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Damage constitutive model of brittle rock considering the compaction of crack

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Abstract. The deformation and strength of brittle rocks are significantly influenced by the crack closure behavior. The relationship between the strength and deformation of rocks under uniaxial loading is the foundation for design and assessment of such scenarios. The concept of relative crack closure strain was proposed to describe the influence of the crack closure behavior on the deformation and strength of rocks. Considering the crack compaction effect, a new damage constitutive model was developed based on accumulated AE counts. First, a damage variable based on the accumulated AE counts was introduced, and the damage evolution equations for the four types of brittle rocks were then derived. Second, a compaction coefficient was proposed to describe the compaction degree and a correction factor was proposed to correct the error in the effective elastic modulus instead of the elastic modulus of the rock without new damage. Finally, the compaction coefficient and correction factor were used to modify the damage constitutive model obtained using the Lemaitre strain equivalence hypothesis. The fitted results of the models were then compared with the experimental data. The results showed that the uniaxial compressive strength and effective elastic modulus decrease with an increase in the relative crack closure strain. The values of the damage variables increase exponentially with strains. The modified damage constitutive equation can be used to more accurately describe the compressive deformation (particularly the compaction stage) of the four types of brittle rocks, with a coefficient of determination greater than 0.9.

Keywords: uniaxial loading; constitutive equation; damage; acoustic emission; compaction coefficient; correction factor

1. Introduction

A rock mass is a complex fractured geological medium containing numerous randomly distributed flaws, such as joints and cracks and it is widely found in deep tunnels (holes), slopes and mining (Panaghi *et al*. 2015, Tan *et al*. 2015, Zhang *et al*. 2017). The constitutive relationship of rock forms the basis for the evaluation and design in geotechnical engineering, and has been one of the core problems facing modern geotechnical-engineering (Pourhosseini and Shabanimashcool 2014, Ahmad *et al*. 2015, Zhang *et al*. 2016, Jin and Chloé 2017, Tan *et al*. 2017, Wang *et al*. 2018). We can obtain rock properties by collecting representative rock samples and conducting laboratory test on the specimens. In addition, numerous laboratory compression test results show that the deformation behavior of intact rocks is mainly related to the

compression, initiation, propagation and coalescence of fractures inside the rocks. And the progressive failure of rock samples can be divided into four stages: a compaction stage, a quasi-linear elastic stage, a yield stage, and a failure stage (Lockner 1993, Diederichs and Martin 2010, Peng *et al*. 2015a, Liu *et al*. 2016, Ning *et al*. 2017), as shown in Fig. 1.

The crack closure behavior is an important part in the whole compression process of rock. Crack closure occurs during the initial loading stage, and the stress-strain response is usually nonlinear (concave), as shown in Fig. 1. Because the nonlinearity is associated with the peak strength and the elastic modulus of the rock, some researchers have studied the crack closure behavior of rocks and established quantitative models to describe the stress-strain relation of rocks in compaction stage (David *et al*. 2012, Peng *et al*. 2015a).

Continuum Damage Mechanics (CDM) based models are widely used to model deformation behaviors of brittle rocks. Many researchers have made contributions to the understanding of the strength and deformability of rocks under uniaxial loading conditions. And constitutive models for rocks considering non-linearity, anisotropy, rheology, and other properties have been established: these have laid solid theoretical foundations for engineering practice (Liu *et al*. 2009, Liu 2014, Wang *et al*. 2016, Cerfontaine *et al*. 2017, Li *et al*. 2017). However, these models neglect the compaction effect of crack and simplify the curve before the yield phase into a straight line, so that the stress-strain relation described by the models is still in error with the actual situation, as shown in Fig. 1. Peng *et al*. (2015b) combined the crack closure model (Peng *et al*. 2015a) with the phenomenological damage model (Liu 2014) to capture complete stress-strain curves of rocks and achieved good results.

To built a constitutive model for rocks with high

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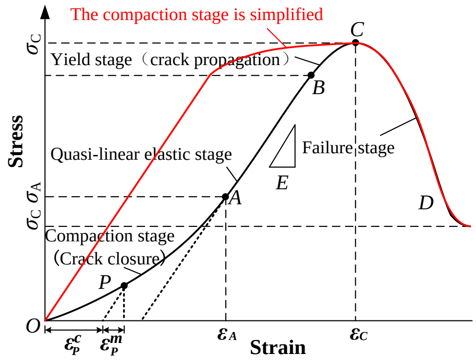


Fig. 1 Schematic diagram of the stage of typical rock failure process

accuracy under uniaxial loading. First, based on the uniaxial compression and AE tests of four types of brittle rocks, the effects of the compaction effect on the deformation and strength were analyzed. Second, the damage variable based on accumulated AE counts was introduced, and the evolution equation was calculated; Third, the compressive coefficient K_1 and deficiency factor K_2 (correcting the error due to the effective elastic modulus instead of the elastic modulus of intact rock) were introduced. The existing rock damage constitutive equation was then modified. Finally, the experimental curves were used to verify the results.

2. Experimental studies

2.1 Sample preparation

The rocks used in the experiment were obtained from the roof of 3-5# coal seam, Tongxin Mine, Shanxi Province, China. Four types of rock samples were considered, wherein, each sample includes three standard specimens, such as fine sandstone FS1-3, medium sandstone MS1-3, calcilutite C1-3, and sandy mudstone SM1-3, as shown in Fig. 2. The same types of rocks were obtained from the same rock formation. The rock samples were cut into standard cylindrical samples in accordance with the Chinese Standard “Standard Test Methods for Engineering Rock” (GB/T 50266-2013) (Hu *et al*. 2017). By drilling, cutting, and grinding, a sample could be processed into a standard specimen. The diameter and height of the prepared samples are 50 mm and 100 mm, respectively. Table 1 lists the specimen size and mechanical parameters.

2.2 Experimental equipment and procedure

The experiment was conducted using the Shimadzu AG-X250 electronic universal testing machine system, which loading speed is 0.0005~500 mm/min and the maximum load is 250 kN. The characteristics of the AE signals of the rocks were obtained using the PCI-2 AE system. The uniaxial loading of the standard rock specimens was conducted using the testing machine. The load system is controlled via the displacement control mode, and the loading speed is 0.001 mm/s. In the process of loading, four sensors were used to monitor the AE signal of the test piece.

Table 1 Basic mechanical parameters of test specimen

Lithology	Specimen number	Diameter/ height/ (mm)	Density/ (kg/m^3)	Crack closure strain/ 10^{-3}	Uniaxial compressive strength /MPa		Elastic modulus /GPa	
					Measured value	Average value	Measured value	Average value
Fine sandstone	FS-1	49.4/101.2	2640.6	2.81	117.8	115.4	8.64	8.50
	FS-2	49.4/100.9	2553.5	1.75	117.2		8.75	
	FS-3	49.6/103.0	2474.7	3.23	111.3		8.10	
Medium sandstone	MS-1	49.4/100.8	2488.9	3.26	59.4	53.5	6.91	6.70
	MS-2	49.4/100.4	2466.1	3.99	49.1		6.45	
	MS-3	49.5/100	2475.1	3.83	52.1		6.74	
Calcilutite	C-1	49.4/100.1	2803.0	3.22	88.0	104.2	8.43	8.39
	C-2	49.4/100.9	2637.6	4.00	114.0		7.78	
	C-3	49.4/100.4	2737.8	3.71	110.7		8.67	
Sandy mudstone	SM-1	49.4/100.8	2609.6	2.91	85.4	103.7	7.46	7.95
	SM-2	49.4/100.8	2595.0	3.52	115.2		8.41	
	SM-3	49.4/100.4	2520.8	3.27	110.6		7.98	

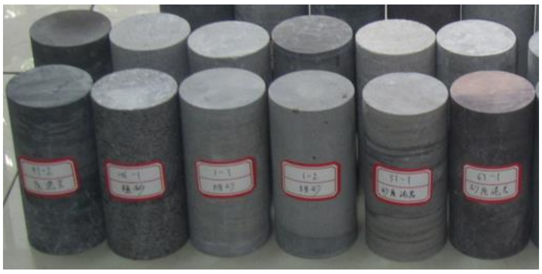


Fig. 2 Part of the standard rock specimen

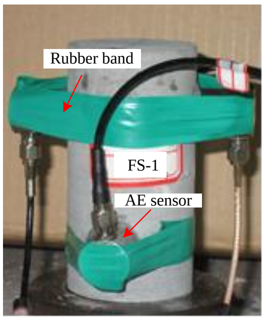


Fig. 3 AE sensor arrangement

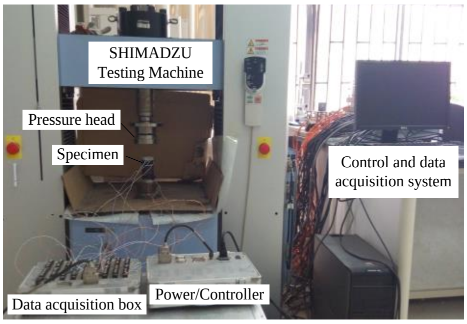
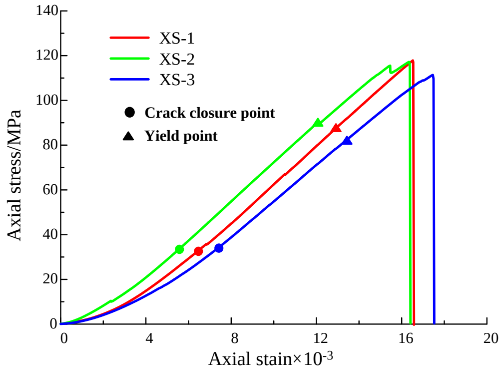
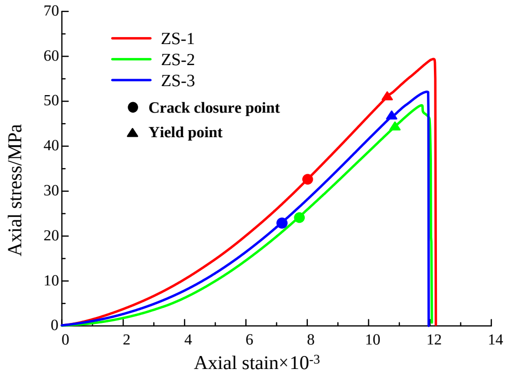


Fig. 4 Loading test system

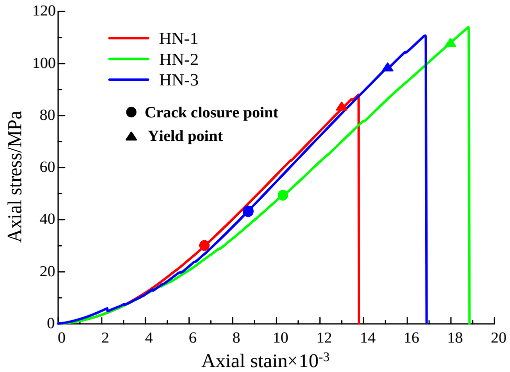
Vaseline was smeared between the sensor and the specimen to ensure the coupling effect. The sensors were fixed on the surface of the test piece using tape, as shown in Fig. 3. The AE monitoring system was synchronized with the loading system. The AE signals, loads, deformation and time of the rock specimen were monitored in the process of uniaxial loading, as shown in Fig. 4.



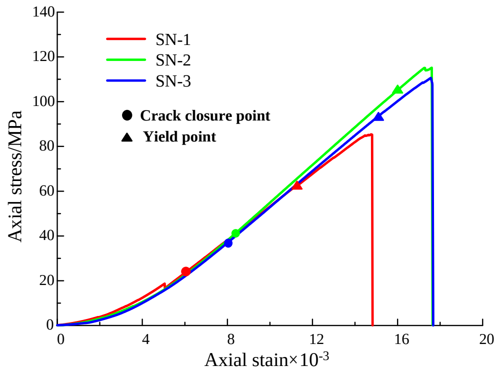
(a) Fine sandstone



(b) Medium sandstone



(c) Calcilutite



(d) Sandy mudstone

Fig. 5 Stress-strain curves of four types of brittle rocks

3. Experimental results

3.1 Strength and deformation behavior

Fig. 1 shows the schematic of the different stages of the typical rock failure process. The figure shows that the rock failed in four different stages: a compaction stage (OA), a quasi-linear elastic stage (AB), a yield stage (BC), and a failure stage (CD). Fig. 5 shows the stress-strain relationship of the four types of rocks. The figure shows that the compaction stage is concave and the four types of rock have experienced different crack compaction process under uniaxial loading. The compaction stage of the fine sandstone is generally short, and the compaction stage of the medium sandstone is generally longer. The linear degree of the yield stage of the four types of rocks is relatively high, indicating that a slight damage is produced in the yield stage. When the peak strain is reached, the stress drops rapidly with the increasing strain. The yield strains of the fine sandstone and medium sandstone are similar. The yield strains of the calcilutite and sandy mudstone are different.

The compressive strength and elastic modulus of the four types of rock were list in Table 1. The uniaxial compressive strength and effective elastic modulus of the fine sandstone are the highest, with averages of 115.4 MPa and 8.50 GPa, respectively. The uniaxial compressive strength and effective elastic modulus of the medium sandstone are the lowest, with averages of 53.5 MPa, 6.70 GPa, respectively. The uniaxial compressive strength and effective elastic modulus of the calcilutite and sandy mudstone are close and slightly lower than that of the fine sandstone. The uniaxial compressive strength and effective elastic modulus of the calcilutite are 104.2 MPa and 8.39 GPa, respectively. The uniaxial compressive strength and effective elastic modulus of the sandy mudstone are 103.7 MPa and 7.95 GPa, respectively.

3.2 Strength and deformation behavior

The presence of microcracks in the rock can significantly affect the deformation process of the rock, including the nonlinearity of the stress-strain curve at the initial stage, peak-strength and decrease in the effective elastic modulus. Based on the effective medium theory, the strain at any point P on the stress-strain curve of the rock before the microcracks are completely closed can be decomposed into matrix axial strain and crack axial strain (Peng *et al*. 2015a), as shown in Fig. 1.

$$\varepsilon_p = \varepsilon_p^c + \varepsilon_p^m \tag{1}$$

In the one-dimensional stress state, the matrix axial strain is linearly related to the effective elastic modulus after the microcracks are completely closed. Hence, the crack axial strain can be expressed as follows.

$$\varepsilon_p^c = \varepsilon_p - \varepsilon_p^m = \varepsilon_p - \frac{\sigma_p}{E} \tag{2}$$

Here, E is the effective elastic modulus of the rock, the

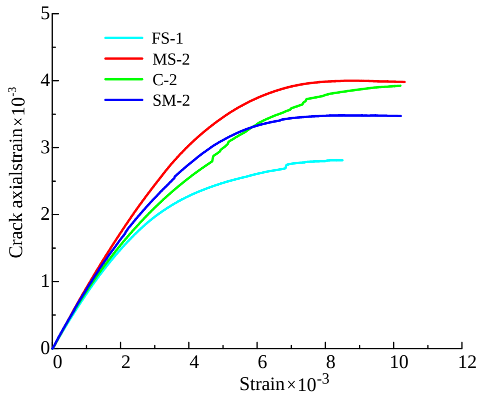
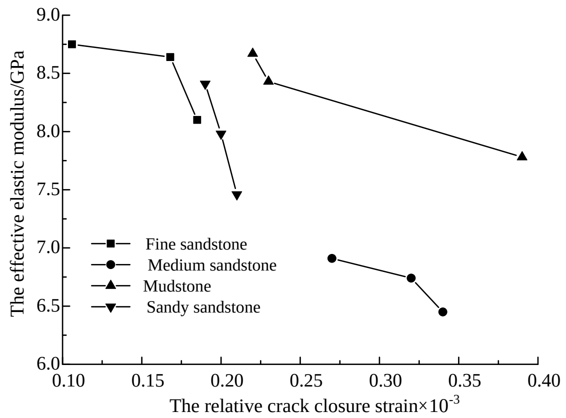
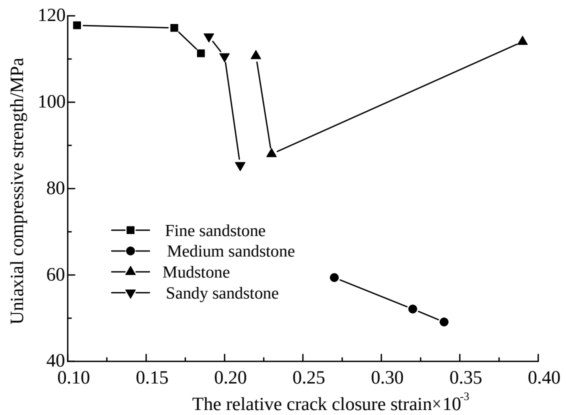


Fig. 6 Relationship between crack strain and total strain in crack compression stage



(a) Relationship between relative crack closure strain and effective elastic modulus



(b) Relationship between relative crack closure strain and uniaxial compressive strength

Fig. 7 Effect of relative closure crack strain on mechanical properties of rocks

magnitude of which is equal to the slope of the stress-strain curve at the elastic stage.

Fig. 6 shows the graph of the relationship between the crack strain and the total strain in the crack compaction stages of the rock specimens. It shows that the crack strain of the rock increases with increasing total strain. In the early stage of the crack compaction, the growth trend in the crack strain of the four types of rocks is the same, indicating that the rock strain at the initial stage of the crack compaction is dominated by the crack strain. In the later stage of the crack compaction, the difference of the crack strain increases gradually, indicating that the strain of the rock largely depends on the matrix strain.

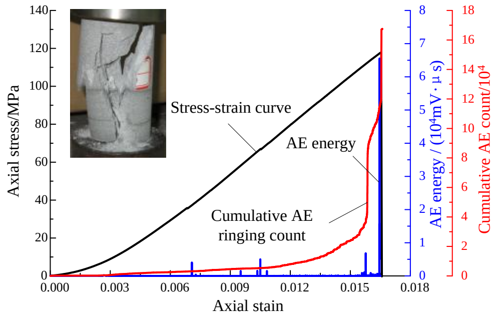
For the same type of rock, the extent of the primary fractures in different specimens (density, length, etc.) is different; moreover, the uniaxial compressive strength and effective elastic modulus are often different. To measure the influence of the rock fissures and compaction process on the mechanical properties of the rock, the concept of the relative closed crack strain was proposed. It is defined as the ratio of the crack strain at point A (see Table. 1) to the strain at point C (peak-point). Fig. 7(a) shows the relationship between the relative crack closure strain and the effective elastic modulus. Fig. 7(b) shows the relationship between the relative crack closure strain and the uniaxial compressive strength.

Based on the stress-strain relationship of the four types of rocks (Fig. 5), when the relative crack closure strain is less than 1.6×10^{-4}, the compaction stage of the rock is shorter and the effects of the crack on the compressive strength and elastic modulus are negligible. When the relative crack closure strain is more than 1.6×10^{-4}, in addition to calcilutite, the compressive strength and effective elastic modulus of the rock decrease with increasing crack closure strain, and the decreasing trend gradually increases. The strength and deformation behaviors of rocks are significantly influenced by the presence of microcracks in the rocks, such as the number, length, and angle of the crack (Peng *et al*. 2015a). For the calcilutite, the relative crack closure strain of C-3 is the highest, and the effective elastic modulus is the lowest; however, the uniaxial compressive strength is the highest. The reason may be that the original fissures in C-3 are more, but the fissure development is lower. The rock can be easily compressed, but the cracks cannot easily expand, resulting in higher rock strength.

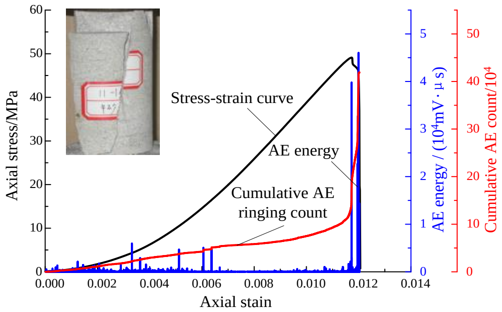
3.3 AE characteristics of four types of rocks

Under the external force, the evolution of the primary and new fissure can be monitored in the AE experiments. The selected AE characterization parameters are AE count and AE energy in this study. The AE count is defined as the number of times the ringing pulse exceeds the threshold value, which is the external acoustic performance of the internal structure of the rock. The accumulated AE counts are defined as the total number of rings before a certain moment, and the AE energy is the energy of the AE event. As the AE count and energy characteristics are regularity similar, the accumulated AE counts can also reflect the variation characteristics of the AE count during the compression process of the rock sample (Jiang *et al*. 2017, Ai *et al*. 2012). In this study, one sample of each type of rock is selected, and the results of the AE energy and accumulated AE counts are given, as shown in Fig. 8.

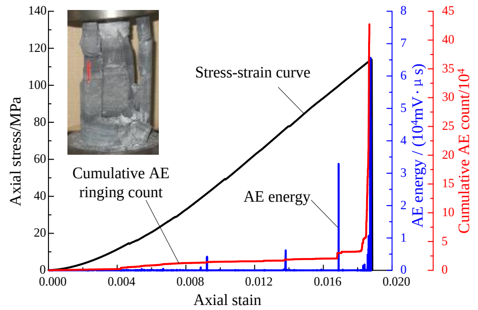
In the compaction stage of the rock fracture, for samples FS-1 and MS-2, the number of AE events is less and the AE energy is low. However, for samples C-2 and SM-2, the number of AE events is relatively more and the AE energy is relatively higher; the accumulated AE counts are 5.4e4 and 2.9e4, respectively, and the maximum energy released are 0.60e4mV·μs, 0.95e4mV·μs, respectively. It is indicated that micro destruction occurred in the compaction stage of MS-2 and SM-2. Samples FS-1 and C-2 occasionally release a large amount of energy in the elastic stage; the maximum value are 0.50e4mV·μs and 3.32e4mV·μs, respectively. The results show that there is still a modest damage and failure in the elastic stage of the rock. Hence, to establish the damage constitutive equation of brittle rocks, in addition to considering the compaction effect of the rock fracture, the damage variation in the entire compression process should be considered.



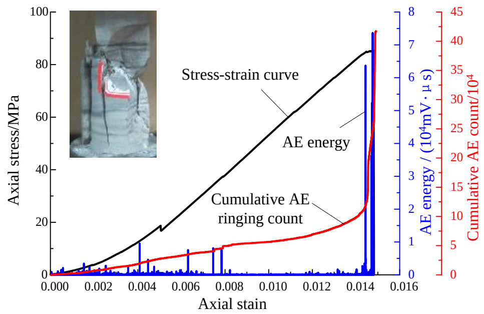
(a) FS-1



(b) MS-2



(b) C-2



(d) SM-2

Fig. 8 Stress-strain and AE curves of rocks

4. Damage constitutive model

4.1 Damage variable and evolution equations

Studies show that the ringing count is one of the characteristic parameters that can reflect the change in the material properties. This is because it is proportional to the strain energy released by the dislocation and fracture of structures and crack propagation in the material (Xiao *et al*. 2013, Daniela *et al*. 2017).

The damage variable is defined as follows (Kachanov 1958).

$$D = \frac{A_d}{A} \tag{3}$$

where: A_d is the area of the new defect on the bearing surface, and A is the effective area of the bearing surface. If the bearing surface is completely destroyed, the cumulative AE ringing count is C_0. Accordingly, the AE ringing count of the micro damage per unit area C_w is given as follows.

$$C_w = \frac{C_0}{A} \tag{4}$$

When the damage area of the bearing surface reaches A_d, the cumulative AE ringing count C_d can be obtained as follows.

$$C_d = C_w A_d = \frac{C_0}{A} A_d \tag{5}$$

Hence,

$$D = \frac{C_d}{C_0} \tag{6}$$

where: C_d is the the sum of the AE ringing count of the rock sample under the uniaxial loading condition.

During the test, if the stiffness of the testing machine is insufficient or modes of the rock failure are different, the test machine is made to shut down before the rock mass is completely destroyed (i.e., the value of C_0 in this case is smaller than that the bearing surface is completely destroyed). Therefore, the damage variable can be modified as follows.

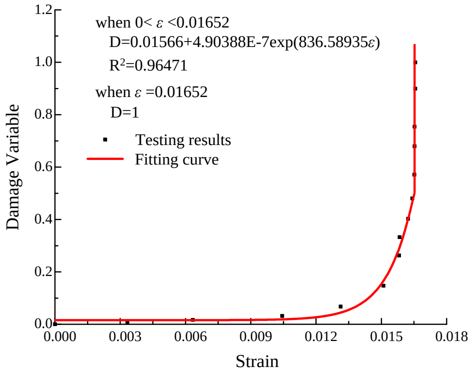
$$D = D_U \frac{C_d}{C_0} \tag{7}$$

The calculation steps are simplified elsewhere. D_u is defined as the damage variable when the rock goes into the failure stage. It can be expressed as follows (Liu *et al*. 2009).

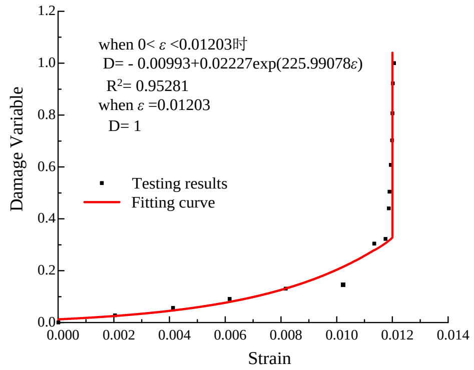
$$D_U = 1 - \frac{\sigma_C}{\sigma_P} \tag{8}$$

where: σ_P is the peak-strength, and σ_c is the residual strength.

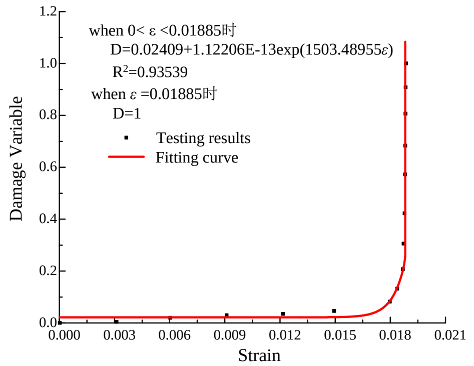
Fig. 9 shows the calculated damage variables of the rocks and their fitted curves. The figure shows the damage variable based on the cumulative AE ringing count



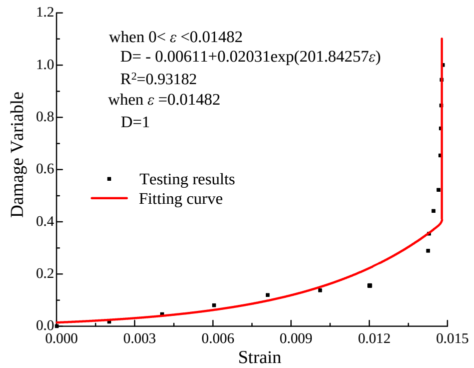
(a) FS-1



(b) MS-2



(c) C-2



(d) SM-2

Fig. 9 Calculated damage variables and fitting curves of rocks

increased exponentially with respect to the strain. The damage variable of the four types of rock could be fitted accurately using an exponential function (shown in Fig. 9), with a coefficient of determination greater than 0.93. Combined with Fig. 5, the damages of FS-1 and C-2 are negligible in the compression and elastic stages; moreover, MS-2 and SM-2 exhibit some damages in the compression stage, and the new damage in the elastic stage is negligible. However, as shown in Fig. 8, the fitting curves of the damage variables of MS-2 and SM-2 in the elastic phase do not reflect the real situation of the rock damage.

4.2 Damage constitutive model

Based on the hypothesis of the strain equivalence and characteristics of AE, the model for rock damage under uniaxial compression can be expressed as follows.

$$\sigma=(1-D)E_0\varepsilon=\left(1-D_U\frac{C_d}{C_0}\right)E_0\varepsilon \tag{9}$$

where: E_0 is the elastic modulus of the non-destructive material; herein referred as the elastic modulus of the rock without new damage.

The new damage to the rock is due to the external force. The damage variable based on the AE count increases fromzero and the carrying capacity of the rock reduces. At the same time, because of the compression closure of the primary fissures of the rock, the deformation of the fractured rock is greater than that of the intact rock under the same stress level. However, in the existing damage model of the rock based on the AE characteristics, the effect of the compaction of the cracks on rock deformation (strength) is neglected (Zhao *et al*. 2017, Huang *et al*. 2018). Hence, the concept of a compaction coefficient, K_1, was proposed to quantify the extent of the compaction of the rocks during loading. It is defined as the ratio of the crack strain to the crack closure strain of the rock; the value is unity in the quasi-linear elastic, yield, and failure stages. Fig. 9 shows that the compaction coefficient during the compaction stages increased logarithmically with strain. Hence, the compaction coefficient can be expressed as follows.

$$\sigma=(1-D)E_0\varepsilon=\left(1-D_U\frac{C_d}{C_0}\right)E_0\varepsilon \tag{10}$$

where n is a constant obtained experimentally; ε_c is the crack strain of the rock at the compaction stage; ε_A is the crack closure strain of the rock.

Considering the new damage of the rock in the compaction and elastic stages, the effective elastic modulus E obtained in the experiment is not equal to the elastic modulus E_0 of the rock without new damage. Moreover, E_0 can not be obtained from the experiment. Hence, the existing uniaxial compression rock damage model based on the AE characteristics is generally expressed as follows (Xiao *et al*. 2013).

$$\sigma=(1-D)E\varepsilon=\left(1-D_U\frac{C_d}{C_0}\right)E\varepsilon \tag{11}$$

To compensate for the effective elastic modulus E instead of the elastic modulus E_0 in Eq. (11) and consider the damage effect, the correction factor K_2 is established.

The greater the new damage in the compaction and elastic stages, the lower the effective elastic modulus E obtained from the experiment. Fig. 8 shows that the damage variable of the rock increases exponentially, during the compaction and quasi-linear elastic stages, with strain. Hence, the correction factor K_2 can be expressed as follows.

$$K_2 = \begin{cases} \dfrac{1}{Ae^{B\varepsilon}+C} & \varepsilon < \varepsilon_s \\ \dfrac{1}{1-D_s} & \varepsilon \geq \varepsilon_s \end{cases} \tag{12}$$

where A, B, and C are the undetermined coefficients, obtained by fitting the test results; ε_s is the strain corresponding to the yield stress; D_s is the damage corresponding to the ε_s.

Thereafter, we obtained a damage constitutive model based on the AE characteristics for rocks under uniaxial loading as follows.

$$\sigma = K_1K_2(1-D)E\varepsilon = K_1K_2\left(1-D_U\frac{C_d}{C_0}\right)E\varepsilon \tag{13}$$

5. Verification and discussion

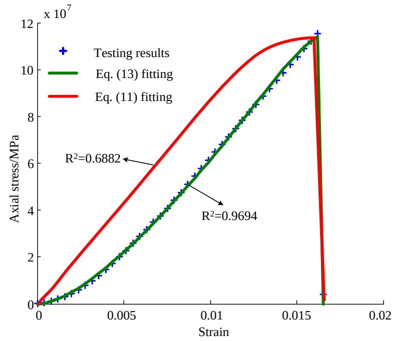
5.1 Verification schemes and results

The crack closure strain ε_A of the rock specimens FS-1, MS-2, C-2 and SM-2 are shown in Table 1. Based on the ratio of the crack strain ε_c and crack closure strain ε_A of the rock in the compaction stage, the undetermined constant n of the compaction coefficient K_1 is obtained (see Table 2). The uniaxial loading curves for the four types of rocks were fitted by using the damage constitutive model (Eq. (11)) and the amended damage constitutive model (Eq. (13)), respectively. The fitting parameters A, B, and C of the correction coefficient K_2 are obtained. The results are shown in Table 2.

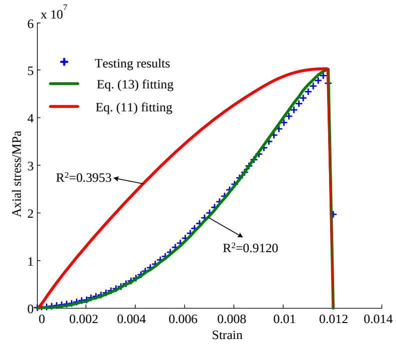
The stress-strain curves were fitted by Eqs. (11) and (13), respectively, then compared to the experimental data and the comparison is shown in Fig. 10. At the compression stage, the stress-strain curves of XS-1 and C-2 fitted by Eq. (11) were linear and the stress-strain curves of MS-2 and SM-2 fitted by Eq. (11) were convex, and quite different from those found experimentally. At the quasi-linear elastic stage, the difference between the fitting results and the experimental results of FS-1 and C-2 is largely the same. The difference between the fitting results and the experimental results of MS-2 and SM-2 decreases gradually. At the yield stage, the difference between the fitting results and the experimental results of the four types of rocks continuously decrease when the rock strain reaches the yield strain; the fitting results are coincident and consistent with the test data. The coefficients of determination R2 of the four rock samples are between 0.3953 and 0.7179. The stress-strain curves fitted by Eq. (13) are concave. The fitting curves agree well with the test curves, and the coefficients of determination R2 is above 0.90. The constitutive model of the rock damage based on the AE characteristics can be satisfactorily used to describe

Table 2 Parameters of four constitutive equations of rocks

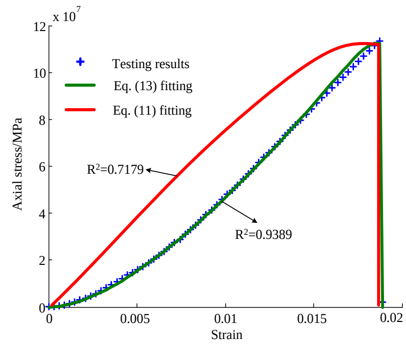
Specimen number	Yield-point strain /ε_s	Yield-point damage /D_s	n	A	B	C
FS-1	0.01297	0.04065	79.8000	-2.27451E-7	882.34733	1.01904
MS-2	0.01062	0.16544	26.0128	-0.01768	246.09739	1.00317
C-2	0.01735	0.04095	10.1967	-5.30460E-16	1801.77306	0.97124
SM-2	0.01376	0.17728	21.4819	-0.04296	148.93618	1.08368



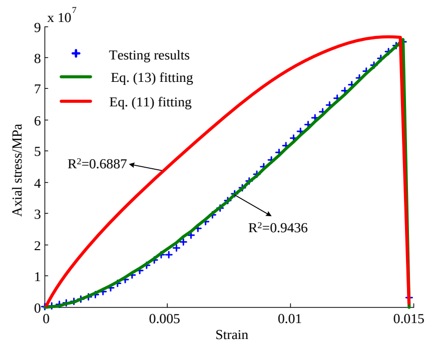
(a) FS-1



(b) MS-2



(c) C-2



(d) SM-2

Fig. 10 Comparison of uniaxial loading test and fitting curves the stress-strain relationship of the brittle rock under uniaxial loading.

5.2 Discussion

Rock containing primary fissures is widely found in deep tunnels (holes), slopes and mining, and their strength and deformability under uniaxial loading are the foundation for design and assessment of their engineering performance. This research aimed to provide a constitutive model for rocks under uniaxial loading with convenient field application and high accuracy.

(1) The damage constitutive models of brittle rocks considering compaction effect of fracture can describe stress-strain relationship of rock more accurately. But most of these models neglect the compaction effect of rock fractures, and cannot reflect the influence of crack compaction effect on rock mechanical properties. In this paper, the concept of relative crack closure strain was put forward, and the law of the influence of the relative closed crack strain on rock mechanical parameters was analyzed.

(2) The development of rock fissure was indirectly obtained based on AE Technology. And the damage constitutive equation of rock was established by using cumulative AE count as damage variables. We defined the compaction coefficient here to quantify the extent of the compaction of the rocks and used it to amend the damage constitutive model obtained by the Lemaitre strain equivalence hypothesis. What's more, considering the influence of rock damage during compaction and elastic stage on the effective elastic modulus, we defined the correction factor and continue to amend the model. Compared with the fitting curve of the damage constitutive model in the literature (Xiao *et al*. 2013), the amended model can accurately describe the stress-strain relationship of the rock, with a coefficient of determination greater than 0.9.

(3) It should be mentioned that this study was based on a few uniaxial loading tests on only four types of rocks with high porosity and a large amount of initial cracks. More tests on other types of rocks, such as granite or even other materials, should be conducted to verify the proposed models. In addition, we need to classify rocks and summarize the characteristics of rocks that the crack compaction needs to be taken into account.

6. Conclusions

The aim of this research was to lay a foundation for the evaluation and design processes used in rock engineering, by building a damage constitutive model for rocks under uniaxial loading. By comparing with the previous studies, this work contains at least two original aspects.

Four types of brittle rock, namely, fine sandstone, medium sandstone, calcilutite and sandy mudstone, exhibit different crack compaction processes under uniaxial loading. To measure the influence of primary crack compaction effect on the mechanical properties of the rock, the concept of relative crack closure strain was first proposed. The larger the relative crack closure strain, the lower are the uniaxial compressive strength and effective elastic modulus of the rock.

AE event can reflect well the development process of cracks in the rock. A damage variable based on the the accumulated AE counts was established, and the damage evolution equation for the four types of brittle rocks were fitted by a piecewise function. The coefficient of determination R2 is above 0.93. The damage variables of the fine sandstone and the calcilutite in the compaction and elastic stages are less. In the compaction stage, the medium sandstone and sandy mudstone exhibit some damage. In the elastic stage, the new damage is insignificant.

The concepts of compaction coefficient and correction factor (correcting the error due to the effective elastic modulus instead of the elastic modulus of intact rock) were proposed. The existing damage constitutive equation of the rock was then modified. The model verification shows that the damage constitutive equation modified can describe the compressive deformation (particularly the compaction stage) of the four types of brittle rocks more accurately, with a coefficient of determination greater than 0.9.

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