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Theoretical explanation of rock splitting based on the micromechanical method

Houxu Huang*[1], Jie Li[1], Yiqing Hao[2] and Xin Dong[1]

[1] *State Key Laboratory of Disaster Prevention and Mitigation of Explosion and Impact,PLA University of Science and Technology, Nanjing, China*
[2] *High-Tech Institute, Fan Gong-ting South Street on the 12th, Qingzhou, Shandong, China*

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Abstract. In this paper, in order to explain the splitting of cylindrical rock specimen under uniaxial loading, cracks in cylindrical rock specimen are divided into two kinds, the longitudinal crack and the slanting crack. Mechanical behavior of the rock is described by elastic-brittle-plastic model and splitting is assumed to suddenly occur when the uniaxial compressive strength is reached. Expression of the stresses induced by the longitudinal crack in direction perpendicular to the major axis of the crack is deduced by using the Maxwell model. Results show that the induced stress is tensile and can be greater than the tensile strength even before the uniaxial compressive strength is reached. By using the Inglis's formula and simplifying the cracks as slender ellipse, the above conclusions that drawn by using the Maxwell model are confirmed. Compared to shearing fracture, energy consumption of splitting seems to be less, and splitting is most likely to occur when the uniaxial loading is great and quick. Besides, explaining the rock core disking occurred under the fast axial unloading by using the Maxwell model may be helpful for understanding that rock core disking is fundamentally a tensile failure phenomenon.

Keywords: rock splitting; uniaxial loading; fast axial unloading; Maxwell model; Griffith's criterion

1. Introduction

As shown in Fig. 1, when subjected to uniaxial loading, the brittle rock specimen (red sandstone) will quickly split into two major parts and the newly generated surfaces are generally parallel to the axis of the specimen. Besides, the newly generated surfaces are found to be rough and no shear loci are observed, which mean that the splitting should be caused by the tensile stress. However, during the uniaxial loading process, there is no tension applied in the direction that perpendicular to the axis of the specimen.

A lot of effort has been done to find explanations for rock splitting. Some researchers attribute it to the wedging of rock fragments of broken specimens. It is certain that the wedging can cause rock splitting, however, not all splitting are induced by wedging. In fact, splitting failure is different from wedging failure to some degree, because wedging failure is mainly the result of external effect, while rock splitting is a failure phenomenon which owing to internal effect caused by tensile stress. They can be respectively referred to as intrusion fractures and internal fractures (Brace 1964). Besides, some researchers attribute it to the specimen's lateral expansion, i.e., they think that the Poisson's effect is the cause of splitting (Shemyakin *et al.* 2000). However, as we know that Poisson's phenomenon is a deformation characteristic that occurs in almost all materials, but splitting appears only in heterogeneous material, such as brittle rock.

Bauch and Lempp (2004) have successfully simulated splitting in laboratory by using the brittle rock specimen, their study showed that with abrupt unloading mode, splitting occurs even the strain is far less than 0.1%, while with slow unloading mode, splitting won't occur even the strain is far greater than 1.5%. Therefore, the strength theory of maximum deformation is unsuitable for splitting judgment of brittle rock. What's more, the most widely used failure criteria such as the Hoek-Brown criterion and the Mohr-Coulomb criterion have also been proved to be unsuitable for explanation of the rock splitting (Bauch and Lempp 2004, Lim and Martin 2010).

Based on the above statements, in order to reveal the mechanism of rock splitting, we have to pay attentions to the internal structure of the rock and analyze the internal stress state of the rock with considering its internal structure. In fact, the rocks are typical heterogeneous material and contain numerous cracks in different sizes; the internal structure of the rocks has decisive influence on its mechanical behaviour (Qi *et al*. 2014, 2016). In-situ investigations as well as experimental and theoretical studies have all shown that the fracture mechanism of rocks is governed by the laws of Maxwell model (Landau *et al*. 2011).

In this paper, based on that the rocks are quasi-brittle heterogeneous materials and contain numerous cracks, in the deformation and fracture processes, the plastic deformation is insignificant, the deformation and fracture processes of rocks are mainly governed by elastic deformations and cracking (Qi *et al*. 2014, 2016). Therefore, we use the Maxwell model to derive the tensile

∗Corresponding author, Ph.D.
E-mail: wuhanhp14315@163.com

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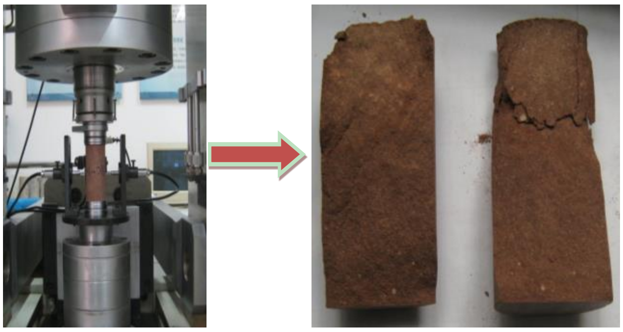


Fig.1 Rock splitting under uniaxial loading

stress that caused by the cracks; besides, the expression of the maximum tensile stress at crack tips is also derived based on simplifying the crack as small slender ellipse, the crack propagation mode which is not included in Griffith's criterion is revealed; Our work may be helpful for the understanding of rock splitting which occurs under uniaxial loading. Besides, the attempts on explaining the rock core disking which occurs under fast axial unloading may be also helpful for understanding the phenomena that: 1. when core disking occurs the axial strain is usually tiny, and 2. the radial stress seems to have no effect on rock core disking.

2. Derivation of the induced tensile stress based on Maxwell model

The rock is heterogeneous and contains large numbers of cracks with different scales. In this paper, we introduce the concept of additional stress, which refers to the stress component that caused by the cracks and exists around the cracks. Unlike the elastic stress is related to the reversible deformation linearly, the additional stress belongs to inelastic stress. During the deformation process of the specimen, there are additional stress's concentration and relaxation processes in the rock, therefore, the value of the additional stress is simultaneously determined by the two processes and can be expressed by the following Maxwell model (Landau *et al*. 2011)

$$\frac{\mathrm{d}\Delta s_{ij}^{l}}{\mathrm{d}t} = 2\rho c_s^2 \dot{e}_{ij} - \upsilon \frac{\Delta s_{ij}^{l}}{l} \tag{1}$$

where Δs_{ij}^{l} is the additional stress corresponding to the cracks in size l; $\dot{e}_{ij}$ is the deviatory strain rate component corresponding to the given loading condition; ρ is the density of the rock; υ is the relaxation velocity of single or multiple cracks depending on the loading condition; c_s is the propagation velocity of the elastic shear wave; l/υ is understood as the relaxation time needed for the relaxation of cracks in size l; Besides, in order to simplify our analyses, all the additional stresses are assumed to relax with the same relaxation time. The first term on the right-hand side of Eq.(1) describes the elastic loading, while the second term on the right-hand side depicts the relaxation of the additional stress due to the crack propagation process.

For any given constant strain rate, the corresponding additional stress Δs_{ij}^{l} can be expressed as

$$\Delta s_{ij}^{l} = 2\rho c_s^2 \dot{e}_{ij} \frac{l}{\upsilon}\left(1 - \mathrm{e}^{\upsilon t/l}\right) \tag{2}$$

When splitting occurs, there will be macroscopic fracture in the rock specimen, and for the occurrence of macroscopic fracture, it is necessary that the loading time t is greater than the relaxation time τ, i.e., $t>\tau$ (Qi *et al*. 2014). Therefore, Eq. (2) gives

$$\Delta s_{ij}^{l} \approx 2\rho c_s^2 \dot{e}_{ij}\tau = 2\rho c_s^2 \dot{e}_{ij}\frac{l}{\upsilon} \tag{3}$$

Define the intensity of additional stress as $\Delta\sigma_{ij} = \sqrt{3\Delta s_{ij}^{l}\Delta s_{ij}^{l}/2}$, and by substituting Eq. (3) into it, we obtain

$$\Delta\sigma_{ij} \approx 3\rho c_s^2 \dot{\varepsilon}_{ij}\frac{l}{\upsilon} \tag{4}$$

where $\dot{\varepsilon}_{ij} = \sqrt{2\dot{e}_{ij}\dot{e}_{ij}/3}$ is the intensity of the strain-rate.

Fig. 2 shows the splitting of cylindrical rock specimen under uniaxial loading. The solid red line *OA* represents the elastic deformation process of the specimen, during this process, it is assumed that the induced tensile stress increases gradually while the shape of the cracks remains unchanged; and at point *A*, the uniaxial loading reaches the compressive strength σ_c and splitting suddenly occurs. The stress concentration and energy accumulation processes during the elastic deformation stage *OA* are regarded as the basis for the suddenly splitting of brittle rock at point *A*. Based on Eq. (4), the intensity of the additional stress perpendicular to the axis of the specimen is

$$\Delta\sigma_3 \approx 3\rho c_s^2 \dot{\varepsilon}_3 \frac{D}{\upsilon} \tag{5}$$

where D is the scale of the specimen. Let's take the granite as an example, according to Qi. *et al.* (2014), the relationship between scale D and the relaxation velocity $\upsilon(D)$ for granite is given as

$$\upsilon(D) = 112.59D^2 - 1403D + 6056.6 \tag{6}$$

Substituting D=5 cminto Eq. (5), we obtain $\upsilon(D)$=1856 m/s, and the loading duration $\tau \sim 10^{-5}$ s, which is quite short.

As aforementioned, the rock is quasi-brittle material, plastic deformations in rock is not significant in deformation and fracture process. As shown in Fig.2, we adopt the elastic-brittle-plastic model to describe the mechanical behavior of the brittle rock. It is assumed that the rock undergoes elastic deformation before σ_1 reaching the uniaxial compressive strength σ_c. Besides, it is also assumed that the applied uniaxial loading makes the following deformation process possible, that is, the rock undergoes elastic deformation during the period $0 < t < \tau$, and splitting suddenly occurs at time $t = \tau$. In this paper, we adopt the convention that the signs for compression and compressive strain are positive, while negative for tension and tensile strain. Therefore, with considering the relationship $E = 2(1+v)\rho c_s^2$, $\varepsilon_3 = -v\varepsilon_1 = -v\sigma_1/E$, Eq. (6) can be rewritten as

$$\Delta\sigma_3 \approx 3\rho c_s^2 \dot{\varepsilon}_3 \frac{D}{\upsilon} = 3\rho c_s^2 \varepsilon_3 = -\frac{3}{2}\frac{\nu}{1+\nu}\sigma_1 \tag{7}$$

Eq. (7) shows that the crack will cause tensile stress in direction perpendicular to the major axis of the crack. Ignoring the variation of the specimen volume, i.e., $\nu = 0.5$, as shown in Fig. 2, from point *O* to *A*, the induced tensile stress $\Delta\sigma_3$ and its extremum are respectively

$$\Delta\sigma_3 = -\frac{1}{2}\sigma_1 \tag{8}$$

$$\Delta\sigma_{3\max} = -\frac{1}{2}\sigma_c \tag{9}$$

considering that $\sigma_c \approx 8\sigma_t$ (Griffith 1924), where σ_c and σ_t are respectively the compressive strength and tensile strength, then

$$\Delta\sigma_{3\max} \approx 4\sigma_t \tag{10}$$

According to Eq. (10), at the end of the elastic state (corresponding to point *A* in Fig. 2), the induced tensile stress will be almost four times of the tensile strength, which is large enough to cause tensile failure in direction perpendicular to the axis of the specimen while without confining pressure or with low confining pressure. Besides, the assumptions that during the elastic deformation process, although the induced tensile stress and the energy consumed by the specimen increase gradually, the shape of the crack kept unchanged till splitting suddenly occurs in the condition that the compressive strength is reached, are helpful for understanding the phenomenon that splitting fracture is quite quickly and suddenly.

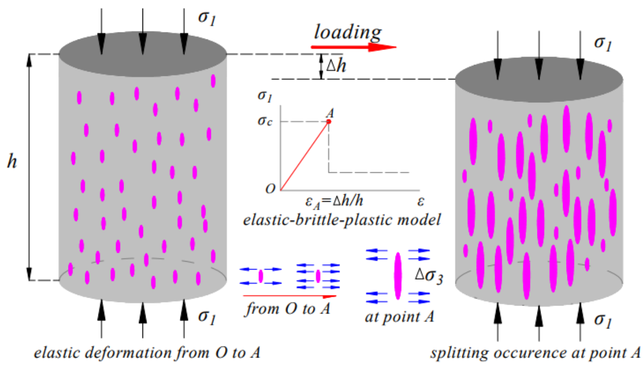


Fig. 2 Model for splitting under uniaxial loading

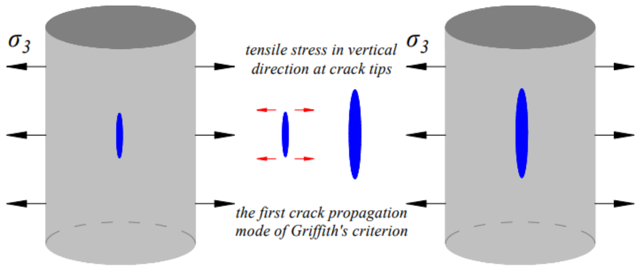


Fig. 3 First crack propagation mode of Griffith's criterion(take one longitudinal crack as an example*)

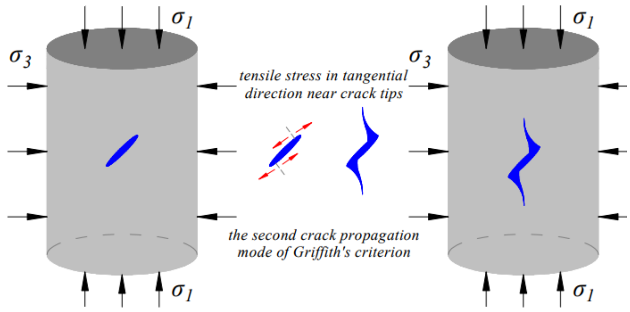


Fig. 4 Second crack propagation mode of Griffith's criterion

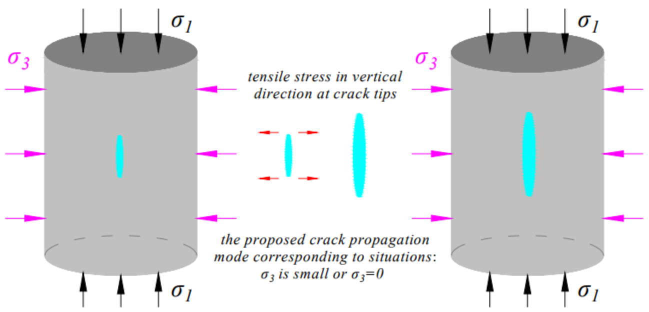


Fig. 5 Proposed crack propagation mode (confining stress is small or null)

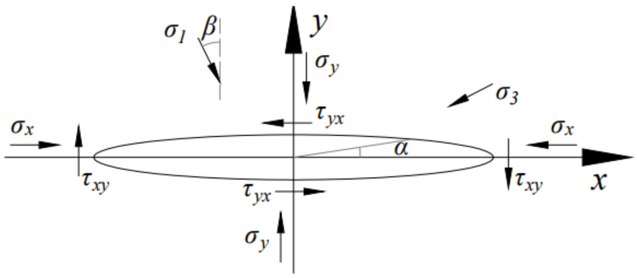


Fig. 6 Stress state of Griffith crack

3. Derivation of induced tensile stress based on Inglis's formula

Unlike the Hoek-Brown criterion and the Mohr-Coulomb criterion, the Griffith's criterion can consider the effect of cracks in rock through analyses on maximum tensile stress on periphery of the crack (Hoek and Martin 2014). The plane Griffith's criterion has the form as

$$\begin{cases} \sigma_3 = -\sigma_t & (\sigma_1 + 3\sigma_3 < 0) \\ \dfrac{(\sigma_1-\sigma_3)^2}{4(\sigma_1+\sigma_3)} = 2\sigma_t & (\sigma_1 + 3\sigma_3 \ge 0) \end{cases} \tag{11}$$

where σ_1 and σ_3 are respectively the maximum and minimum principle stress components. The above equation includes two forms of crack fracture, of which, one is the tensile fracture at crack tips along the major axis of the crack that caused by the tensile stress arising from the tension or unloading in direction perpendicular to the major axis of the crack (as shown in Fig. 3); another is the tensile fracture near the crack tips in tangential direction on periphery, the maximum tensile stress near the crack tips in this condition can arise from various compressive states including biaxial compression and uniaxial compression (as shown in Fig. 4).

In order to simplify the analyses, we only take on crack as an example. As shown in Fig. 3, the longitudinal crack refers to the one with its major axis paralleled to the axis of the specimen, while the other as shown in Fig. 4 is called as slanting crack. According to Griffith's criterion, when subjected to tension as shown in Fig. 3, the maximum induced tensile stress is at the tip of the longitudinal crack, while under compressive state as shown in Fig. 4, the maximum induced tensile is near the tip of the slanting crack. Therefore, the induced tensile stresses as denoted by the red arrows in Figs. 3 and 4 are respectively regarded as the internal causes for the failure of the specimen under the given stress conditions. Besides, based on Griffith's criterion, splitting as shown in Fig. 3 only occurs when the specimen is subjected to tension, and under the compressive state, the specimen will undergo shearing fracture.

However, the analyses based on the Maxwell model show that if the major axis of the crack and the axis of the cylindrical specimen are parallel, under the uniaxial loading, there will be tensile stress in direction perpendicular to the major axis of the crack. However, this kind of fracture is not included in Griffith's criterion, therefore, it is necessary for us to investigate whether it is possible for the crack to propagate along its major axis when the specimen is under uniaxial compressive state.

The Griffith's criterion indicates that while under the compressive state (uniaxial compressive state or biaxial compressive state with small confining pressure), the maximum tensile stress on periphery is near the crack tip and the propagation direction of the crack is $\gamma = -2\beta$ or $\gamma = \pi - 2\beta$.

As shown in Fig. 6, γ is the angle between the crack propagation direction and the major axis of crack; β is the angle between the minor axis of the crack and the direction of the maximum principle stress σ_1; α is the central angle of the ellipse, it is determined by the shape of the ellipse and also represents the location of the maximum tangential stress on the periphery (Griffith 1924).

In Griffith's criterion, all the cracks in rock are assumed to be randomly distributed slender ellipses; the inclination of the cracks is the master variation of tangential stress on periphery. According to Inglis's formula, the tangential stress on the periphery of a slender ellipse can be expressed as

$$\sigma_b = \frac{\sigma_y\left[m(m+2)\cos^2\alpha - \sin^2\alpha\right] + \sigma_x\left[(1+2m)\sin^2\alpha - m^2\cos^2\alpha\right] - \tau_{xy}\cdot 2(1+m)^2\sin\alpha\cos\alpha}{m^2\cos^2\alpha + \sin^2\alpha} \quad (12)$$

As shown in Fig. 6, $m=b/a$ is the ratio of the minor axis b to major axis a of the ellipse, for the slender crack, α is infinitely small, therefore, $\sin\alpha \to \alpha$ and $\cos\alpha \to 1$, then, Eq. (12) can be simplified as

$$\sigma_b = \frac{\sigma_y\left[m(m+2) - \alpha^2\right] + \sigma_x\left[(1+2m)\alpha^2 - m^2\right] - \tau_{xy}\cdot 2(1+m)^2\alpha}{m^2 + \alpha^2} \quad (13)$$

Let $\lambda = \sigma_3/\sigma_1\ (\sigma_1 \neq 0)$, and considering the following Eq. (14)

$$\begin{cases} \sigma_y = \dfrac{\sigma_1+\sigma_3}{2} + \dfrac{\sigma_1-\sigma_3}{2}\cos 2\beta \\ \sigma_x = \dfrac{\sigma_1+\sigma_3}{2} - \dfrac{\sigma_1-\sigma_3}{2}\cos 2\beta \\ \tau_{xy} = -\dfrac{\sigma_1-\sigma_3}{2}\sin 2\beta \end{cases} \quad (14)$$

By substituting Eq. (14) into Eq. (13), we will have

$$\sigma_b = \frac{\sigma_1}{m^2+\alpha^2}\left[m(1+\alpha^2)(1+\lambda) + (1+m)(m-\alpha^2)(1-\lambda)\cos 2\beta + \alpha(1+m)^2(1-\lambda)\sin 2\beta\right] \quad (15)$$

Under the uniaxial loading, when the major axis of the crack and the axis of the cylindrical specimen are parallel, the values of several parameters in Eq. (15) are as follows

$$\beta = -\pi/2,\ \ \alpha = 0,\ \ \sigma_3 = 0,\ \ \sigma_1 \neq 0,\ \ \lambda = 0 \quad (16)$$

and then the stress extremum σ_{tt} of Eq. (15) is

$$\sigma_{tt} = -\sigma_1 \quad (17)$$

As shown in Fig. 2, by using the elastic-brittle-plastic model, during the elastic deformation process, although the tangential stress on the periphery will increase, the shape of the crack is assumed to be unchanged, during the elastic deformation process, the tensile stress at the tips of the longitudinal crack can reach the same value as the axial loading. There are stress concentration and energy accumulation processes in the elastic deformation process (from the point *O* to *A*), which are regarded as the basis for the sudden rock splitting at point *A*. Besides, when the compressive strength is reached, the extremum of Eq. (17) is $\sigma_{tt\max} = -\sigma_c \approx -8\sigma_t$, which is large enough to cause the rock splitting.

4. Discussion on the condition required for rock splitting under uniaxial loading

In the above analyses, we assumed that the major axis of the crack and the axis of the cylindrical specimen are parallel (as shown in Fig. 2), by describing the mechanical behavior of the rock with elastic-brittle-plastic model, expressions of the tensile stress induced by the longitudinal crack in direction perpendicular to the major axis of the crack are respectively derived by using the Maxwell model and the Inglis's formula. Therefore, the possibility of rock splitting which is caused by the longitudinal crack under uniaxial loading is confirmed. However, as we may see that both in uniaxial compressive and biaxial compressive state, the most common failure phenomenon for cylindrical rock specimen is shearing failure, the splitting failure seems to be rare, therefore, here comes the question, what special conditions does it really required for the occurrence of splitting in uniaxial loading state.

In fact, according to Inglis's formula and Griffith's criterion, although in the same compressive state as shown in Fig. 7, for the cracks with the same shape, the induced tensile stress σ_{t1} at tip of longitudinal crack is smaller than σ_{t2} near the tip of the slanting crack, i.e., $\sigma_{t1} < \sigma_{t2}$. Therefore, under the compressive state, the fracture of the slanting crack will always happen first, but this kind of fracture is limited due to the fact that as the fracture of the slanting

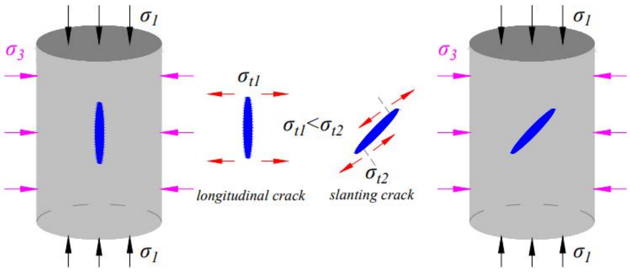


Fig. 7 Induced tensile stress on periphery of longitudinal crack and slanting crack (confining pressure is small or null)

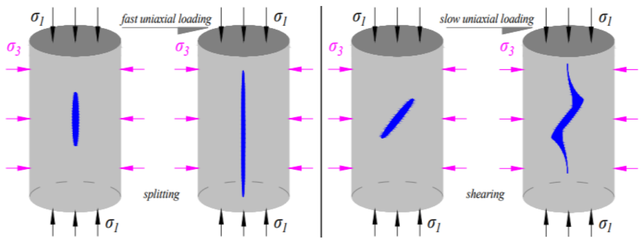
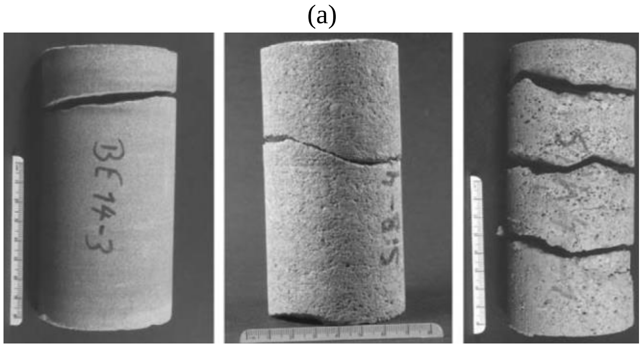


Fig. 8 Two failure model corresponding to different uniaxial loading model (confining pressure is small or null)

(a)



(b)

Fig. 9 (a) Rock core disking during core drilling in URL (Lim and Martin 2010) and (b) Rock splitting under axial unloading (Bauch and Lemmp 2004)

crack extends further, the original stretching fracture model will change and more energy consumption is required (Germanovich *et al*. 1994). However, with the increase of the uniaxial loading, this kind of fracture will turn into shearing, which means under the uniaxial loading state, the fracture of the specimen is generally shearing and the fracture plane is inclined (Hoek and Martin 2014).

The above analyses show that in usual uniaxial compressive experiment, that the fracture of the specimen will be shearing is most likely due to the fact that the loading on the specimen increases gradually and the absorbed energy of the specimen increases continually, therefore, these can also be regarded as the conditions required for shearing fracture. However, if the uniaxial loading is great and the loading duration is short, the tensile stress at the tip of the longitudinal crack can be great enough to exceed the tensile strength of the rock. Although at the same time, the fracture of the slanting crack and the longitudinal crack will both occur, but the development of the shearing fracture requires further energy consumption and longer loading duration, therefore, the fracture of the slanting crack and the shearing process will be limited. Splitting is a typical brittle fracture process, under the fast uniaxial loading, when the loading is great, the induced tensile stress at the tip of longitudinal crack can be far greater than the tensile strength, and the fracture of the longitudinal crack will happen. For a brittle rock specimen subjected to uniaxial loading, compared with the energy consumption of shearing fracture which is caused by the fracture of the slanting crack, the energy consumption required for the splitting which is induced by the fracture of the longitudinal crack is less. Therefore, under the uniaxial loading state, splitting is more likely to occur when the loading is great and the loading duration is short.

As shown in Fig.8, when the uniaxial loading is great and the loading duration is short, splitting is most likely to occur; while the uniaxial loading increases gradually and the loading duration is long, with the continually supplied energy, shearing fracture is most likely to occur. Compared to the shearing failure, when splitting occurs, the fracture process is quite quickly and the energy consumption is less.

5. Rock core disking under axial unloading

Fig. 9(a) shows the rock splitting (rock core disking) occurred during the core drilling in URL (Underground Research Laboratory) (Lim and Martin 2010), the newly generated fracture surfaces are approximately parallel to each other and perpendicular to the drilling direction. In fact, the phenomenon of rock core disking under axial unloading has already been observed and realized by laboratory tests (Bauch and Lemmp 2004). The laboratory tests conducted by Bauch and Lemmp, as shown in Fig.9(b) show that, in the situation that the given initial stresses beyond the critical stress required for rock splitting, rock core disking is more likely to occur when the unloading duration is short. Their tests also show that for the cylindrical rock specimen, the rock core disking which occurs under fast axial unloading is only related to the initial stress in the axial direction of the specimen, the stress in the radial direction seems to have no effect on rock core disking.

In this section, based on the conclusion that the rock core disking which occurs in the fast axial unloading process is mainly related to the initial axial stress, we will establish the following mechanical model, as shown in Fig. 10, to attempt to give an explanation for rock core disking which occurs under the fast axial unloading.

Fig. 10 shows the rock core disking of cylindrical rock specimen which occurs in fast axial unloading process. In the initial stage, the cylindrical rock specimen is under axial stress $\sigma_1(\sigma_{1\leq}\sigma_c)$, and the specimen is within its elastic deformation range. The solid red line *BC* represents the

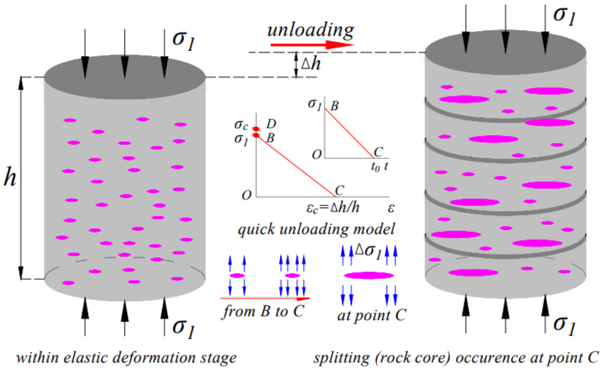


Fig. 10 Model for splitting under axial unloading

axial unloading process of the specimen (during the unloading process, from time t=0 to t=t_0, the axial stress quickly decreases from σ_1 to 0, at the same time, the specimen quickly recovers to its initial length). In order to simplify the analysis, it is assumed that during the fast unloading process *BC*, the induced tensile stress increases gradually while the shape of the cracks remains unchanged; At point *C*, the axial stress decrease to 0, the specimen recovers to its initial length, and rock core disking suddenly occurs. The stress concentration process during the unloading process *BC* is regarded as the basis for the suddenly splitting of brittle rock at point *C*. Based on Eq.(4), the intensity of the additional stress $\varDelta\sigma_1$ that parallels to the axis of the specimen can be expressed as

$$\varDelta\sigma_1 \approx 3\rho c_s^2 \dot{\varepsilon}_1 \frac{D}{\upsilon} \tag{18}$$

Still taking the granite (typical brittle rock) as an example, then, during the axial unloading process, the scale of the specimen is represented by its length.

During the axial unloading process, the plastic deformation in rock is not significant, the brittle rock specimen will mainly undergo the elastic deformation and fracture process. As shown in Fig. 10, for the brittle rock, it is assumed that the rock undergoes elastic deformation before σ_1 decreases to 0. Besides, it is also assumed that during the axial unloading process, the rock specimen undergoes elastic deformation during the period $0<t<t_0$, and splitting suddenly occurs at time t=t_0. As shown in Fig. 10, at time t=0, the axial strain of the specimen is $\varepsilon_c = \sigma_1/E = \varDelta h/h$, while at time t=t_0, the axial strain of the cylindrical specimen is assumed to be 0. If the axial strain of the specimen at the initial stage is taken as 0, then, at the end of the unloading process, the axial strain of the specimen will be $\varepsilon_1 = -\varDelta h/h$. Therefore, with considering the relationship $E = 2(1+\nu)\rho c_s^2$, and the total axial strain during the whole unloading process is $\varepsilon_1 = -\varDelta h/h = -\sigma_1/E$, then Eq. (7) can be rewritten as

$$\varDelta\sigma_1 \approx 3\rho c_s^2 \dot{\varepsilon}_1 \frac{D}{\upsilon} = -3\rho c_s^2 \varepsilon_1 = -\frac{3}{2}\frac{\sigma_1}{1+\nu} \tag{19}$$

Eq. (19) also shows that the crack will cause tensile stress in direction perpendicular to the major axis of the crack. Ignoring the variation of the specimen volume during the unloading process, as shown in Fig. 10, at point *C*, the induced tensile stress $\varDelta\sigma_1$ can be expressed as

$$\varDelta\sigma_1 = -\sigma_1 \tag{20}$$

If the initial axial stress $\sigma_1 = \sigma_c$, then the extremum of the induced tensile stress at the crack tips at the end of the unloading process will be

$$\varDelta\sigma_{1\max} = -\sigma_c \tag{21}$$

Also considering the relation that $\sigma_c \approx 8\sigma_t$ (Griffith 1924), where σ_c and σ_t are respectively the axial compressive strength and tensile strength, then

$$\varDelta\sigma_{1\max} \approx 8\sigma_t \tag{22}$$

According to Eq. (22), at the end of the unloading process (corresponding to point *C* in Fig. 10), the induced tensile stress will be almost eight times of the tensile strength, which is large enough to cause tensile failure in direction that parallels to the axis of the specimen. Besides, the assumptions that during the unloading process, although the induced tensile stress increases gradually, the shape of the crack kept unchanged till splitting suddenly occurs in the condition that the axial stress decreases to 0, are helpful for understanding the phenomenon that rock core disking is a fierce phenomenon.

Based on our analyses above, during the fast axial unloading process, the rock core disking is assumed to occur at the moment that the specimen recovers to its initial length (or the axial stress decreases to 0). It is necessary to note that at that moment the induced tensile stress is almost eight times of the tensile strength, which means, in fact, during the axial unloading process, the rock core disking will occur even before the specimen recovers to its initial length, i.e., during the unloading process, when core disking occurs, the axial strain is very small. These analyses may be helpful for understanding that rock core disking is fundamentally a tensile failure phenomenon (Lim and Martin 2010), and the phenomenon observed in the laboratory tests by Bauch and Lemmp (2004), that is, during the fast unloading process, rock core disking occurs even the axial strain is tiny.

6. Conclusions

In this paper, attentions are paid to the heterogeneous internal structure of the rock, the splitting phenomena under uniaxial compressive state and fast axial loading state are both explained by the conclusion that there are tensile stresses in direction perpendicular to the major axis of the crack. Based on our study, the following conclusions are drawn

- The rocks are heterogeneous material and contain numerous randomly distributed cracks with different scales, under uniaxial compressive state, there are tensile stresses caused by the longitudinal crack, which are in direction perpendicular to the major axis of the longitudinal crack.
- During the uniaxial compressive process, the expressions of the tensile stress induced by the longitudinal crack in direction perpendicular to the major axis of the crack are derived both by using the Maxwell model and the Inglis's formula, which all show that the induced tensile

stress can be far greater than the tensile strength even before the uniaxial compressive strength is reached.

• Under the uniaxial compressive state, the extremum of the induced tensile stress often appears in two situations, of which one is near the tip of the slanting crack, and another is at the tip of the longitudinal crack. Therefore, under the uniaxial loading, the rock may either undergo splitting failure or shearing failure, but the conditions required for the two failure modes are different.

• Generally speaking, under the uniaxial loading, when the loading is great and the loading duration is short, splitting is more likely to occur; however, if the loading increases gradually and the loading duration is long, shearing is most likely to happen. Compared to the shearing failure, splitting is quite quickly and less energy consumed.

• Attempts on explaining the rock core disking by using the Maxwell model show that during the fast axial unloading process, in the direction that parallels to the specimen axis, there are induced tensile stresses that are greater than the tensile strength. Which is helpful for understanding that: 1. rock core disking is fundamentally a tensile failure; 2.the axial strain required for rock core disking is tiny and 3. The radial stress seems to have no obvious effect on rock core disking.

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