Geomechanics and Engineering, *Vol. 13, No. 3 (2017) 505-515*
DOI: https://doi.org/10.12989/gae.2017.13.3.505

Effect of surface bolt on the collapse mechanism of a shallow rectangular cavity

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(*Received October 16, 2014, Revised March 20, 2017, Accepted April 05, 2017*)

Abstract. Based on the collapse characteristics of a shallow rectangular cavity, a three-dimensional failure mechanism which can be used to study the collapsing region of the rock mass above a shallow cavity roof is constructed. Considering the effects of surcharge pressure and surface bolt on the collapsing block, the external rate of works produced by surcharge pressure and surface bolt are included in the energy dissipation calculation. Using variational approach, an analytic expression of surface equation for the collapsing block, which can be used to study the collapsing region of the rock mass above a shallow cavity roof, is derived in the framework of upper bound theorem. Based on the analytic expression of surface equation, the shape of the collapsing block for shallow cavity is drawn. Moreover, the changing law of the collapsing region for different parameters indicates that the collapsing region of rock mass decreases with the increase of the density of surface bolt. This conclusion can provide reference for practicing geotechnical engineers to achieve an optimal design of supporting structure for a shallow cavity.

Keywords: shallow rectangular cavity; collapsing block; upper bound theorem; surface bolt

1. Introduction

The surface subsidence caused by the collapsing of a cavity roof is a common accident in geotechnical and civil engineering, especially in an area where there is a cavity excavated in the shallow strata. As the overburden of the shallow cavity is thin, the collapsing surface of the cavity will extend from the cavity roof to the ground surface. So, the study of the possible collapse surface of a cavity roof will contribute to predicting surface subsidence range which is a rather complex issue encountered by geotechnical engineers. However, due to the random variability of the mechanical properties of the geotechnical material, it is difficult to determine the shape and region of the collapsing surface of a cavity roof.

In recent years, many scholars have tried to solve this problem and their efforts are focused on the collapsing surface of a cavity roof. Osman *et al.* (2006) proposed a simple plastic deformation

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http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

mechanism to investigate the pattern of deformation around a shallow unlined tunnel. Based on this mechanism, the width of a subsurface settlement trough for a shallow circular tunnel was determined in the framework of upper bound theorem. Using finite element limit analysis, Yamamoto *et al*. (2011) studied the stability of circular tunnels in cohesive-frictional soils subjected to surcharge loading. Their achievements not only provided rigorous lower- and upper-bound solutions for the ultimate surcharge loading, but also demonstrated the collapsing surface which extends from the tunnel roof to the ground surface.

Since the collapsing surface of a deep tunnel is different from a shallow tunnel, some failure mechanisms in connection with deep tunnels have been proposed in previous works. Fraldi and Guarracino (2009) constructed a collapsing mechanism of a deep cavity which is composed of a detaching curve. Using this mechanism, they studied the shape of the collapsing block above a deep cavity roof in the framework of limit analysis theorem and the Hoek-Brown failure criterion. Later on, Fraldi and Guarracino (2010) employed the same method to predict the potential collapse region in tunnels with arbitrary excavation profiles. Since their solutions have not been compared with other numerical or analytical methods, Fraldi and Guarracino (2011) made a comparison between numerical and analytical approaches to modeling collapse in circular tunnels. The good agreement between numerical and analytical approaches shows that the method proposed by Fraldi and Guarracino (2011) is an effective one for evaluating the collapsing mechanism above a deep cavity roof. As a roof collapse by progressive failure is widespread, Fraldi and Guarracino (2012) studied the occurrence of progressive roof failure in Hoek–Brown rock cavities. As the analytical approach proposed by Fraldi and Guarracino (2009) is effective, recent researches have been focused on the development of this method (Yang and Huang 2011, Huang and Yang 2011). Considering the destructive process of rocks in practical projects is a three-dimensional evolutional process, Yang and Huang (2013) proposed a three-dimensional stability analysis method for the rocks over a deep cavity roof by constructing a three-dimensional failure mechanism.

Current studies of the collapsing surface of a shallow tunnel in limit analysis are conducted by using two-dimensional failure mechanisms. As the three-dimensional failure mechanism can reflect three-dimensional destructive characteristics of the possible collapse of a shallow cavity roof, it is necessary to develop a three-dimensional stability analysis method for evaluating the stability of a shallow cavity roof. In this work, we study the shape and range of the collapsing block above a shallow cavity roof excavated in 'low quality' rock masses following the Hoek–Brown failure criterion. In this process, the 2D failure mechanism proposed by Yang and Huang (2011) has been extended to three dimensions, which describes the three-dimensional failure features of the rock mass above a shallow cavity roof. Based on this three-dimensional failure mechanism, an analytical expression of the surface equation for the collapsing block is obtained by using the upper bound theorem of limit analysis in conjunction with a variational approach.

2. Three-dimensional failure mechanism of a shallow cavity roof

In accordance with the mechanical characteristics and actual collapsing situation of a shallow tunnel, Yang and Huang (2011) constructed a curved 2D failure mechanism which is composed of two detaching curves extending from the circular tunnel roof to the ground. To develop a three-dimensional stability analysis method for the rock mass above a shallow cavity roof, this paper presents a three-dimensional rotational failure mechanism by extending the 2D failure mechanism

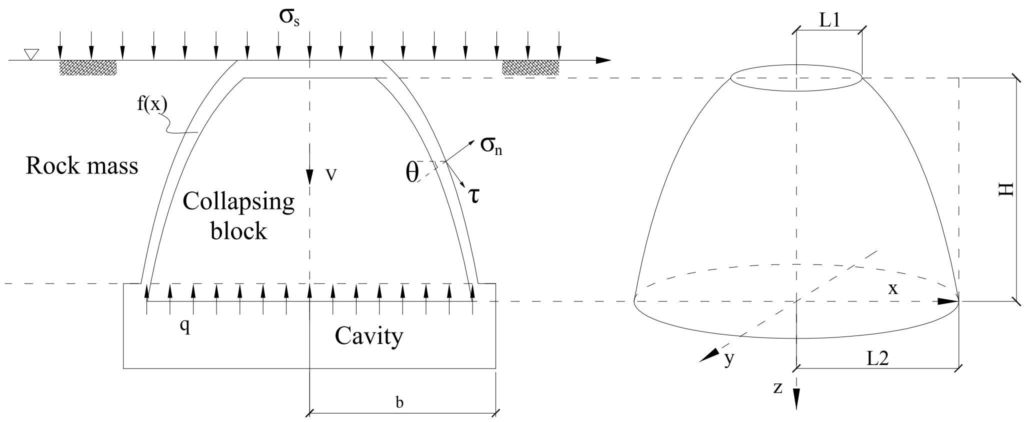

Fig. 1 Three-dimensional failure mechanism of a shallow cavity

proposed by Yang and Huang (2011) to three dimensions. As illustrated in Fig. 1, the velocity discontinuity occurs along an unknown detaching curve $f(x)$, and a two dimensional kinematically admissible velocity field is formed in the XOZ plane. Assuming that the detaching curve $f(x)$ is rotated around the Z-axis by 360 degrees, a rotational solid which is symmetrical with respect to the Z-axis is formed.

3. Hoek-Brown failure criterion

When geomaterials reach a plastic yield surface under the plastic limit load, the plastic flow will occur on the interface between the collapsing block and surrounding rock. Furthermore, an energy dissipation induced by the plastic flow between the collapsing block and surrounding rock can be calculated in the framework of the upper bound theorem in limit analysis. Therefore, the choice of a failure criterion controls the occurrence of the plastic flow in the kinematically admissible velocity field, which is a key factor in energy dissipation calculation. As the energy dissipation between the collapsing block and surrounding rock consists of two components: the normal energy dissipation induced by a normal stress–strain and the shear energy dissipation induced by a shear stress–strain, we can use the Hoek–Brown failure criterion represented by normal and shear stresses to achieve the energy dissipation calculation, which can be demonstrated as Hoek and Brown (1997)

$$\tau = A\sigma_{ci}\left(\frac{\sigma_n - \sigma_{tm}}{\sigma_{ci}}\right)^B \tag{1}$$

where σ_n is the normal stress, τ is the shear stress, A and B are material constants, σ_{ci} is the uniaxial compressive strength and σ_{tm} is the tensile strength of the rock mass. Hoek and Brown (1997) thought this criterion can be applied to a variety of rock masses including very poor quality rocks, which could almost be classed as engineering soils. Thus, the Hoek–Brown failure criterion can be used to investigate the stability of cavity excavated in shallow stratum.

4. Upper bound solution of collapsing block for shallow cavity

The upper bound theorem states that when the velocity boundary condition is satisfied, the load derived by equating the external rate of work to the rate of the energy dissipation in any kinematically admissible velocity field is no less than the actual collapse load (Chen 1975). So, to obtain the upper solution of the surface equation for the three-dimensional collapsing block, it is necessary to compute the internal dissipation of energy and the external rate of work at first. The energy dissipation at a random point on the velocity discontinuity curve caused by a normal stress–strain and a shear stress-strain has been derived by Fraldi and Guarracino (2009), which is expressed as

$$\begin{aligned} D &= \sigma_n \dot{\varepsilon}_n + \tau \dot{\gamma}_n \\ &= \left\{ \sigma_{ci} \left[ABf'(x) \right]^{\frac{1}{1-B}} \left(1 - B^{-1}\right) - \sigma_{tm} \right\} \frac{1}{\sqrt{1 + f'(x)^2}} v \end{aligned} \tag{2}$$

where $\dot{\varepsilon}_n$ and $\dot{\gamma}_n$ are the normal and shear plastic strain rates respectively, $f(x)$ is the equation of velocity discontinuity curve, $f'(x)$ is the first derivative of $f(x)$, and v is the velocity of the collapsing block. By integrating the energy dissipation at a random point on the lateral face of the three-dimensional rotational failure mechanism, we can obtain the energy dissipation of the whole velocity discontinuity surface. Based on the definite integral calculation, the internal energy dissipation of the whole failure mechanism can be written as

$$P_D = 2\pi \int_{L_1}^{L_2} \left\{ \sigma_{ci} \left[ABf'(x) \right]^{\frac{1}{1-B}} \left(1 - B^{-1}\right) - \sigma_{tm} \right\} xv dx \tag{3}$$

where L_1 and L_2 are the radius of the circular top and bottom of the rotational solid respectively. Moreover, the work rate of the collapsing block produced by its weight can also be computed by a definite integral calculation

$$P_\gamma = \gamma \int_{L_2}^{L_1} \pi x^2 f'(x) dx v \tag{4}$$

where γ is the unit weight of the rock mass. The work rate of supporting pressure can be expressed as

$$P_q = \pi L_2^{\ 2} q v \cos \pi \tag{5}$$

where q is the supporting pressure of the shallow cavity. As the collapsing surface of the cavity extends to the ground surface, the surcharge pressure at the ground surface has great effect on the range of the collapsing block. So, the work rate of the surcharge pressure is considered in the energy dissipation calculation, which is written as

$$P_{\sigma_s} = \pi L_1^{\ 2} \sigma_s v \tag{6}$$

where σ_s is the surcharge pressure. To study the stability of the rock mass above a shallow cavity roof, the upper bound solution of the three-dimensional shape of the collapsing block should be determined at first. Moreover, the upper bound theorem states that the solution closest to the real solution is the extremum among the upper solutions which are determined by equating the rate of energy dissipation to the external rate of work. Thus, it is necessary to establish an objective function composed of the rate of energy dissipation and the external rate of work to search the extremum. The objective function ξ which is represented by the difference of the rate of energy dissipation and the whole external rate of work can be written as

$$\xi\left[f(x), f'(x), x\right] = P_D - P_\gamma - P_q - P_{\sigma_s} \tag{7}$$

Substituting Eqs. (3)-(6) into Eq. (7), the expression of ξ is determined

$$\begin{aligned} \xi &= 2\pi\int_{L_1}^{L_2}\left\{\left\{\sigma_{ci}\left[ABf'(x)\right]^{\frac{1}{1-B}}\left(1-B^{-1}\right)-\sigma_{tm}\right\}x+\frac{\gamma}{2}x^2 f'(x)\right\}vdx \\ &\quad + \pi q L_2{}^2 v - \pi\sigma_s L_1{}^2 v \\ &= 2\pi\int_{L_1}^{L_2}\psi\left[f(x), f'(x), x\right]vdx + \pi q L_2{}^2 v - \pi\sigma_s L_1{}^2 v \end{aligned} \tag{8}$$

where ψ is a functional, which can be written as

$$\begin{aligned} &\psi\left[f(x), f'(x), x\right] \\ &= \left\{\sigma_{ci}\left[ABf'(x)\right]^{\frac{1}{1-B}}\left(1-B^{-1}\right)-\sigma_{tm}\right\}x+\frac{\gamma}{2}x^2 f'(x) \end{aligned} \tag{9}$$

Since ξ is determined by ψ, the calculation of upper bound solution of the collapsing block shape above a shallow cavity roof converts to searching extremum of ψ. When ψ reaches the extremum, the equation of the velocity discontinuity curve $f(x)$ corresponding to ψ is the optimized upper solution we seek, which can be used to draw the three-dimensional shape of the collapse region for a shallow cavity roof. Furthermore, it is found that ψ is a functional whose extremum can be derived by variational calculation. On the basis of the variational principle, when ψ reaches the extremum. Euler's equation, which is illustrated as follows, is satisfied

$$\begin{aligned} &\gamma x - \frac{1}{B}\sigma_{ci}\left(AB\right)^{\frac{1}{1-B}}\left[f'(x)\right]^{\frac{B}{1-B}} \\ &-\frac{1}{1-B}\sigma_{ci}\left(AB\right)^{\frac{1}{1-B}}\left[f'(x)\right]^{\frac{2B-1}{1-B}} f''(x)x = 0 \end{aligned} \tag{10}$$

It is evident that Eq. (10) is a nonlinear second-order homogeneous differential equation with a constant coefficient. According to Yang and Huang (2011), the analytical solution of $f(x)$ for this differential equation can be derived by the method of variation of constants, which is expressed as

$$f(x) = A^{-\frac{1}{B}}\left(\frac{\gamma}{2\sigma_{ci}}\right)^{\frac{1-B}{B}} x^{\frac{1}{B}} - c_1 \tag{11}$$

where c_1 is an integration constant. By substituting Eq. (11) into Eq. (8), the expression of objective function ξ is obtained

$$\xi = \left\{ \frac{2\pi B}{1+2B} A^{-\frac{1}{B}} \sigma_{ci}^{\frac{B-1}{B}} \left(\frac{\gamma}{2}\right)^{\frac{1}{B}} \left(L_2^{\frac{1+2B}{B}} - L_1^{\frac{1+2B}{B}} \right) \right. \\ \left. - \pi\sigma_{tm} \left(L_2^2 - L_1^2 \right) + \pi q L_2^2 - \pi\sigma_s L_1^2 \right\} v \tag{12}$$

As the collapsing surface of cavity will extend from the cavity roof to the ground surface and the buried depth of a shallow cavity is known, two geometric equations can be found from Fig. 1, which can be demonstrated as

$$\begin{cases} f\left(x = L_1\right) = -H \\ f\left(x = L_2\right) = 0 \end{cases} \tag{13}$$

Substituting Eq. (11) into Eq. (13), the relationship between L_1 and L_2 is determined

$$L_1 = \left[L_2^{\frac{1}{B}} - H \left(\frac{\gamma}{2\sigma_{ci}} \right)^{\frac{B-1}{B}} A^{\frac{1}{B}} \right]^B \tag{14}$$

On the basis of the upper bound theorem, the external rate of work is equal to the internal energy dissipation when the kinematically admissible velocity field is at the limit state. Thus, the following equation is obtained by equating Eq. (12) to zero

$$\frac{2\pi B}{1+2B} A^{-\frac{1}{B}} \sigma_{ci}^{\frac{B-1}{B}} \left(\frac{\gamma}{2}\right)^{\frac{1}{B}} \left(L_2^{\frac{1+2B}{B}} - L_1^{\frac{1+2B}{B}} \right) \\ -\pi\sigma_{tm} \left(L_2^2 - L_1^2 \right) + \pi q L_2^2 - \pi\sigma_s L_1^2 v = 0 \tag{15}$$

Combining Eqs. (14) and (15), we can derive a nonlinear equation. Using the numerical method the numerical solutions of L_1 and L_2 are determined. Then, substituting the numerical solutions of L_1 and L_2 into Eq. (11), the analytic expression of surface equation for the three-dimensional collapsing block is obtained

$$z = \left(\frac{\gamma}{2\sigma_{ci}} \right)^{\frac{1-B}{B}} A^{-\frac{1}{B}} \left(x^2 + y^2 \right)^{\frac{1}{2B}} - \left(\frac{\gamma}{2\sigma_{ci}} \right)^{\frac{1-B}{B}} A^{-\frac{1}{B}} L_2^{\frac{1}{B}} \tag{16}$$

5. Effect of different parameters on the shape of collapsing block

From the upper bound analysis of the collapsing block for a shallow cavity, it is found that the collapsing block extends from the cavity roof to the ground surface. Therefore, the surcharge pressure at the ground surface has great influence on the region of the collapsing block for a cavity roof. The surface subsidence caused by surcharge pressure is a common accident when a shallow cavity is excavated, especially in areas where there are many high-rise buildings. If the region of the collapsing block with determined surcharge pressure can be predicted in time, certain reinforcement measures can be adopted to prevent the possible occurrence of surface subsidence above the shallow cavity roof. So, it is important to study the effect of surcharge pressure on the potential collapsing region for a shallow cavity. To do this, the potential three-dimensional collapsing block of the parameters corresponding to A = 2/3, B = 0.6, σ_{ci} = 10 MPa, $\sigma_{tm} = \sigma_{ci}/100$, γ = 25 kN/m^3, q = 20 kPa, σ_s = 0, 50, 100 kPa, H = 10 m is presented in Fig. 2. From this figure, it can be seen that the surcharge pressure has significant influence on the region of the potential collapsing block. The region of the potential collapsing block increases remarkably with the increase of the value of surcharge pressure. Obviously, when there is great surcharge pressure on the ground surface, surface reinforcement measures should be adopted to prevent the

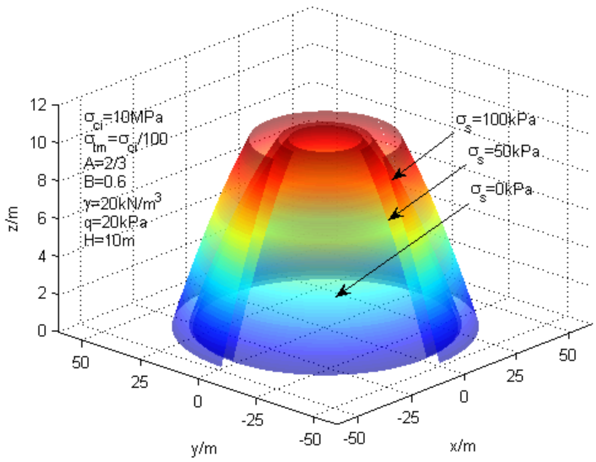

Fig. 2 Effect of surcharge pressure σ_s on the shape of the collapsing block
(A = 2/3, B = 0.6, σ_{ci} = 10 MPa, $\sigma_{tm} = \sigma_{ci}/100$, γ = 25 kN/m^3, q = 20 kPa, H = 10 m)

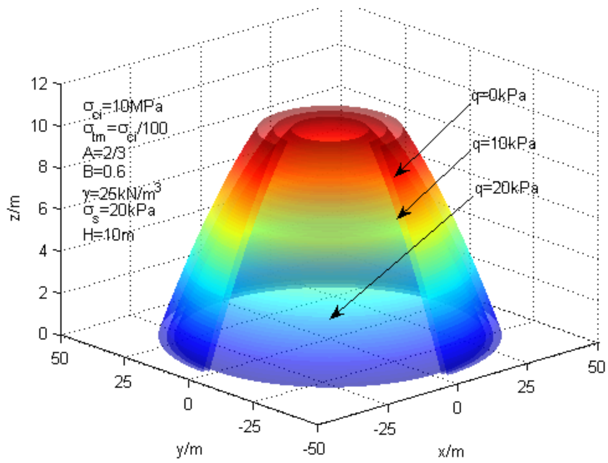

Fig. 3 Effect of supporting pressure q on the shape of the collapsing block
(A = 2/3, B = 0.6, σ_{ci} = 10 MPa, $\sigma_{tm} = \sigma_{ci}/100$, γ = 25 kN/m^3, σ_s = 20 kPa, H = 10 m)

occurrence of surface subsidence induced by the excavated cavity.

In addition, the supporting structure of a shallow cavity also has influence on the region of potential collapsing block. However, the work rate of supporting structure is difficult to compute in upper bound analysis. To analyze the effect of the supporting structure on the region of potential collapsing block, the supporting structure is simplified as an uniform pressure which distrubutes evenly on the cavity roof. Based on the upper bound calculation, the potential collapsing blocks for different values of supporting pressure are drawn when other parameters are constants. It can be noted from Fig. 3 that the region of potential collapsing block decreases with the increase of the value of supporting pressure.

6. Effect of surface bolt on the shape of collapsing block

As the collapse of a cavity roof may cause accident in tunnel engineering, how to prevent the occurrence of a collapse above a cavity roof is a problem that many geotechnical engineers have to face. At present, the surface bolt pre-reinforcement is an effective method to control surface subsidence and cavity roof collapse which is widely applied in tunnel engineering. Using suspension effect and composite beam effect, the surface bolt makes the collapsing block combine into the whole surrounding rock. Consequently, the surface bolt pre-reinforcement can not only help the surrounding rock to stabilize itself, but also reduce the surface subsidence induced by tunneling.

To study the effect of surface bolt pre-reinforcement on the shape of a collapsing block, the failure mechanism of a shallow cavity roof with surface bolts acting evenly on the ground surface is established, which is illustrated in Fig.4. Similar to the calculation process mentioned above, the external rate of work and the rate of the energy dissipation for this mechanism should be computed to derive the analytic expression of surface equation for the collapsing block. To make the calculation simpler, some assumptions are proposed, which are as follows: (1) the row and column distances of the surface bolts n are fixed values, and are distributed evenly as a square; (2) the collapsing block is suspended in the surrounding rock by the axial force of the surface bolt and the

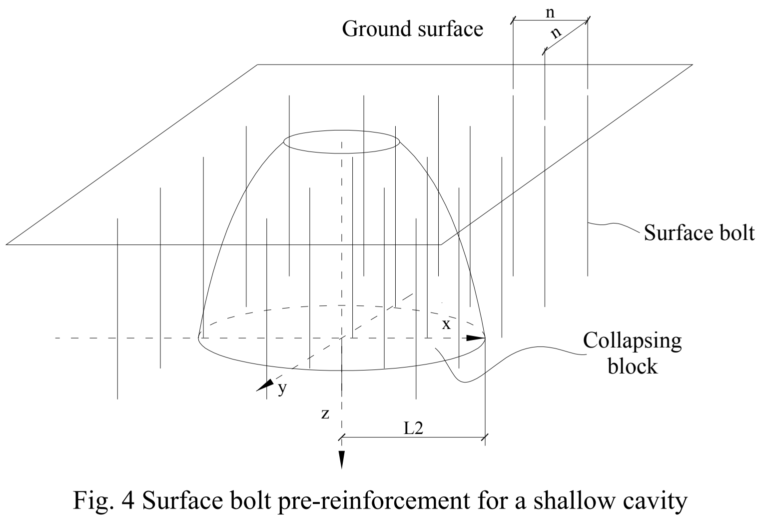

Fig. 4 Surface bolt pre-reinforcement for a shallow cavity

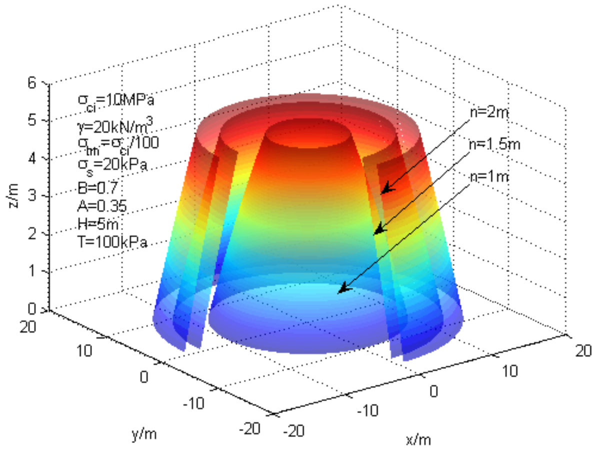

Fig. 5 Effect of surface bolt on the shape of the collapsing block
(A = 0.35, B = 0.7, σ_{ci} = 10 MPa, $\sigma_{tm} = \sigma_{ci}/100$, γ = 20 kN/m^3, σ_s = 20 kPa, H = 10 m, T = 100 kPa)

shear force is insignificant; (3) the axial force of each bolt is the same. Based on these assumptions, the work rate of the bolt is calculated and the expression of surface equation for the collapsing block is obtained. The shapes of the collapsing block for parameter values of A = 0.35, B = 0.7, σ_{ci} = 10 MPa, $\sigma_{tm} = \sigma_{ci}/100$, γ = 20 kN/m^3, σ_s = 20 kPa, H = 5 m, n = 1, 1.5, 2 m are plotted in Fig. 5. Obviously, the surface bolts have significant influence on the region of the collapsing block for a shallow cavity. The region of potential collapsing block decreases with the decrease of the distance of surface bolt. Since the distance of a surface bolt reflects the density of the surface bolt, it can be concluded that the region of potential collapsing block decreases with the increase of the density of surface bolt. Therefore, an increase of the density of surface bolt is an effective measure to achieve the stability of the surrounding rock above a shallow cavity roof in practical engineering.

As the volume of the collapsing block reflects the size of the collapsing block more intuitively, the volume of the collapsing block is computed for different bolt distance. The analytical expression of the volume of the collapsing block can be derived from integral calculation, which is

$$V = \int_{L_2}^{L_1} \pi x^2 f'(x)\,dx \tag{17}$$

Then, substituting the expression of $f'(x)$ and the numerical solutions of L_1 and L_2 into Eq. (17), the volume of the collapsing block is obtained

$$V = \frac{\pi}{1+2B} A^{-\frac{1}{B}} \left(\frac{\gamma}{2\sigma_{ci}}\right)^{\frac{1-B}{B}} \left(L_2^{\frac{1+2B}{B}} - L_1^{\frac{1+2B}{B}}\right) \tag{18}$$

From Table 1, it can be seen that the volume of the collapsing block decreases with the decrease of the distance of surface bolt. This may be explained by the fact that the density of the bolt increases with the decrease of bolts distance. Since the density of the bolt contributes to the stability of the surrounding rock above a shallow cavity roof, the decrease of the distance of surface bolt makes the decrease of the volume of the collapsing block.

Table 1 The volume of the collapsing block for different bolt distance n

n (m)	1	1.5	2
V (m^3)	1041.4	2369.6	3344.5
$A = 0.35$, $B = 0.7$, $\sigma_{ci} = 10$ MPa, $\sigma_{tm} = \sigma_{ci}/100$, $\gamma = 20$ kN/m^3			

7. Conclusions

Based on the destructive features of the surrounding rock above a shallow cavity roof, a three-dimensional failure mechanism is constructed in the framework of the upper bound theorem of limit analysis. With the help of integral calculation and variational approach, the upper bound solution of the surface equation for the collapsing block is obtained. By analyzing the shape of the collapsing block for different parameters, the following conclusions can be drawn:

- The surcharge pressure σ_s and supporting pressure q have significant influence on the region of potential collapsing block above a shallow cavity roof. The region of potential collapsing block increases with the increase of the value of surcharge pressure σ_s but decreases with the increase of the value of supporting pressure q.
- The region of potential collapsing block decreases with the increase of the density of surface bolt. So, an increase of the density of surface bolt is an effective measure to achieve the stability of the surrounding rock above a shallow cavity roof in practical engineering.

Acknowledgments

The preparation of this paper received financial support from the National Natural Science Foundation (51308072 and 51678071), the National Basic Research 973 Program of China (2013CB036004) and Innovation Research Project of Priority Key Disciplines at Changsha University of Science and Technology (no. 15ZDXK13). The financial supports are greatly appreciated.

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