Geomechanics and Engineering, *Vol. 15, No. 1 (2018) 711-719*
DOI: https://doi.org/10.12989/gae.2018.15.1.711

Free vibration and buckling analysis of orthotropic plates using a new two variable refined plate theory

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(Received June 14, 2017, Revised November 19, 2017, Accepted November 27, 2017)

Abstract. The present work presents a free vibration and buckling analysis of orthotropic plates by proposing a novel two variable refined plate theory. Contrary to the conventional higher order shear deformation theories (HSDT) and the first shear deformation theory (FSDT), the proposed theory utilizes a novel displacement field which incorporates undetermined integral terms and involves only two unknowns. The governing equations are obtained from the dynamic version of principle of virtual works. The analytical solution of a simply supported orthotropic plate has been determined by using the Navier method. Numerical investigations are performed by employing the proposed model and the obtained results are compared with the existing HSDTs.

Keywords: two variable refined plate theory; vibration, buckling; orthotropic plate

1. Introduction

Classical plate theory (Kirchhoff, 1850ab) overestimates natural frequencies and buckling loads and underestimates the vertical displacement. This is due to not considering the influence of transverse shear and transverse normal stresses. The errors in natural frequencies and buckling loads are quite considerable for plate fabricated from composite materials.

First order shear deformation theory (FSDT) is developed to improve the classical plate theory (CPT). Reissner (1944, 1945) was the first to propose consistent stress-based plate model, which introduces the influence of shear deformation; whereas Mindlin (1951) developed kinematic based first order shear deformation theory. In these models, the transverse shear strain variation is considered to be constant within the plate thickness and therefore, shear correction factor is needed to account for the strain energy because of the shear deformation (Meksi 2015, Bellifa 2016, Bouderba 2016). In general, these shear correction coefficients are problem dependent.

The limitations of CPT and FSDT stimulated the development of higher order shear deformation theories (HSDTs), to introduce influence of cross sectional warping and to provide the realistic distribution of the transverse shear strains and stresses across the thickness of plate. For HSDTs, primarily two types of formulations are employed. In one formulation, the stresses are considered as primary variables. In the other formulation, displacements are considered as primary variables. Reddy (1984) has proposed well known HSDT for the investigation of laminated plates assuming polynomial function in-terms of thickness co-ordinate to introduce influence of transverse shear deformation. Many review articles are available on displacement based plate models (Ghugal and Shimpi 2002, Chen and Zhen 2008, Kreja 2011, Eltaher *et al.* 2012, Sobhy 2013, Tounsi *et al.* 2013, Han *et al.* 2015, Hadji *et al.* 2015, Barati and Shahverdi 2016, Rahmani *et al.* 2017, Aldousari 2017). Recently many novel refined theories (Shimpi 2002, Shimpi *et al.* 2007, Ghugal and Sayyad 2010, Bouderba *et al.* 2013, Zidi *et al.* 2014, Belkorissat *et al.* 2015, Attia *et al.* 2015, Zemri *et al.* 2015, Mahi *et al.* 2015, Taibi *et al.* 2015, Bounouara *et al.* 2016, Beldjelili *et al.* 2016, Becheri *et al.* 2016, Javed *et al.* 2016, Ahouel *et al.* 2016, Boukhari *et al.* 2016, Bousahla *et al.* 2016, Bellifa *et al.* 2017a, Mouffoki *et al.* 2017, Klouche *et al.* 2017, Zidi *et al.* 2017, Amar *et al.* 2017) are developed for the investigation of isotropic and orthotropic plates and beams. Ghugal and Sayyad (2011) have presented trigonometric shear deformation theory including influence of transverse shear and transverse normal strain/stress for the dynamic analysis of thick orthotropic plates. Shimpi and Patel (2006) have proposed a two variable refined plate theory for orthotropic plate analysis. Ghugal and Pawar (2011) have studied static flexure behavior of isotropic and orthotropic plates by hyperbolic shear deformation theory. Karama *et al.* (2003) has employed exponential function to predict the mechanical response multilayered laminated composite

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http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

beams. Meziane *et al.* (2014) considered an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Yahia *et al.* (2015) studied the wave propagation in FG plates with porosities applying various HSDTs of four unknowns. Kar and Panda (2015) studied the free vibration responses of temperature dependent FG curved panels under thermal environment by using a refined HSDT. Akavci (2014, 2016) presented new HSDTs to study the mechanical behavior of FG plates. Ahmed *et al.* (2014) examined the post-buckling response of sandwich beams with functionally graded faces using a consistent higher order theory. Kar *et al.* (2015) studied the nonlinear flexural analysis of laminated composite flat panel under hygro-thermo-mechanical loading. Draiche *et al.* (2016) developed a refined plate theory with stretching effect for the flexure analysis of laminated composite plates. Saidi *et al.* (2016) presented a simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations. Bennoun *et al.* (2016) proposed a novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates. Baseri *et al.* (2016) presented an analytical solution for buckling of embedded laminated plates based on HSDT. Chikh *et al.* (2017) proposed a simplified HSDT to investigate the thermal buckling response of cross-ply laminated plates. Bourada *et al.* (2016) presented a buckling analysis of isotropic and orthotropic plates using a novel four variable refined plate theory. Recently, Karami *et al.* (2017) studied effects of triaxial magnetic field on the anisotropic nanoplates.

In the present work, an efficient HSDT is presented for buckling and free vibration analysis of orthotropic plate analysis. The displacement model contains undetermined integral terms in addition to classical plate theory terms. The numbers of unknown variables are lower as that of FSDT. Governing equations are found from the dynamic version of principle of virtual works. The Navier type solution is used for solving the governing equations of simply supported orthotropic plates. The critical buckling loads and natural frequencies of orthotropic plates for various modular and aspect ratios are investigated and discussed in detail.

2. Orthotropic plate under consideration

In this study, a rectangular plate of length a, width b, and a constant thickness h is considered for examination. The plate is subjected to in-plane compressive loads (N^0_x, N^0_y and N^0_{xy}). The structure occupies (in O–x–y–z right-handed Cartesian coordinate system) a region

$$0 \le x \le a\ ; \quad 0 \le y \le b\ ; \quad -h/2 \le y \le h/2 \tag{1}$$

2.1 Kinematics

In this study, some simplifying suppositions are employed to the existing HSDT so that the number of variables is reduced. The displacement field of the existing HSDT is given by

$$u(x,y,z,t) = -z\frac{\partial w_0}{\partial x} + f(z)\phi_x(x,y,t) \tag{2a}$$

$$v(x,y,z,t) = -z\frac{\partial w_0}{\partial y} + f(z)\phi_y(x,y,t) \tag{2b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{2c}$$

where w_0, φ_x and φ_y are five generalized displacements, $f(z)$ is the shape function representing the variation of the transverse shear strains and stresses within the thickness. By adopting that $\varphi_x = \int \theta(x,y)dx$ and $\varphi_y = \int \theta(x,y)dy$, the kinematic of the proposed theory can be expressed in a simpler form as (Hebali *et al.* 2016, Meksi *et al.* 2017, Besseghier *et al.* 2017, El-Haina *et al.* 2017, Fahsi *et al.* 2017, Meftah *et al.* 2017, Menasria *et al.* 2017, Khetir *et al.* 2017, Bellifa *et al.* 2017b, Sekkal *et al.* 2017)

$$u(x,y,z,t) = -z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y,t)\,dx \tag{3a}$$

$$v(x,y,z,t) = -z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y,t)\,dy \tag{3b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{3c}$$

where $w_0(x,y)$ and $\theta(x,y)$ are the two unknown displacement functions of middle surface of the plate. The constants k_1 and k_2 depends on the geometry. The integrals utilized are undetermined.

In this work, the present HSDT is obtained by putting

$$f(z) = z\left(\frac{5}{4} - \frac{5z^2}{3h^2}\right) \tag{4}$$

The strains associated with the kinematic in Eq. (3) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{5}$$

where

$$\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\int \theta\,dx + k_2\dfrac{\partial}{\partial x}\int \theta\,dy \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_1\int \theta\,dy \\ k_2\int \theta\,dx \end{Bmatrix}, \tag{6a}$$

and

$$g(z) = \frac{df(z)}{dz} \tag{6b}$$

The integrals used in the above expressions shall be resolved by a Navier solution and can be expressed by

$$\frac{\partial}{\partial y}\int \theta\,dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \quad \frac{\partial}{\partial x}\int \theta\,dy = B'\frac{\partial^2\theta}{\partial x\partial y}, \quad \int \theta\,dx = A'\frac{\partial\theta}{\partial x}, \quad \int \theta\,dy = B'\frac{\partial\theta}{\partial y} \tag{7}$$

where the parameters A' and B' are defined according to the type of solution employed, in this case via Navier. Hence,

A' and B' are expressed by

$$A' = -\frac{1}{\alpha^2},\quad B' = -\frac{1}{\beta^2}, k_1 = \alpha^2, k_2 = \beta^2 \tag{8}$$

where α and β are defined in Eq. (23).

2.2 Constitutive equations

The constitutive relations for orthotropic materials can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{9}$$

where Q_{ij} are the plane stress-reduced stiffnesses, and are expressed as

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}};\ Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}};\ Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}};\ Q_{66} = G_{12};\ Q_{44} = G_{23};\ Q_{55} = G_{13} \tag{10}$$

2.3 Equation of motions

Hamilton's principle is used to deduce the equations of motion

$$0 = \int_0^t (\delta U + \delta W - \delta K)\, dt \tag{11}$$

where δU is the variation of strain energy; δW is the virtual potential energy due to constant in-plane compressive and shear forces ($N^0{}_x$, $N^0{}_y$ and $N^0{}_{xy}$) and δK is the variation of kinetic energy.

The variation of strain energy of the plate is given by

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz} \right] dV \\ &= \int_A \left[M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \right. \\ &\quad \left. + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta \gamma_{yz}^s + S_{xz}^s \delta \gamma_{xz}^0 \right] dA = 0 \end{aligned} \tag{12}$$

where A is the top surface and the stress resultants M, and S are given by

$$\left(M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (z, f)\sigma_i dz,\ (i = x, y, xy) \text{ and } \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g(\tau_{xz}, \tau_{yz}) dz \tag{13}$$

Substituting Eqs. (5) and (9) into Eq. (13) and integrating through the thickness of the plate, the stress resultants are related to the generalized displacements (w_0 and θ) by the relations

$$\begin{Bmatrix} M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 \\ D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66} & 0 & 0 & D_{66}^s \\ D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 & H_1^s & H_{22}^s & 0 \\ 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s \end{bmatrix} \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \\ k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{14a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{14b}$$

and stiffness components are given by

$$(D_{ij}, D_{ij}^s, H_{ij}^s) = \int_{-h/2}^{h/2} Q_{ij} \left(z^2, z\, f(z), f(z)^2 \right) dz,\ (i, j = 1, 2, 6), \tag{15a}$$

$$A_{ij}^s = \int_{-h/2}^{h/2} Q_{ij} \left(g(z) \right)^2 dz,\ (i, j = 4, 5) \tag{15b}$$

The work done by applied forces can be expressed as

$$\delta V = -\int_A \left(N_x^0 \frac{\partial w_0}{\partial x} \frac{\partial \delta w_0}{\partial x} + 2N_{xy}^0 \frac{\partial w_0}{\partial x} \frac{\partial \delta w_0}{\partial y} + N_y^0 \frac{\partial w_0}{\partial y} \frac{\partial \delta w_0}{\partial y} \right) dA \tag{16}$$

The variation of kinetic energy of the plate can be computed by

$$\begin{aligned} \delta K &= \int_V [\dot{u} \delta \dot{u} + \dot{v} \delta \dot{v} + \dot{w} \delta \dot{w}]\, \rho(z) dV \\ &= \int_A \left\{ I_0[\dot{w}_0 \delta \dot{w}_0] + I_2 \left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta \dot{w}_0}{\partial y} \right) + K_2 \left((k_1 A')^2 \left(\frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta \dot{\theta}}{\partial x} \right) + (k_2 B')^2 \left(\frac{\partial \dot{\theta}}{\partial y} \frac{\partial \delta \dot{\theta}}{\partial y} \right) \right) \right. \\ &\quad \left. - J_2 \left((k_1 A') \left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta \dot{w}_0}{\partial x} \right) + (k_2 B') \left(\frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta \dot{\theta}}{\partial y} + \frac{\partial \dot{\theta}}{\partial y} \frac{\partial \delta \dot{w}_0}{\partial y} \right) \right) \right\} dA \end{aligned} \tag{17}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; ρ is the mass density of the material; and (I_i, J_i, K_i) are mass inertias calculated by

$$(I_0, I_2) = \int_{-h/2}^{h/2} \left(1, z^2\right) \rho(z) dz \text{ and } (J_2, K_2) = \int_{-h/2}^{h/2} \left(z\, f, f^2\right) \rho(z) dz \tag{18}$$

Substituting the relations for δU, δV, and δK from Eqs. (12), (16) and (17) into Eq. (11) and integrating by parts, and collecting the coefficients of δw_0, and $\delta\theta$, the following equations of motion for the plate are deduced as follows

$$\begin{aligned} \delta w_0 :\ & \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N_x^0 \frac{\partial^2 w_0}{\partial x^2} + 2 N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + N_y^0 \frac{\partial^2 w_0}{\partial y^2} = \\ & I_0 \ddot{w}_0 - I_2 \nabla^2 \ddot{w}_0 + J_2 \left(k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) \end{aligned} \tag{19a}$$

$$\begin{aligned} \delta \theta :\ & -k_1 M_x^s - k_2 M_y^s - (k_1 A + k_2 B) \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = \\ & -K_2 \left((k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + (k_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) + J_2 \left(k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right) \end{aligned} \tag{19b}$$

Substituting Eq. (14) into Eq. (19), the governing equations of the plate in term of generalized displacements are as follows

$$\begin{aligned} & -D_{11} d_{1111} w_0 - 2(D_{12} + 2D_{66}) d_{1122} w_0 - D_{22} d_{2222} w_0 + \left(D_{11}^s k_1 + D_{12}^s k_2\right) d_{11}\theta \\ & + 2\left(D_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{1122}\theta + \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22}\theta + N_x^0 d_{11} w_0 + 2 N_{xy}^0 d_{12} w_0 + N_y^0 d_{22} w_0 = \\ & I_0 \ddot{w}_0 - I_2 \left(d_{11} \ddot{w}_0 + d_{22} \ddot{w}_0\right) + J_2 \left(k_1 A' d_{11} \ddot{\theta} + k_2 B' d_{22} \ddot{\theta}\right) \end{aligned} \tag{20a}$$

$$\begin{aligned} & + \left(D_{11}^s k_1 + D_{12}^s k_2\right) d_{11} w_0 + 2 \left(D_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{1122} w_0 + \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22} w_0 \\ & - H_{11}^s k_1^2 \theta - H_{22}^s k_2^2 \theta - 2 H_{12}^s k_1 k_2 \theta - \left(\left(k_1 A' + k_2 B'\right)^2 H_{66}^s\right) d_{1122}\theta + A_{44}^s \left(k_2 B'\right)^2 d_{22}\theta + A_{55}^s \left(k_1 A'\right)^2 d_{11}\theta = \\ & J_2 \left(k_1 A' d_{11} \ddot{w}_0 + k_2 B' d_{22} \ddot{w}_0\right) - K_2 \left(\left(k_1 A'\right)^2 d_{11} \ddot{\theta} + \left(k_2 B'\right)^2 d_{22} \ddot{\theta}\right) \end{aligned} \tag{20b}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j},\quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},\quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m},\quad d_i = \frac{\partial}{\partial x_i},\ (i, j, l, m = 1, 2). \tag{21}$$

3. Analytical solutions for orthotropic plates

The Navier method is considered to present the analytical solutions of the partial differential equations in Eq. (20) for simply supported plates.

Based on the Navier procedure, the following solutions of displacements are employed to automatically respect the simply supported boundary conditions of plate

$$\begin{Bmatrix} w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} W_{mn} \sin(\alpha\, x)\sin(\beta\, y)\, e^{i\omega\, t} \\ X_{mn} \sin(\alpha\, x)\sin(\beta\, y)\, e^{i\omega\, t} \end{Bmatrix} \tag{22}$$

where W_{mn} and X_{mn} are coefficients; ω is the natural frequency of the system; and α and β are expressed as

$$\alpha = m\pi / a \ \text{ and } \ \beta = n\pi / b \tag{23}$$

The governing equations of plate in the case of static buckling are obtained by setting $N_x^0 = -\gamma_1 N_{cr}$, $N_y^0 = -\gamma_2 N_{cr}$, $N_{xy}^0 = 0$ in Eq. (20).

Substituting Eq. (22) into Eq. (20), the Navier solution of orthotropic plates can be deduced from equations

$$\left(\begin{bmatrix} S_{33}+k & S_{34} \\ S_{34} & S_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{33} & m_{34} \\ m_{34} & m_{44} \end{bmatrix} \right) \begin{Bmatrix} W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{24}$$

where

$$\begin{aligned} &S_{33} = -\left(D_{11}\alpha^4 + 2(D_{12}+2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right), \\ &S_{34} = -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2(k_1 A' + k_2 B')\, D_{66}^s\alpha^2\beta^2 - k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right), \\ &S_{44} = -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - (k_1 A' + k_2 B')^2 H_{66}^s\alpha^2\beta^2 - k_2\left(H_{12}^s k_1 + H_{22}^s k_2\right) - (k_1 A')^2 A_{55}^s\alpha^2 - (k_2 B')^2 A_{44}^s\beta^2 \\ &k = N_{cr}\left(\gamma_1\,\alpha^2 + \gamma_2\beta^2\right) \\ &m_{33} = -I_0 - I_2(\alpha^2+\beta^2) \quad m_{34} = J_2\left(k_1 A'\alpha^2 + k_2 B'\beta^2\right), m_{44} = -K_2\left(\left(k_1 A'\right)^2\alpha^2 + \left(k_2 B'\right)^2\beta^2\right) \end{aligned} \tag{25}$$

4. Numerical results and discussion

4.1 Free vibration analysis

The considered orthotropic plate has following materials properties

$$E_1/E_2 = 0.52500,\ G_{12}/E_2 = 0.26293,\ G_{13}/E_2 = 0.15991,\ G_{23}/E_2 = 0.26681,\ \nu_{12} = 0.44046,\ \nu_{21} = 0.23124 \tag{26}$$

The flexure mode and shear mode frequencies of plate are presented in the non-dimensional form

$$\bar{\omega}_{mn} = \omega_{mn} h \sqrt{\frac{\rho}{Q_{11}}} \tag{27}$$

In the present work, dynamic investigation of a simply supported orthotropic plate is considered. Natural predominantly bending mode ($\bar{\omega}_w$) and thickness shear mode ($\bar{\omega}_\theta$) frequencies of square plate are determined for thickness ratio 10. Tables 1 to 3 give non-dimensional frequencies of simply supported square plate and the results are compared with exact solution of Srinivas *et al.* (1970), HSDT of Reddy (1984), trigonometric shear deformation theory (TSDT) of Ghugal and Sayyad (2011), exponential shear deformation theory (ESDT) of Ghugal and Sayyad (2014), refined late theory (RPT) of Shimpi and Patel (2006), first shear deformation theory (FSDT) of Mindlin (1951) and CPT.

Table 1 Comparison of non-dimensional natural predominantly bending mode frequencies $\bar{\omega}_w$ of simply-supported orthotropic square plate (a/h=10)

(m,n)	Present	Exact	ESDT	HSDT	TSDT	RPT	FSDT	CPT
(1,1)	0.0477	0.0474	0.0474	0.0474	0.0474	0.0477	0.0474	0.0497
(1,2)	0.1040	0.1033	0.1033	0.1033	0.1031	0.1040	0.1032	0.1120
(1,3)	0.1898	0.1888	0.1888	0.1888	0.1793	0.1898	0.1884	0.2154
(1,4)	0.2980	0.2969	0.2969	0.2969	0.2932	0.2980	0.2959	0.3599
(2,1)	0.1198	0.1188	0.1190	0.1189	0.1196	0.1198	0.1187	0.1354
(2,2)	0.1722	0.1694	0.1697	0.1695	0.1696	0.1722	0.1692	0.1987
(2,3)	0.2520	0.2475	0.2480	0.2477	0.2478	0.2520	0.2459	0.3029
(2,4)	0.3534	0.3476	0.3482	0.3479	0.3468	0.3534	0.3463	0.4480
(3,1)	0.2197	0.2180	0.2191	0.2184	0.2199	0.2197	0.2178	0.2779
(3,2)	0.2675	0.2624	0.2637	0.2629	0.2671	0.2675	0.2619	0.3418
(3,3)	0.3407	0.3320	0.3337	0.3326	0.3326	0.3407	0.3310	0.4470
(4,1)	0.3344	0.3319	0.3351	0.3330	0.3346	0.3344	0.3311	0.4773
(4,2)	0.3774	0.3707	0.3743	0.3720	0.3727	0.3774	0.3696	0.5415

Table 2 Comparison of non-dimensional natural predominantly thickness shear mode frequencies $\bar{\omega}_\theta$ of simply-supported orthotropic square plate (a/h=10)

(m,n)	Present	Exact	ESDT	HSDT	TSDT	FSDT
(1,1)	1.5137	1.6530	1.6448	1.6550	1.6530	1.6647
(1,2)	1.6791	1.7160	1.7105	1.7209	1.7145	1.7307
(1,3)	1.8073	1.8115	1.8052	1.8210	1.8044	1.8307
(1,4)	1.9418	1.9306	1.9249	1.9466	1.9121	1.9562
(2,1)	1.5155	1.6805	1.6728	1.6827	1.6817	1.6922
(2,2)	1.6784	1.7509	1.7462	1.7562	1.7513	1.7657
(2,3)	1.8310	1.8523	1.8418	1.8622	1.8458	1.8717
(2,4)	1.9795	1.9749	1.9701	1.9912	1.9524	2.0004
(3,1)	1.6478	1.7334	1.7274	1.7361	1.7373	1.7452
(3,2)	1.7691	1.8195	1.8068	1.8255	1.8255	1.8343
(3,3)	1.9115	1.9289	1.9203	1.9395	1.9301	1.9418
(4,1)	1.8414	1.8458	1.8437	1.8583	1.7163	1.7267
(4,2)	1.9306	1.9447	1.9351	1.9514	1.9568	1.9588

The predominantly bending mode frequencies of orthotropic square plate are presented in Table 1 for various modes of vibration. It is seen that the proposed model provides excellent results for all modes of vibration. From Table 2 it is seen that, thickness shear modes frequencies of orthotropic square plate determined by the proposed model are in good agreement with other results.

4.2 Buckling analysis

In this section, buckling response of an orthotropic square and rectangular plate is examined. Three types of in-plane loading conditions are considered in this study: (1) uniaxial compression along the x-axis; (2) uniaxial compression along y-axis; and (3) biaxial compression. For the comparison studies, numerical results are also compared

Table 3 Comparison of non-dimensional buckling load factors (N_{cr}) for simply-supported orthotropic square plate under uniaxial compression (γ_1=-1, γ_2=0, m=n=1)

a/h	Model	Non-dimensional critical buckling load factor (N_{cr})				
		Modular ratio E_1/E_2				
		3	10	20	30	40
5	Present	3.9587	6.3478	8.3967	9.6821	10.578
	ESDT	3.9650	6.3014	8.0946	9.2166	10.049
	HSDT	3.9434	6.2072	7.8292	8.7422	9.3472
	TSDT	4.0572	6.3212	7.9324	8.8418	9.4502
	FSDT	3.9386	6.1804	7.7450	8.5848	9.1084
	CPT	5.4248	11.163	19.383	27.606	35.830
10	Present	4.9637	9.3732	14.563	18.772	22.258
	ESDT	4.9612	9.2998	14.080	17.748	20.676
	HSDT	4.9568	9.2772	14.001	17.577	20.386
	TSDT	5.0128	9.3646	14.116	17.711	20.534
	FSDT	4.9562	9.2734	13.982	17.532	20.304
	CPT	5.4248	11.163	19.383	27.606	35.830
20	Present	5.3016	10.653	17.898	24.690	31.069
	ESDT	5.3004	10.625	17.681	24.146	30.094
	HSDT	5.2994	10.621	17.664	24.108	30.025
	TSDT	5.3194	10.653	17.714	24.175	30.107
	FSDT	5.2994	10.620	17.662	24.102	30.014
	CPT	5.4248	11.163	19.383	27.606	35.830
50	Present	5.4047	11.078	19.129	27.094	34.972
	ESDT	5.4044	11.072	19.087	26.982	34.758
	HSDT	5.4040	11.072	19.085	26.976	34.748
	TSDT	5.4116	11.081	19.098	26.993	34.769
	FSDT	5.4046	11.072	10.085	26.976	34.748
	CPT	5.4248	11.163	19.383	27.606	35.830
100	Present	5.4197	11.141	19.319	27.477	35.612
	ESDT	5.4196	11.400	19.308	27.447	35.554
	HSDT	5.4192	11.139	19.307	27.466	35.553
	TSDT	5.4250	11.145	19.314	27.453	35.562
	FSDT	5.4206	11.142	19.309	27.448	35.554
	CPT	5.4248	11.163	19.383	27.606	35.830

Table 4 Comparison of non-dimensional buckling load factors (N_{cr}) for simply-supported orthotropic square plate under biaxial compression (γ_1=-1, γ_2=-1, m=n=1)

a/h	Model	Non-dimensional critical buckling load (N_{cr})				
		Modular ratio E_1/E_2				
		3	10	20	30	40
5	Present	1.9793	3.1739	4.1984	4.8411	5.2892
	ESDT	1.9825	3.1507	4.0473	4.6083	5.0246
	HSDT	1.9717	3.1036	3.9146	4.3711	4.6736
	TSDT	2.0281	3.1606	3.9662	4.4209	4.7251
	FSDT	1.9693	3.0902	3.8725	4.2924	4.5542
	CPT	2.7124	5.5814	9.6917	13.8034	17.9154

Table 4 Continued

a/h	Model	Non-dimensional critical buckling load factor (N_{cr})				
		Modular ratio E_1/E_2				
		3	10	20	30	40
10	Present	2.4818	4.6866	7.2816	9.3862	11.1291
	ESDT	2.4806	4.6499	7.0402	8.8741	10.3380
	HSDT	2.4784	4.6386	7.0002	8.7885	10.1929
	TSDT	2.5064	4.6823	7.0582	8.8558	10.2674
	FSDT	2.4781	4.6367	6.9910	8.7662	10.1522
	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
20	Present	2.6508	5.3267	8.9490	12.3448	15.5343
	ESDT	2.6502	5.3124	8.8405	12.0731	15.0470
	HSDT	2.6497	5.3101	8.8320	12.0540	15.0127
	TSDT	2.6597	5.3266	8.8574	12.0875	15.0537
	FSDT	2.6497	5.3100	8.8311	12.0513	15.0070
	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
50	Present	2.7023	5.5390	9.5646	13.5470	17.4859
	ESDT	2.7022	5.5364	9.5437	13.4911	17.3791
	HSDT	2.7020	5.5360	9.5424	13.4884	17.3744
	TSDT	2.7058	5.5407	9.5490	13.4969	17.3849
	FSDT	2.7023	5.5362	9.5425	13.4885	17.3745
	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
100	Present	2.7099	5.5707	9.6596	13.7384	17.8060
	ESDT	2.7098	5.5700	9.6542	13.7238	17.7779
	HSDT	2.7096	5.5697	9.6533	13.7230	17.7767
	TSDT	2.7124	5.5727	9.6571	13.7269	17.7811
	FSDT	2.7103	5.5710	9.6544	13.7241	17.7772
	CPT	2.7124	5.5814	9.6917	13.8034	17.9154

to those reported by HSDT of Reddy (1984), trigonometric shear deformation theory (TSDT) of Ghugal and Sayyad (2011), exponential shear deformation theory (ESDT) of Ghugal and Sayyad (2014), first shear deformation theory (FSDT) of Mindlin (1951) and CPT.

Table 5 Comparison of non-dimensional buckling load factors (N_{cr}) for simply-supported orthotropic rectangular plate under uniaxial compression along x-axis (a/h=5, γ_1=-1, γ_2=0, m=n=1)

E_1/E_2	Model	Non-dimensional critical buckling load factor (N_{cr})						
		Aspect ratio b/a						
		1.0	1.5	2	2.5	3.0	3.5	4.0
10	Present	6.3478	5.3284	5.0109	4.8706	4.7961	4.7518	4.7232
	ESDT	6.3014	5.3026	5.0148	4.8939	4.8317	4.7953	4.7723
	HSDT	6.2072	5.2245	4.9412	4.8223	4.7611	4.7253	4.7026
	TSDT	6.3212	5.2923	4.9940	4.8682	4.8033	4.7654	4.7412
	FSDT	6.1804	5.2025	4.9205	4.8021	4.7412	4.7056	4.6831
	CPT	11.163	9.3549	8.8428	8.6270	8.5154	8.4500	8.4083

Table 5 Continued

E_1/E_2	Model	Non-dimensional critical buckling load factor (N_{cr})						
		Aspect ratio b/a						
		1.0	1.5	2	2.5	3.0	3.5	4.0
25	Present	9.1039	7.9408	7.5409	7.3561	7.2558	7.1952	7.1559
	ESDT	8.7062	7.8373	7.6007	7.5047	7.4562	7.4281	7.4109
	HSDT	8.3394	7.4929	7.2631	7.1701	7.1231	7.0961	7.0792
	TSDT	8.4398	7.5414	7.2929	7.1909	7.1391	7.1091	7.0905
	FSDT	8.2199	7.3805	7.1530	7.0610	7.0154	6.9883	6.9713
	CPT	23.495	21.690	21.179	20.964	20.854	20.783	20.744
40	Present	10.579	9.2342	8.7587	8.5368	8.4158	8.3426	8.2950
	ESDT	10.049	9.2310	9.0145	8.9282	8.8853	8.8608	8.8454
	HSDT	9.3472	8.5541	8.3455	8.2628	8.2217	8.1983	8.1837
	TSDT	9.4502	8.6015	8.3719	8.2791	8.2324	8.2056	8.1888
	FSDT	9.1084	8.3237	8.1178	8.0363	7.9958	7.9728	7.9585
	CPT	35.830	34.027	33.516	33.300	33.189	33.124	33.082

Table 6 Comparison of non-dimensional buckling load factors (N_{cr}) for simply-supported orthotropic rectangular plate under uniaxial compression along y-axis (a/h=5, γ_1=0, γ_2=-1, m=n=1)

E_1/E_2	Model	Non-dimensional critical buckling load factor (N_{cr})						
		Aspect ratio b/a						
		1.0	1.5	2	2.5	3.0	3.5	4.0
10	Present	6.3478	11.989	20.044	30.441	43.165	58.210	75.572
	ESDT	6.3014	11.930	20.059	30.587	43.485	58.743	76.356
	HSDT	6.2072	11.755	19.765	30.139	42.849	57.885	75.242
	TSDT	6.3212	11.907	19.975	30.426	43.229	58.375	75.859
	FSDT	6.1804	11.705	19.682	30.013	42.670	57.644	74.929
	CPT	11.163	21.048	35.371	53.918	76.638	103.51	134.53
25	Present	9.1039	17.867	30.164	45.976	65.302	88.141	114.494
	ESDT	8.7062	17.634	30.403	46.904	67.107	90.999	118.57
	HSDT	8.3394	16.859	29.052	44.813	64.110	86.931	113.27
	TSDT	8.4398	16.968	29.171	44.943	64.253	87.089	113.44
	FSDT	8.2199	16.606	28.611	44.131	63.132	85.604	111.54
	CPT	23.495	48.803	84.716	131.02	187.66	254.63	331.92
40	Present	10.578	20.777	35.035	53.355	75.742	102.20	132.72
	ESDT	10.049	20.769	36.058	55.801	79.968	108.55	141.42
	HSDT	9.3472	19.246	33.382	51.642	73.995	100.42	130.93
	TSDT	9.4502	19.353	33.487	51.744	74.092	100.52	131.02
	FSDT	9.1084	18.728	32.471	50.226	71.962	97.667	137.33
	CPT	35.830	76.560	134.06	208.12	298.69	405.76	529.31

Tables 3 and 4 present the comparison of non-dimensional critical buckling load for square plate under uniaxial and biaxial compression with the variation of modular and thickness ratios. It can be observed that the proposed model with only two unknowns provides good results for all thickness and modular ratios. The difference between the proposed model and HSDT of Reddy (1984) will slightly increases as the modular ratios increase. CPT overestimates the values of critical buckling load for all thickness ratios and modular ratios.

Table 7 Comparison of non-dimensional buckling load factors (N_{cr}) for simply-supported orthotropic rectangular plate under biaxial compression (a/h=5, γ_1=-1, γ_2=-1, m=n=1)

E_1/E_2	Model	Non-dimensional critical buckling load factor (N_{cr})						
		Aspect ratio b/a						
		1.0	1.5	2	2.5	3.0	3.5	4.0
10	Present	3.1739	3.6889	4.0087	4.1988	4.3165	4.3932	4.4454
	ESDT	3.1507	3.6710	4.0118	4.2189	4.3485	4.4334	4.4915
	HSDT	3.1036	3.6170	3.9530	4.1571	4.2849	4.3687	4.4260
	TSDT	3.1606	3.6639	3.9952	4.1967	4.3230	4.4057	4.4623
	FSDT	3.0902	3.6017	3.9364	4.1398	4.2671	4.3505	4.4076
	CPT	5.5814	6.4765	7.0743	7.4371	7.6638	7.8122	7.9137
25	Present	4.5519	5.4974	6.0327	3.3415	6.5302	6.6522	6.7349
	ESDT	4.3531	5.4258	6.0806	6.4696	6.7107	6.8678	6.9750
	HSDT	4.1697	5.1874	5.8105	6.1811	6.4110	6.5609	6.6631
	TSDT	4.2199	5.2210	5.8343	6.1991	6.4253	6.5728	6.6734
	FSDT	4.1099	5.1096	5.7224	6.0870	6.3132	6.4607	6.5613
	CPT	11.757	15.016	16.943	18.072	18.767	19.217	19.524
40	Present	5.2892	6.3929	7.0069	7.3593	7.5742	7.7130	7.8070
	ESDT	5.0246	6.3907	7.2116	7.6967	7.9968	8.1920	8.3251
	HSDT	4.6736	5.9221	6.6764	7.1231	7.3995	5.5796	7.7023
	TSDT	7.7251	5.9549	6.6875	7.1372	7.4092	7.5863	7.7071
	FSDT	4.5542	5.7626	6.4942	6.9278	7.1963	7.3711	7.4903
	CPT	17.915	23.557	26.813	28.707	29.870	30.623	31.136

The comparison of non-dimensional critical buckling load for rectangular plate is presented in Tables 5 to 7. From these Tables, it can be seen that the non-dimensional critical buckling load diminishes with increase in aspect ratios (b/a) when the plate is under uniaxial compression along x-axis whereas increases when the plate is under uniaxial compression along y-axis and biaxial stability.

5. Conclusions

A novel two variable refined plate theory is proposed in this work for stability and dynamic analysis of orthotropic plates. The theory considers the transverse shear effect and parabolic variation of the transverse shear strain across the thickness of the plate. From the obtained results it can be concluded that, the frequencies computed by the proposed theory are accurate as observed from the comparison with exact results specially in the case of natural bending mode. Also, it is confirmed from this study that the present theory can accurately predict the critical buckling loads of the orthotropic plates.

An improvement of present study will be considered in the future work to take into account the thickness stretching effect by using quasi-3D shear deformation models (Bessaim *et al.* 2013, Bousahla *et al.* 2014, Belabed *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Meradjah *et al.* 2015, Chaht *et al.* 2015, Hamidi *et al.* 2015, Bourada *et al.* 2015, Bennoun *et al.* 2016, Draiche *et al.* 2016, Benbakhti *et al.* 2016, Benahmed *et al.* 2017, Atmane *et al.* 2017, Bouafia *et al.* 2017, Benchohra *et al.* 2018, Abualnour *et al.* 2018).

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