. It follows from Fig. 4 that the imperfectness between the constituents of the system decreases the wave propagation velocity of both modes for this initial stress case. Moreover, for small values of wavenumber, for example *kh*<1 the wave propagation velocity is more sensitive to the imperfectness of the contact conditions and any small changes in interface conditions change the wave propagation velocity dramatically, especially for the first mode of propagation. But, for high wavenumbers as $kh\to\infty$, the wave propagation velocity does not depend on the imperfectness of the contact conditions.

Figs. 5 and 6 illustrate the effect of existence of the initial compressive stresses in vertical direction (case 2) and initial compressive stresses in vertical direction and the initial tensile stresses in horizontal direction (case 3) on wave propagation velocity of the first and second modes, respectively. The graphs show that, the character of the effect of the imperfectness of the contact conditions, i.e., of the parameters $F_1$ and $F_2$ on the influence of the initial stresses in the covering layer and the half-space on the wave propagation velocity depends on the values of the dimensionless wavenumber *kh*. So, it follows from these graphs that, before (after) a certain value of the *kh*, the imperfectness of the contact conditions causes to increase (decrease), the wave propagation velocity.

Finally, Fig. 7 shows the dispersion curves for initial stress combinations in case 4 for the different values of the imperfect interface parameters. As mentioned above, here

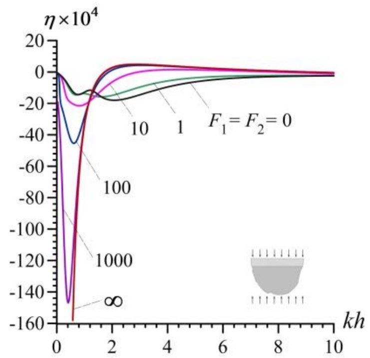



(a) Mode 1

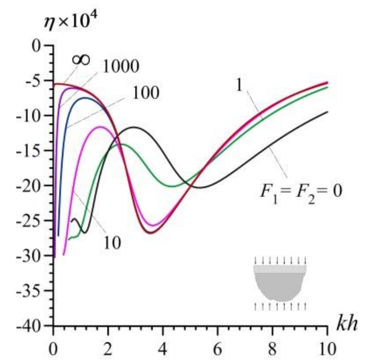



(b) Mode 2

Fig. 4 Influence of the contact imperfections on the dependence between $\eta$ and $kh$ for initial stress combinations in Case 1. Imperfection both in transverse and normal directions

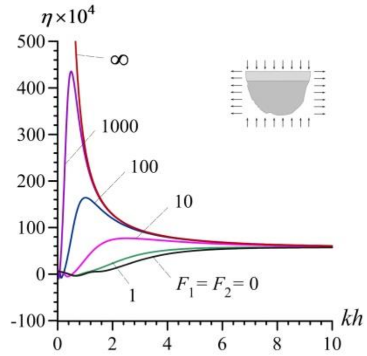



(a) Mode 1

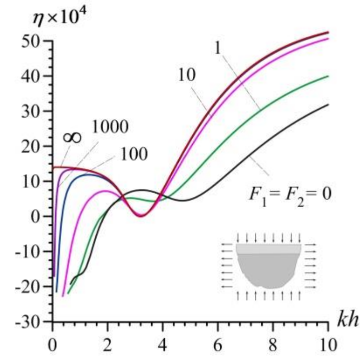



(b) Mode 2

Fig. 5 Influence of the contact imperfections on the dependence between $\eta$ and $kh$ for initial stress combinations in Case 2. Imperfection both in transverse and normal directions

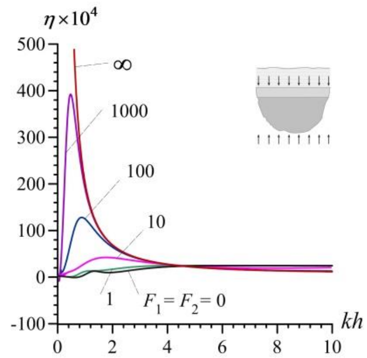



(a) Mode 1

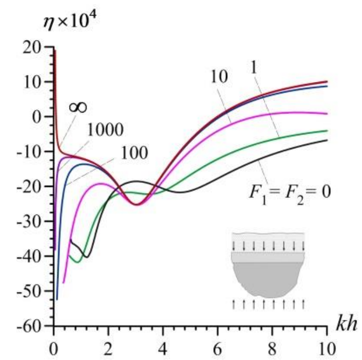



(b) Mode 2

Fig. 6 Influence of the contact imperfections on the dependence between $\eta$ and $kh$ for initial stress combinations in Case 3. Imperfection both in transverse and normal directions




(a) Mode 1




(b) Mode 2

Fig. 7 Influence of the contact imperfections on the dependence between $\eta$ and $kh$ for initial stress combinations in Case 4. Imperfection both in transverse and normal directions we assume that the external forces are “follower forces”. Thus, in this case the external forces not only cause the initial stresses, but they constrain the wave propagation in the system. It follows from Fig. 7 that first of all, the influence of the contact conditions on the wave propagation velocity is complicated and in general depends on the values of the wavenumber. For example, from Fig. 7(b) we can see that the wave propagation velocity before/after some values of *kh* starts to decreases/increases for the different values of the imperfect interface parameters. Then, as Fig. 7(a) shows the imperfectness of the contact conditions increases the wave propagation velocity of the first mode of the propagation for this initial stress case (i.e., “follower forces”). This is completely different from the results we got for “dead forces” as we considered for initial stress combinations in case 2 (Fig. 5(a)). Consequently, presence of the follower forces completely changed the behavior of the wave propagation velocity, i.e., “dead” and “follower” forces affect the wave propagation velocity in opposite directions, as under the action of dead forces the wave propagation velocity decreases, while under the action of the follower forces the velocity increases for this mode. This conclusion raises the significance of considering the effect of follower type forces when we study dispersion of the seismic surface waves propagating under the bottom of the oceans.

## 4. Conclusions

In this study propagation of the seismic Rayleigh waves in an elastic half-space covered by an elastic layer for different initial stress combinations and imperfect contact conditions is investigated. Three dimensional linearized theory of elastic waves in piecewise homogeneous bodies with initial stresses is used. It is assumed that the covering layer and the half-space are bounded together through two shear-spring and normal-spring type imperfect interface. Numerical results for different initial stress combinations are presented and discussed. To study dispersion of the seismic waves propagating on the ocean floors the case where the external forces are assumed to be “follower forces” or “pressure loading” is considered as well. The following main conclusions can be drawn:

• In the absence of initial stresses the imperfectness of the contact conditions cause the propagation velocity of the generalized Rayleigh wave to decrease.

• For small values of wavenumber Generalized Rayleigh wave propagation velocity is very sensitive to imperfectness of the contact conditions and any small change in the interface conditions highly affects the wave propagation velocity. However, for high wavenumbers the wave propagation velocity does not depend on the imperfectness of the contact conditions at all.

• In general, the influence of the contact conditions on the wave propagation velocity is complicated and depends on the values of the wavenumber, i.e., the velocity of wave propagation before/after some values of *kh* decreases/increases.

• The presence of the follower forces changes the dispersion curves quantitatively. For example comparing Fig. 5 and Fig. 7 shows that the “dead” and “follower” forces affect the wave propagation velocity in opposite directions, as under the action of dead forces the wave propagation velocity decreases, while under the action of the follower forces the velocity increases.

These results provide some theoretical basis for characterization of the structure of the soil of the bottom of the oceans, study of the dispersion of seismic surface waves propagating on the ocean floors and also study of the near-surface wave propagation in layered mechanical systems with a liquid upper layer.

## References

Akbarov, S.D. (2007), “Recent investigations on the dynamical problems of the elastic body with initial (residual) stresses (review)”, *Appl. Mech.*, **43**(12), 3-27.

Akbarov, S.D. (2012), “The influence of third order elastic constants on axisymmetric wave propagation velocity in the two-layered pre-stressed hollow cylinder”, *Comput. Mater. Continua*, **32**(1), 29-60.

Akbarov, S.D. and Ipek, C. (2010), “The influence of the imperfectness of the interface conditions on the dispersion of the axisymmetric longitudinal waves in the pre-strained compound cylinder”, *Comput. Model. Eng. Sci.*, **70**(2), 93-121.

Akbarov, S.D. and Ipek, C. (2012), “Dispersion of axisymmetric longitudinal waves in the pre-strained imperfectly bonded bi-layered hollow cylinder”, *Comput. Mater. Continua*, **30**(2), 99-144.

Akbarov, S.D. and Negin, M. (2017), “Near-surface waves in a system consisting of a covering layer and a half-space with imperfect interface under two-axial initial stresses”, *J. Vib. Control*, **23**(1), 55-68.

Akbarov, S.D. and Ozisik, M. (2003), “The influence of the third order elastic constants to the generalized Rayleigh wave dispersion in a pre-stressed stratified half-plane”, *J. Eng. Sci.*, **41**(17), 2047-2061.

Akbarov, S.D., Agasiyev, E.R. and Zamanov, A.D. (2011), “Wave propagation in a pre-strained compressible elastic sandwich plate”, *Eur. J. Mech. A Solids*, **30**(3), 409-422.

Badsar, S.A., Schevenels, M., Haegeman, W. and Degrande, G. (2010), “Determination of the material damping ratio in the soil from SASW tests using the half-power bandwidth method”, *Geophys. J.*, **182**(3), 1493-1508.

Bigoni, D. and Gei, M. (2001), “Bifurcations of a coated, elastic cylinder”, *J. Solid. Struct.*, **38**(30), 5117-5148.

Bigoni, D., Gei, M. and Movchan, A.B. (2008), “Dynamics of a prestressed stiff layer on an elastic half space: Filtering and band gap characteristics of periodic structural models derived from long-wave asymptotics”, *J. Mech. Phys. Solid.*, **56**(7), 2494-2520.

Bigoni, D., Ortiz, M. and Needleman, A. (1997), “Effect of interfacial compliance on bifurcation of a layer bonded to a substrate”, *J. Solid. Struct.*, **34**(33-34), 4305-4326.

Biot, M.A. (1965), *Mechanics of Incremental Deformations: Theory of Elasticity and Viscoelasticity of Initially Stressed Solids and Fluids, Including Thermodynamic Foundations and Applications to Finite Strain*, John Wiley & Sons, New York, U.S.A.

Eringen, A.C. and Suhubi, E.S. (1975), *Elastodynamics*, Volume I, *Finite Motions*, Academic Press, New York, U.S.A.

Eringen, A.C. and Suhubi, E.S. (1975), *Elastodynamics*, Volume II, *Linear Theory*, Academic Press, New York, U.S.A.

Foti, S. (2002), “Numerical and experimental comparison between two-station and multistation methods for spectral analysis of

surface waves", *Rivista Italiana di Geotecnica*, **36**(1), 11-22.
Foti, S. (2004), "Using transfer function for estimating dissipative properties of soils from surface wave data", *Near Surf. Geophys.*, **2**(4), 231-240.
Foti, S., Parolai, S., Albarello, D. and Picozzi, M. (2011), "Application of surface-wave methods for seismic site characterization", *Survey. Geophys.*, **32**(6), 777-825.
Gei, M., Bigoni, D. and Franceschni, G. (2004), "Thermoelastic small-amplitude wave propagation in nonlinear elastic multilayers", *Math. Mech. Solid.*, **9**(5), 555-568.
Goldstein, R.V. and Maugin, G.A. (2005), *Surface Waves in Anisotropic and Laminated Bodies and Defects Detection*, Kluwer Academic Publishers, Dordrecht, The Netherlands
Gupta, S., Majhi, D.K. and Vishwakarma, S.K. (2012), "Torsional surface wave propagation in an initially stressed non-homogeneous layer over a non-homogeneous half-space", *Appl. Math. Comput.*, **219**(6), 3209-3218.
Guz, A.N. (1999), *Fundamentals of the Three-dimensional Theory of Stability of Deformable Bodies*, Springer-Verlag, Berlin, Germany.
Guz, A.N. (2002), "Elastic waves in bodies with initial (residual) stresses", *Appl. Mech.*, **38**(1), 23-59.
Guz, A.N. (2004), *Elastic Waves in Bodies with Initial (Residual) Stresses*, A.S.K, Kiev, Ukraine (in Russian).
Guz, A.N. (2005), "On foundations of the ultrasonic non-destructive method of determination of stresses in near-the-surface layers of solid bodies", *Comput. Model. Eng. Sci.*, **8**(3), 217-230.
Guz, A.N. and Makhort, F.G. (2000), "Physical principles of ultrasonic non-destructive method of determination of stresses in rigid solids", *Appl. Mech.*, **36**, 3-34.
Lai, C.G., Rix, G.J., Foti, S. and Roma, V. (2002), "Simultaneous measurement and inversion of surface wave dispersion and attenuation curves", *Soil Dyn. Earthq. Eng.*, **22**(9-12), 923-930.
Leungvichcharoen, S. and Wijeyewickrema, A.C. (2003), "Dispersion effects of extensional waves in pre-stressed imperfectly bonded incompressible elastic layered composites", *Wave Motion*, **38**, 311-325.
Malagnini, L., Herrmann, R.B., Biella, G. and de Frando, R. (1995), "Rayleigh waves in quaternary alluvium from explosive sources: determination of shear-wave velocity and Q structure", *Bull. Seism. Soc. Am.*, **85**(3), 900-922.
Mantič, V. (2013), *Mathematical Methods and Models in Composites*, Imperial College Press.
Martin, P.A. (1992), "Boundary integral equations for the scattering of elastic waves by elastic inclusions with thin interface layers", *J. Nondestruct. Eval.*, **11**(3-4), 167-174.
Melkumyan, A. and Mai, Y.W. (2008), "Influence of imperfect bonding on interface waves guided by piezoelectric/piezomagnetic composites", *Philos. Mag.*, **88**(23), 2965-2977.
Negin, M. (2015), "Generalized Rayleigh wave propagation in a covered half-space with liquid upper layer", *Struct. Eng. Mech.*, **56**(3), 491-506.
Negin, M., Akbarov, S.D. and Erguven, M.E. (2014), "Generalized Rayleigh wave dispersion analysis in a pre-stressed elastic stratified half-space with imperfectly bonded interfaces", *Comput. Mater. Continua*, **42**(1), 25-61.
Ogden, R. and Singh, B. (2011), "Propagation of waves in an incompressible transversely isotropic elastic solid with initial stress: Biot revisited", *J. Mech. Mater. Struct.*, **6**(1), 453-477.
Ogden, R.W. (1984), *Non-linear Elastic Deformations*, Courier Corporation.
Reissner, E. and Goland, M. (1944), "The stress in cemented joints", *J. Appl. Mech.*, **66**, A17-A27.
Rix, G.J., Lai. C.G. and Spang, A.W. (2000), "In situ measurements of damping ratio using surface waves", *J. Geotech. Geoenviron. Eng.*, **126**(5), 472-480.
Rogerson, G.A. and Fu, Y.B. (1995), "An asymptotic analysis of the dispersion relation of a pre-stressed incompressible elastic plate", *Acta Mechanica*, **111**(1-2), 59-74.
Rokhlin, S.I. and Wang, Y.J. (1991), "Analysis of boundary conditions for elastic wave interaction with an interface between two solids", *J. Acoust. Soc. Am.*, **89**(2), 503-515.
Shams, M. and Ogden, R.W. (2014), "On Rayleigh-type surface waves in an initially stressed incompressible elastic solid", *IMA J. Appl. Math.*, **79**(2), 360-376.
Socco, L.V., Foti, S. and Boiero, D. (2010b), "Surface wave analysis for building near surface velocity models: Established approaches and new perspectives", *Geophys.*, **75**(5), 75A83-75A102.
Sotiropoulos, D.A. (1998), "Interfacial waves in pre-stressed compressible elastic media", *Comput. Mech.*, **21**(4-5), 293-299.
Strobbia, C. and Cassiani, G. (2007), "Multilayer ground-penetrating radar guided waves in shallow soil layers for estimating soil water content", *Geophys.*, **72**(4), 17-29.
Tolstoy, I. and Usdin, E. (1953), "Dispersive properties of stratified elastic and liquid media: A ray theory", *Geophys.*, **18**(4), 844-870.
Van der Kruk, J.R., Jacob, W. and Vereecken, H. (2010), "Properties of precipitation-induced multilayer surface waveguides derived from inversion of dispersive TE and TM GPR data", *Geophys.*, **75**(4), WA263-WA273.
Vishwakarma, S.K., Gupta, S. and Verma, A.K. (2014), "Torsional wave propagation in Earth's crustal layer under the influence of imperfect interface", *J. Vib. Control*, **20**(3), 355-369.
Wijeyewickrema, A., Ushida, Y. and Kayestha, P. (2008), "Wave propagation in a pre-stressed compressible elastic layer with constrained boundaries", *J. Mech. Mater. Struct.*, **3**(10), 1963-1976.
Xia, J., Miller, R.D., Park, C.B. and Tian, G. (2002), "Determining Q of near-surface materials from Rayleigh waves", *J. Appl. Geophys.*, **51**(2-4), 21-129.
Zhang, R., Pang, Y. and Feng, W. (2014), "Propagation of Rayleigh waves in a magneto-electro-elastic half-space with initial stress", *Mech. Adv. Mater. Struct.*, **21**(7), 538-543.
Zhang, X.M. and Yu, J.G. (2013), "Effects of initial stresses on guided waves in unidirectional plates", *Arch. Mech.*, **65**(1), 3-26.

*JS*