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Determination of elastic parameters of the deformable solid bodies with respect to the Earth model

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Abstract. The study of behavior and values of deformations in the geological medium makes the scientific basis of the methodology of synthesis of true values of parameters of its physico-mechanical and density properties taking into account the influence of geodynamic impacts. The segments of continuous variation of homogeneous elastic uniform deformations are determined under overall compression of the medium. The limits of these segments are defined according to the criteria of instability (on geometric form changes and on “internal” instability). Analytical formulae are obtained to calculate current and limiting (critical) values of deformations within the framework of various variants of small and large initial deformations of the non-classically linearized approach of non-linear elastodynamics. The distribution of deformation becomes non-uniform in the medium while the limiting values of deformations are achieved. The proposed analytical formulae are applicable only within homogeneous distribution of deformations.
Numerical experiments are carried out for various elastic potentials. It is found that various forms of instability can precede phase transitions and destruction. The influence of these deformation phenomena should be removed while the physico-mechanical and density parameters of the deformed media are determined. In particular, it is necessary to use the formulae proposed in this paper for this purpose.

Keywords: nonlinear elastodynamics; instability of the equilibrium state; elastic potentials; physico-mechanical and density parameters

1. Introduction

The studies of the internal structure and distribution of physico-mechanical parameters in them relate to the main problems of the Earth’s theoretical models (www.sciencedirect.com/..., https://ds.iris.edu/...). Analysis of results of direct observations, model theoretical and experimental studies show that deformation processes play an important role in the formation and development of various structural elements, internal dynamics and the Earth as a whole.

The changes of equilibrium state, phase transitions and medium destruction occur at elastic, elastoplastic and at subsequent stages of strain. The sequence of their implementation significantly depends on the substantial content of the geological medium, its state of stress, geometry of structures, etc.

Stress values corresponding to phase transitions (partial melting) are determined in laboratory experiments. Depths are theoretically calculated. Based on data of the medium density the depth in the Earth’s interior is theoretically calculated where phase transitions can be implemented. The dividing borders in the Earth’s interior are defined comparing the experimental and theoretical results. The models of the internal structure are constructed additionally requiring the fulfillment of integral criteria of mechanics with respect to the average moment of inertia of the rotating spherical body, total mass of the Earth and intervals of its free oscillations (Bullen 1963, Bullen 1975, Dziewonski and Anderson 1981, Anderson 2007, Zharkov 2012, Molodenskii and Molodenskaya 2009, 2015, Molodenskii and Molodenskii 2015, www.sciencedirect.com/..., https://ds.iris.edu/...). Their improvement is continued by adjusting the parameters of models to data of seismic tomography, deep seismic sounding, the Earth’s free-oscillations, geochemistry, petrology, etc. (Navrotsky 1994, Anderson 1995, Kalinin 2000, Adushkin and Vityazev 2007, Molodenskii and Molodenskaya 2009, 2015, Molodenskii and Molodenskii 2015, Akbarov *et al.* 2016). Correlations are defined between conditions P, T (P-pressure, T-temperature) and processes of phase transformations for a number of ultramafic rocks in Green, Ringwood, Akimito, Liu and other’s well-known experimental studies (Bullen 1975, Ringwood 1981, Zharkov 2012). It follows from the results of these studies that stress values corresponding to phase transitions are less than stress values corresponding to strength limits of the

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considered rocks, i.e., process of phase transformations is implemented long before the destruction process. Homogeneous samples are tested on composition in experiments. Uniform homogeneous strains are implemented in samples within overall compression. Unlike the experimental conditions, the geological medium has various impurities, inclusions in the Earth’s interior, disturbances in the structure, etc. Furthermore, inhomogeneity of chemical composition (mineral associations) of rocks and ascending flows (mainly gas) of interstitial elements contribute the geological medium to become at least a two-phase system with various correlations of crystalline and amorphous phases (Gufeld 2013, Liu and Lin 2014). Therefore, strain processes of rocks significantly differ in natural conditions and samples in laboratory. This difference becomes even more significant due to non-uniformity of strain within the conditions of various homogeneous and inhomogeneous states of stress.

The advanced technologies are used in modern experiments (Altshuler *et al.* 2004, Tateno *et al.* 2010, Mao *et al.* 2012, Liu and Lin 2014, Li and Tao 2015) to obtain thermobaric conditions corresponding to deeper entrails of the Earth. According to them, the experiments are conducted using dynamic method in quite short time intervals. Therefore, it is not possible to observe lengthy strain processes in general. The concept on fast kinetics used in method of shock-wave compression is based on shear strain of substance. But these strains are caused to record the results of experimental measurements, and don't relate to deformations occurred in natural evolutionary process of the Earth. More graphical presentation on deformation processes is drawn conducting the analogy with mechanics of strain of fibrous and layered composite materials. It is known that (Guz 1989, Akbarov and Guz 2000) small and large curvatures of the reinforcing fibers and layers occur in their structure as a result of strain of composite materials. As a result, significant changes occur in distribution of deformations and stress in the medium. In its turn, these changes are reflected on mechanical properties of the material. Such studies relate to a priority area in the theory of composite materials, and are developed intensively (Guz 1989, Akbarov and Guz 2000, Akbarov 2013, Akbarov *et al.* 2016, 2017).

More interesting results are provided in (Liu and Lin 2014). Anomalies in velocity and rheological characteristics of some basalts and silicates are observed in laboratory experiments at high pressure. The local changes in the structures in the process of compression are considered one of the main reasons of these anomalies. Experimental results of crystallization of the molten material are obtained within similar conditions of inner solid core (Tateno *et al.* 2010). These and numerous similar experimental data are source in modeling the inner core as a solid body consisting of iron alloys.

Similar but more complex, diverse and scaled strain processes are implemented in the layered composite structures of the Earth that can’t only be described by overall strains (Guliyev 2010, Guliyev 2011). In principle, the correct consideration of the role of strain processes in the structural and geodynamic-tectonic evolution of the Earth will also allow more progress in better understanding of the problem (the Earth's dynamics) (Adushkin and Vityazev 2007).

It’s necessary to make the following remark. Various criteria are suggested in the mechanics of standard materials and constructions on the basis of the theory of strength, limit states, plasticity, mechanics of destruction of the fractured bodies, durability and the theory of stability to determine the limit of allowable strains. As a usual, these strains don’t exceed 1-2% at elastic stage. The rubber-like materials that can undergo high strains make an exception. The problem on values of strain is different in geology. The substances are strained for a long geological time in the Earth’s interior, and strain values may be different. The achievement of strain values in local zones of any limit values determined within the above mentioned theories don’t mean that the loss of strength occurs or bearing capacity of the Earth's composite structure is exhausted as a whole. These limit values indicate possible processes in local zones. Depending on structural, geometric features of these zones, physico-mechanical properties of substances, types and values of impacts in the Earth’s interior different strain processes such as the curvature of the layered structures, formation of faults and cavities, expansion of the existed faults, plastic flow, delamination, loosening, consolidation of the media, partial melting, etc. can occur. As a result, significant redistribution of stress and strains occur in the considered zones. Such changes can create the condition of implementation of fracture and formation of shear and layered zones even at extremely high normal pressure (at a depth of 700 km and greater) as mechanical processes. It is mentioned in some publications (Guefeld 2013) based on the conception of overall deformation of homogeneous medium that the indicated mechanical processes cannot be implemented at such depths. Moreover, it is not necessary that these processes should be directly related to processes of interaction of the lower fields of the mantle with the outer liquid core. Seismotomographic studies show that large deconsolidated zones and diverse canals of mass flow are located in the deep entrails of the Earth far from borders of the outer core (Van der Hilst and Karason 1999, Ritsema 2005, Trampert *et al.* 2004).

Summarizing of the above mentioned, it should be noted that on one hand, the geological medium acts as a material, but on the other hand as an element of structure. Behavior of strain of homogeneous, composite material and elements of structure differ significantly. It is necessary to properly consider this fact in theoretical models of the Earth’s development and in the interpretation of the results of experimental studies.

Great successes in geophysics, physics of high pressure and temperature, seismotomography, deep seismic sounding, the Earth’s free-oscillations, tidal strains and nutations, surface waves, geomechanical and petrological studies gave a lot of information on the internal structure and on the occurred processes there. An enormous array of data is accumulated on various parameters of the Earth’s interior structure, and it continues filling by flows of new data on various directions. The parameters of the generated theoretical models are clarified by accumulation of new data. A key component of all models is the solution of problems of density distribution and determining the law of state (correlation between density and pressure) (Bullen 1975, Hofmeister 1993, Anderson 1995, Zharkov 2012, www.sciencedirect.com/, https://ds.iris.edu/) of substance on depth.

There are errors and faults in the obtained results for objective and subjective reasons. The errors and faults enter in final data in the form of inaccuracies in experimental and observational data, approximations of theoretical basis of processing and interpretation, and uncertainties in our scientific concepts. In principle, it is not possible to completely assess the full-scale validity of observational and experimental (the setting of which are primarily based on our subjective assumptions) data. Furthermore, the consistency of data aggregate of various studies (geological, geophysical, physico-chemical, mechanical, etc.) and their compliance with experimental (invented by us) results and current scientific concepts are required. This is the essence of an idea on reliability of observational (direct and indirect) data. There is a need to develop a systematic approach for processing and interpretation this, an ever-renewing array of different nature and varying degree of data validity. It is necessary to develop an approach based on the minimum amount of observational and experimental data that most completely cover the assumed mechanisms of process flow in order to minimize the number of faults and errors in results. Numerous studies are carried out (Bullen 1975, Navrotsky 1994, Anderson 1995, Kalinin 2000, Adushkin and Vityazev 2007, Molodenskii and Molodenskaya 2009, 2015, Zharkov 2012, Molodenskii and Molodenskayii 2015, Akbarov *et al.* 2016, 2017) within this framework. The areas of allowable values of some mechanical parameters in the lithosphere, mantle, outer liquid and inner solid core are determined in them.

The main idea of study of problems on distribution of various fundamental parameters of theoretical models of the Earth's development consists of their consideration at present time. The history of their formation is excluded. Despite the fact that such an approach significantly reduces the number of variants, uncertainty and diversity still remain in solving this fundamental problem of the Earth sciences.

In case of overall compression, the problem on distribution of density of the Earth's substance on depth was studied by Birch (Birch 1952) using the Eulerian method within the theory of finite strains. As a result, the Birch-Murnaghan equation of state is suggested (Birch 1952, Bullen 1975). Further, it is shown (Knopoff 1963) that significantly different result is obtained in case of applying the Lagrangian method for this purpose to describe strain process. Henceforth, different authors discussed this problem. The bibliography of these studies is provided in (Anderson 1995). Theoretical results of both approaches are applied for interpretation of various observational and experimental data and the supposed mechanisms of formation of structures within model representations. Limits of applicability of various state equations are determined. Experimentally determined restrictions imposed on the physico-mechanical properties of substances (e.g,. shear and pressure moduli should not be negative, etc.) are used for this purpose. The consistency of calculation data with model representations are considered as a measure of result reliability. The disadvantages of such approaches are mainly related with two reasons. Firstly, theoretical correlations of various methods of description of deformation processes are used to describe strain processes within one and the same model. Secondly, multiple details on mineralogical compositions, physico-mechanical, petrological, geochemical and other properties of substances are required. Uncertainty is high in the degree of homogeneity of mineral composition of rocks. It is considered that values of various observational and experimental data should be within intervals of stable change of fundamental physico-mechanical and chemical parameters (Anderson 1995). Such approaches are time-consuming and related with great risks. At the same time, probability of involving significant errors and inaccuracies is quite high in results.

Dependencies between results of the Lagrangian and Eulerian method had already been obtained for a long time in the mechanics of the deformed solids to describe strain processes. There are simple analytical correlations connecting the principal values of Green's strain tensor (the Lagrangian method) and Almansi (the Eulerian method) (Truesdell 1972, Guz 1989). Regarding the convenience in the use and procedures of mathematical simplifications, methods of description of nonlinear processes of strain have their advantages and disadvantages in various problems. For instance, it is preferable to use the Lagrangian method in studies of problems of stability of equilibrium state. In this case, the state and form of the deformable body is known in natural state that allows simplifying the solution of quite difficult mathematical problem. From this point of view, it is also convenient to apply the Lagrangian method in problem of distribution the density of the Earth's substance as the density of substance is known in the unstrained state. Moreover, the medium density behind the front of wave compression and in front of it are known (directly measured) in case of experimental studies of density using shock-wave of compression.

There is no method allowing determining the countless number of parameters of deformable system (impact components and impact objects) in more or quite less complete volume in practice within real conditions (i.e., in various unreachable depths of the Earth's interior considering the geological time). Despite the fact that quite acceptable results describing certain aspects of the given problem is obtained in some concrete cases, in general, such case has led to creation of numerous models of the Earth and obtaining ambiguous and uncertain conclusions (Bullen 1975, Dziewonski and Anderson 1981, Anderson 1995, Anderson 2007, www.sciencedirect.com/, https://ds.iris.edu/).

Finally, the studies (Guliyev 2016, 2017) show that the solution of the problem of the Earth's internal structures and the distribution of elastic parameters in them is not only sufficient on the basis of integral criteria. The sufficient local conditions follow from the requirements of the mechanics of continuous medium, and it is necessary to achieve the fulfillment in the solution of structural problems and in the problems on the distribution of elastic parameters.

It is proposed to base on the problems of nonlinear theory of mechanics of continuous medium set in strains to achieve valid and possibly unambiguous (certain) results on distribution of density of substance and other parameters of the Earth's model within the current geodynamics in this manuscript. Non-classical linearized approach (NLA) of nonlinear theory of straining of solid bodies is used as a theoretical apparatus considering small and large (finite) strains within the Lagrangian method of their description.

The strains consider all-possible forms and natures of impact, degree of the medium exposure to impacts, physico-mechanical, petrophysical, geochemical, thermal and other characteristics of deformable system in a complete scale. At the same time, in case of considering problems in strains, there is no need in concrete data on the above mentioned properties as well as in equations of state. The strain is the most universal parameter in system of impact, in the object of impact and in the results of impact. It’s necessary to evaluate this parameter in deformable system correctly and to use it in theoretical, experimental studies and interpretations properly. Consequently, first of all, it is necessary to define fields of stable continuous change (in case of compression-increase) of strain and then to determine other necessary parameters and conduct interpretations of different data within it. Critical values of strains determining intervals of conserving the given condition is achieved within different processes, and their implementation mechanisms can be caused by impacts of various nature.

2. Medium density and high strains

It is obtained in nonlinear theory of deformable solid bodies using general theoretical dependencies of determination of geometric objects (length, areas and volume) and law of mass conservation (Guz 1989)

$$\frac{\rho}{\rho_0} = I_3^{-\frac{1}{2}};$$
$$I_3 = 1 + 2A_1 + 2(A_1^2 - A_2) + \frac{4}{3}(2A_3 - 3A_2A_1 + A_1^3); \tag{1}$$
$$A_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3; \quad A_2 = \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2; \quad A_3 = \varepsilon_1^3 + \varepsilon_2^3 + \varepsilon_3^3.$$

where ε_i (i=1,2,3) are the principal values of Green’s strain tensor in describing strains using the Lagrangian method; ρ is medium density in the current state; ρ_0 is medium density in natural (unstrained) state.

Eq. (1) in (Guliyev 2013) are used to study the density distribution of the Earth’s substances considering the nonuniformity of homogeneous strains.

It follows from Eq. (1) that it is necessary to know the numerical value of its initial density ρ_0 and the principal values of Green’s strain tensor ε_i in the current state to determine the distribution of density of any solid deformable body. Knowledge on types of the stressed states, on their nature, state equations, parameters of state equations, etc. are not required in using this universal correlation.

In case of homogeneous strains, it is possible to assume without loss of generality that

$$\varepsilon_1 = \alpha\varepsilon_0, \quad \varepsilon_2 = \beta\varepsilon_0, \quad \varepsilon_3 = \gamma\varepsilon_0, \tag{2}$$

where ε_0 is parameter of overall strain, α, β, γ are substance numbers. It is possible to consider different non-uniform homogeneous strained states presetting specific values on these numbers. Considering Eq. (2) in Eq. (1), we obtain a more convenient equation for calculations

$$\frac{\rho}{\rho_0} = \left(1 + A\varepsilon_0 + B\varepsilon_0^2 + C\varepsilon_0^3\right)^{-\frac{1}{2}}, \quad A = 2(\alpha + \beta + \gamma),$$
$$B = 4(\alpha\beta + \alpha\gamma + \beta\gamma), \quad C = 8\alpha\beta\gamma \tag{3}$$

Eqs. (1)-(3) cover stages of small and large strains.

In case of overall strain (α=β=γ=1) Eq. (3) is simplified and takes quite simple form

$$\frac{\rho}{\rho_0} = (1 + 2\varepsilon_0)^{-\frac{3}{2}} \tag{4}$$

Eq. (4) takes the following form for an isotropic elastic body under compression

$$\frac{\rho}{\rho_0} = (1 - x)^{-\frac{3}{2}}; \quad x = \frac{2(1 - 2\nu)}{E}P, \tag{5}$$

where P is pressure per unit of the area in the unstrained state (in case of small elastic strains); in case of large strains, pressure can also be correlated to the unit of the area of the initial strained state; ν is Poisson's ratio; E is Young’s modulus of elasticity.

It follows from Eqs. (1)-(4) that with the definition of the density of the medium at any stage of deformation (e.g., at atmospheric pressure), values of pressure parameter are determined at subsequent changes of deformation without using data on state equation and on system of external impacts. In contrast, it's seen from Eq. (5) that there is an additional necessity in data on physico-mechanical properties of the medium (Poisson’s ratio ν and elasticity modulus E) while defining stress (pressure).

Table 1 shows the numerical values of variation of the density parameter depending on variation of nonuniform strains. Table 1 shows the numerical values of variation depending on variation of nonuniform strains.

The following equation is obtained using Euler’s method (Birch 1952, Bullen 1975)

It was obtained by Birch using the Eulerian method (Birch 1952, Bullen 1975)

$$\frac{\rho}{\rho_0} = (1 - 2\varepsilon_0)^{\frac{3}{2}}. \tag{6}$$

The Birch-Murnaghan equation of state is suggested in case of finite overall strains on the basis of Eq. (6)

$$\frac{2}{3K_0^{'}}P = \left(\frac{\rho}{\rho_0}\right)^{\frac{7}{3}} - \left(\frac{\rho}{\rho_0}\right)^{\frac{5}{3}}. \tag{7}$$

Similarly, state equation is obtained in case of using Eq. (4) (Knopoff 1963)

$$\frac{2}{3K_0^{'}}P = \left(\frac{\rho}{\rho_0}\right)^{-\frac{1}{3}} - \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{3}}. \tag{8}$$

$K_0^{'}$ is isothermal modulus of compression in these equations.

The problems on determination of areas of applicability

Table 1 Numerical values $\frac{\rho}{\rho_0}$ in relation to ε_0 on (3)

α,β,γ / ε_0	$\alpha=\beta=\gamma=1$	$\alpha=\beta=1$, $\gamma=0{,}1$	$\alpha=\beta=1$, $\gamma=0{,}5$	$\alpha=\beta=1$, $\gamma=2$
-0,4	11.18034	5.2129	6.4549	-
-0,2	2.1517	1.701	1.8634	3.7268
0	1	1	1	1
0,2	0.6037	0.7004	0.6521	0.5324
0,4	0.4141	0.5346	0.4695	0.3445

Note. It is known that the density of the medium is determined as the ratio of mass unit to volume unit. At the same time, according to the law of mass conservation, the mass unit remains constant. The volume changes in the process of deformation. Naturally, the volume unit can also change. This fact led to commonly encountered mistakes. The density of the medium and materials are determined relative to the volume unit of the unstrained state, and just this value being the nominal data characterizes the density properties. It is necessary to be careful while choosing a volume unit in the process of finite deformations. The current volume values depending on method of strain description (above mentioned) correlate to the volume unit of the unstrained state or the deformed state. Non-linear correlations are obtained in both cases. If we expand these correlations in a Taylor series on the initial density, so, only the first component (ratio of mass unit to volume unit of the unstrained state) will be responsible for the density (according to preliminary agreement) of the medium. The subsequent components being the characteristics of nonlinear strain should not be called the density of the medium, in spite of the fact that they are density values on the measurement unit. In such cases, it is possible to speak of either a quasi-density or an effective density of the medium at the best cases. In principle, the expression “the change of density depending on the change of strain” is unsuccessful. This remark also remains in force for all other nominal data (for example, elastic) of the medium. In this connection, the terms of density parameters, elasticity parameters, etc. are used instead of density and elastic properties in order to avoid confusion in the appropriate places in this manuscript.

of various tectonophysical parameters, as well as correlations (4) are solved below through assessment of limits of change of strains of overall compression from the point of view of three-dimensional theory of elastic stability of equilibrium states.

3. Validity intervals

3.1 The “Internal” instability

The problems on stability of strain process are one of intensively studied fields of the mechanics of the strained solids (Guz 1977, 1989, 1999, Biot 1965, Akbarov 2013, Guliyev 2010).

The elastic equilibrium state of the strained isotropic body (within the compressible material model of the medium) is stable (Guz 1999) irrespective of its geometric shape in the interval

$$\lambda_1^* < \lambda_1 < 1 \tag{9}$$

in case of overall compression by nonconservative (follower) external load. Limit values of elongation (shortening) coefficient determining the intervals of stability define concrete structures (forms) of elastic potentials. In case of modeling of strain process using harmonic elastic potential

$$\lambda_1^* = \frac{1+\nu}{2-\nu}, \tag{10}$$

λ_1 is elongation along the coordinate axes, ν is Poisson’s ratio.

It is known that $0<\nu<0.5$. Consequently, we obtain the value $0.5<\lambda_1^*<1$ for critical elongation λ^*_1 for all possible materials. In case of homogeneous initial finite strains (Guz 1999)

$$\lambda_1^2 = 1+2\varepsilon_0 . \tag{11}$$

We determine areas of its change using Eqs. (10) and (11) for the parameter of deformations in the area of compression

$$\varepsilon_0^* < \varepsilon_0 < 0;\ \ \varepsilon_0^* = \frac{3}{4}\frac{2\nu-1}{(2-\nu)^2}, \tag{12}$$

where the equilibrium state is stable.

λ^*_1 In case of modeling of strain process using quadratic elastic potential λ^*_1 is determined in the form

$$\lambda_1^* = \left(\frac{1+\nu}{2-\nu}\right)^{\frac{1}{2}} \tag{13}$$

Using Eqs. (11) and (13), we determine the critical parameter of deformation in the form

$$\varepsilon_0^* = \frac{1}{2}\frac{2\nu-1}{2-\nu} . \tag{14}$$

The stability area is also determined in the form of inequality (12) considering Eq. (14) in this case.

We obtain on the basis of these results to determine possible intervals of density change of the Earth's substances in cases of harmonic and quadratic elastic potentials of finite strains respectively

$$1 \le \frac{\rho}{\rho_0} \le \left(\frac{\rho}{\rho_0}\right)^*;\ \ \left(\frac{\rho}{\rho_0}\right)^* = \left(1+2\varepsilon_0^*\right)^{-\frac{3}{2}} = \left(\frac{2-\nu}{1+\nu}\right)^3, \tag{15}$$

$$1 \le \frac{\rho}{\rho_0} \le \left(\frac{\rho}{\rho_0}\right)^*;\ \ \left(\frac{\rho}{\rho_0}\right)^* = \left(\frac{2-\nu}{1+\nu}\right)^{\frac{3}{2}} . \tag{16}$$

The achievement of shortening and strains of their critical values in Eqs. (10), (13) and Eqs. (12), (14) corresponds to “internal” instability (Guz 1999). All points of the unlimited homogeneous isotropic medium (within the compressible material model) get the finite disturbances in the achievement of these critical values. These values of strains determine the theoretical limit of strength of the considered materials (Guz 1999). It is considered that the body not undergone any geometric form changes is destructed in all points simultaneously. Consequently, in this case, Eqs. (1), (3)-(5) correctly describe changes of the density parameter of the media up to the value ε_0^*. Consolidation process of the medium is changed into deconsolidation process in the achievement of this critical value of deformation.

Thus, Eqs. (4) and (7) interpreting experimental and observation data should be applied to describe density distribution and state equation of the Earth's substance conserving the condition put on strain $\varepsilon_0^* < \varepsilon_0 < 0$. Limits of these inequalities are concretized using Eqs. (12) and (14)-(16) in case of applying harmonic and quadratic elastic potentials. It is also possible to derive the corresponding correlations for other forms of elastic potentials of finite strains similar to Eqs. (12) and (14)-(16).

3.2 The instability of elastic equilibrium state on form changes

The above mentioned inequalities allow theoretically determining the possible limits of variation of the studied parameters within the accepted conditions. These intervals may be different in the current practice due to properties of deformation process even at elastic stage. If there are inhomogeneities of physico-mechanical and geometric origin in the medium, more complex strain processes can occur in it. Nonuniform distribution of strains can be caused by the process of change of elastic equilibrium state within the overall compression in certain cases, i.e., due to the buckling.

Let's consider the case of overall strain of the medium influenced by conservative ("dead") external loads. The state of elastic equilibrium body loses the stability at such impact within values $(\varepsilon_0)_*$ lower than critical deformations ε_0^*. Let's consider the stability of elastic equilibrium state of half-space in the vicinity of a vertical cylindrical cavity of circular cross-section as a specific example. The "dead" loads are defined on the cylindrical surface of the cavity, intensity of which is equal to the value of intensity of the external load acting on the "infinity" along horizontal planes. In this case, state of homogeneous strain is implemented in the form

$$u_m^0 = \delta_{im}\left(\lambda_i - 1\right) x_i , \qquad (17)$$

where $x_i \equiv x^i$ ($i = 1, 2, 3$) are the Lagrangian coordinates which coincide with the Cartesian coordinates in the undeformed state; δ_{im}-Kronecker's symbols; $u^0{}_m$-constituents of displacement vector in the initial state; λ_i-elongation along coordinate axis; direction of coordinate lines coincide with the principal directions of strain tensor.

The values of the strain tensor don't depend on the coordinates in the homogeneous state. In the case of overall homogeneous deformation, their values are equal between themselves.

It should be noted that the invariants $s_j\,(j = 1, 2, 3)$ of the Green's strain tensor are used in the nonlinear theory of elasticity of finite strains for bodies with an elastic potential of harmonic type. They are determined in the form (Guz 1999)

$$\begin{aligned} s_1 &= \left(\lambda_{(1)} - 1\right) + \left(\lambda_{(2)} - 1\right) + \left(\lambda_{(3)} - 1\right); \\ s_2 &= \left(\lambda_{(1)} - 1\right)^2 + \left(\lambda_{(2)} - 1\right)^2 + \left(\lambda_{(3)} - 1\right)^2; \\ s_3 &= \left(\lambda_{(1)} - 1\right)^3 + \left(\lambda_{(2)} - 1\right)^3 + \left(\lambda_{(3)} - 1\right)^3. \end{aligned} \qquad (18)$$

where $\lambda_{(n)}$-the principal elongations (elongations along the principal directions of the Green's strain tensor); ε_n - the principal values of the Green's strain tensor. On the analogy of nonlinear theory

$$\lambda_{(n)} = \sqrt{1 + 2\varepsilon_n}; \quad \delta_n = \lambda_{(n)} - 1 \qquad (19)$$

In the general case, in the nonlinear theory, $\lambda_{(n)}$ doesn't coincide with λ_n. In the case of the homogeneous state (17) according to the chosen coordinate system, the values λ_n and $\lambda_{(n)}$ (in the particular case of the nonlinear state corresponding to the homogeneous state) coincide. In this connection, the following expressions are used for invariants s_j^0 in the initial state

$$\begin{aligned} s_1^0 &= \left(\sqrt{1+2\varepsilon_1^0} - 1\right) + \left(\sqrt{1+2\varepsilon_2^0} - 1\right) + \left(\sqrt{1+2\varepsilon_3^0} - 1\right) \\ s_2^0 &= \left(\sqrt{1+2\varepsilon_1^0} - 1\right)^2 + \left(\sqrt{1+2\varepsilon_2^0} - 1\right)^2 + \left(\sqrt{1+2\varepsilon_3^0} - 1\right)^2 \\ s_3^0 &= \left(\sqrt{1+2\varepsilon_1^0} - 1\right)^3 + \left(\sqrt{1+2\varepsilon_2^0} - 1\right)^3 + \left(\sqrt{1+2\varepsilon_3^0} - 1\right)^3. \end{aligned} \qquad (20)$$

The Poisson's ratio of the material is determined within the linear theory of elasticity. In the nonlinear theory, especially at high strains as well as in using the harmonic type of the elastic potential, problems can arise while using the Poisson's ratio. This problem has been the subject of discussion for the longest time in geophysics. A satisfactory solution of this problem was obtained in (Kuliev 2000). The remark on this problem is made at the end of the second section of the manuscript.

Three-dimensional nonlinear problem of stability is studied in such formulation in (Kuliev 1988). Unfortunately, unlike the case of "follower" loads, it is not possible to make conclusions on stability and instability in general form irrespective of forms of elastic potentials and body shape in the considered variant. Therefore, the problems are separately studied using different elastic potentials. It is shown that elastic equilibrium state is unstable in the vicinity of the cylindrical cavity in case of defining conservative forces on its surface. The following analytical equations are obtained for critical values of elongation corresponding to the buckling of the equilibrium state with geometric form changes:

in case of harmonic potential

$$(\lambda_1)_* = -\frac{(1+\nu)(1+2\nu)}{2(2+\nu-4\nu^2)}\left\{1-\left[1+\frac{8(2+\nu-4\nu^2)}{(1+2\nu)^2}\right]^{\frac{1}{2}}\right\}. \qquad (21)$$

in case of quadratic potential

$$(\lambda_1)_* = \left(\frac{3}{3-2x}\right)^{\frac{1}{2}}; \quad x = -\frac{3(5-4\nu)}{16(1+\nu)}\left\{1-\left[1-\frac{16(1-2\nu)}{(5-4\nu)^2}\right]^{\frac{1}{2}}\right\}. \qquad (22)$$

The corresponding values of critical strains $(\varepsilon_0)_*$ are determined using Eq. (11) considering Eqs. (21) and (22). The body loses the stability and gets the geometric form through the changing of the existed geometric shape into more stable state of equilibrium in achieving values of strains of critical values $(\varepsilon_0)_*$. The distribution of stress and strains doesn't become overall in a new state. Therefore, the values of change of the medium density determined using Eqs. (1), (3)-(5) will be equal up to critical values of strains $(\varepsilon_0)_*$ in each of its point. The values of change of the medium density in various points of the body will differ after the buckling.

The problems of stability in the vicinity of cylindrical cavities have also been studied for more complex models of deformable media (Wei and Yan 2014).

Numerical results are shown in Table 2 for critical values of shortening and strains at various data of Poisson's ratio calculated on Eqs. (10), (12)-(14), (21) and (22). The results corresponding to harmonic potential are given in numerator and the results corresponding to quadratic potential are given in the denominator. It follows from these results that the buckling of elastic equilibrium state that precedes the "internal" instability occurs in the vicinity of the cylindrical cavity in the considered case. Stable intervals of change of strains and density parameter instead of inequalities (12), (13) and (15), (16) are determined from the following inequalities in such cases

$$(\varepsilon_0)_* < \varepsilon_0 < 0; \quad \varepsilon_0^* < (\varepsilon_0)_*;$$
$$1 \le \frac{\rho}{\rho_0} \le \left(\frac{\rho}{\rho_0}\right)_*; \quad \left(\frac{\rho}{\rho_0}\right)_* \le \left(\frac{\rho}{\rho_0}\right)^*; \quad \left(\frac{\rho}{\rho_0}\right)_* = \left[1+2(\varepsilon_0)_*\right]^{-\frac{3}{2}}; \qquad (23)$$

$$(\varepsilon_0)_* = \frac{1}{2}\left[(\lambda_1)_*^2 - 1\right]. \qquad (24)$$

Thus, changes of equilibrium state are implemented by buckling in case of influence of conservative forces on the cylindrical surface in bodies described by harmonic and quadratic elastic potentials. The body undergone geometric form change in the local vicinity of cavity gets more stable curved geometric form before the beginning of destruction process. In its turn, this form change leads to nonuniform distribution of stress and strains in the vicinity of cavity. This conclusion is not only related with solutions of the considered specific problem but also carries a general character. The fact is that according to three-dimensional theory of stability of the deformed solid bodies, the

Table 2 Critical values ε_0 and $\frac{\rho}{\rho_0}$

ν	0	0.1	0.2	0.3	0.4	0.5
$(\lambda_1)_*$	$\frac{0.78}{0.89}$	$\frac{0.82}{0.91}$	$\frac{0.84}{0.94}$	$\frac{0.89}{0.95}$	$\frac{0.94}{0.98}$	$\frac{1}{1}$
λ_1^*	$\frac{0.50}{0.71}$	$\frac{0.58}{0.76}$	$\frac{0.67}{0.82}$	$\frac{0.77}{0.87}$	$\frac{0.87}{0.94}$	$\frac{1}{1}$
$(\varepsilon_0)_*$	$\frac{-0.1958}{-0.1040}$	$\frac{-0.1638}{-0.0860}$	$\frac{-0.1472}{-0.0582}$	$\frac{-0.1040}{-0.0488}$	$\frac{-0.0582}{-0.0198}$	$\frac{0}{0}$
ε_0^*	$\frac{-0.375}{-0.2480}$	$\frac{-0.3318}{-0.2112}$	$\frac{-0.2756}{-0.1638}$	$\frac{-0.2036}{-0.1216}$	$\frac{-0.1216}{-0.0582}$	$\frac{0}{0}$
$\left(\frac{\rho}{\rho_0}\right)_*$	$\frac{2.1073}{1.4185}$	$\frac{1.8137}{1.3270}$	$\frac{1.6872}{1.2040}$	$\frac{1.4185}{1.1664}$	$\frac{1.2040}{1.0625}$	$\frac{1}{1}$
$\left(\frac{\rho}{\rho_0}\right)^*$	$\frac{8}{2.7940}$	$\frac{5.1253}{2.2780}$	$\frac{3.3249}{1.8137}$	$\frac{2.1904}{1.5186}$	$\frac{1.5186}{1.2040}$	$\frac{1}{1}$

buckling of equilibrium state (local in case of model of the unrestricted bodies; general forms in cases of the restricted bodies) with geometric form changes are necessarily implemented in their vicinity due to increase of deformation values within the presence of lines or conservative forces in the deformed system. In all such cases, initial conditions of homogeneous distribution of strain in the body are violated and distribution of strain becomes non-homogeneous.

Before discussing the numerical results, it should be noted that the obtained results can also be generalized in this section in case of elastoplastic stage of strain. In case of elastoplastic strain, nonclassical linearization creates new opportunities to achieve approximate solutions. Difficult nonlinear problems are solved assuming that these discharge zones may only appear in the initial-strained state and considering the processes of active loading in the disturbed state. Such an approach is designated as a generalized conception of continuous loading in the theory of elastoplastic strain (Guz 1989).

4. Numerical results and discussions

Numerical values of critical values of density corresponding to the "internal" instability and buckling of equilibrium state on geometric form changes are given in Table 2 along with the above mentioned discussion. They show that in the considered case for all values of Poisson's ratio, the "internal" instability precedes the buckling of equilibrium state on geometric form changes, i.e., the beginning of destruction process. Similar processes of local buckling of the equilibrium state occur in the vicinity existed in the medium of inclusions in the form of rods, ribbons, plates, etc. Therefore, the use of inequalities (23) and (24) is more reasonable and correct in practice, especially in studies of the Earth's crust and the upper mantle. When the structural inhomogeneity is included to the medium by the cylindrical rod or band from more rigid media, it is known (Guz 1999, Kuliev 1987) that rectilinear form of rod becomes instable and gets more stable more stable curved shape within overall compression in quite small strains by conservative forces. In this case, the critical force of buckling is two times less than the Eulerian's force corresponding to uniaxial compression along the rod line. This force is many times less than the pressure of partial melting and phase transitions for different substances of the

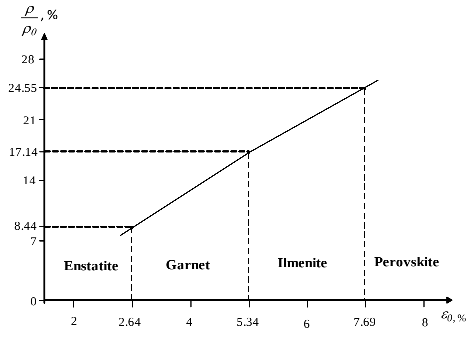

Fig. 1 $\frac{\rho}{\rho_0}$ dependence on ε_0 and sequence of phase transitions of orthopyroxene

Earth. Depending on geometric dimensions of the inclusion, their curvatures may lead to significant local changes (dissimilarity from overall strain) of strains and stresses in a large scale.

Such local strain processes will influence on further change of density and other tectonophysical parameters in large geometric scales and for a geological time. In particular, processes of partial melting and phase transitions will not occur at the same depth levels of the Earth's interior as in overall deformation. In real conditions, these processes will be implemented at different levels of the Earth's interior depending on the nature of strain distribution.

It's seen from the structure of Eqs. (10), (12)-(16), (21), (22) that, it's necessary to know only values of Poisson's ratio to determine critical values of elongation (shortening) and strains. That fact is of great practical importance. There are different geophysical methods that allow determining this important physico-mechanical parameter of the medium within conditions of unreachable deep entrails of the Earth. It is possible to conduct assessments of various theoretical and observation results and to determine their degree of reliability using these data and inequalities suggested in this manuscript.

Hofmeister (1993) analyzed problems of applicability of theoretical results to determine an area of reliability of the Birch-Murnaghan equation of state (B-M EoS) achieved within the Eulerian's statement of finite strain problems. Strains are modeled using equivalent repulsive interatomic potentials. As the criterion of applicability (validity) of results, the condition of conserving the interval of change of isothermal modulus of compression is accepted within the framework of which the potential structure remains stable. Based on experimental results of studies (Webb and Jackson 1990), it is concluded that, B-M EoS theoretical model is unreliable for some solids as orthopyroxene. Let's consider this problem from the point of view of the above obtained inequalities.

It is determined in the known Lin-Gun Liu's (Ringwood 1981, Zharkov 2012) experiments that as a result of overall strain of sample of orthopyroxene (90% $MgSiO_3 \cdot Al_2O_3$), phase transition from enstatite into garnet occurs in a relative decrease of the volume by 7,8%. The decrease in the volume by 8,0% causes a new phase transition from garnet into ilmenite. The subsequent decrease in the volume by 6,9% leads to phase transition from ilmenite into perovskite. Using Eq. (4) and these experimental data, numerical calculations are conducted and their results are reflected in Fig. 1.

Results show that enstatite undergone overall strain of compression in its 2.64%, 5.34% and 7.69% values is sequently changed into garnet, ilmenite and into perovskite at last. An increase in the density parameter of substance by 8.45%, 8.7% and 7.4% corresponds to these values of strains. Comparison of the results with data in Table 2 for ε_0^*, show that sequence of phase transitions of orthopyroxene precedes the destruction in all values of Poisson's ratio as a result of continuous strain (in its modeling by harmonic potential). A similar conclusion is also obtained for a quadratic potential except of variation interval of the values of Poisson's ratio $\nu > 0.38$. Comparison of results on parameter $(\varepsilon_0)_*$ show that the local buckling of the elastic equilibrium state can precede separate phase transitions in the vicinity of inclusions in the form of the cylindrical cavity for a range of changes of Poisson's ratio $\nu > 0.38$ (harmonic potential) and $\nu \geq 0.12$ (quadratic potential). In such case, the obtained results cannot be considered reliable due to violation the condition of overall uniformity of strain process. It is known (Prodaivoda *et al.* 2012), that the value of Poisson's ratio averaged in Voigt-Reuss-Hill's approximation changes within $0{,}19 \leq \nu \leq 0{,}21$ in the interval of temperature $25^0C \leq T \leq 700^0C$ for orthopyroxene. Thus, results obtained within overall compression should be corrected in case of presence the inhomogeneity as inclusion the form of cylindrical cavity in orthopyroxene medium. The given conclusion relates to the case when strain process is modeled by quadratic elastic potential. As a result of local instability, uniform character of distribution of compression strains is disturbed in the medium. Therefore, nature of density distribution and other tectonophysical parameters will differ from analogical nature corresponding to the case of overall uniform strain in case of overall compression. Furthermore, it follows from results in Fig. 1 that the parameters of physico-mechanical properties of orthopyroxene are undergone significant changes in the strained state due to the implemented phase transitions. Numerical values of parameters of physico-mechanical properties of these rocks differ among themselves significantly. This example shows the difficulties in assessing the confidence intervals of state equations on criteria of fundamental moduli of the elasticity quite clearly. Apparently, conclusions on the unreliability of model B-M EoS are related with the mentioned facts here to describe experimental data of orthopyroxene.

The results in Table 1 show that one and the same density changes can occur in a variety of combinations between the principal values of Green's strain tensor differing from overall compression.

Thus, even in homogeneous strains, disturbance of compression uniformity, implementation on various mechanisms of instability of elastic equilibrium state (instability of equilibrium state can also be implemented at stages of elastic-plastic, elastic-viscous and other stages of strains) and phase transitions may have a significant impact on distribution of parameter of the medium density, depth of implementation of phase transitions and other parameters of theoretical models of the Earth. Hence, determination of substance and other parameters of theoretical models can lead to inaccurate conclusions only on the basis of results of experimental researches on separate physico-mechanical characteristics and phase transitions in mineral associations under the conditions of overall deformation.

The conditions put on strains are "strong". Parameters, first and foremost, elastic, theoretical models of the Earth should be determined within the suggested variation intervals of strains. In other approaches based on conditions of satisfaction of restrictions put on separate parameters of state equations (whether the equations are so successful), compliance of their results with the indicated intervals of strains aren't checked and true mechanisms of strain aren't taken into account in the considered problems. Consequently, "weak" restrictions on separate parameters included in various correlations of theoretical models of the Earth's development are formulated within such approaches. It means that the required conditions can be implemented on separate parameters but they will not provide unambiguity of interpretation and uncertainties in results.

5. Conclusions

Reasonable differential criteria of reliability for determining various physico-mechanical parameters of theoretical models of the Earth's development are achieved on the basis of the NLA of nonlinear theory of small and finite strains in the form of homogeneous strain intervals. Thus, the considered problems in theoretical models corresponding to various depths of the Earth along with integral criteria should be solved taking into account these differential criteria on strains. Apparently, these criteria are the most universal, simple and convenient to apply.

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