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Advanced discretization of rock slope using block theory within the framework of discontinuous deformation analysis

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Abstract. Rock is a heterogeneous material, which introduces complexity in the analysis of rock slopes, since both the existing discontinuities within the rock mass and the intact rock contribute to the degradation of strength. Rock failure is often catastrophic due to the brittle nature of the material, involving the sliding along structural planes and the fracturing of rock bridge. This paper proposes an advanced discretization method of rock mass based on block theory. An in-house software, GeoSMA-3D, has been developed to generate the discrete fracture network (DFN) model, considering both measured and artificial joints. Measured joints are obtained from the photogrammetry analysis on the excavation face. Statistical tools then facilitate to derive artificial joints within the rock mass. Key blocks are searched to provide guidance on potential reinforcement measures. The discretized blocky system is subsequently implemented into a discontinuous deformation analysis (DDA) code. Strength reduction technique is employed to analyze the stability of the slope, where the factor of safety can be obtained once excessive deformation of slope profile is observed. The combined analysis approach also provides the failure mode, which can be used to guide the choice of strengthening strategy if needed. Finally, an illustrated example is presented for the analysis of a rock slope of 20 m height inclined at 60° using combined GeoSMA-3D and DDA calculation.

Keywords: rock slope; stability analysis; block theory; strength reduction technique; discontinuous deformation analysis (DDA); artificial joints

1. Introduction

Fractures distribute randomly within the rock mass due to complex geological conditions (e.g., weathering, stress history and temperature variations, etc.). On the basis of size, structural planes can be categorized into two groups, where faults and dykes are large-scale geological structures and joints and bedding planes are medium-scale discontinuities. These discontinuities partition the rock mass into multiple blocks with various size and shape. The strength of jointed rock mass is

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therefore reduced significantly compared to that of intact rock (Ni *et al*. 2016a), since individual blocks have the potential to move and introduce stress concentration to rock bridges. The behavior of jointed rock mass is dominated by the distribution of joints, where their spatial randomness adds complexity for stability analysis of rock structures. Geomaterials often present material nonlinearity at large deformations (i.e., involving excessive mesh distortion for mesh-based analysis). It is difficult to select an appropriate constitutive model for rock mass as it presents heterogeneity and anisotropy heavily due to joints. In addition, the failure mechanism of rock structures is usually unpredictable (i.e., catastrophic or brittle). All these features make conventional numerical methods (i.e., standard finite element approaches) largely unsuitable for the analysis of rock engineering projects, such as slope stability.

Numerical modeling technique has been improved significantly over the past years, to take into account large deformation problems for rock slopes. For example, Zhao *et al*. (2008) modeled the influence of wave propagation on fractured rock mass using the universal distinct element code (UDEC) and summarized the parameter calibration process. However, the UDEC technique is relative new, and further assessment of its efficacy by field observations or laboratory tests is needed. Wang *et al*. (2013a) presented an investigation using 2D numerical models based on the discrete element code PFD2D for analyzing acoustic emission of intact rock. Their numerical calculations have been found to be comparable to those measured in the laboratory. However, it only demonstrated that element tests could be reproduced well. Further application of discrete element code in slope stability analysis requires calibration of numerical models using experiment evidence. Improved finite element schemes are more theoretically sound to investigate problems of stress/fracture propagation within the rock mass. The developed methods include the Fast Lagrangian Analysis of Continua, FLAC (Ni *et al*. 2016b), the Realistic Failure Process Analysis (RFPA) code (Wang *et al*. 2009) and the implementation of the strength reduction method with displacement finite-element method (SRFEA) in Plaxis (Tschuchnigg *et al*. 2015a, b). A hybrid finite-/discrete-element approach (e.g., ELFEN) can also provide reasonable evaluation for rock slopes (Stead *et al*. 2006), but the involved complexity may hinder its application in practice.

Rock mass consists of both intact rock medium and discontinuities, so a method that can simulate blocky systems is of significant interest to engineers. After the initial work of Goodman and Shi (1985), Lin and Fairhurst (1988) extended the block theory by considering the rotational motion of blocks. A discrete fracture network simulation has been performed subsequently to analyze the stability of tunnels (Shi and Goodman 1989). A novel automatic identification system was developed for 3D multi-block systems (Ikegawa and Hudson 1992), and similar studies can be referred to the key-group method (Yarahmadi Bafghi and Verdel 2003, 2004). Block theory has been applied to identify key block classification based on polyhedral block modeling (Elmouttie *et al*. 2010a, b), which can further be used in discrete fracture network modeling (Elmouttie and Poropat 2012, Ni *et al*. 2016a) for accurate representation of the joints/fractures for use in limit equilibrium analysis. An alternative of discontinuous deformation analysis (DDA) proposed by Shi (1992) has become a popular analytical approach in geotechnical engineering in recent years. Kim *et al*. (1999) studied the effect of water, excavation and rock reinforcement using the DDA technique. The effectiveness of DDA has been evaluated by Hatzor and Feintuch (2001) for use in prediction of dynamic block displacement. The ability of DDA to capture the kinematics of rock slopes has also been demonstrated (MacLaughlin *et al*. 2001).

In this paper, a novel approach is developed based on the combination of block theory and discontinuous deformation analysis technique. This concept has been initially proposed by Ning *et al*. (2011), where they incorporated artificial joints in rock mass to generate an assembly of small rock blocks. This blocky system was subsequently analyzed using a DDA code. The strength reduction technique was used to lower the mechanical strength of joints. However, their program was evaluated against continuum-based deformation calculations for static problems. A generalized definition of parameters for static problems was then considered and employed directly to analyze rock-blasting problems. Block theory was not considered during the advanced discretization process at all (Ning *et al*. 2011), where artificial joints could introduce bias in the calculation if mesh dependency was not studied for different problem (sensitive analysis should be conducted to determine mesh refinement). The current study is to deal with the issue of mesh dependency of Ning *et al*. (2011), where block theory is employed to generate the blocky system using an in-house software GeoSMA-3D (Geotechnical Structure and Model Analysis). Therefore, the 3D fracture attributes (i.e., persistence and density) can be extrapolated from 2D attributes (scan-line mapping) or 1D attributes (borehole logging) based on photogrammetry technique, although sampling bias may affect measurements of 1D and 2D attributes. For a rock slope, key blocks with a factor of safety less than unity could be identified on the slope surface based on limit equilibrium analysis. At these locations, urgent support and reinforcement should be suggested. Apart from these critical spots, 2D plane strain analysis is conducted using the DDA code for the blocky system obtained from the GeoSMA-3D analysis. Strength reduction technique facilitates to determine the corresponding factor of safety. In the end, an illustrative example is presented to show the ability of the proposed strategy of combined analysis.

2. Basic principles of block theory

Block theory (Goodman and Shi 1985) is developed based on geometric methods, such as topology and set theory, to identify blocks within the rock mass. Mapping of joints on the rock surface is required. However, the partition process of blocky system not only needs the information of mapped joint trace, but is also facilitated by mathematical modeling of artificial joints within the rock medium. The removability of individual blocks is subsequently assessed based on limit equilibrium analysis, which provides the calculation for stability. The identification of key blocks (those with a factor of safety less than unity) on the slope surface is critical, since measures can then be proposed and implemented at specific locations to improve the stability of the slope.

During the joint mapping process, if the total number of identified discontinuities is n, there will be at least $2n$ blocks (i.e., intersected regions between joints) within the rock mass. The dimension of blocks needs to be evaluated, where the finiteness theorem facilitates to determine blocks with finite size for subsequent analysis. A finite block is defined when the intersection of the joint pyramid (*JP*) and the excavation pyramid (*EP*) is empty.

$$JP \cap EP = \Phi \tag{1}$$

Furthermore, the joint pyramid (*JP*) needs to be a subset of the space pyramid (*SP*)

$$JP \subset SP \quad (2$$

Once all finite blocks are screened, the Riemann’s theorem on removable singularities is used to calculate the removability of these finite blocks.

$$\left.\begin{array}{l} JP \neq \Phi \\ EP \bigcap JP = \Phi \end{array}\right\} \tag{3}$$

Alternatively, the Riemann's theorem for removability of blocks can be written in vector form

$$\left.\begin{array}{l} n_i \cdot S \geq 0, (i = 1, \cdots, N) \\ w \cdot S > 0 \end{array}\right\} \tag{4}$$

where S is the potential displacement vector if the block is removable; N is the number of fractures that form the block; n_i is the unit normal vector (towards the interior) of the block and w is the direction of driving force.

In the following, the limit equilibrium method (Chugh 1986) is adopted to calculate the factor of safety for all blocks. If the computed factor of safety of a block is less than a specified value (e.g., unity), this block is considered as a 'key block'.

3. Fracturing characterization for discretization of blocky system

The excavation face of a rock slope intersects with existing structural planes, so discrete blocks of different size and shape could be exposed on the slope face. Therefore, there is a possibility that a newly excavated rock slope fails due to the instability of these 'exposed' rock blocks (Goodman and Shi 1985, Shi and Goodman 1989). The geometry of individual blocks formed by discontinuities within the rock mass and the slope face becomes critical (Kalenchuk *et al*. 2006). Nevertheless, existing methods for simulation and characterization of fractures are limited by survey techniques (Ferrero *et al*. 2011). Sampling bias could influence the accuracy of 1D or 2D measurements (e.g., scan-line mapping or borehole logging), which may cause unrealistic estimates of 3D fracture attributes (i.e., persistence and density) from extrapolation. New techniques, such as photogrammetry (Lambert *et al*. 2012), are therefore attractive to perform comprehensive characterization of joints for simulation of blocky system. Moreover, advanced 3D modeling of jointed rock mass is of great value for rock slope engineering, such as polyhedral modeling technique (Elmouttie *et al*. 2010a, b).

In the previous work of the authors (Wang *et al*. 2011), the photogrammetry technique (a non-contact approach) has been used to characterize fractures from pairs of photos of the excavation face captured at different angle. A numerical modeling package – Geotechnical Structure and Model Analysis (GeoSMA-3D) system – is developed based on block theory to discretize the blocky system. An interactive interface enables the users to input and extract data easily in the pre- and post-processing mode, respectively. Geometrical and physical parameters of structural discontinuities within the rock mass can be defined for further simulation of artificial joints. Limit equilibrium analysis is then conducted for screening removable blocks. Key blocks are finally identified and visualized. The developed software package has been successfully applied to different projects comparing to field measurements, such as rock slopes (Wang and Ni 2014) and tunnels (Wang *et al*. 2013b). In this study, determination of key blocks is an initial step for slope stability analysis. The generated blocky system from GeoSMA-3D will be implemented in a discontinuous deformation analysis (DDA) code for further investigation, which corresponds to an advanced discretization process. In the following, the discretization of blocky system from

GeoSMA-3D will be introduced briefly, and further details of the software can be found elsewhere (Wang *et al*. 2011, 2013b, Wang and Ni 2014).

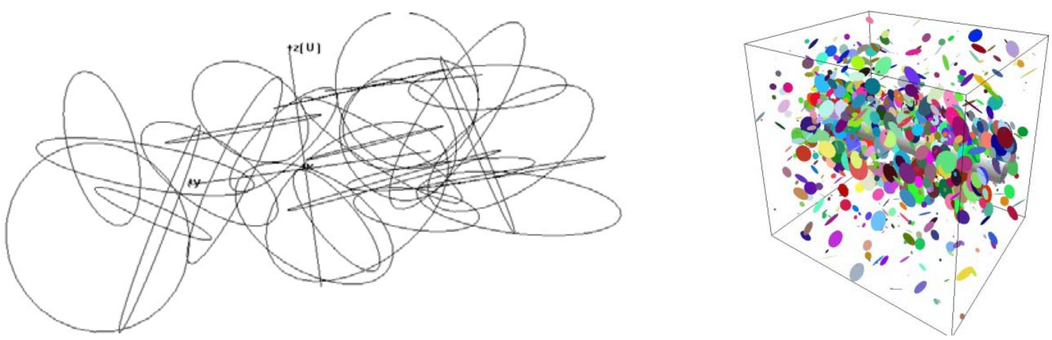
3.1 Discrete fracture network (DFN) simulation

Previous studies (Goodman and Shi 1985, Shi and Goodman 1989) usually assumed that structural discontinuities spread infinitely within the jointed rock mass. This is, however, not true, as joints have finite size in reality. The simplified interpretation of spatial distribution of joints could result in significant difference if finite size of joints is considered correctly during the analysis, since infinite size joint models divide the rock mass into much more blocks. To model joints as realistic as possible, GeoSMA-3D adopts finite size joints, following a disk-shaped distribution (Baecher *et al*. 1977). Hence, a structural discontinuity can be described geometrically in terms of dip direction, dip angle, radius and center coordinates.

Joints on the excavation face can be determined accurately through survey techniques (Mauldon 1998). Statistical tools are normally used to fit the observed results (i.e., joint size, orientation, spatial location) with a mathematical model, which can then be used to interpret the spatial distribution of discontinuities within the rock mass. These random joints are generally described using a set of geometric parameters following a probability distribution function (i.e., uniform distribution, negative exponential distribution, normal distribution and logarithmic normal distribution) (Shen and Abbas 2013). The Monte Carlo simulation method is subsequently employed to generate randomly distributed joints based on probability models (Ni *et al*. 2016a), where the main statistical characteristics of the rock mass should be considered and represented (see Fig. 1).

3.2 Simulation of blocky system

An unbounded DFN model is generated first, which is larger than the size of the rock slope. This is to avoid unrealistic distribution (i.e., rare) of joints at the edge of the simulated blocky system (Ni *et al*. 2016a). The randomly distributed joints (shown in Fig. 1) divide the rock mass into small blocks, which can then be merged to form more complex blocks. Colors are used to distinguish joint groups with different geometric parameters. These random-disk joint models essentially correspond to a number of intersection faces. The joint network needs to be identified from these joint models. Two structural discontinuities produce an intersection edge, while vertices



(a) Spatial distribution of joint center coordinates (b) Random-disk DFN model

Fig. 1 Example of generating a random-disk discrete fracture network (DFN) model in GeoSMA-3D

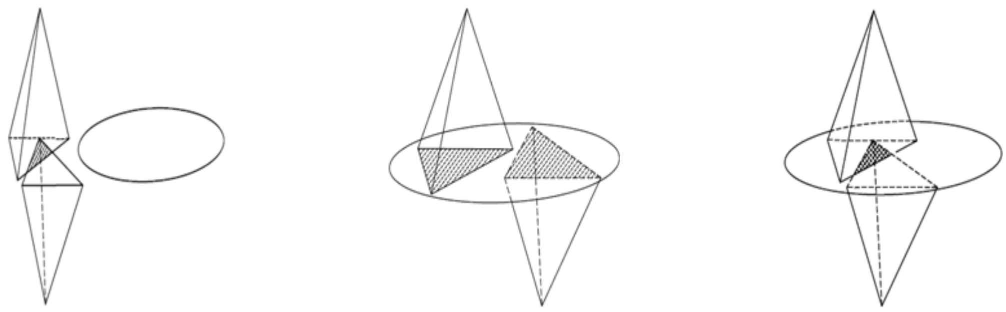


Fig. 2 Different scenarios during the block merging process

are formed by intersection of three or more structural discontinuities. These geometric features finally form polyhedrons or blocks. The following criteria are used in GeoSMA-3D to determine whether a set of structural planes can form a block, and further details of DFN model simulation can be found elsewhere (Wang *et al*. 2011, 2013b, Wang and Ni 2014).

- The number of vertices (V), faces (F) and edges (E) for a given block should satisfy the Euler equation

$$V + F - E = 2 \tag{5}$$

- Every vertex of a block should be formed by intersection of at least 3 faces.
- Every face of a block should intersect with at least 3 other faces.
- Except at vertices, any two edges can never intersect with each other at any point.
- Except those on the face, all other vertices should be on one side of a face (i.e. concave blocks cannot be generated).

After all blocks have been defined, individual blocks can be merged to form more complex blocks, such as concave blocks. The block merging process ensures that structural planes are finite throughout the modeling process. Two adjacent blocks must share at least one common face (i.e., fully or partially) as shown in Figs. 2(a) and (c). Furthermore, the common face of two neighboring polyhedrons does not belong to a disk-shaped structural discontinuity (see Fig. 2(a)). Different scenarios during the block merging process is given in Fig. 2, where blocks in Fig. 2(a) can be merged and blocks in Figs. 2(b) and (c) cannot.

3.3 Identification of key blocks

Removable blocks (i.e., key blocks) are those blocks exposed on the excavation face that have potential to move (i.e., slide) in a certain direction (due to insufficient resistance compared to the driving force). Key blocks must be geometrically removable. For instance, triangular pyramid or pentagonal pyramid cannot move even though the driving force exceeds the resistance, if the block is restrained geometrically with only one vertex exposed. All fundamental principles of block theory must be considered in searching removable blocks, such as the finiteness theorem and the Riemann’s theorem on removable singularities.

After key blocks are identified, algorithms are applied within the GeoSMA-3D to analyze all blocks for a number of properties, including: (1) the number of sliding faces, (2) the area of each

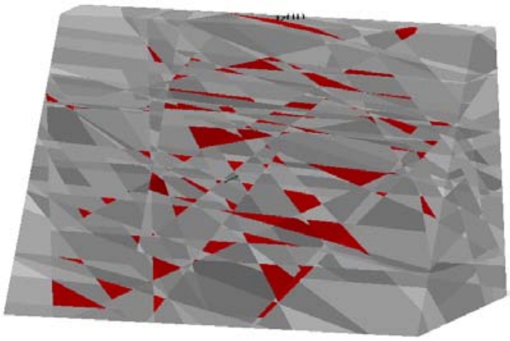


Fig. 3 Example of identified key blocks (in red) for a rock slope

sliding face, and (3) the block volume. Based on the limit equilibrium analysis, the factors of safety for all blocks can be calculated to compare with the allowable value defined in the design guidelines for rock slopes. Fig. 3 shows an example of GeoSMA-3D analysis for a rock slope, where key blocks are identified and denoted in red for ease of differentiation.

4. Discontinuous deformation analysis (DDA) of blocky system based on strength reduction technique

4.1 Basic principles of DDA

Discontinuous deformation analysis (Shi 1992) follows similar principles of finite element method (FEM), where a medium is discretized into multiple small elements/blocks for analysis. The generated discrete fracture network (DFN) model in GeoSMA-3D can be used as the blocky system to be implemented in a DDA code. Deformations/strains are allowed to occur within individual blocks. Besides, each block can slide and move as a rigid body along boundaries/joints. DDA has notable advantages over FEM, namely (a) the discretization of DDA is not continuous, which makes the DDA more suitable for the analysis of large deformation problems; (b) individual blocks can be of any geometry, such as convex or concave; and (c) geometric nonlinearity can be taken into account properly, which may result in a more realistic failure mode with a lower factor of safety.

4.1.1 Displacement variables

The displacements of an arbitrary point within a block can be represented by six displacement variables

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -(y-y_0) & (x-x_0) & 0 & (y-y_0)/2 \\ 0 & 1 & (x-x_0) & 0 & (y-y_0) & (x-x_0)/2 \end{bmatrix} \cdot (u_0, v_0, \gamma_0, \varepsilon_x, \varepsilon_y, \gamma_{xy})^T \tag{6}$$

where u_0 and v_0 are the rigid body displacements of the point (x_0, y_0) within the block; γ_0 is the rotation angle, and ε_x, ε_y and γ_{xy} are the normal and shear displacements of the block respectively.

4.1.2 General equation of blocky system

DDA uses displacement method to solve the force equilibrium by minimizing the total potential energy as a function of external forces and displacements of blocky system. The general equation is thus established as follows

$$\begin{bmatrix} K_{11} & K_{12} \cdots K_{1n} \\ K_{21} & K_{22} \cdots K_{2n} \\ \vdots & \vdots \ \ddots \ \vdots \\ K_{n1} & K_{n2} \cdots K_{nn} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ \vdots \\ D_n \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{Bmatrix} \tag{7}$$

where K_{ij} is the element stiffness matrix (i.e., the global stiffness matrix is 6 by 6, as there are three degrees of freedom: horizontal displacement, vertical displacement and in-plane rotation), D_i is the deformation of block i, and F_i is the load (force) on block i.

4.1.3 Constitutive models for contacts between blocks

For a blocky system, individual blocks must always be in contact with adjacent blocks (i.e., zero tension at contacts). At the same time, two blocks cannot occupy the same space. Three modes of movement at contacts can be considered in terms of separation, sliding and interlocking. DDA simulates the rock mass as a series of individual blocks connected with spring elements at contacts/joints. At each interactive step, the following criteria must be met at contacts:

- No normal tensile stress is allowed (i.e., $\sigma_n \leq 0$);
- The calculation of shear stress follows the Mohr-Coulomb criterion: $|\sigma_s| \leq c + \sigma_n \tan \phi$, where c is the cohesion and φ is the friction angle.
- Friction between blocks depends on the coefficient of friction (μ), which is a function of the tangent of friction angle (i.e., $\tan \varphi$).

4.2 Strength reduction method

Zienkiewicz *et al*. (1975) initially conceived the idea of using the strength reduction method to evaluate the performance of slopes. This technique has been subsequently used as a common approach in several numerical codes for slope stability analysis, such as finite element method (Griffiths and Lane 1999, Chang and Huang 2005, Tschuchnigg *et al*. 2015a, b) and finite difference method (Dawson *et al*. 1999, Ni *et al*. 2016b). The factors of safety could be estimated based on the ratios by which the cohesion and the coefficient of friction reduced (Zienkiewicz *et al*. 1975). They also pointed out that the calculations using the strength reduction approach were not much influenced by the assumption of flow rules (associated or non-associated).

This study adopts the idea of Ning *et al*. (2011) to implement the strength reduction method in discontinuous deformation analysis (DDA), with the novelty of using advanced discretization technique from GeoSMA-3D based on block theory. Following the previous work of the authors (Ni *et al*. 2016b), numerical analyses need to be conducted multiple times using progressively reduced strength parameters. The characteristic strength (alternatively, the failure strength) in terms of c_f and μ_f can then be determined when a sufficient number of individual blocks yield along a shear band, so the rock mass above the band initiates to slide. The ratio between the actual strength (i.e., c and μ) and the characteristic strength (i.e., c_f and μ_f) is an indicative of the factor of safety. The definition is equivalent to that in traditional limit equilibrium analysis, where the factor of safety is usually calculated as the ratio of restoring to driving moments. In the DDA calculation, the reduction factor can start from 1.0 as the base analysis, which increases afterwards. The lowest value computed corresponds to the factor of safety (i.e., FoS) for the rock slope. The failure mechanism and slip mode can also be observed from the analysis.

$$\mathrm{FoS} = \frac{\mu}{\mu_f} = \frac{\tan\varphi}{\tan\varphi_f} = \frac{c}{c_f} \tag{8}$$

A failure criterion must be defined in the analysis to identify the occurrence of slope failure. In the literature, different techniques have been employed, such as excessive deformation of slope profile, continuous plastic zone (Liu *et al*. 2005), a limiting shear stress, exceedance of shear strains (Ni *et al*. 2016b), non-convergence of the calculation (Griffiths and Lane 1999), second order work (Laouafa and Darve 2002), acoustic emission events (Li *et al*. 2009), and maximum equivalent plastic strains (Zheng *et al*. 2009). In this paper, the criterion of excessive deformation of slope profile is used to indicate progressive failure of rock slope.

5. Illustrative example

In this section, an illustrative example is presented to show how the GeoSMA-3D software is used to discretize a rock slope model for use in discontinuous deformation strength reduction analysis. The advanced process of model discretization includes mapping joints on the excavation face, statistical analysis of joints, Monte Carlo simulation of artificial joints based on interpreted spatial distribution of joints, generating discrete fracture network (DFN) model and block merging. In addition, the calculation of GeoSMA-3D also provides the information on key blocks, where strengthening measures are recommended to improve the performance of the slope. A cross-section (not at key blocks) of the generated 3D blocky system is then implemented into a DDA code. During the DDA calculation, different strength parameters should be assigned to mapped and artificial joints. This is because artificial joints are not deterministic, but are generated based on statistical models. Therefore, the strength of intact rock is used for artificial joints. Strength reduction technique is used to lower strength for all joints. The slope becomes unstable once the joint strength is small enough, so individual blocks initiate to collapse due to gravity. Finally, the stability of the slope is evaluated, as well as the failure mode.

5.1 Key blocks and advanced discretization

A rock slope of 20 m high inclined to the horizontal at an angle of 60° is considered in this case study. The joint mapping process analyzed using the photogrammetry technique is not illustrated here, which provides two groups of dominant joints. Joint Group 1 features an average dip angle of 40° with an average spacing of 3.94 m, whereas Joint Group 2 has a mean dip angle of 142° and a mean spacing of 3.15 m. The strike of both joint groups is parallel to the slope surface. Except these dominant joints, two additional fractures are also included. Fig. 4 depicts the generated discrete fracture network (DFN) model with measured joints. The length of the DFN model is set to 30 m. This is to eliminate boundary effects, as the main features of joints are far from the lateral sidewalls (i.e., boundaries).

The intact rock is modeled as a standard Mohr-Coulomb material, with a Young's modulus of 10 GPa, a Poisson's ratio of 0.25, a friction angle of 40° and a cohesion of 10 MPa. At measured joints, a lower strength should be used to characterize the degradation effect. Therefore, the friction angle and cohesion are set to 34° and 0.24 MPa respectively to represent the joint strength.

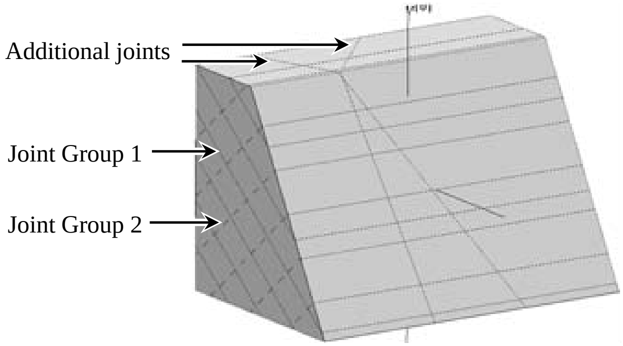


Fig. 4 Discrete fracture network (DFN) model with measured joints

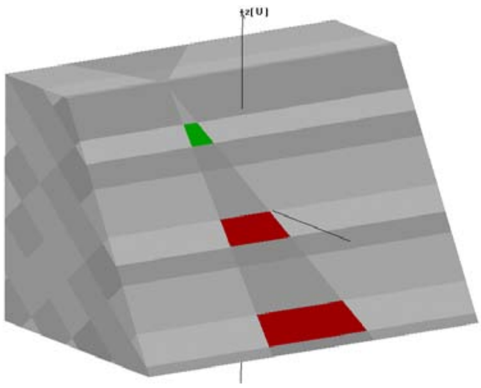
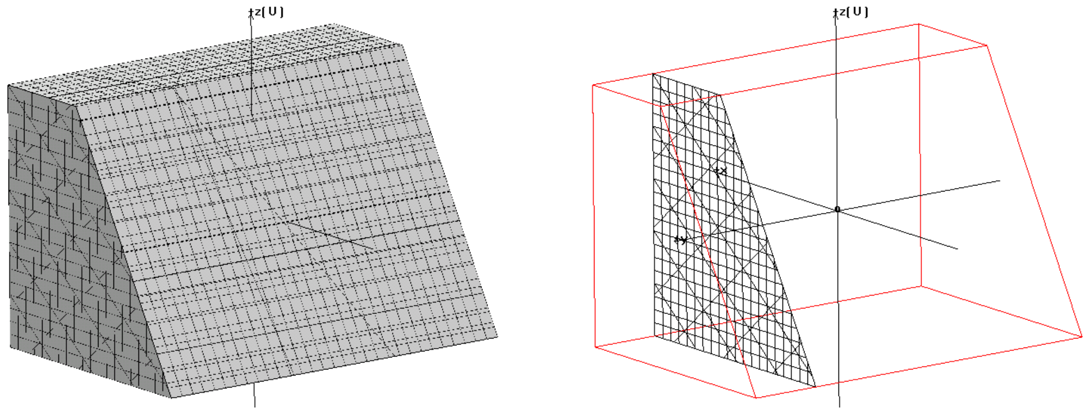


Fig. 5 Identified key blocks for the DFN model with measured joints

Limit equilibrium analysis is then performed in GeoSMA-3D to calculate the factors of safety for all generated blocks. Key blocks are determined for those with a low factor of safety less than unity. Fig. 5 identifies the key blocks using red color. It is interesting that the two obtained key blocks are bounded within the two additional fractures. In Fig. 5, green color is employed to illustrate the block that has a factor of safety slightly larger than unity, which is also in the narrow band between the two additional fractures. Therefore, it is suggested to take mitigation measures within the two additional joints (see location in Fig. 4) to stabilize the slope, such as adding reinforcement of rock bolts, rock dowels, shear pins, injectable resin/epoxy and shotcrete.

The joint mapping process can only provide information of fractures on the excavation face, but the distribution of joints within the rock mass is still unknown. Statistical models can be used to model artificial joints within the rock mass for Monte Carlo simulation to reduce bias in the calculation. Three orthogonal sets of artificial joints with a spacing of 1 m are then simulated in GeoSMA-3D, and the generated discrete fracture network (DFN) model is shown in Fig. 6(a). The identification of key blocks and the associated mitigation measures offer important guidance on the selection of 2D cross-section in discontinuous deformation analysis (DDA). As the strengthening strategy could significantly improve the stability of the slope between the two additional factures, further analysis of a cross-section at these locations will not be representative. Therefore, a cross-section outside the two additional fractures is used as the characteristic plane for discontinuous deformation analysis (DDA). The advanced discretization can be seen in Fig. 6(b).



(a) DFN model with artificial joints (b) A 2D discretization

Fig. 6 Discrete fracture network (DFN) model with artificial joints for use in advanced discretization

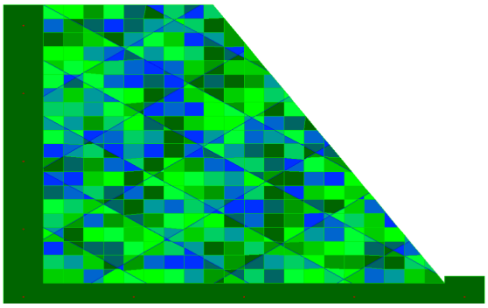


Fig. 7 Numerical model used in discontinuous deformation analysis (DDA)

5.2 Discontinuous deformation strength reduction analysis

The discretization obtained from GeoSMA-3D is imported into a DDA code. The same strength parameters of intact rock are assigned to both the intact rock and artificial joints (as they are not deterministic), while the joint strength is used for measured joints. Fig. 7 illustrates the numerical model in discontinuous deformation analysis (DDA) implemented from GeoSMA-3D. The rock slope should remain stable due to self-weight, since the GeoSMA-3D calculation has indicated that there is no key block involved in the model.

All joints (i.e., measured and artificial) divide the 2D model into 498 blocks. A stabilization simulation is conducted first with no reduction of strength parameters. This process allows stress redistribution to occur, along which partial rebound deformations take place near the slope crest. This is because there is no surcharge load to restrict rebound deformations. Cracks can be seen at the artificial joints, so that some blocks have the potential to slide along the measured structural planes. Finally, all blocks are stabilized after completion of stress redistribution as presented at Fig. 8. The numerical model can then be used for further stability analysis of the slope using strength reduction technique.

A strength reduction factor of 1.0 is used in the stabilization calculation for the blocky system. The stabilized model is subsequently analyzed for a range of joint strength (i.e., the increment in

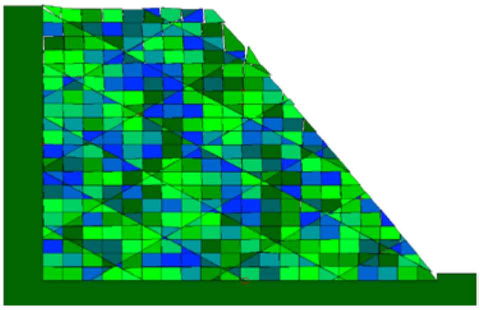
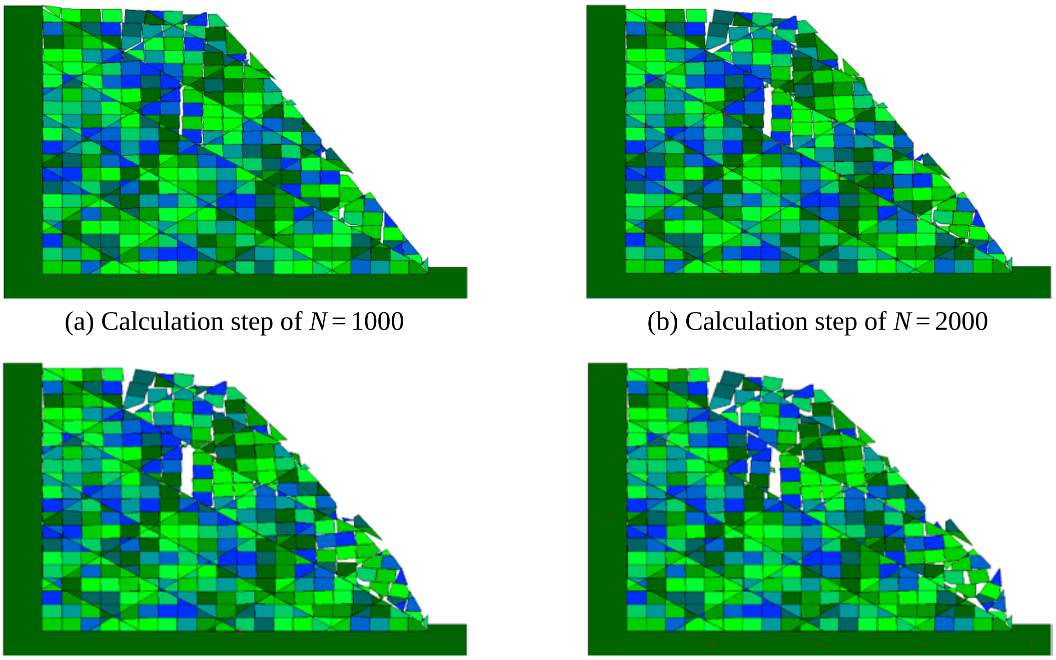


Fig. 8 Stabilized rock slope model after completion of stress redistribution



(a) Calculation step of $N = 1000$

(b) Calculation step of $N = 2000$

(c) Calculation step of $N = 4000$

(d) Calculation step of $N = 5000$

Fig. 9 Failure mode of the slope analyzed using a strength reduction factor of 1.3

strength reduction factor is 0.1). The strength of both measured and artificial joints needs to be reduced in the calculation. For example, the initial values of cohesion and coefficient of friction at measured joints are $c = 0.24$ MPa and $\mu = \tan\varphi = \tan 34° = 0.67$, respectively, which reduce to $c_f = 0.185$ MPa and $\mu = \tan\varphi = \tan 27.5° = 0.52$, when a strength reduction factor of 1.3 is used. Similarly, the strength of artificial joints (i.e., intact rock) changes from $c = 10$ MPa and $\mu = \tan\varphi = \tan 40° = 0.84$ to $c_f = 7.692$ MPa and $\mu = \tan\varphi = \tan 33° = 0.65$. At this strength level, the number of iterations is increased significantly to get a converged result at each calculation step. Slope failure has occurred finally, since excessive deformation of slope profile is observed. The failure mode of the blocky system is given in Fig. 9. Therefore, the factor of safety is determined as 1.3 for the rock slope in this case study. Similarly, Ni *et al.* (2016b) reported that the factor of safety was 1.33 for a soil slope with a height of 10 m and an inclination of 45° using finite difference strength reduction technique. The two investigations produced very similar results for the factor of safety. However, the observed failure mechanisms between the two studies differed significantly, where the soil slope was governed by a shallow toe mechanism (Ni *et al*. 2016b) and the rock slope was failed due to block toppling. The difference is probably caused by the material of the slope, but not by the geometry of the problem. This calculation is consistent with the observation of Shi (2007) that toppling is the dominant failure mechanism for rock slopes.

In Fig. 9, it is clear that stress redistribution induced rebound deformations occurred in the early stages of the analysis. The reduced joint strength did not alter the deformation mode of the slope at $N = 1000$ (see Fig. 9(a)) too much compared to that in the stabilization calculation at Fig. 9. Failure can be observed mainly near the slope crest. Cracks initiated from measured joints, and propagated to some artificial joints. Finally, these cracks formed polygons (including several blocks) could start to move during subsequent calculation steps. A brittle failure – rock toppling – happened in the end. Although the factor of safety is estimated as 1.3, prevention measures still need to be implemented to enhance the stability of the slope. The location with excessive block deformations should be given priority to be reinforced. Rock toppling would influence a greater depth, so shallow measures of injectable resin/epoxy and shotcrete may not work. The influencing zone can indicate the selection of reinforcements, such as rock bolts, rock dowels and shear pins.

6. Conclusions

In this paper, a combined approach based on block theory and discontinuous deformation analysis has been proposed to analyze rock slopes. A 3D numerical software package, GeoSMA-3D, is introduced briefly, which is capable of simulating structural planes within the rock mass efficiently. Photogrammetry technique is used to evaluate the distribution of joints measured on the excavation face. Statistical analysis of these joints helps to generate artificial joints within the rock mass. The strength parameters of intact rock are employed for artificial joints, so that they can behave as intact rock if there is no strength reduction involved. The calculation of GeoSMA-3D provides information on key blocks based on limit equilibrium analysis, which offers guidance on the selection of strengthening measures if needed.

The GeoSMA-3D method represents a great improvement in simulating the blocky system, where all coordinates of joints become available for use in further implementation in discontinuous deformation analysis (DDA). The stability of discretized blocky system is then evaluated in a DDA code, where strength reduction technique is used to lower the strength for both measured and artificial joints. The discretization of blocky system in GeoSMA-3D has bright prospects, since the simulation of artificial joints allows cracks to occur at these locations to model the failure of rock bridge. Excessive deformation of slope profile is considered as the criterion to determine the occurrence of slope failure. A series of analyses using different strength reduction factor (starting from 1.0, with an increment of 0.1) can be conducted. Once the slope fails, the lowest strength reduction factor corresponds to the factor of safety for the analyzed slope.

An illustrative example is presented to show the ability of the proposed strategy of combined GeoSMA-3D and DDA calculation to assess the stability of a rock slope with a height of 20 m and an inclination of 60°. The identified key blocks are located within two additional joints. Mitigation measures are therefore suggested to reinforce the slope, such as rock bolts, rock dowels, shear pins, injectable resin/epoxy and shotcrete. The 3D blocky system obtained from GeoSMA-3D is then analyzed in a DDA code. A representative cross-section is selected outside the two additional fractures to perform 2D plane strain calculation in DDA. Strength reduction method evaluates a factor of safety of 1.3, which is line with similar investigations. The failure mode is found as rock toppling, which is different from a typical toe failure for a soil slope. The difference lies in the material of the slope, instead of the geometry of the problem. Prevention measures are still recommended to enhance the performance of the slope. Based on the failure mechanism observed, a measure that can penetrate to a great depth should serve the purpose, such as rock bolts, rock dowels and shear pins.

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