

Experimental study on crushable coarse granular materials during monotonic simple shear tests

Sihong Liu¹, Hangyu Mao^{*1}, Yishu Wang¹ and Liping Weng²

¹College of Water Conservancy and Hydropower, Hohai University, No.1, Xikang Road, Nanjing 210098, China

²Business School of Hohai University, No.1, Xikang Road, Nanjing 210098, China

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Abstract. To investigate the crushing behaviour of coarse granular materials, a specifically designed, large-scale simple shear apparatus with eight-staged shearing rings was developed. A series of monotonic simple shear tests were conducted on two kinds of coarse granular materials under different vertical stresses and large shear strains. The evolution of the particle breakage during the compression and simple shearing processes was investigated. The results show that the amount of particle breakage is related to the particle hardness and the state of the stresses. The amount of particle breakage is greater for softer granular materials and increases with increasing vertical stresses. Particle breakage may tend towards a critical value during both the compression and the shearing processes. Particle breakage mainly occurs during the processes of confined compression and contraction.

Keywords: simple shear tests; particle breakage; coarse granular materials; compression

1. Introduction

The mechanics of particle breakage are of interest to many research disciplines, including powder technology, geophysics, geomechanics, and minerals and mining engineering. Prior studies mainly aimed at characterizing the evolution of the particle size distribution with increases in stress and strain (Marsal 1967, Hardin 1985, Lade *et al.* 1996, Einav 2007, Kumara and Hayano 2016). Based on laboratory tests, researchers have determined the general factors influencing the breakage of coarse granular materials, such as particle strength, particle size, angularity, particle size distribution, and initial porosity (Hardin 1985, Lade *et al.* 1996, McDowell and Bolton 1998, Nakata *et al.* 2001, Feda 2002, Arslan *et al.* 2009, Wu *et al.* 2013). A larger particle possesses a lower crushing strength because it has a higher probability of having defects and is also subjected to higher contact forces. Angular particles experience more breakage than rounded particles because angular particles concentrate stress. A specimen with a uniform gradation exhibits more crushing than a well-graded specimen with the same maximum particle size, due to an increased number of contacts points between particles that decreases the average contact stress. Breakage is also more severe in a loose packing than in a dense packing.

It is believed that particle breakage contributes to the reduction of the dilatancy of sand in a drained condition and to the increase of pore pressure in the undrained condition, resulting in a lower shear strength (Ueng and Chen 2000, Yu 2017). The volume change or dilatancy resulting from

particle sliding and breakage during shearing can be measured and investigated with shearing tests such as triaxial compression, ring shear, and simple shear tests. However, laboratory test results have shown different findings regarding the influence of particle breakage on macro mechanical behaviour. Coop and Lee (1993) found that the straight part of the critical state line in the plot of the void ratio against the logarithm of the mean effective stress was parallel to the isotropic and one-dimensional compression lines. However, this simple critical state type of framework could be doubted because of the research conducted by Luzzani and Coop (2002). They found that breakage continued through very large strains in the ring shear and shear box tests, with no evidence of a stable gradation being reached within the range of strains used. Coop *et al.* (1993) thought that the critical state concept (Schofield and Wroth 1968) was merely a balance between particle rearrangement and volumetric compression, resulting in a stable volume. Due to the measurement of the critical state line for granular material was problematic, the application of the critical state concept to sands was less successful (Ghafghazi *et al.* 2014). The critical state, as determined by means of conventional triaxial or shear box testing, would only be approximate because of the limited strains (Liu 2010, Xiao *et al.* 2015). With the development of modern laboratory techniques, the measurement problems were apparently resolved. Grain crushing continued to occur until the shear strain was much larger than the range that triaxial tests could achieve. As the particle breakage phenomenon is more obvious in large shear strain conditions, the simple shear test was adopted to examine grain crushing in this study. The aim of this study was to investigate the evolution of grain crushing during simple shear tests. A large-scale simple shear apparatus with

*Corresponding author, Ph.D.
E-mail: hymao@hhu.edu.cn

eight-staged shearing rings was developed. A series of simple shear tests were conducted on two kinds of coarse granular materials with different particle strengths under different vertical stresses and large shear deformations. The changes in the particle breakage during compression and simple shearing were investigated. The existence of the critical state of particle breakage is discussed further.

2. Materials and test setups

2.1 Simple shear apparatus

Fig. 1 shows the large-scale simple shear apparatus used in this research, which was composed of the vertical loading system, the horizontal shearing system, the eight-staged shearing rings, and the data acquisition and processing computer system.

The vertical loading system consisted of a servo motor, a normal force actuator, and a controller. The normal force actuator was fixed on a reaction frame with sufficient strength and stiffness. The normal force produced by the actuator pressed on the sample through the compression bar and was measured by a vertical pressure sensor connected to the compression bar.

The horizontal shearing forces were applied by a horizontal bar connected to the bottom box. The horizontal bar could be driven by the servo motor installed on the reaction frame. A row of scroll wheels was set up between the bottom box and the base. A force sensor was fixed on the horizontal bar to measure the horizontal shear force.

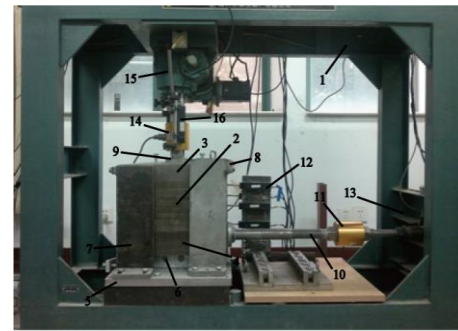
The eight-staged shearing rings were externally square (with side lengths of 36 cm) and internally circular (with diameters of 30 cm). The height of each ring was 2 cm. Several ball bearings were set between each ring, along the shearing direction, to reduce friction. Two connecting plates were placed on the front and back of the apparatus to connect the eight shearing rings. Ball bearings also existed between the connecting plates and the staged rings.

Four horizontal displacement sensors were installed at different heights on the staged shearing rings to measure the horizontal shear displacements of the sample. Two vertical displacement sensors were placed on the top of the compression roof to explore the characteristics of volumetric contraction (i.e., negative dilatancy) and volumetric expansion (i.e., positive dilatancy) of the coarse granular materials.

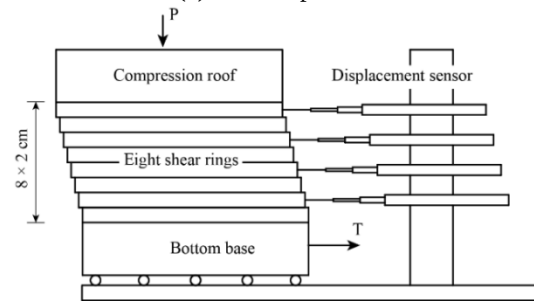
The data acquisition and processing computer system was responsible for the measurements of the force sensor, the displacement sensor, the control of the vertical force and the shearing velocity. All data were recorded and exported into the EXCEL file format.

2.2 Materials

The test materials in this research were two kinds of coarse granular materials with different compressive strengths. The first material (coarse aggregate 1) was silt mudstone, with a particle density of 2.74 g/cm^3 , a saturated compressive strength of 21.1 MPa, and a softening coefficient of 0.45 (the ratio of the ultimate compressive strength of the material under saturated state to the ultimate



(a) Overall picture



(b) Schematic view

Fig. 1 Large-scale simple shear apparatus with eight-staged shearing rings, 1-reaction frame, 2-shear rings, 3-compression roof, 4-bottom box 5-base, 6-contact roller, 7-protection board, 8-connecting plate, 9-cover, 10-horizontal bar, 11-horizontal force sensor, 12-horizontal displacement sensor, 13-horizontal motor, 14-vertical pressure sensor, 15-vertical power-generator and 16-vertical displacement sensor

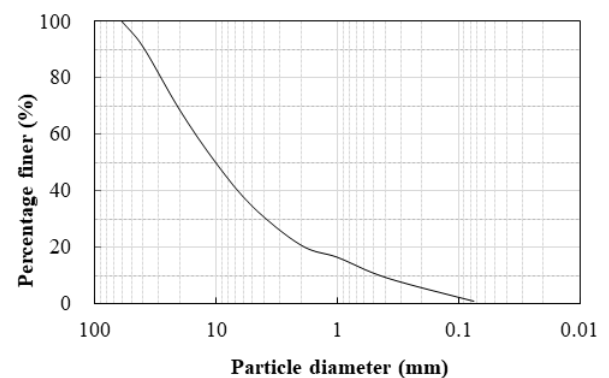


Fig. 2 Initial gradation of the two artificially prepared samples

compressive strength under the wind and dry state). The other material (coarse granular materials 2) was weakly weathered sandstone, with a particle density of 2.61 g/cm^3 , a saturated compressive strength of 45.5 MPa, and a softening coefficient of 0.74. Both materials were classified as soft rocks. They were also delicate and broke easily under loads. Fig. 2 shows the particle size distribution (PSD) of the two materials before testing. The largest grain size was always $d_{max}=60 \text{ mm}$. The average particle diameter of the sample was $d_{50}=10 \text{ mm}$, the uniformity coefficient was $C_u=31$, and the curvature factor was $C_c=2$.

2.3 Experimental procedures

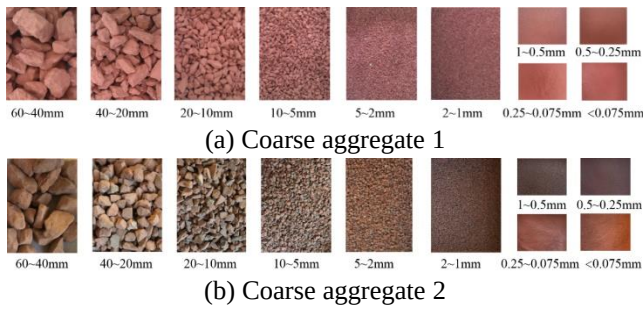


Fig. 3 Ten sieve intervals of the two coarse granular materials

The complete simple shear tests in this study included the confined compression test and the simple shear test. To study the evolution of particle breakage during the compression and shearing processes under different vertical stresses, the tests were conducted on the two coarse granular materials with the sieve interval 0.075–60 mm.

The procedures of the laboratory experiments were as follows:

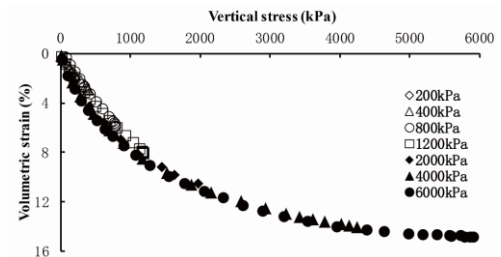
(1) Preparation of the samples: A well-graded sample was created with grains from each of the 10 sieve intervals –60~40 mm, 40~20 mm, 20~10 mm, 10~5 mm, 5~2 mm, 2~1 mm, 1~0.5 mm, 0.5~0.25 mm, 0.25~0.075 mm, and <0.075 mm (Fig. 3). The proportions of each grain size were chosen according to the gradation shown in Fig. 2. Then, the 10 prepared grain materials were mixed well. The initial relative density (RD) of both the coarse aggregate 1 and the coarse aggregate 2 was set to be 0.9, with the dry densities of 1.96 g/cm^3 and 1.92 g/cm^3 , respectively. With the eight shearing rings fixed by vertical scupper plugs, the samples were prepared in six layers, using hand-tamping of each layer to avoid voids, until the surface layer was rolled to smooth.

(2) Tests with grain crushing during compression: Different vertical stresses were applied to the samples prepared in step (1). The final values of the vertical stress were set to be 200 kPa, 400 kPa, 800 kPa, 1200 kPa, 2000 kPa, 4000 kPa, and 6000 kPa. During the loading process, the relationship between the normal stress and axial strain was measured. The specimen sieved after the compression deformation was stable under a given vertical stress. The ultimate PSD for the confined compression test was obtained.

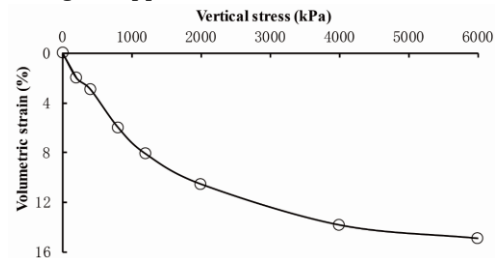
(3) Tests with grain crushing during shearing: Simple shear tests were conducted on the two coarse granular materials under different vertical stresses at 200 kPa, 400 kPa, 800 kPa, and 1200 kPa. Because of the limited horizontal tensioning capacity of the apparatus, only simple shear tests with vertical stresses under 2 MPa were conducted. The new specimen, prepared according to step (1), underwent horizontal shearing as soon as the compression deformation became stable for the given vertical stress. The shearing velocity was set to be 0.2 mm/min. The simple shear tests stopped at shear displacements of 10 mm, 20 mm, 30 mm, and 40 mm to study the evolution of particle breakage. During the shearing process, the volumetric change and shear stress of the specimen were measured. For a given vertical stress, four specimens were needed to complete the shearing processes.

3. Stress-strain behaviour of coarse granular materials

Fig. 4(a) and Fig. 5(a) show the volumetric strains of the coarse aggregate 1 sample and the coarse aggregate 2 sample during the confined compression, respectively. Fig. 4(b) and Fig. 5(b) give the ultimate volumetric strains under different vertical stresses for the two samples. It can be seen that the volume strain for both the two samples gradually increases with the increasing vertical stress when the vertical stress is less than 2 MPa and the rate of the increase in the volume strain becomes small when the vertical stress exceeds 2 MPa.

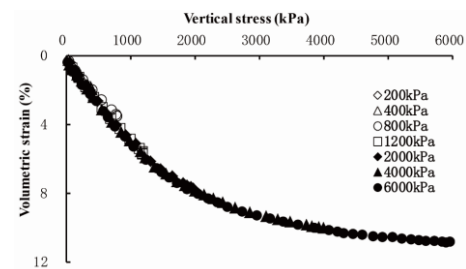


(a) During the application of different vertical stresses

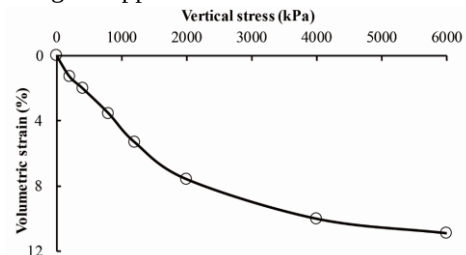


(b) Ultimate volumetric strains

Fig. 4 Volumetric strains of the coarse aggregate 1 sample during confined compression at different vertical stresses

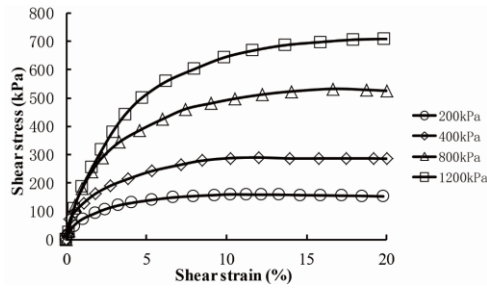


(a) During the application of different vertical stresses

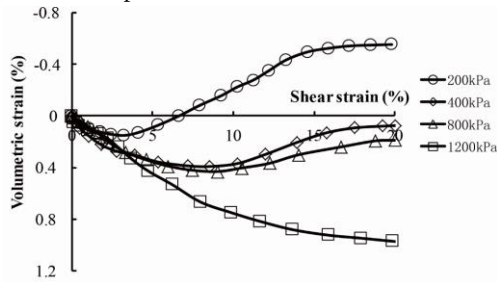


(b) Ultimate volumetric strains

Fig. 5 Volumetric strains of the coarse aggregate 2 sample during confined compression at different vertical stresses

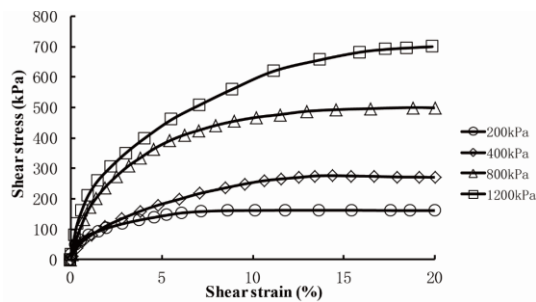


(a) Relationship between shear stress and shear strain

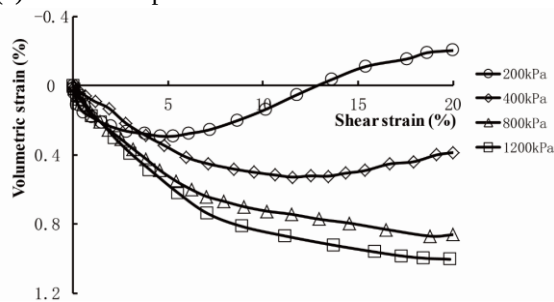


(b) Relationship between volume strain and shear strain

Fig. 6 Stress-strain relationships of the coarse aggregate 1 sample during simple shearing, under different vertical stresses



(a) Relationship between shear stress and shear strain



(b) Relationship between volume strain and shear strain

Fig. 7 Stress-strain relationships of the coarse aggregate 2 sample during simple shearing, under different vertical stresses

Fig. 6 and Fig. 7 present the relationship between stress and strain during the shearing process under different vertical stresses. Fig. 6(a) and Fig. 7(a) show that there was a rapid growth in the shear stress during the initial stage of the shearing process. The shear stress finally tended towards a stable value with increasing shear strain. The maximum shear stress increased with the increase of vertical stress. Fig. 6(b) and Fig. 7(b) show that when the vertical stress was small, the volume change first

corresponded to contractive behaviour and then to dilatant behaviour. The volumetric strain finally tended towards a stable value when the shear strain was large enough. When the vertical stress was large enough, the volume change observed during the shearing process was characterized as contractive behaviour. The volume strain also tended towards a stable volume.

4. Quantification of particle breakage

Several investigations have used a single number or measure to represent the amount of particle breakage that has taken place during loading (Marsal 1967, Hardin 1985, Lee and Farhoomand 1967, Lade *et al.* 1996, Einav 2007, Huang *et al.* 2017). In particular, Hardin (1985) suggested the use of a relative breakage property based on the relative position of the current cumulative distribution to the initial cumulative distribution, with an arbitrary cut-off value of 'silt' particle size (0.074 mm). In this study, the amount of particle breakage was quantified by means of Hardin's relative breakage, Br , the definition of which is given in Fig. 8.

4.1 Particle breakage during the compression process

Table 1 gives the mass fraction within each grain size interval and the corresponding relative breakage, Br , under different vertical stresses. From Table 1, we can see that the proportion of the grain size above 20 mm decreased while the proportion of the grain size below 20 mm increased because of particle breakage during the compression process. The variation grew larger with increasing vertical stress. Fig. 9 shows the evolution of the relative breakage, Br , with vertical stress during confining compression. We can reach the following conclusions: (1) Particle breakage is related to the compressive strengths of the coarse granular materials. The Hardin relative breakage, Br , of the silt mudstone with the lower compressive strength (coarse aggregate 1) is greater than that of the weakly weathered sandstone with the higher compressive strength (coarse aggregate 2). The similar results have also been found by Hardin *et al.* (1985) and Lade *et al.* (1996). (2) When the vertical stress is less than 2 MPa, the relative breakage, Br , increases with increasing vertical stress. However, when the vertical stress is greater than 2 MPa, the rate of the increase in the relative breakage, Br , decreases along with the vertical stress increases. The value of Br tends to converge to a constant value, meaning that the grain crushing would not continue without limit, even with increasing vertical stress. Coop *et al.* (1993) and Wood D M *et al.* (2008) have confirmed the existence of a critical particle breakage. Particle breakage will tend towards a critical value when the vertical stress increases adequately. This tendency occurs because, with an increasing number of small particles, the large particles surrounded by small particles reduce the grain deviatoric state and create a hydrostatic effect. The evolution of Br with vertical stresses shown in Fig. 9 is consistent with the evolution of the volumetric strain shown in Fig. 4(b) and Fig. 5(b).

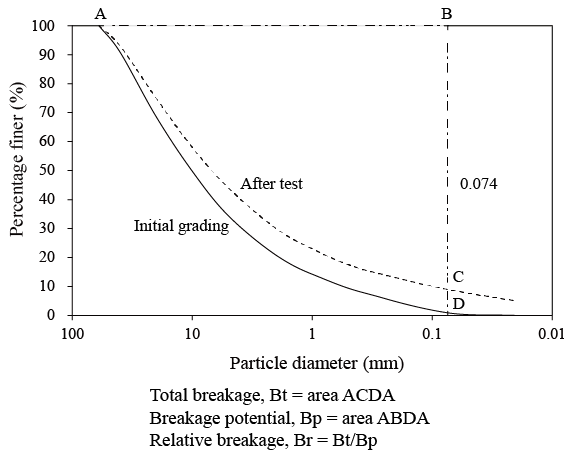
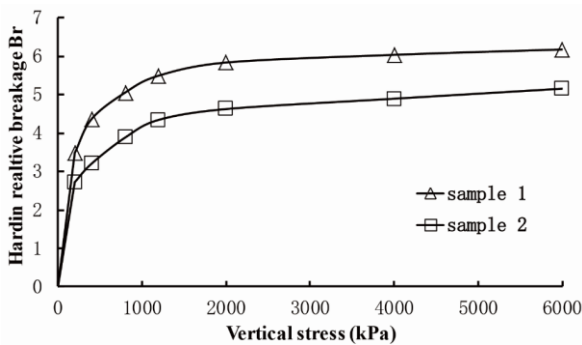
Fig. 8 Definition of Hardin's relative breakage, Br

Table 1 Particle breakage during compression process

Sample	Status of tests	Vertical stress (kPa)	Mass fraction within each grain size interval (mm)/%							Br/%
			60~40	40~20	20~10	10~5	5~2	<2		
Coarse aggregate 1	initial		9.11	22.26	18.83	15.38	13.61	20.81		
	ultimate	200	8.66	21.99	18.72	15.61	13.61	21.41	3.48	
		400	8.22	21.83	19.17	15.50	13.44	21.84	4.36	
		800	8.06	21.01	19.73	15.73	13.67	21.80	5.05	
		1200	7.96	21.05	19.30	15.96	13.60	22.13	5.49	
		2000	7.14	20.42	19.42	16.41	13.84	22.77	5.83	
		4000	6.93	20.25	19.43	16.57	14.02	22.80	6.03	
		6000	6.82	20.21	19.35	16.53	14.13	22.96	6.17	
Coarse aggregate 2	initial		9.11	22.26	18.83	15.38	13.61	20.81		
	ultimate	200	8.94	22.02	18.88	15.44	13.60	21.12	2.72	
		400	8.75	21.96	19.11	15.32	13.48	21.38	3.20	
		800	8.68	21.88	19.13	15.24	13.53	21.54	3.88	
		1200	8.50	21.52	19.23	15.28	13.67	21.80	4.34	
		2000	7.61	20.68	19.30	16.19	13.72	22.50	4.62	
		4000	7.50	20.61	19.33	16.24	13.73	22.59	4.89	
		6000	7.42	20.53	19.34	16.27	13.73	22.71	5.15	

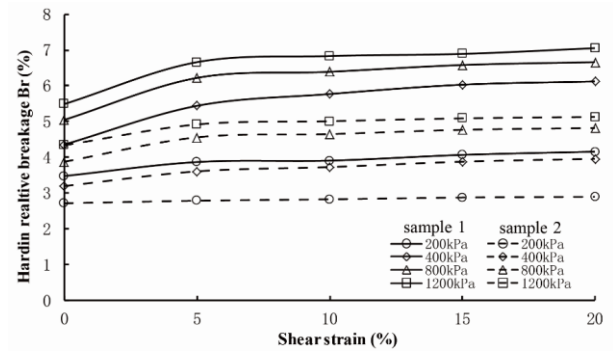
Fig. 9 Evolution of Br during confined compression

4.2 Particle breakage during the shearing process

Simple shear tests were conducted on the two coarse granular materials under different vertical stresses at 200 kPa, 400 kPa, 800 kPa, and 1200 kPa. The specimen was

Table 2 Particle breakage of the two coarse granular materials under different shear strains

Sample	Vertical stresses/kPa	Relative breakage Br under different shear strains/%				
		0	5	10	15	20
Coarse aggregate 1	200	3.48	3.87	3.91	4.07	4.16
	400	4.36	5.44	5.77	6.03	6.12
	800	5.05	6.22	6.39	6.58	6.66
	1200	5.49	6.66	6.83	6.89	7.05
Coarse aggregate 2	200	2.72	2.79	2.83	2.88	2.90
	400	3.20	3.61	3.73	3.88	3.96
	800	3.88	4.56	4.65	4.77	4.82
	1200	4.34	4.92	5.01	5.09	5.13

Fig. 10 Relationship between relative breakage, Br , and shearing strain

sieved at shear displacements of 10 mm, 20 mm, 30 mm, and 40 mm, under each of the vertical stresses. The mass fraction of each particle size group was measured and the relative breakage, Br , under the different shear strains was calculated. The results for the relative breakage, Br , are listed in Table 2. Because the shearing processes were started after the compression was stable, the relative breakage, Br , listed in Table 2 includes the value obtained during the compression processes. Fig. 10 shows the relationship between the relative breakage, Br , and the shearing displacement, under different vertical stresses. From Fig. 10, we can observe that particle breakage mainly occurred during the initial stage of the shearing process and tended towards a stable value when the shear strain was large enough. That is, particle breakage may tend towards a critical value during the shearing process.

Particle breakage occurred mainly in the contraction process during simple shearing, corresponding to the relationship between volume strain and shear strain shown in Fig. 6(b). Under the vertical stress of 200 kPa, the coarse aggregate 1 changed from contraction to dilatancy when the shear strain reached 5% with the increase of the relative breakage Br from 3.48% to 3.87% (the increment $\Delta B_r = 0.39\%$). At the shear strain of 5%, the sample remained contractive under the other vertical stresses (400 kPa to 1200 kPa) with the increment of the relative breakage ΔB_r ranging from 1.08% to 1.17%. When the shear strain exceeded 5%, the increase in the relative breakage, ΔB_r , under a vertical stress of 200 kPa was notably smaller than that under the other vertical stresses. The relative breakage, Br , of the silt mudstone with the lower compressive strength (coarse aggregate 1) was

greater than that of the weakly weathered sandstone with the higher compressive strength (coarse aggregate 2).

5. Analysis of particle breakage

5.1 Analysis of particle breakage during the compression and shearing processes

Fig. 9 shows the evolution of the relative breakage, Br , during the compression process. Table 3 shows the proportion of the crushing due to compression during the entire test process. The following two points can be concluded from Table 3.

(1) The amount of particle breakage was much greater during the compression process than that during the shearing process for both coarse granular materials. This is because particles with large fractures and low local strength are more easily broken during the compression process (see Fig. 11(b)). When the sample starts shearing, the larger particles that are easily broken have already been crushed into smaller particles during the compression process. These smaller particles tear, rub, scratch, and even bounce against each other (Schofield and Wroth 1968) during contraction, which may result only in the rough edges being ground to round. Fig. 11 illustrates the microscopic model of particle breakage in the simple shear tests. When the sample begins to dilate (Fig. 11(d)), the interparticle forces decrease, which results in less possibility of crushing. Therefore, the amount of particle breakage is relatively small during shearing compared to the compression process.

(2) Greater value of the proportion shown in Table 3 corresponds to less significant particle breakage during shearing. The proportion of the crushing due to compression under the vertical stress of 200 kPa was much greater than that under the other vertical stresses, because an obvious dilatancy behaviour occurred under the vertical stress of 200 kPa. As the vertical stress increased, the dilatancy behaviour became weak, and the change in the volume strain decreased. These observations further support the fact that particle breakage mainly occurs during the compression and contraction process, because the coarse granular materials become more compact and the contact forces between the particles increase.

5.2 Particle breakage during the contraction and dilatancy processes

The results of the simple shear tests under different vertical stresses indicate that particle breakage mainly occurs during the volume contraction stage. Monkul (2013) point out that particle breakage results in more fine particles and fine particles increase the volumetric contraction of specimens. The contraction of volume leads to an increase in relative density. Yoshimoto *et al.* (2016) found that a rise in relative density intensifies the dilation behaviour. To verify this phenomenon, a detailed analysis of the particle breakage was conducted. Several samples with similar initial properties were prepared and were sieved when sheared at different shearing strains. By deducing the component of the relative breakage during the confined compression, the increase of the relative breakage, ΔBr , of the tested two coarse aggregates during the simple shearing under different vertical stresses was calculated. Fig. 12 and

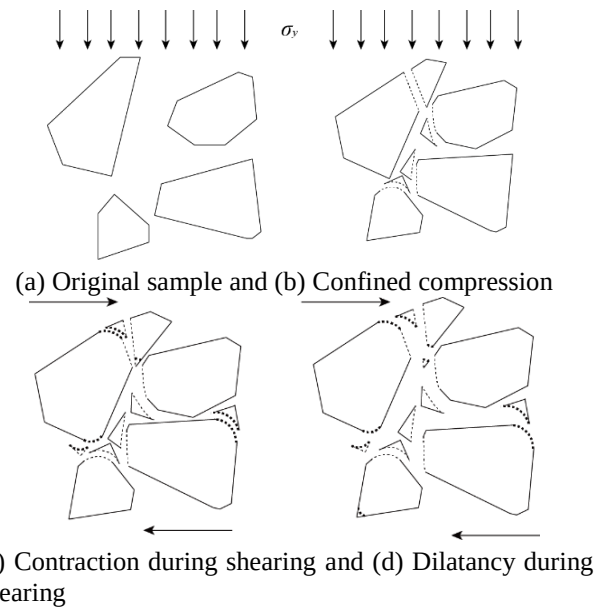


Fig. 11 The microscopic model of particle breakage in the simple shear tests

Table 3 Proportion of the crushing due to compression

Sample	Relative breakage Br (%)	Vertical stress (kPa)			
		200	400	800	1200
Coarse aggregate 1	whole test process	4.16	6.12	6.66	8.13
	compression process	3.48	4.36	5.05	5.49
	proportion	83.67	71.27	75.83	67.55
Coarse aggregate 2	whole test process	2.90	3.96	4.82	5.13
	compression process	2.72	3.20	3.88	4.34
	proportion	93.79	80.81	80.50	84.60

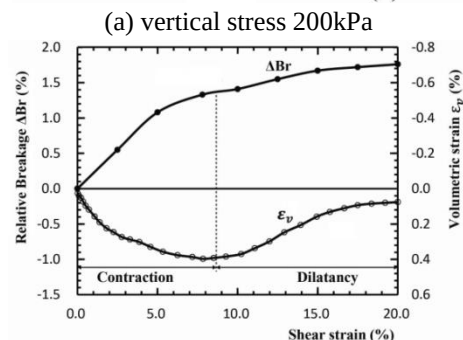
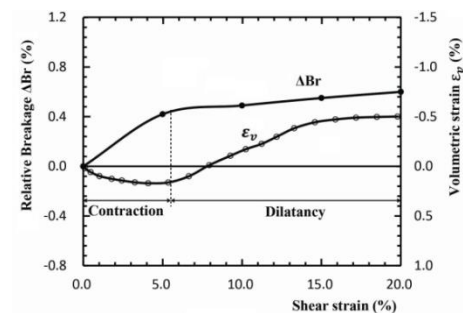
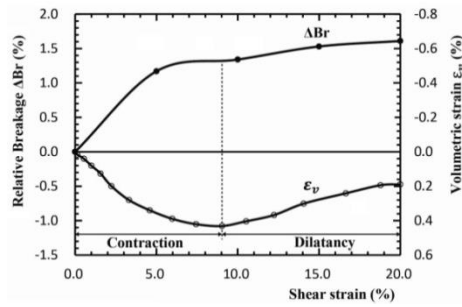
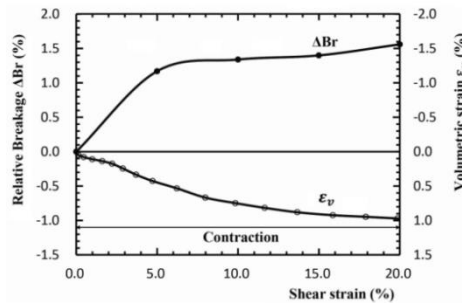


Fig. 12 Increase of the relative breakage of the coarse aggregate 1 accompanying with the volumetric strain during simple shearing

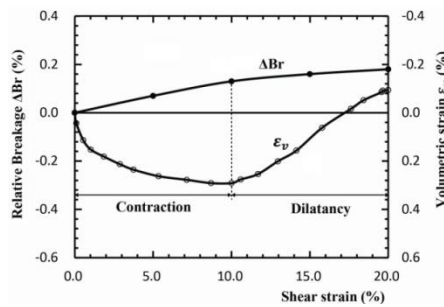


(c) vertical stress 800kPa

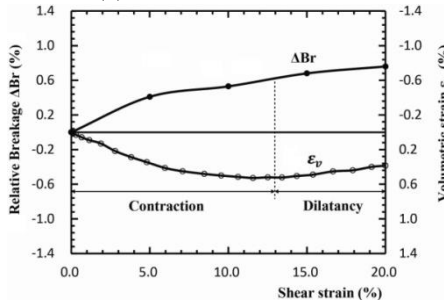


(d) vertical stress 1200kPa

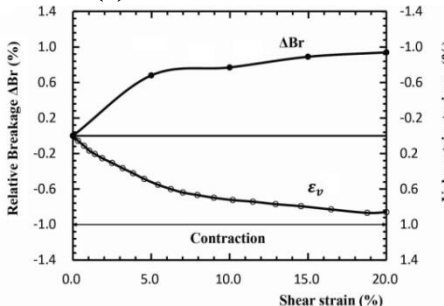
Fig. 12 Continued



(a) vertical stress 200kPa

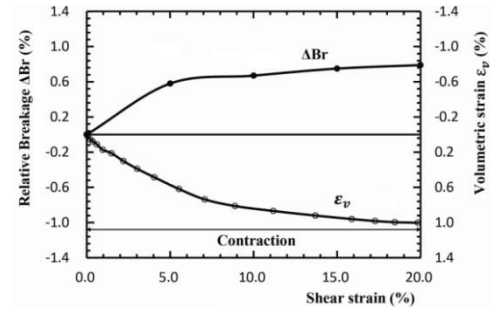


(b) vertical stress 400kPa



(c) vertical stress 800kPa

Fig. 13 Increase of the relative breakage of the coarse aggregate 2 accompanying with the volumetric strain during simple shearing



(d) vertical stress 1200kPa

Fig. 13 Continued

Fig. 13 gives the evolution of the increase of the relative breakage ΔBr and the volumetric strains of the two coarse granular materials against the shear strain, respectively. We can observe that the relative breakage was much greater during the contraction process. As the sample changed from contraction to dilatancy, the increase of particle breakage became small and tends to converge to a constant value.

6 Conclusions

Based on a series of simple shear tests, an investigation of the particle breakage of two different coarse granular materials with different compressive strengths was conducted under the conditions of different vertical stresses and large shear deformations. From this study, the following conclusions can be drawn:

1) Particle breakage of the coarse granular materials was related to the compressive strengths. The Hardin relative breakage, Br , of the silt mudstone (with the lower compressive strength) was greater than that of the weakly weathered sandstone (with the higher compressive strength).

2) During the compression process, there was an obvious increase in grain crushing during the initial application of a small vertical stress value, but after a certain compression stage, grain crushing began to reach a plateau value for a fixed vertical stress. During the shearing process, the relative breakage increased with increasing vertical stress for the same shear strain. There was also a larger amount of grain crushing when the shear strain was small. The grain crushing tended towards a stable value when the shear strain was large enough. That is, the particle breakage may tend towards a critical value during either the compression or the shearing process.

3) Particle breakage occurred mainly during the processes of confined compression and the shear contraction, because with the increase in the packing density and the contact forces between particles, the particles were more easily crushed.

As we know, particle breakage is closely related to many factors. In this study, we mainly examined the influences of compressive strength, vertical stress, and shear strain on particle breakage. Since the simple shear test is inevitably carried out under a vertical stress, the test consists of confined compression and simple shearing processes. The stress states in both processes are the

combination of hydrostatic (compressive) stress and deviatoric (shear) stress. It is difficult to distinguish the effects of the compressive stress and shear stress on particle breakage. In this study, we made an assumption that the compressive stress is dominant during the confined compression and the shear stress is dominant during the simple shearing, as adopted in several studies (e.g., Hagerty *et al.* 1993, Coop *et al.* 2004, Ciantia *et al.* 2016). Thus, the existence and mechanism of the critical value of relative breakage, Br , concluded from the experimental results needs further research.

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