Geomechanics and Engineering, *Vol. 13, No. 2 (2017) 237-254*
DOI: https://doi.org/10.12989/gae.2017.13.2.237

Assessment of slope stability using multiple regression analysis

Balendra M. Marrapu [a] and Ravi S. Jakka [*]

Department of Earthquake Engineering, Indian Institute of Technology Roorkee, Roorkee 247667, India

(*Received July 30, 2016, Revised December 22, 2016, Accepted February 20, 2017*)

Abstract. Estimation of slope stability is a very important task in geotechnical engineering. However, its estimation using conventional and soft computing methods has several drawbacks. Use of conventional limit equilibrium methods for the evaluation of slope stability is very tedious and time consuming, while the use of soft computing approaches like Artificial Neural Networks and Fuzzy Logic are black box approaches. Multiple Regression (MR) analysis provides an alternative to conventional and soft computing methods, for the evaluation of slope stability. MR models provide a simplified equation, which can be used to calculate critical factor of safety of slopes without adopting any iterative procedure, thereby reducing the time and complexity involved in the evaluation of slope stability. In the present study, a multiple regression model has been developed and tested its accuracy in the estimation of slope stability using real field data. Here, two separate multiple regression models have been developed for dry and wet slopes. Further, the accuracy of these developed models have been compared and validated with respect to conventional limit equilibrium methods in terms of Mean Square Error (MSE) & Coefficient of determination (R^2). As the developed MR models here are not based on any region specific data and covers wide range of parametric variations, they can be directly applied to any real slopes.

Keywords: slope stability; multiple regression; landslides

1. Introduction

Normally, Slope instability failures, which arise from the disturbances in hilly regions, pose serious threat to structures, as these failures may lead to great loss of lives and property. Limit equilibrium methods are commonly used methods for identifying these slope instabilities. There are quite a large number of slope failures occurring in hilly regions across the world every year. Analyzing stability of huge number of slopes using the convectional limit equilibrium methods is difficult, as these methods take significant amount of time for the development of slope models. Now a days, soft computing approaches like Artificial Neural Network (ANN), Fuzzy Logic have been in use for the estimation of slope instabilities (Sakellariou and Ferentinou 2005, Das *et al*. 2011, Erzin 2009, Erzin and Cetin 2012, Mohamed *et al*. 2012). However, these methods suffer due to their black box approach. A better alternative to these methods is Multiple Regression (MR) methods. MR is a statistical technique and provides a simplified equation that can be used to calculate critical factor of safety of slopes without adopting any iterative procedure, thereby reducing the time and complexity involved in the evaluation of slope stability. Further, the

∗Corresponding author, Associate Professor, E-mail: rsjakka@gmail.com
[a] Ph.D. Student, E-mail: royces2gather@gmail.com

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http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

developed MR equation provides a clear and transparent relationship between the independent and dependent parameters unlike soft computing approaches.

The use of multiple regression models have become common across a wide variety of engineering disciplines including geotechnical engineering for the estimation and prediction of various parameters. Zhang and Goh (2013) applied Multivariate adaptive regression splines for geotechnical engineering systems. Sayed *et al*. (2012) and Esmaeili *et al*. (2014) used multiple regression in the prediction of backbreak in blasting operation of rocks. Zhang *et al*. (2015) used linear regression analysis for predicting displacement of embankment slopes. Yilmaz and Yuksek (2009) used multiple regression for the estimation of the strength and elasticity modulus of gypsum, while Yilmaz and Kaynar (2011) used in the prediction of swell potential of clayey soils, Samui and Karup (2012) used Multivariate adaptive regression spline for prediction of undrained shear strength of clay. Samui (2011) used Multivariate regression spline for prediction of friction capacity of driven piles. Kumar *et al*. (2013) used multiple regression for prediction rock properties. Zhang and Goh (2016) used multivariate regression for evaluating liquefaction potential. Sah *et al*. (1994) and Erzin and Cetin (2013) developed regression equations for the prediction of slope stability. Sah *et al*. (1994) developed an empirical relationship for the estimation of slope stability using maximum likelihood method. However, the developed relationship is based on limited data (46 cases) and is applicable to only few failure modes of slopes. The multiple regression equation developed by Erzin and Cetin (2013) for slope stability prediction was based on limited soil parametric ranges (675 cases) and did not test its accuracy for real field cases.

In the present study, MR models have been developed for the calculation of critical FOS using huge number of slopes (a total of 29112 cases) which covers all the possible slope configurations and soil characteristics. Two separate equations have been developed for homogeneous dry, and saturated & partially saturated slopes (i.e., wet slopes). The developed models are further validated by applying to the real field data. IBM SPSS software is used for the development of these MR models.

2. Preparation of data set

In order to develop multiple regression model, huge quantity of raw data is used. Huge number of slopes (a total of 29112 cases) with all the possible configurations and soil characteristics have been considered. Details of the considered data ranges are presented in Table 1 for dry cases (14112 cases) and in Table 2 for wet cases (15000 cases). For all the considered slopes, stability condition expressed in terms of FOS is assessed. Calculating FOS for this huge number of cases

Table 1 Data ranges considered for the study of dry cases

Parameter	Range	Interval	No. of cases
Cohesion, C (kPa)	10-45	5 kPa	7
Angle of friction, ϕ	10°-40°	5°	6
Angle of inclination, β	15°-50°	5°	7
Unit weight, γ (kN/m^3)	15-24	1.5 (kN/m^3)	6
Height H (m)	6-54	6	8
Total number of cases = 7 × 6 × 7 × 6 × 8 =			14112

Table 2 Data ranges considered for the study of wet cases

Parameter	Range	Interval	No. of cases
Cohesion, C (kPa)	10-45	7 kPa	5
Angle of friction, ϕ	10°-40°	6°	5
Angle of inclination, β	15°-50°	7°	5
Unit weight, γ (kN/m^3)	15-23	2 (kN/m^3)	4
Height H (m)	6-54	8	6
Γ_u	0-0.5	0.1	5
Total number of cases = 5 × 5 × 5 × 4 × 6 × 5 =			15,000

using convectional limit equilibrium methods is not an easy task due to complex and time consuming nature of the method. Hence, we relied up on stability chart method for the estimation of FOS. Michalowski stability chart method implementation in MATLAB code is used for stability analysis of this huge number of slopes. Michalowski stability chart method uses lower limit kinematic analysis and is thoroughly validated Michalowski (2002). Here in this section, 14112 cases of dry slopes and 15000 cases of wet slopes are analyzed for developing the MR model.

3. Multiple regression model

Multiple Regression model (MR) is a statistical method, which is traditionally used to predict an indirect estimation of the given problem using empirical equations. The main purpose of MR is to learn more about the underlying relationship between several independent variables and a dependent variable. A dependent variable is modeled as a function of several independent variables with corresponding coefficients, along with a constant term. Development of multiple regression requires two or more independent variables. The generalized multiple regression equation takes the following form

$$Y = \beta_1^* X_1 + \beta_2^* X_2 + ...+ \beta_n^* X_n + C$$

Here, β_1, β_2,…, β_n are the regression coefficients, which represent the value at which the dependent variable changes when the independent variables changes.

In the present study, MR is used in the stability assessment of slopes for dry as well as saturated & partially saturated cases (i.e., wet cases). For partially saturated & saturated cases, the independent variables are geotechnical properties of soils (C, ϕ, β, γ, H, Γ_u) and dependent variable is FOS, while (C, ϕ, β, γ, H) are independent and FOS is dependent variable for dry case. The accuracy of the developed MR equation is measured in terms of R^2 and MSE. R^2 measures the association, which indicates the percentage of overlap between the predictor variables and the calculated variable. MSE measures the average of the squares of the errors, that is, the difference between the predictor value and calculated value. For the accurate and better performance of MR model, the following four assumptions are to be satisfied (Osborne and Waters 2002).

(1) The relationship between the independent and dependent variables needs to be linear. Also it is important to check for outliers as MR is sensitive to outlier effects. This assumption can be tested using scatter plots.

(2) Multiple regression analysis requires all the dependent variables to be normally distributed and this can be checked using a histogram and a fitted normal curve.
(3) Multiple regression assumes that there is little or no multicollinearity in the data. Multicollinearity occurs when the independent variables are not independent from each other. This can be identified using correlation matrix. The correlation coefficients among all the independent variables in the Pearson's Bivariate matrix need to be smaller than 0.19.
(4) Multiple regression analysis requires is homoscedasticity. The scatter plot is good way to check whether homoscedasticity (that is the error terms along the regression line are equal) is given. This means residual errors should be uniformly distributed.

3.1 Development and application of MR model for dry slopes

In this section, MR model for assessing critical FOS for dry slopes has been developed using

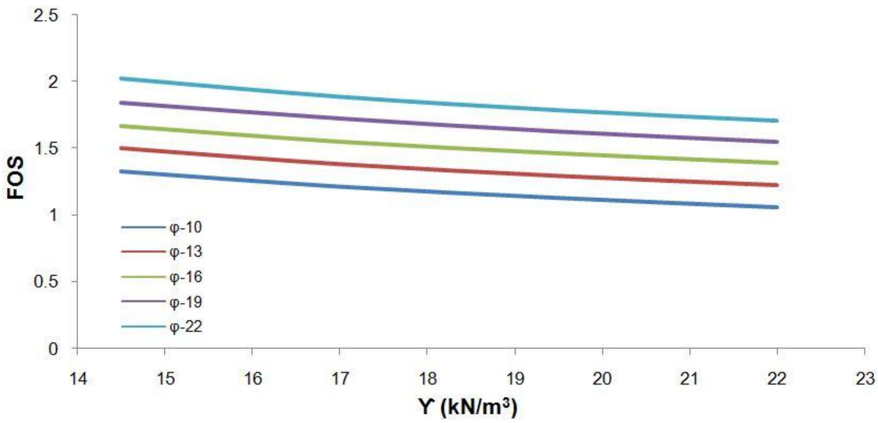


Fig. 1 Relation between stability of slopes and unit weight of soils

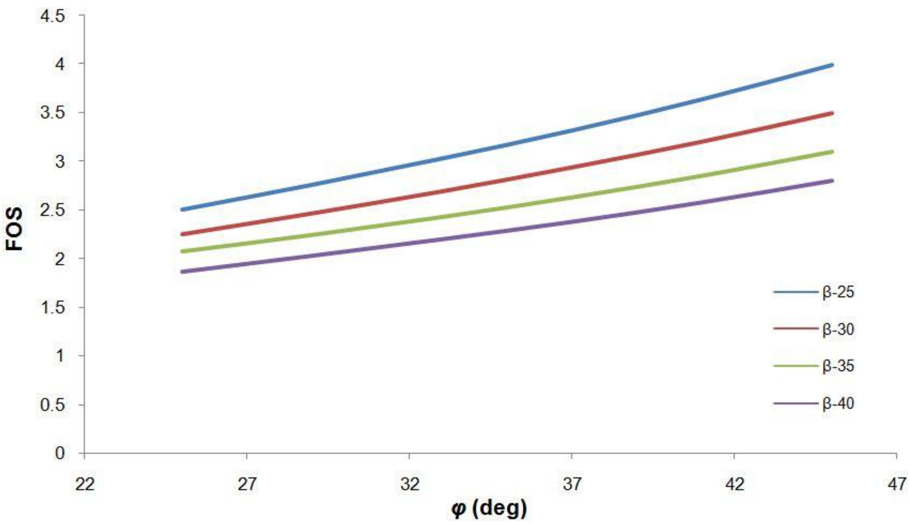


Fig. 2 Relation between stability of slopes and angle of internal friction of soils ϕ

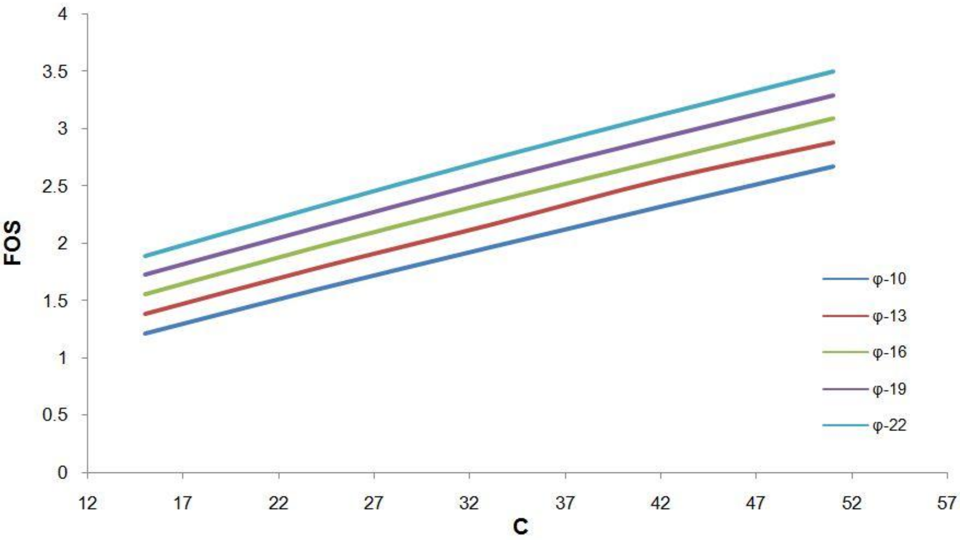


Fig. 3 Relation between stability of slopes and cohesion of soils

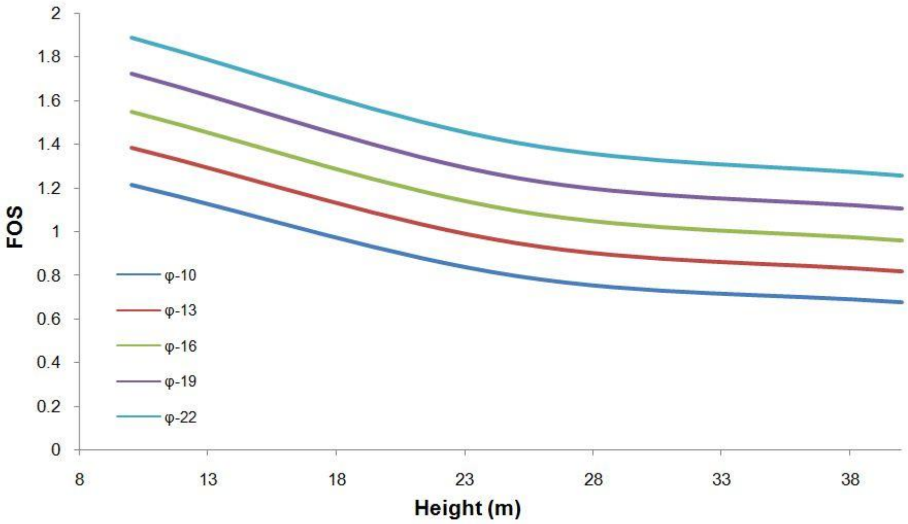


Fig. 4 Relation between stability of slopes and height of the soils

the raw data prepared using Michalowski stability chart. As the development of MR model requires huge raw data, data ranges from Table 1 are considered for the development of MR equation. Before developing the equation, it is first required to verify whether the key assumptions of MR equation are satisfied or not.

First, to check the linearity assumption between dependent and independent variables, scatter plots have been prepared. Figs. 1-5 show the linear effects between dependent and independent variables. Here the dependent variable is FOS and the independent variables are C, ϕ, β, γ and H. Fig. 1 shows the relation between FOS and γ by varying ϕ while keeping the other independent variables C = 15, β = 25, H = 10 constant. Similarly Fig. 2 is show the relation between FOS and ϕ by varying β while keeping the other independent variables C = 10, γ = 14, H = 5 constant. Fig. 3 shows the relation between FOS and C by varying ϕ while keeping the other independent variables γ = 17, β = 25, H = 10 constant. Fig. 4 shows the relation between FOS and H by varying ϕ while keeping the other independent variables C = 15, β = 25, γ = 17 constant. Fig. 5 shows the relation between FOS and β by varying ϕ while keeping the other independent variables C = 24, γ = 17, H = 10 constant. It can be seen from the figures (Figs. 1-5) that all the independent variables are linear to the dependent variable, thereby the first assumption is satisfied.

To verify the second assumption, histogram with a fitted normal curve is plotted and is shown

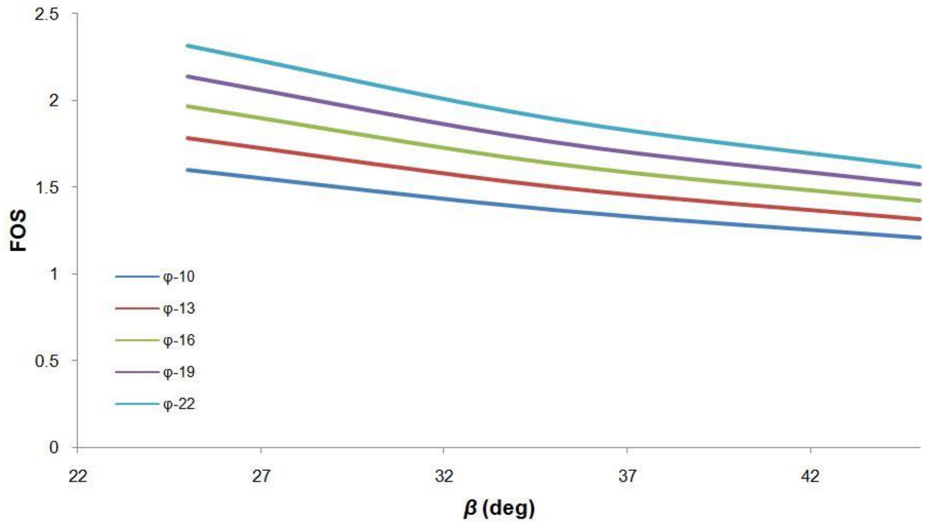


Fig. 5 Relation between stability of slopes and slope angles

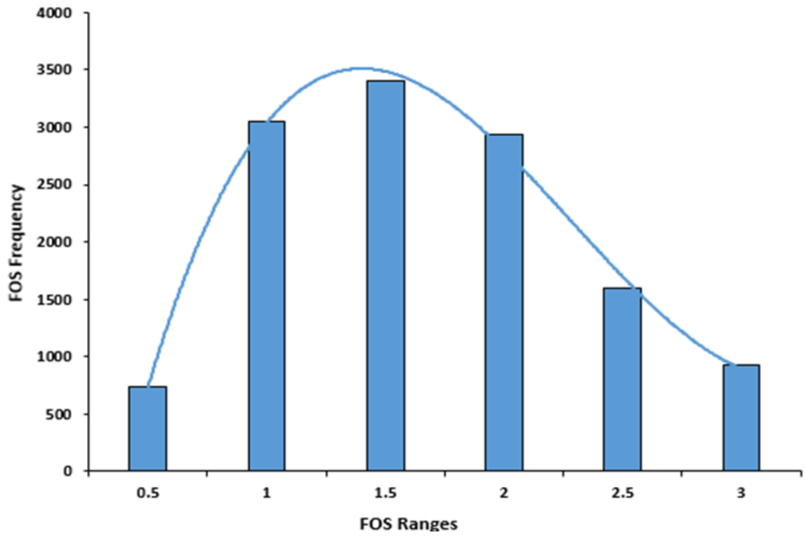


Fig. 6 Normal distribution plot for dry cases

Table 3 Descriptive statistics of the dependent variable

Descriptive statics	
Mean	1.418
Standard error	0.006
Median	1.318
Mode	1.248
Standard deviation	0.628
Sample variance	0.394
Kurtosis	-0.678
Skewness	0.464
Minimum	0.3
Maximum	3.12
Count	14112

in Fig. 6. It can be seen from the figure that the dependent variable FOS is normally distributed. The descriptive statistic details of this dependent variable are presented in Table 3.

Next to check the presence of multicollinearity in the data (i.e., 3rd assumption), Pearsons matrix is used. Multi collinearity occurs when the independent variables are not independent from each other. The collinearity ranges are between 1 to -1. The value ±1 implies that the variables are strongly related whereas ‘0’ implies that there no relation between the variables. Whereas the negative sign implies strong correlation between those variables but in opposite direction. Correlation coefficients among all the independent variables are obtained and given in Table 4. It can be seen from the table that there is negligible or very less multicollinearity between all the independent variables.

The last assumption of homoscedasticity refers to equal variance of errors across all the levels of the independent variables (Osborne and Waters 2002). It means, the researchers assume that the errors spread out consistently between the variables. Homoscedasticity can be checked by visual examination of a plot of the standardized residuals with the regression standardized predicted values (Osborne and Waters 2002). Ideal distribution or random distribution of residuals around zero (the horizontal line) represents even distribution (Osborne and Waters 2002). The scatter plot for the considered dry cases is shown in Fig. 7. From the figure, it can be seen that all the residual errors are uniformly scattered around zero thus verifying the condition.

After verifying all the key assumptions, MR model for dry slopes has been developed using SPSS software. The developed MR model is presented in Eq. (1).

Table 4 Multicollinearity of independent variables

	γ	ϕ	C	H	β
γ	1.000	-0.010	-0.026	0.021	0.026
ϕ	-0.010	1.000	.049	-0.085	-0.169
C	-0.026	0.049	1.000	-0.104	-0.061
H	0.021	-0.085	-0.104	1.000	0.113
β	0.026	-0.169	-0.061	0.113	1.000

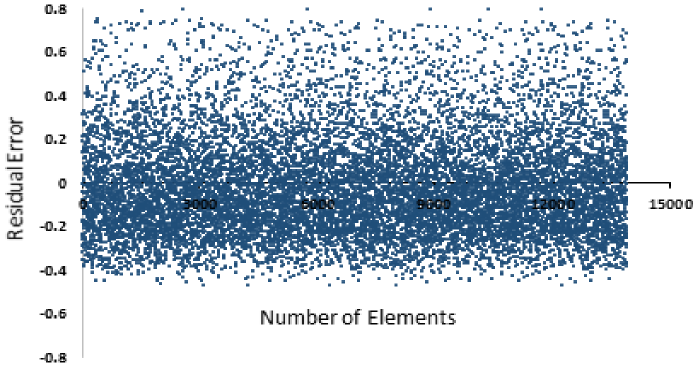


Fig. 7 Homoscedasticity residual plot for dry cases

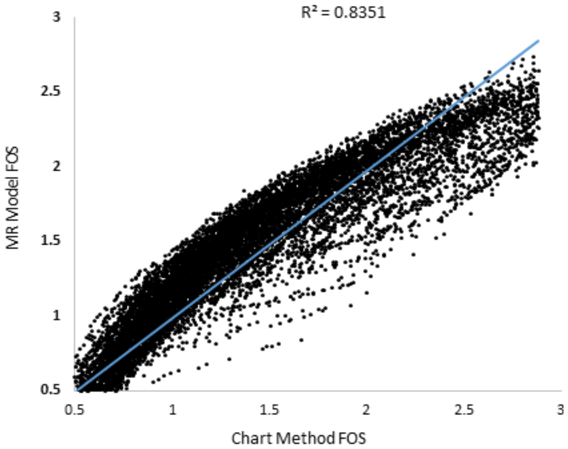


Fig. 8 Correlation plot of training data for dry cases

$$FOS = C * 0.0169 + \phi * 0.0208 - \beta * 0.0371 - H * 0.0371 + \gamma * 0.0208 + 2.4727 \quad (1)$$

Performance of the developed multiple regression model is verified using the data that is used in developing the MR model (Table 1) and the correlation plot is plotted. Fig. 8 shows the correlation plot between FOS obtained from the developed MR model and the FOS determined using stability chart method.

The statistical estimate, Coefficient of determination R^2, shows the correlation between the two FOS and is calculated using Eq. (2). R^2 value of 1 indicates that the FOS estimated using MR model is exactly same as the FOS obtained from the chart method. However this is an ideal case and as per Smith (1986) any R^2 value greater than 0.8 is considered to be good.

$$R^2 = \left[\frac{N\sum y * y' - (\sum y)(\sum y')}{\sqrt{[N\sum y^2 - (\sum y)^2][N\sum y'^2 - (\sum y')^2]}}\right]^2 \quad (2)$$

$$MSE = \frac{1}{N}\sum_{N}^{1}(y - y')^2 \quad (3)$$

Value Account For (VAF) implies variations, indices is calculated to control the performance of the prediction capacity of developed model in the study if VAF is close to 100% its error is less and is calculated using Eq. (4).

$$VAF = 100\left[1 - \frac{var(y - y')}{var(y)}\right] \tag{4}$$

Root mean square error (RMSE) is a frequently used to measure the difference between values predicted by a developed model and the values actually observed from the real field that is being observed. These individual differences are also called residuals, and the RMSE serves to aggregate them into a single measure, it also amplifies the error, it clearly elevates errors and is calculated using Eq. (5).

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y - y')^2} \tag{5}$$

Where, y = FOS from MR Method; y' - FOS from Stability chart method; N - Total no of cases; var () is the variance.

The calculated Coefficient of determination R^2 value is found to be 0.835, which is indicating good correlation between the two values thus verifying the developed MR model (Smith 1986).

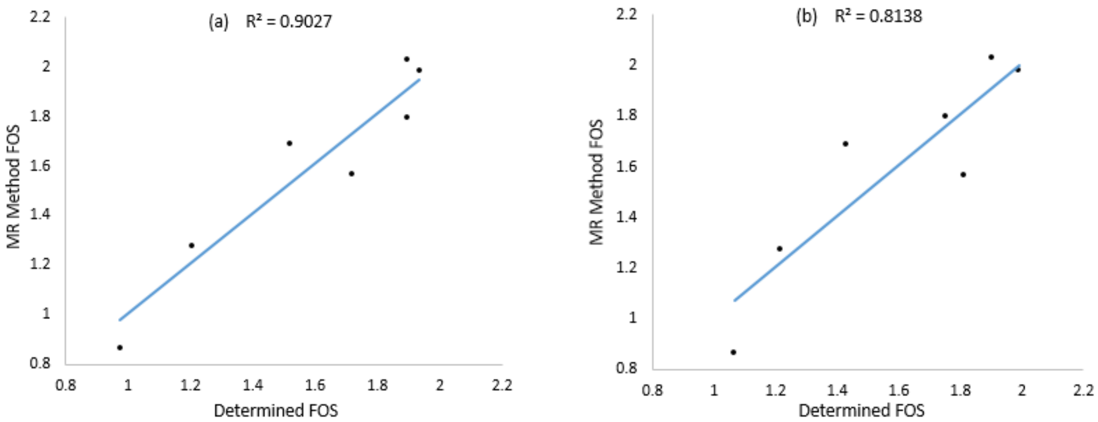
3.1.1 Application of MR model to real field data

In this section, the effectiveness of the developed MR model has been validated for real-world application using the real field data collected from the literature (Wang *et al*. 2005). Details of the real field data is presented in Table 5. The predicted FOS values using the developed MR model are given in last column of Table 5. In order to validate the model, the FOS using possible convectional limit equilibrium methods is recalculated and then compared it with the predicted FOS using the developed MR model. The data correlation plot for the both M.P method and Chart

Table 5 Real field data properties collected from literature for dry slopes

S. No	H (m)	β^0	C (kPa)	ϕ^0	γ (kN/m^3)	FS_{Bis}	FS_{Fel}	$FS_{M.P}$	FS_{Jan}	FS_{Chart}	FS_{MR}
1	8.23	35	26.34	15	18.68	1.789	1.733	1.783	1.7	1.72	1.516
2	10	30	10	35	22.4	1.908	1.814	1.903	1.796	1.897	2.126
3	20	30	10	30.3	21.4	1.433	1.377	1.43	1.37	1.518	1.768
4	30.5	20	14.36	25	18.84	1.752	1.704	1.751	1.7	1.894	1.847
5	40	30	16.28	26.5	20.6	1.22	1.172	1.216	1.17	1.206	1.346
6	50	45	20	36	22	1.07	1.029	1.067	1.02	0.975	1.002
7	12	40	20	40	21	2	1.909	1.99	1.886	1.938	1.978

FS_{MR}: Factor of safety predicted by multiple regression model
FS_{Chart}: Factor of safety obtained from stability chart
FS_{Fel}: Factor of safety calculated using Fellenius Method
FS_{Bis}: Factor of safety calculated using Bishop Method
FS_{Jan}: Factor of safety calculated using Janbu Method
$FS_{M.P.}$: Factor of safety calculated using Morgenstern Price Method



(a) FOS by chart method (b) FOS by M-P method

Fig. 9 Correlation plot between the FOS estimated by MR and other methods for real field data for dry cases

method are shown in Fig. 9. The calculated Coefficient of determination R^2 values is found to be 0.90 & 0.813, which is indicating good correlation between the two values thus verifying the developed MR model.

4. Development and application of MR model to saturated & partially saturated slopes

A separate MR equation has been developed for saturated and partially saturated cases. A total of 15000 cases have been considered for the development of MR equation for both the cases. Before developing the multiple regression equation, the data is verified for all the four key assumptions mentioned in the Multiple Regression Model section.

From the Figs. 1-5 and 10, it can be seen that the relation between the dependent variable FOS

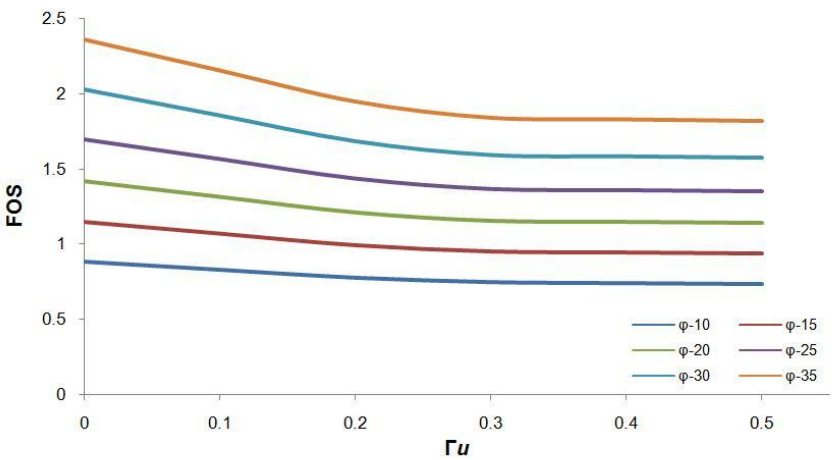


Fig. 10 Relation between stability of slopes and pore water pressure Γ*u*

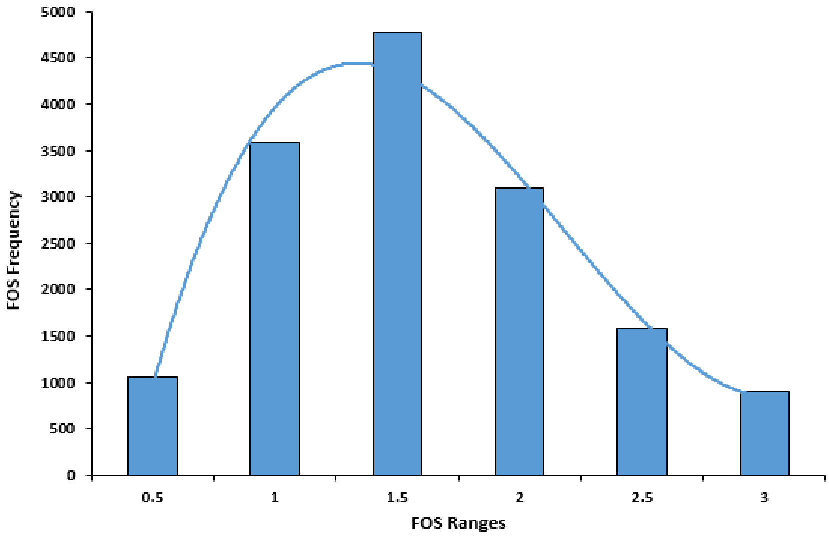


Fig. 11 Normal distribution for wet case

Table 6 Descriptive statics of dependent variable

Descriptive statics	
Mean	1.304
Standard error	0.005
Median	1.178
Mode	0.813
Standard deviation	0.641
Sample variance	0.41
Kurtosis	-0.544
Skewness	0.603
Minimum	0.3
Maximum	3.183
Count	15000

and the independent variables is considerably linear and thus satisfying the required linearity assumption. Fig. 10 shows the relation between FOS and Γ_u by varying ϕ, while keeping the other independent variables $C = 15$, $\beta = 25$, $\gamma = 17, H = 22$ constant. Normality of a dependent variable is verified using normal distribution curve. Fig. 11 shows the normal distribution plot of the dependent variable. It can be clearly seen from the figure that the dependent variable, FOS is normally distributed and there by satisfying the second assumption. Descriptive statistics of the dependent variable are also obtained and are presented in Table 6.

Next Pearson's matrix is used to verify multicollinearity assumption. The correlation coefficients among all the independent variables are obtained and tabulated in Table 7. As per (Osborne and Waters 2002) these values must be lower than 0.19 to neglect dependency among the variables. It can be seen from the table (Table 7) that there is negligible multicollinearity between

Table 7 Multi collinearity of independent variables

	H	β	C	ϕ	γ	Γ_u
H	1.000	0.025	0.033	-0.070	-0.110	0.111
β	0.025	1.000	0.013	-0.041	-0.035	0.049
C	0.033	0.013	1.000	-0.021	-0.030	0.014
ϕ	-0.070	-0.041	-0.021	1.000	0.053	-0.130
γ	-0.110	-0.035	-0.030	0.053	1.000	-0.068
Γ_u	0.111	0.049	0.014	-0.130	-0.068	1.000

all the independent variables and there by verifying the third assumption of multicollinearity.

To verify homoscedasticity assumption, scatter plot for the considered cases is plotted and shown in Fig. 12. It can be seen from the figure that all the residual errors are uniformly scattered around zero, representing even distribution and thus verifying our last assumption.

After validating all the four key assumptions, MR equation for wet cases has been developed and is provided in Eq. (6).

$$FOS = C * 0.0169 + \phi * 0.0334 - \beta * 0.0371 - H * 0.02046 \\ -\gamma * 0.0208 - \Gamma_u * 0.6409 + 2.4727 \quad (5)$$

To verify the developed MR equation, the FOS of the data (15,000 cases) that is used for developing the MR equation (Table 2) is calculated using chart method and a correlation plot is plotted against the FOS obtained using the developed MR model. The correlation plot is shown in Fig. 13. The statistical estimate, Coefficient of determination R^2 value is calculated and is found to be 0.818, which is indicating good correlation between the two FOS and there by verifying the developed MR model.

4.1 Application of developed MR equation to real field data

After successful development of the MR model, it is further validated for real-world application using real field data (18 cases) collected from the literature (Sakellariou and Ferentinou 2005). Details of the real field data for wet cases is presented in Table 8. The predicted FOS values using

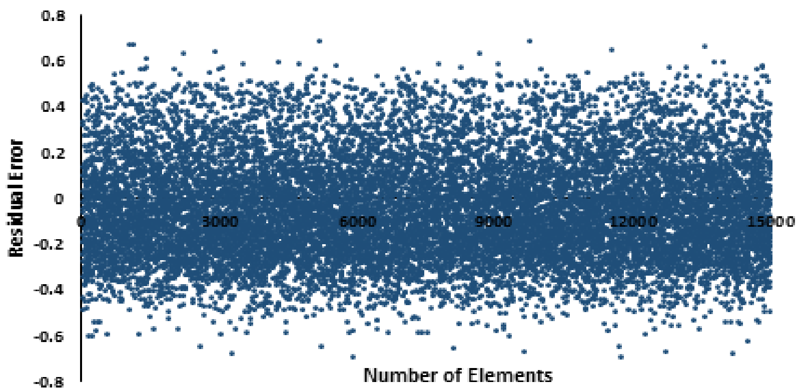


Fig. 12 Homoscedasticity residual plot for wet cases

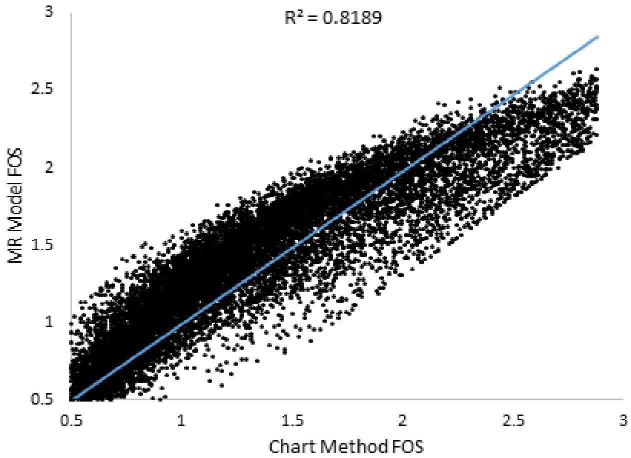
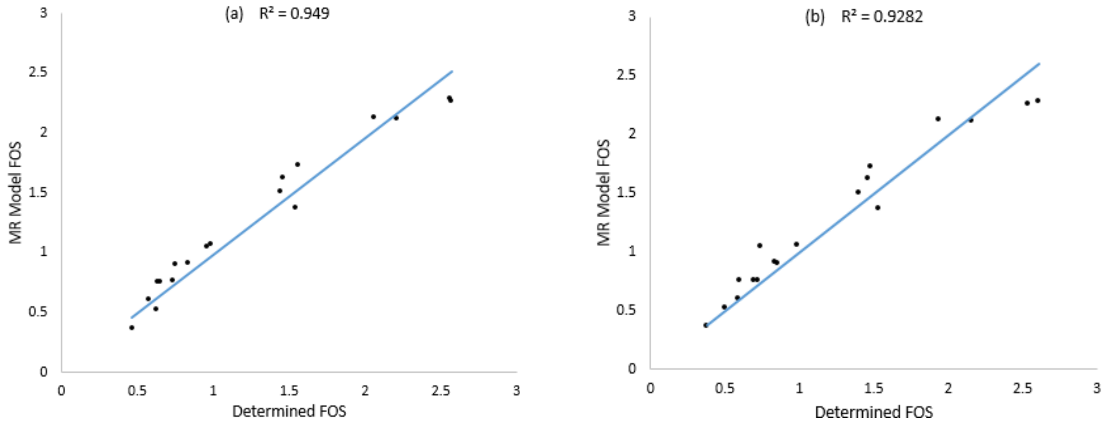


Fig. 13 Correlation plot for training data for wet cases

Table 8 Real field data collected from literature for testing wet soil slopes

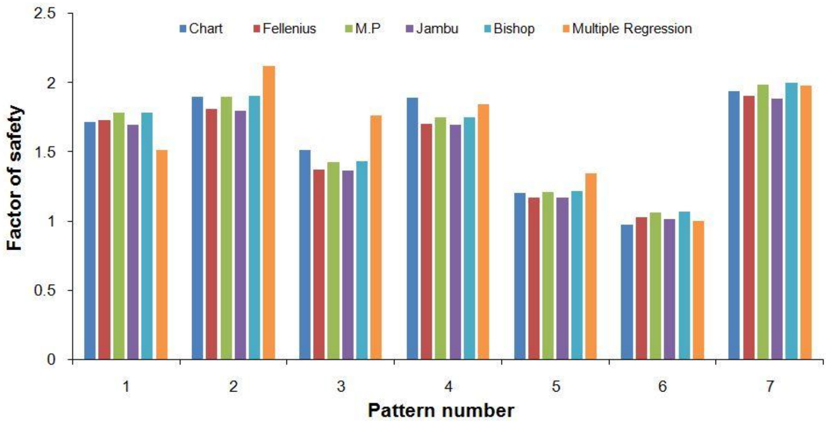
S. No	H (m)	β^0	$C\ (kPa)$	ϕ^0	$\gamma(\text{kN}/m^3)$	Γ_u	FS $_{\text{Bis}}$	FS $_{\text{Fell}}$	FS $_{\text{M.P}}$	FS $_{\text{jamb}}$	FS $_{\text{Chart}}$	FS $_{\text{MR}}$
1	12	49	45	25	21	0.3	1.53	1.494	1.53	1.529	1.544	1.377
2	12	40	30	35	21	0.4	1.479	1.36	1.474	1.363	1.562	1.728
3	12	40	35	28	21	0.5	1.401	1.315	1.397	1.317	1.443	1.514
4	6	34	10	29	20	0.3	1.461	1.318	1.459	1.327	1.457	1.618
5	15	30	40	30	20	0.3	1.944	1.786	1.94	1.775	2.064	2.125
6	14	25	45	25	18	0.3	2.614	2.536	2.61	2.492	2.556	2.29
7	11	35	30	35	19	0.2	2.165	2.021	2.161	1.999	2.212	2.104
8	10	40	40	40	20	0.2	2.54	2.398	2.534	2.382	2.572	2.254
9	37	29.2	24.8	21.3	18.85	0.5	0.74	0.67	0.74	0.688	0.956	1.051
10	37	34	10.34	21.3	18.85	0.3	0.588	0.506	0.597	0.523	0.631	0.756
11	50	25	30	10	18.8	0.1	0.855	0.836	0.854	0.826	0.755	0.909
12	50	25	25	10	18.8	0.2	0.72	0.7	0.72	0.68	0.654	0.760
13	50	25	20	10	18.8	0.3	0.593	0.572	0.592	0.572	0.576	0.611
14	50	25	10	10	19.1	0.4	0.38	0.358	0.38	0.362	0.464	0.371
15	50	30	30	20	18.8	0.1	0.986	0.925	0.984	0.915	0.982	1.058
16	50	30	25	20	18.8	0.2	0.838	0.774	0.836	0.774	0.834	0.909
17	50	30	20	20	18.8	0.3	0.692	0.622	0.692	0.634	0.733	0.76
18	50	30	10	20	19.1	0.4	0.492	0.417	0.496	0.445	0.622	0.520

FS_{MR}: Factor of safety predicted by multiple regression model
FS_{Chart}: Factor of safety obtained from stability chart
FS_{Fel}: Factor of safety calculated using Fellenius Method
FS_{Bis}: Factor of safety calculated using Bishop Method
FS_{Jan}: Factor of safety calculated using Janbu Method
$FS_{M.P.}$: Factor of safety calculated using Morgenstern Price Method

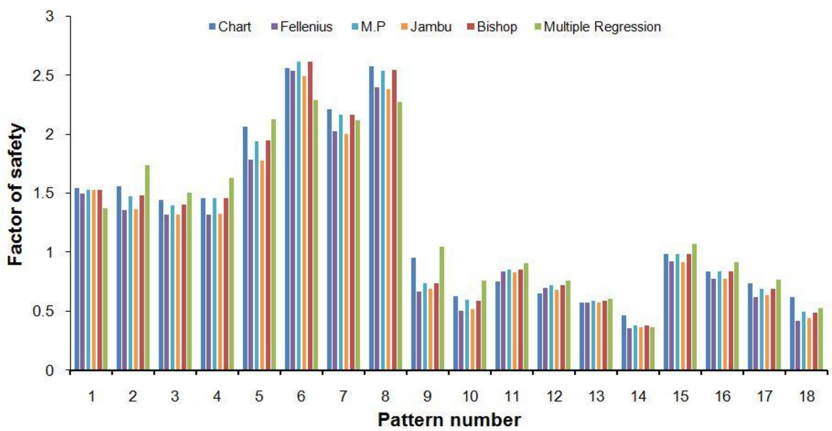


(a) FOS by chart method (b) FOS by M-P method

Fig. 14 Correlation plot between the FOS estimated by MR and other methods for real field data for wet cases



(a) For dry cases



(b) For wet cases

Fig. 15 Variation of the determined factor of safety using different various methods the developed MR model (Eq. (6)) for all the real field cases are given in the last column of Table 8. In order to validate the model, the FOS for all the real field cases is recalculated using possible conventional limit equilibrium methods and then compared with the predicted FOS using the developed MR model. The calculated FOS values using different LEMs are also given in Table 8. For comparison, the data correlation plot of the FOS obtained using the developed MR model with the chart method and M.P method has been plotted and is shown in Fig. 14. The calculated Coefficient of determination R^2 values is found to be 0.949 & 0.928, which is indicating good correlation between the two values thus verifying the developed MR model. Further to see the results more clearly, a comparison of variation of the determined factor of safety using different methods for dry as well as wet cases is shown in Figs. 15(a)-(b) respectively.

5. Conclusions

To demonstrate potential of the developed multiple regression models (both dry and wet cases) in predicting stability condition of the slopes, they are applied to the corresponding real field cases of dry as well as wet cases collected from the literature (Tables 5 and 8). For comparison, the FOS of the real field cases is also calculated using different limit equilibrium methods and chart method. Correlation plots are shown for the FOS estimated using the developed MR equation and FOS determined using Chart method & Morgenstern Price method (Figs. 9 and 14). To verify the accuracy of the developed MR models, statistical estimates Coefficient of determination and Mean Squared Error (MSE) are used.

Lower coefficient of determination is observed for M-P method in comparison to Chart method, for both dry and wet cases. This is primarily due to the fact that the developed multiple regression equation is based on the data prepared using Chart method. The calculated coefficient of determination for both dry and wet cases with M-P method is found to be 0.813 and 0.928 respectively. These correlation coefficient values showing an excellent correlation between the developed multiple regression model and Morgenstern Price method for both the cases and there by validating the potential of the developed regression models to assess the stability condition of real slopes.

The statistical estimates R^2, MSE, VAR and RMSE for the developed MR model with respect to various limit equilibrium methods and chart method for dry as well as wet slopes have been calculated using the Eqs. (3)-(4) and (5) and presented in Tables 9 and 10 respectively. Also the variation of all these statistical estimates between various limit equilibrium methods have been calculated and given in Table 11.

From the MSE comparison tables of dry and wet cases (Table 9 and 10), the minimum MSE is observed between MR model and chart method in both the cases. MSE obtained in both the cases between MR and other methods is also considerably good. From the MSE variation comparison table between various standard LEMs (Table 11), the maximum variation of MSE is observed between Fellenius and chart method which is 0.02 while minimum variation is observed between Bishop and M.P method which is 0.0002. And the MSE observed between Fellenius and chart method is less than the MSE observed between the developed MR models and chart method which is 0.01. However, from the Tables 9, 10 and 11, it can be clearly seen that the MSE of the developed MR models with respect to limit equilibrium methods for both dry and wet cases is considerably good. From the comparison of VAF variation of dry and wet cases (Tables 9 and 10), the maximum VAF is observed between MR model and chart method in both the cases. VAF

Table 9 Comparison of the developed MR model with conventional LEMs for dry cases

MR to LEM methods		MSE	RMSE	R^2	VAF
MR	Bishop	0.0271	0.1646	0.7337	68.67
MR	M.P	0.0271	0.1646	0.7363	68.72
MR	Fellenius	0.0330	0.1816	0.7185	63.97
MR	Jambu	0.0338	0.1838	0.7381	64.76
MR	Chart	0.0142	0.1191	0.8275	82.03

Table 10 Comparison of the developed MR model with conventional methods for wet cases

MR to LEM methods		MSE	RMSE	R^2	VAF
MR	Bishop	0.0274	0.1655	0.9278	94.52
MR	M.P	0.0272	0.1649	0.9282	94.54
MR	Fellenius	0.0432	0.2078	0.8995	93.24
MR	Jambu	0.0413	0.2032	0.907	93.48
MR	Chart	0.0202	0.1421	0.949	95.64

Table 11 Comparison between various conventional LEM methods

LEM methods		MSE	RMSE	R^2	VAF
Fellenius	Bishop	0.006	0.0774	0.997	99.53
Bishop	M.P	0.0002	0.0141	1	99.99
M.P	Fellenius	0.007	0.0836	0.9968	99.59
Chart	Fellenius	0.020	0.1414	0.9757	97.87
M.P	Jambu	0.008	0.0894	0.9975	99.46

obtained in both the cases between MR and other methods is also considerably good. From the comparison of VAF variation between various standard LEMs (Table 11), the minimum variation of VAF is observed between Fellenius and chart method which is 97.87. However, from the Tables 9, 10 and 11, it can be clearly seen that the VAF of the developed MR models with respect to limit equilibrium methods for both dry and wet cases is considerably good.

From the comparison of RMSE variation of dry and wet cases (Tables 9 and 10), the minimum RMSE is observed between MR model and chart method in both the cases. RMSE obtained in both the cases between MR and other methods is also considerably good. From the RMSE variation comparison table between various standard LEMs (Table 11), the maximum variation of RMSE is observed between Fellenius and chart method which is 0.1414 while minimum variation is observed between Bishop and M.P method which is 0.014. And the RMSE observed between Fellenius and chart method is less than the RMSE observed between the developed MR models and chart method which is 0.1191. However, from the Tables 9, 10 and 11, it can be clearly seen that the RMSE of the developed MR models with respect to limit equilibrium methods for both dry and wet cases is considerably good.

From these results, we can see that the developed MR models for both dry as well as wet cases are able to well predict the FOS of slopes similar to the conventional limit equilibrium methods and there by validating the developed models for the estimation of critical FOS to identify any slope instabilities of real world problems. Further the developed MR models here are not based on any region specific data and cover wide range of parameter variations. And hence, the developed MR models can be directly applied to any real slopes.

Although non linearity is observed from the regression plots of the training data for both the cases as seen from Figs. 8 and 13, the effect of nonlinearity is not considered in this study. In the present study we are restricted to develop linear multiple regression equation. However, considering the effect of these nonlinearity’s into account, it can further be improved as future scope of the study.

Acknowledgments

The research described in this paper was financially supported by the Ministry of Human Resources Development (MHRD), Government of India.

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