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Collapse mechanism of tunnel roof considering joined influences of nonlinearity and non-associated flow rule

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Abstract. Employing non-associated flow rule and Power-Law failure criterion, the failure mechanisms of tunnel roof in homogeneous and layered soils are studied in present analysis. From the viewpoint of energy, limit analysis upper bound theorem and variation principle are introduced to study the influence of dilatancy on the collapse mechanism of rectangular tunnel considering effects of supporting force and seepage force. Through calculation, the collapsing curve expressions of rectangular tunnel which are excavated in homogeneous soil and layered soils respectively are derived. The accuracy of this work is verified by comparing with the existing research results. The collapsing surface shapes with different dilatancy coefficients are draw out and the influence of dilatancy coefficient on possible collapsing range is analyzed. The results show that, in homogeneous soil, the potential collapsing range decreases with the decrease of the dilatancy coefficient. In layered soils, the total height and the width on the layered position of possible collapsing block increase and the width of the falling block on tunnel roof decrease when only the upper soil's dilatancy coefficient decrease. When only the lower soil's dilatancy coefficient decrease or both layers' dilatancy coefficients decrease, the range of the potential collapsing block reduces.

Keywords: collapse; non-associated flow rule; Power-Law criterion; tunnel roof; upper bound

1. Introduction

In 1975, Chen (1975) expounded systematically the theory of limit analysis and applied the theory to geotechnical engineering such as slope stability. Then, the limit analysis method has been widely used in geotechnical engineering. The limit analysis theory is more rigorous in contrast to the limit equilibrium method and other methods, its calculation process is simpler and the results are more accurate.

Later, limit analysis theory was used to analyze the stability problems of tunnels by some scholars. Atkinson and Potts (1977) studied the circular tunnel in the cohesionless soil by using limit analysis and obtained the expression of supporting force of shallow tunnel. Through assuming the failure mechanism of shallow tunnel, Davis *et al.* (1980) deduced the upper and lower bound of stability coefficient of tunnel without drainage by using limit analysis method. Leca and Dormieux (1990) built the active and passive 3-D failure mechanisms of shallow tunnel face and obtained the upper bound value of supporting force of tunnel in the cohesive and

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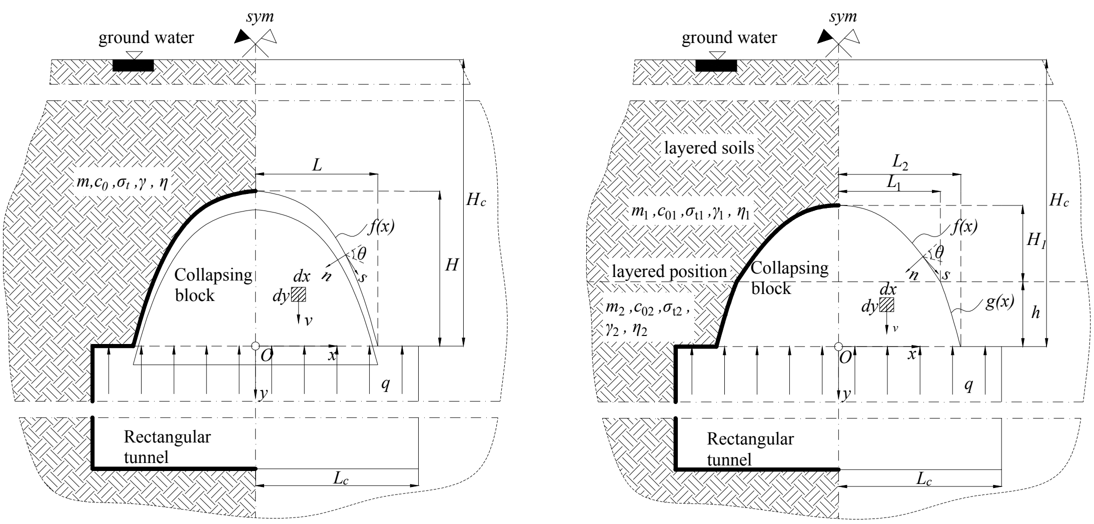
frictional materials based on the failure mechanisms. On the basis of the failure mechanism built by Leca and Dormieux (1990), Soubra (Soubra 2000, 2002, Soubra *et al*. 2008) proposed a three-dimensional failure mechanism consisted of many blocks and calculated the limit supporting force of tunnel face under the active and passive failure modes to analyze the face stability. Later, combined with reliability analysis and limit analysis theory, Mollon *et al*. (2009) analyzed the 3-D face stability of the shallow buried circular tunnel and found that the contribution of serviceability limit state to the system reliability was significant and serviceability limit state can be used alone to assess the tunnel reliability. In order to study collapse mechanisms of cavities and tunnels under the limit state, Fraldi and Guarracino (2009, 2010, 2011, 2012) constructed a kind of curved failure mechanism according to the Hoek-Brown failure criterion and used the numerical simulation method to verify the reliability of the calculation.

At the beginning, using limit analysis to study the stability problems of geotechnical engineering were mainly based on associated flow rule. Later, some scholars began to use non-associated flow rule in the limit analysis so as to obtain more accurate failure loads. Drescher and Detourany (1993) have a substantial contribution to determine the limit load in a translational failure mechanism for non-associative geomaterials with a coaxiality of the principal directions stresses and deformation rates. Kumar (2004) calculated the stability coefficient of soil slope based on associated flow rule and non-associated flow rule respectively and analyzed the influence of dilatancy angle on the slope stability coefficient with coaxial non-associated flow rule and non-coaxial non-associated flow rule. Chaaba *et al*. (2010) studied the ideal plastic material with non-associated flow rule according to the limit analysis method and finite element method and got the limit load of rectangular tunnels. Veiskarami *et al*. (2014) discussed the influence of flow rule on the bearing capacity of strip foundations placed on sand based on limit analysis upper bound method. Combing non-associated flow rule with nonlinear failure criterion, Zhang and Wang (2015) analyzed the surrounding rock stability problem of shallow tunnel by using tangential method, virtual power principle and strength reduction technique. Yang and Yan (2015) discussed the collapse mechanism of tunnel subjected to seepage force in layered soils.



(a) In homogeneous soil (b) In layered soil

Fig. 1 Collapse mechanisms of rectangular tunnel in homogeneous soil and in layered soil

However, there are few studies about the influence of non-associated flow rule on the collapse mechanism of tunnels according to nonlinear failure criterion and limit analysis theory. Therefore, based on limit analysis upper bound theorem and variation principle, collapse mechanisms in rectangular tunnels which is excavated in homogeneous and layered soils were discussed through combining nonlinear failure criterion with non-associated flow rule, as shown in Fig. 1, which can provide theoretical basis and reference for the stability analysis of surrounding rock and the optimization design of supporting system on tunnels and cavities in the future.

2. Definitions and theorems

2.1 Power-Law failure criterion of geotechnical materials following non-associated flow rule

Using the limit analysis method, many efforts have been carried out to analyze the stability of geotechnical engineering on the basis of an associated flow rule. However, a large number of research results show that, based on this assumption, the dilatancy characteristics of geotechnical materials was overestimated. In general, geotechnical materials follow non-associated flow rule and the associated flow rule is just a special case. For the associated flow rule, the dilation angle of geotechnical materials is equal to the internal frictional angle. While according to the non-associated flow rule, the dilation angle is less than the internal frictional angle. The real deformation and failure property of soils can be better simulated by using non-associated flow rule. Therefore, the introduction of non-associated flow rule in upper bound analysis is necessary. For geotechnical materials which satisfy non-associated flow rule and linear Mohr-Coulomb failure criterion, Drescher and Detournay (1993) adopted the following relationships to consider the dilation

$$\begin{cases} c^* = \eta c \\ \tan\varphi^* = \eta \tan\varphi \end{cases} \tag{1}$$

$$\eta = \frac{\cos\psi\cos\varphi}{1-\sin\psi\sin\varphi} \tag{2}$$

in which c and φ are shear strength parameters of geotechnical materials, c^* and φ^* are the modified shear strength parameters of geotechnical materials based on non-associated flow rule, ψ is dilatancy angle varying from zero to the internal friction angle φ, η is dilatancy coefficient and $0 < \eta \leq 1$. It can be noted that the non-associated flow rule is degraded to the associated flow rule when $\eta = 1$. Then, the non-associated flow rule can be introduced to limit analysis theory by modifying the strength parameters of geotechnical materials.

However, experiments have shown that the strength envelops of geotechnical materials have the nature of nonlinearity while the linear failure criterion is just a special case (Agar *et al*. 1985, Baker 2004, Anyaegbunam 2015). The Power-Law failure criterion is used in present study and the expression can be written in the Mohr's plan $\sigma_n - \tau_n$ as

$$\tau_n = c_0\left(1+\sigma_n/\sigma_t\right)^{\frac{1}{m}} \tag{3}$$

where c_0 is initial cohesion, σ_t is axial tensile stress, m is nonlinear coefficient and the parameters values of c_0, σ_t and m can be determined by tests. It's worth noting that when $m = 1$, the Power-Law failure criterion can be degenerated to the well-known linear Mohr-Coulomb criterion

$$\tau_n = c_0 + \sigma_n \frac{c_0}{\sigma_t} \tag{4}$$

Then

$$\begin{cases} c = c_0 \\ \tan\varphi = \dfrac{c_0}{\sigma_t} \end{cases} \tag{5}$$

The physical meaning of Eq. (1) is that, based on linear Mohr-Coulomb criterion, the shear strength parameters of geotechnical materials satisfying non-associated flow rule are modifications of c and $\tan\varphi$ in essence. Similarly, According to Eq. (1), Eq. (5) can be revised as

$$\begin{cases} c^* = \eta c = \eta c_0 \\ \tan\varphi^* = \eta \tan\varphi = \eta \dfrac{c_0}{\sigma_t} \end{cases} \tag{6}$$

As shown in Fig. 2, the modified expression of Power-Law failure criterion of geotechnical materials following non-associated flow rule can be written as

$$\tau_n = \eta c_0 \left(1 + \sigma_n \cdot \left(\eta \frac{c_0}{\sigma_t}\right) \Big/ \eta c_0 \right)^{\frac{1}{m}} = \eta c_0 \left(1 + \sigma_n / \sigma_t\right)^{\frac{1}{m}} \tag{7}$$

2.2 Upper bound theorem considering seepage force

The upper bound theorem can be depicted as: if compatible plastic deformation satisfies the condition on the displacement boundary in any kinematically admissible velocity field, the loads obtained by equating the power which external forces do to the rate of internal dissipation will be no less than the actual limit load. In order to analyze the effect of seepage forces on slope stability with upper bound theorem, the power of seepage forces was introduced to limit analysis as rate of external forces by Saada *et al*. (2012). Then, the upper bound theorem considering seepage forces can be written as

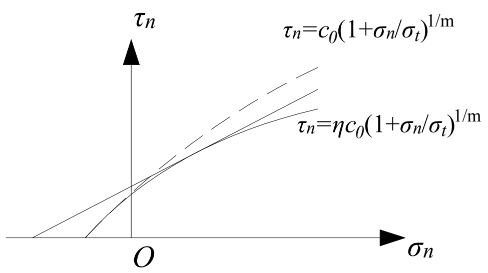


Fig. 2 The modified Power-Law failure criterion of non-associated geotechnical materials

$$\int_V \sigma_{ij}\dot{\varepsilon}_{ij}\mathrm{d}V \geq \int_S T_i v_i \mathrm{d}S + \int_V F_i v_i \mathrm{d}V + \int_V -grad\ u \cdot v_i \mathrm{d}V \tag{8}$$

where σ_{ij} and $\dot{\varepsilon}_{ij}$ are the stress and strain rates, respectively, T_i is the external force acting on the boundary and S is the length of boundary, X_i is body force and V is volume of the mechanism, v_i is the velocity along the velocity discontinuity surface, $-grad\ u$ excess pore pressure.

3. Limit analysis of collapse mechanisms according to non-associated flow rule

Although circular or multiple circular cross section is used in most of underground structure to improve the stress distribution and develop the bearing capacity, rectangular cross section is also employed in many tunnels and cavities. In order to analyze the collapse mechanism of tunnel, Fraldi and Guarracino (2009) established a kind of parabolic failure mechanism according to the arch effect of surrounding rock when collapse occurs in deep tunnel. However, the influence of non-associated flow rule on two-dimensional collapse mechanism is not taken into account in the process of calculation. Then, based on the study of Fraldi and Guarracino, collapse mechanisms of deep tunnel in homogeneous soil and layered soils with non-associated flow rule are analyzed respectively in present study.

3.1 In homogeneous soil

As shown in Fig. 1(a), the falling block is considered to be symmetrical with respect to the y-axis. c_0 is initial cohesion, σ_t is the axial tensile stress, m is the nonlinear coefficient, η is dilatancy coefficient and γ is the weight per unit volume of the soil. $f(x)$ is the expression of the curve of collapse mechanism, and L and H are half-width and height of collapsing block respectively. q is the supporting force of tunnel.

In order to get the plastic potential function of geotechnical material following coaxial non-associated flow rule when yielding occurs, it is assumed that plastic potential surface is coincident with the yield curve of the geotechnical material which strength parameters are revised. Thus the plastic potential is

$$\Im = \tau_n - \eta c_0 \left(1 + \sigma_n / \sigma_t\right)^{\frac{1}{m}} \tag{9}$$

So that the plastic strain rate can be written as follows

$$\begin{cases} \dot{\varepsilon}_n = \lambda \dfrac{\partial \Im}{\partial \sigma_n} = -\lambda \dfrac{\eta c_0}{m\sigma_t}\left(1 + \dfrac{\sigma_n}{\sigma_t}\right)^{\frac{1-m}{m}} \\ \dot{\gamma}_n = \lambda \dfrac{\partial \Im}{\partial \tau_n} = \lambda \end{cases} \tag{10}$$

in which λ is a scalar parameter. By following a purely geometrical line of reasoning, the plastic strain rate components can also be expressed as

$$\begin{cases} \dot{\varepsilon}_n = \dfrac{v}{w}\left[1 + f'(x)^2\right]^{-\frac{1}{2}} \\ \dot{\gamma}_n = -\dfrac{v}{w} f'(x)\left[1 + f'(x)^2\right]^{-\frac{1}{2}} \end{cases} \tag{11}$$

in which λ is a scalar parameter. By following a purely geometrical line of reasoning, the plastic strain rate components can also be expressed as

$$\begin{cases} \sigma_n = -\sigma_t + \sigma_t \left(\dfrac{m\sigma_t}{\eta c_0} \right)^{\frac{m}{m-1}} f'(x)^{\frac{m}{m-1}} \\ \tau_n = \eta c_0 \left(\dfrac{m\sigma_t}{\eta c_0} \right)^{\frac{1}{1-m}} f'(x)^{\frac{1}{1-m}} \end{cases} \tag{12}$$

Then, energy dissipation density on the velocity discontinuity results

$$\dot{D}_i = \sigma_n \dot{\varepsilon}_n + \tau_n \dot{\gamma}_n = \frac{v}{w}\left[1 + f'(x)^2\right]^{-\frac{1}{2}} \left[-\sigma_t + \frac{1-m}{m} (m\sigma_t)^{\frac{1}{1-m}} (\eta c_0)^{\frac{m}{m-1}} f'(x)^{\frac{m}{m-1}} \right] \tag{13}$$

So that the total energy dissipation can be obtained by calculating the integral of $\dot{D}_i$ along the velocity discontinuity by considering the right half of the falling block, results

$$\begin{aligned} W_D &= \int_0^L \dot{D}_i \cdot w\mathrm{d}s = \int_0^L \dot{D}_i w\sqrt{1 + f'(x)^2}\,\mathrm{d}x \\ &= v\int_0^L \left[-\sigma_t + \frac{1-m}{m} (m\sigma_t)^{\frac{1}{1-m}} (\eta c_0)^{\frac{m}{m-1}} f'(x)^{\frac{m}{m-1}} \right] \mathrm{d}x \end{aligned} \tag{14}$$

The work rate produced by self-weight can be expressed as follows

$$W_\gamma = v\int_0^L \gamma' f(x)\mathrm{d}x = v\int_0^L (\gamma - \gamma_w) f(x)\mathrm{d}x \tag{15}$$

in which γ' is submerged unit weight of soil and $\gamma' = \gamma - \gamma_w$, γ_w referring to the unit weight of water. According to research of Saada *et al.* (2012), the distribution of excess pore pressure is expressed as

$$u = p - p_w = r_u \gamma H - \gamma_w H \tag{16}$$

where p is the pore water pressure at the considered point and $p = r_u\gamma H$, in which r_u is pore pressure coefficient, p_w is the hydrostatic distribution for pore pressure which can be derived by $p_w = \gamma_w H$, H referring to the vertical dimension between the tunnel roof and the top of the falling block. Then the work rate produced by seepage forces along the velocity discontinuity surface can be obtained, that is

$$W_u = \int_V -grad\ u \cdot v\mathrm{d}V = v\int_0^L (\gamma_\mathrm{w} - r_u\gamma) f(x)\mathrm{d}x \tag{17}$$

The power of supporting forces can be expressed as

$$W_q = -vqL \tag{18}$$

In order to achieve the optimum upper bound solution, it is necessary to establish an objective function ξ by the work rate of external forces and the total energy dissipation

$$\xi\left[f(x),f'(x),x\right]=W_D-W_\gamma-W_u-W_q=v\int_0^L\Lambda\left[f(x),f'(x),x\right]\mathrm{d}x+vqL \tag{19}$$

in which the expression of $\Lambda[f(x), f'(x), x]$ is

$$\Lambda[f(x),f'(x),x]=-\sigma_t+\frac{1-m}{m}(m\sigma_t)^{\frac{1}{1-m}}(\eta c_0)^{\frac{m}{m-1}}f'(x)^{\frac{m}{m-1}}-(1-r_u)\gamma f(x) \tag{20}$$

In order to get the upper bound solution close to the actual solution, the extremum of the objective function ξ should be obtained. It can be seen from Eq. (19) that the extremum of ξ is completely determined by $\Lambda[f(x), f'(x), x]$. Then the problem transforms into a typical calculus of variations, i.e., to find a function, $y = f(x)$, which makes the Eq. (19) a stationary value under the regularity conditions. According to variation principle, the first variation of the objective function ξ can be written as follows

$$\delta\xi\left[f(x),f'(x),x\right]=0\Rightarrow\frac{\partial\Lambda}{\partial f(x)}-\frac{\partial}{\partial x}\left[\frac{\partial\Lambda}{\partial f'(x)}\right]=0 \tag{21}$$

Substituting Eq. (20) into Eq. (21), the explicit form of Euler-Lagrange equation can be expressed as

$$-(1-r_u)\gamma+\frac{1}{m-1}(m\sigma_t)^{\frac{1}{1-m}}(\eta c_0)^{\frac{m}{m-1}}f'(x)^{\frac{2-m}{m-1}}f''(x)=0 \tag{22}$$

It is obvious that Eq. (22) is a second-order homogeneous nonlinear differential equation which can be solved by integral calculation. Thus the expression of the detaching curve $f(x)$ results

$$f(x)=k\left(x+\frac{s_1}{(1-r_u)\gamma}\right)^m-s_2\ ,\ \ k=\frac{\sigma_t}{\eta c_0}\left[\frac{(1-r_u)\gamma}{\eta c_0}\right]^{m-1} \tag{23}$$

where s_1 and s_2 are two constants which can be determined by boundary conditions. Since the detaching curve $f(x)$ is symmetrical with respect to the y-axis, the equilibrium of the stresses on the plane of $x = 0$ requires that the shear stress vanishes, that is

$$\tau_{xy}(x=0,y=-H)=0 \tag{24}$$

So the concrete expression of equation $f(x)$ can be obtained, that is

$$f(x)=kx^m-H\ ,\qquad k=\frac{\sigma_t}{\eta c_0}\left[\frac{(1-r_u)\gamma}{\eta c_0}\right]^{m-1} \tag{25}$$

It's also worthy noticing that the condition $f(x = L) = 0$ is an implicit constraint. According to the upper bound theorem of limit analysis, the explicit form of L and H can be easily derived by equating the power of external forces to the total energy dissipation

$$\xi=0\Rightarrow H=\frac{(\sigma_t-q)(m+1)}{(1-r_u)\gamma} \tag{26}$$

$$L = \frac{\eta c_0}{(1-r_u)\gamma}\left[\frac{(\sigma_t - q)(m+1)}{\sigma_t}\right]^{\frac{1}{m}} \tag{27}$$

Furthermore, said L_c and H_c are the half-width and the buried depth of the excavation respectively. The collapse of the roof will occur if the inequality $L_c \geq L$ is satisfied. Along the same line of reasoning, a tunnel can be said "deep" if the inequality $H_c >> H$ is satisfied. It can be easily found that deep tunnel considered as 'deep' not only is because of its actual depth but also depends on physical properties of soil.

3.2 In layered soils

In practical engineering, the surrounding rock is frequently not homogeneous, so the collapse mechanism of rectangular tunnel in layered soils with non-associated flow rule is established in this section, as shown in Fig. 1(b). The falling block is also symmetrical with respect to the y-axis. The detaching curve is a smooth curve which is composed of two equations, $f(x)$ and $g(x)$. The 1 and 2 in the subscript of parameters c_0, σ_t, m, γ and η represent the upper soil and lower soil respectively. L_1 and L_2 are the half-width of the falling block on the layered position and on the tunnel roof. H_1 is the height between the top of collapsing block and the layered position, and h s the vertical distance between the layered position and the tunnel roof.

The normal stress of any point on the velocity discontinuity in two layers can be obtained as follows

$$\begin{cases} \sigma_{n1} = -\sigma_{t1} + \sigma_{t1}\left(\dfrac{\eta_1 c_{01}}{m_1\sigma_{t1}}\right)^{\frac{m_1}{m_1-1}} f'(x)^{\frac{m_1}{m_1-1}} \\ \sigma_{n2} = -\sigma_{t2} + \sigma_{t2}\left(\dfrac{\eta_2 c_{02}}{m_2\sigma_{t2}}\right)^{\frac{m_2}{m_2-1}} g'(x)^{\frac{m_2}{m_2-1}} \end{cases} \tag{28}$$

Then the dissipation densities of internal forces on the detaching surface in two layers, $\dot{D}_{i1}$ and $\dot{D}_{i2}$, result

$$\begin{aligned} \dot{D}_{i1} &= \sigma_{n1}\dot{\varepsilon}_{n1} + \tau_{n1}\dot{\gamma}_{n1} \\ &= \frac{v}{w}\left[1+f'(x)^2\right]^{-\frac{1}{2}}\left[-\sigma_{t1} + \frac{1-m_1}{m_1}(m_1\sigma_{t1})^{\frac{1}{1-m_1}}(\eta_1 c_{01})^{\frac{m_1}{m_1-1}} f'(x)^{\frac{m_1}{m_1-1}}\right] \end{aligned} \tag{29}$$

$$\begin{aligned} \dot{D}_{i2} &= \sigma_{n2}\dot{\varepsilon}_{n2} + \tau_{n2}\dot{\gamma}_{n2} \\ &= \frac{v}{w}\left[1+g'(x)^2\right]^{-\frac{1}{2}}\left[-\sigma_{t2} + \frac{1-m_2}{m_2}(m_2\sigma_{t2})^{\frac{1}{1-m_2}}(\eta_2 c_{02})^{\frac{m_2}{m_2-1}} g'(x)^{\frac{m_2}{m_2-1}}\right] \end{aligned} \tag{30}$$

Thus the total energy dissipation results

$$W_D = \int_0^{L_1} \dot{D}_{i1} w\sqrt{1+f'(x)^2}\,\mathrm{d}x + \int_{L_1}^{L_2} \dot{D}_{i1} w\sqrt{1+g'(x)^2}\,\mathrm{d}x \tag{31}$$

$$
\begin{aligned}
&= v\int_0^{L_1}\left[-\sigma_{t1} + \frac{1-m_1}{m_1}\left(m_1\sigma_{t1}\right)^{\frac{1}{1-m_1}}\left(\eta_1 c_{01}\right)^{\frac{m_1}{m_1-1}} f'(x)^{\frac{m_1}{m_1-1}}\right]\mathrm{d}x \\
&+ v\int_{L_1}^{L_2}\left[-\sigma_{t2} + \frac{1-m_2}{m_2}\left(m_2\sigma_{t2}\right)^{\frac{1}{1-m_2}}\left(\eta_2 c_{02}\right)^{\frac{m_2}{m_2-1}} g'(x)^{\frac{m_2}{m_2-1}}\right]\mathrm{d}x
\end{aligned}
\tag{31}
$$

The power produced by self-weight can be obtained by integral calculation

$$
\begin{aligned}
W_\gamma &= v\left\{\int_0^{L_1}\gamma_1'\left[f(x)-(-h)\right]\mathrm{d}x + \int_0^{L_1}\gamma_2'\left[-h-0\right]\mathrm{d}x + \int_{L_1}^{L_2}\gamma_2' g(x)\mathrm{d}x\right\} \\
&= v\left\{\int_0^{L_1}\left[\gamma_1' f(x) + \gamma_1' h - \gamma_2' h\right]\mathrm{d}x + \int_{L_1}^{L_2}\gamma_2' g(x)\mathrm{d}x\right\}
\end{aligned}
\tag{32}
$$

The power produced by self-weight can be obtained by integral calculation

$$
\begin{aligned}
W_u &= v\left\{\int_0^{L_1}\left(\gamma_w - r_u\gamma_1\right)\left[f(x)-(-h)\right]\mathrm{d}x + \int_0^{L_1}\left(\gamma_w - r_u\gamma_2\right)\left(-h-0\right)\mathrm{d}x + \int_{L_1}^{L_2}\left(\gamma_w - r_u\gamma_2\right)g(x)\mathrm{d}x\right\} \\
&= v\left\{\int_0^{L_1}\left(\gamma_w - r_u\gamma_1\right)f(x)\mathrm{d}x + \int_{L_1}^{L_2}\left(\gamma_w - r_u\gamma_2\right)g(x)\mathrm{d}x - \int_0^{L_1} r_u\left(\gamma_1-\gamma_2\right)h\mathrm{d}x\right\}
\end{aligned}
\tag{33}
$$

The power produced by supporting force can be expressed as follows

$$
W_q = -vqL_2 \tag{34}
$$

The power produced by supporting force can be expressed as follows

$$
\zeta = W_D - W_\gamma - W_u - W_q = \zeta_1\left[f(x), f'(x), x\right] + \zeta_2\left[g(x), g'(x), x\right] + vqL_2 \tag{35}
$$

in which

$$
\begin{aligned}
&\zeta_1\left[f(x), f'(x), x\right] = v\int_0^{L_1}\Lambda_1\left[f(x), f'(x), x\right]\mathrm{d}x \\
&= v\int_0^{L_1}\left[-\sigma_{t1} + \frac{1-m_1}{m_1}\left(m_1\sigma_{t1}\right)^{\frac{1}{1-m_1}}\left(\eta_1 c_{01}\right)^{\frac{m_1}{m_1-1}} f'(x)^{\frac{m_1}{m_1-1}} - \left(1-r_u\right)\gamma_1 f(x) - \left(1-r_u\right)\left(\gamma_1-\gamma_2\right)h\right]\mathrm{d}x
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
&\zeta_2\left[f(x), f'(x), x\right] = v\int_{L_1}^{L_2}\Lambda_2\left[g(x), g'(x), x\right]\mathrm{d}x \\
&= v\int_{L_1}^{L_2}\left[-\sigma_{t2} + \frac{1-m_2}{m_2}\left(m_2\sigma_{t2}\right)^{\frac{1}{1-m_2}}\left(\eta_2 c_{02}\right)^{\frac{m_2}{m_2-1}} g'(x)^{\frac{m_2}{m_2-1}} - \left(1-r_u\right)\gamma_2 g(x)\right]\mathrm{d}x
\end{aligned}
\tag{37}
$$

Then the problem also transforms into a typical calculus of variations, i.e., to find two functions, $y = f(x)$ and $y = g(x)$, which make the Eq. (35) a stationary value under the regularity conditions. It can be found that the objective function ζ is determined completely by two functions, ζ_1 and ζ_2. Thus, it is assumed that the extremum of the objective function ζ can be obtained when two functions ζ_1 and ζ_2 obtain extremum simultaneously. According to the variation principle, the first variations of two objective functions ζ_1 and ζ_2 can be written as follows

$$
\frac{\partial\Lambda_1}{\partial f(x)} - \frac{\partial}{\partial x}\left[\frac{\partial\Lambda_1}{\partial f'(x)}\right] = -\left(1-r_u\right)\gamma_1 + \frac{1}{m_1-1}\left(m_1\sigma_{t1}\right)^{\frac{1}{1-m_1}}\left(\eta_1 c_{01}\right)^{\frac{m_1}{m_1-1}} f'(x)^{\frac{2-m_1}{m_1-1}} f''(x) = 0 \tag{38}
$$

$$\frac{\partial \Lambda_2}{\partial g(x)} - \frac{\partial}{\partial x}\left[\frac{\partial \Lambda_2}{\partial g'(x)}\right] = -(1-r_u)\gamma_2 + \frac{1}{m_2-1}(m_2\sigma_{t2})^{\frac{1}{1-m_2}}(\eta_2 c_{02})^{\frac{m_2}{m_2-1}} g'(x)^{\frac{2-m_2}{m_2-1}} g''(x) = 0 \tag{39}$$

Then the problem also transforms into a typical calculus of variations, i.e., to find two functions, $y = f(x)$ and $y = g(x)$, which make the Eq. (35) a stationary value under the regularity conditions. It can be found that the objective function ζ is determined completely by two functions, ζ_1 and ζ_2. Thus, it is assumed that the extremum of the objective function ζ can be obtained when two functions ζ_1 and ζ_2 obtain extremum simultaneously. According to the variation principle, the first variations of two objective functions ζ_1 and ζ_2 can be written as follows

$$f(x) = k_1\left(x + \frac{s_3}{(1-r_u)\gamma_1}\right)^{m_1} - s_4 \ , \ k_1 = \frac{\sigma_{t1}}{\eta_1 c_{01}}\left[\frac{(1-r_u)\gamma_1}{\eta_1 c_{01}}\right]^{m_1-1} \tag{40}$$

$$g(x) = k_2\left(x + \frac{s_5}{(1-r_u)\gamma_2}\right)^{m_2} - s_6 \ , \ k_2 = \frac{\sigma_{t2}}{\eta_2 c_{02}}\left[\frac{(1-r_u)\gamma_2}{\eta_2 c_{02}}\right]^{m_2-1} \tag{41}$$

where s_3, s_4, s_5 and s_6 are integration constants, respectively, which can be determined by boundary conditions. Since the detaching curve is symmetrical with respect to the y-axis, the equilibrium of the stresses on the plane of $x = 0$ also requires that the shear stress vanishes, that is

$$\tau_{xy}(x=0, y=-(h+H_1)) = 0 \tag{42}$$

In order to keep the curve smooth, the equation, $f'(x = L_1) = g'(x = L_1)$, should be satisfied. The condition $g'(x = L_2) = 0$ can also be found according to the geometrical relationship. Thereby, the expressions of the detaching curve $y = f(x)$ and $y = g(x)$ can be obtained

$$f(x) = k_1 x^{m_1} - (h+H_1) \tag{43}$$

$$g(x) = k_2(x+Z)^{m_2} - k_2(L_2+Z)^{m_2} \tag{44}$$

where

$$Z = \left(\frac{m_1 k_1}{m_2 k_2}\right)^{\frac{1}{m_2-1}} L_1^{\frac{m_1-1}{m_2-1}} - L_1 \tag{45}$$

According to the upper bound theorem of limit analysis, by equating the power of external forces to the total energy dissipation, an equation which consists of L_1, L_2 and H_1 can be obtained

$$\begin{aligned} &qL_2 + \left[(1-r_u)\gamma_1 H_1 + (1-r_u)\gamma_2 h - \sigma_{t1}\right]L_1 - \frac{m_1}{m_1+1}k_1(1-r_u)\gamma_1 L_1^{m_1-1} \\ &+\left[(1-r_u)\gamma_2 k_2(L_2+Z)^{m_2} - \sigma_{t2}\right](L_2-L_1) - \frac{m_2}{m_2+1}k_2(1-r_u)\gamma_2\left[(L_2+Z)^{m_2+1} - (L_1+Z)^{m_2+1}\right] = 0 \end{aligned} \tag{46}$$

It also can be found according to the geometrical relationship that

$$f(L_1) = h \tag{47}$$

$$g(L_1) = h \tag{48}$$

It is obvious that $L_c \geq L_2$ and $H_c >> h + H_1$ should also be satisfied. By combining Eq. (46) with Eqs. (47) and (48), a system of nonlinear equations about L_1, L_2 and H_1 can be obtained. The concrete expressions of $f(x)$ and $g(x)$ can be obtained after the values of L_1, L_2 and $H_1$are solved through MATLAB software and the shape of failure surface can be drawn by Eq. (43) and Eq.(44).

4. Calculation and analysis

The problem considered here is the failure mechanism of rectangular tunnel roof with nonlinear failure and non-associated flow rule. The numerical results to this problem have been obtained. Numerical results are summarized in following analysis. Example problems are selected to include the following: (a) considering the effects of associated flow rule, comparisons are made with the published solutions; and (b), the effects of non-associated flow rule in homogeneous soil and layered soils on the failure mechanism of rectangular tunnel roof are studied.

4.1 comparison and analysis

According to the nonlinear failure criterion and associated flow rule, Fraldi and Guarracino (2009) obtained the collapse mechanisms in cavities and tunnels by employing limit analysis and variation principle. In order to compare with Fraldi's results to verify the correctness in this work, the width and height of falling block were calculated when dilatancy coefficient $\eta = \eta_1 = \eta_2 = 1$, pore pressure coefficient $r_u = 0$, supporting force $q = 0$ kPa, initial cohesion $c_0 = c_{01} = c_{02}$, axial tensile stress $\sigma_t = \sigma_{t1} = \sigma_{t2}$, unit weight of soil $\gamma = \gamma_1 = \gamma_2$ and the nonlinear coefficient $m = m_1 = m_2$, namely, the non-associated flow is degraded into associated flow rule, the tunnel isn't affected with seepage forces and supporting force, and the parameters of geotechnical material in layered soil are the same in both layer and no difference with the parameters in homogeneous soil. It can be found through calculation that the values of width L and height H in homogeneous soil is the same as Fraldi's results and the values of width L_2 and total height $h + H_1$ in layered soil is also the same as Fraldi's results which proves the results and calculation accurate.

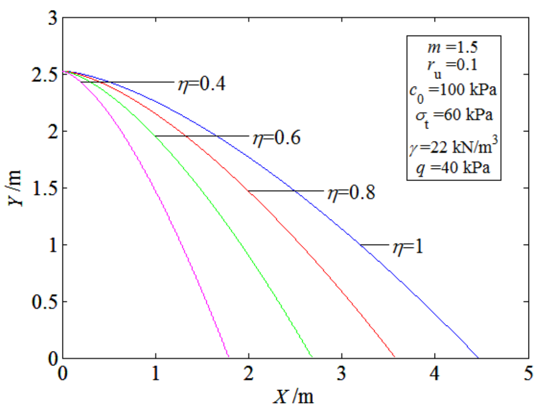


Fig. 3 The effect of dilatancy coefficient η on the range of possible collapsing block

4.2 The influence of dilatancy coefficient η on collapse mechanism in homogeneous soil

To investigate how the collapse mechanism of tunnel roof in homogeneous soil is influenced by dilatancy coefficient η, Fig. 3 illustrates the effects of the dilatancy coefficient η on the range of falling block for pore pressure coefficient r_u = 0.1, supporting force q = 40 kPa, initial cohesion c_0 = 100 kPa, nonlinear coefficient m = 1.5, axial tensile stress σ_t = 60 kPa and soil unit weight γ = 22 kN/m^3 with dilatancy coefficient η varying from 0.4 to 1.0. It can be seen from Fig. 3 that the dilatancy coefficient η has a significant influence on the failure mechanism of tunnel roof in homogeneous soil. When the other parameters remain constant, the height H of collapsing block remains constant and the width L decrease with the dilatancy coefficient η decrease. It follows that with the decrease of dilatancy coefficient η, the potential collapsing range of rectangular tunnel reduced in homogeneous soil.

4.3 The influence of dilatancy coefficient η on collapse mechanism in layered soils

In generally, with the increase of buried depth, the nature of the soil is gradually getting better. Therefore, in the process of analysis, the relationships of parameters m, c_0 and γ are $m_1 > m_2$, $c_{01} \leq c_{02}$ and $\gamma_1 < \gamma_2$. In the process of investigating the influence of dilatancy coefficient η on collapse mechanism in layered soil, the cases that only upper soil follows non-associated flow rule, only lower soil follows and both layers follow are discussed.

(1) Only upper soil follows non-associated flow rule

In order to investigate the influence of dilatancy coefficient η when only upper geotechnical material follows non-associated flow rule, the corresponding parameters are: dilatancy coefficient of lower soil η_2 = 1.0, initial cohesion c_{01} = 100 kPa and c_{02} = 110 kPa, nonlinear coefficient m_1 = 1.7 and m_2 = 1.5, axial tensile stress σ_{t1} = 60 kPa and σ_{t2} = 80 kPa, unit weight γ_1 = 18 kN/m^3 and γ_2 = 22 kN/m^3, the vertical distance between the layered position and the tunnel roof h = 1.5 m, pore pressure coefficient r_u = 0.1 and supporting force q = 50 kPa. The numerical results with dilatancy coefficient η_1 varying from 0.7 to 1.0 is obtained and illustrated in Fig. 4. It can be seen from Fig. 4, when other parameters remain constant, the height H_1 and the width L_1 of collapsing block increase with the decrease of dilatancy coefficient η_1. Meanwhile the width L_2 on the tunnel roof reduces.

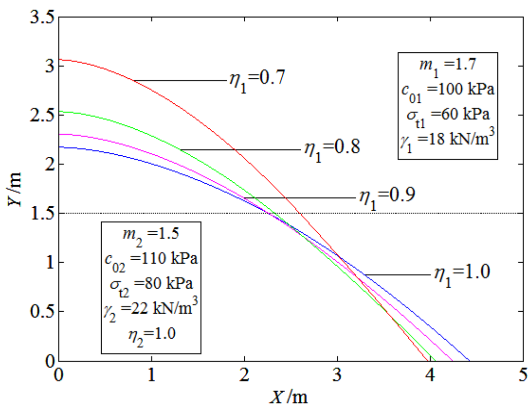


Fig. 4 The effect of dilatancy coefficient η_1 on the range of possible collapsing block

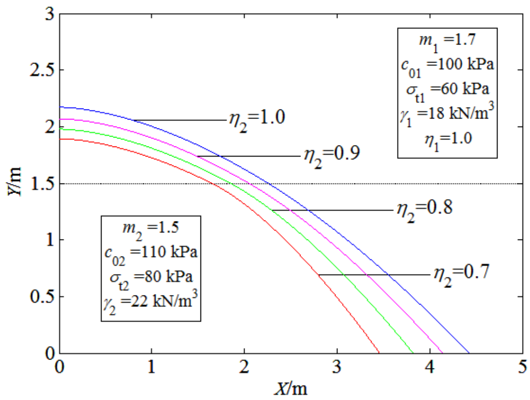
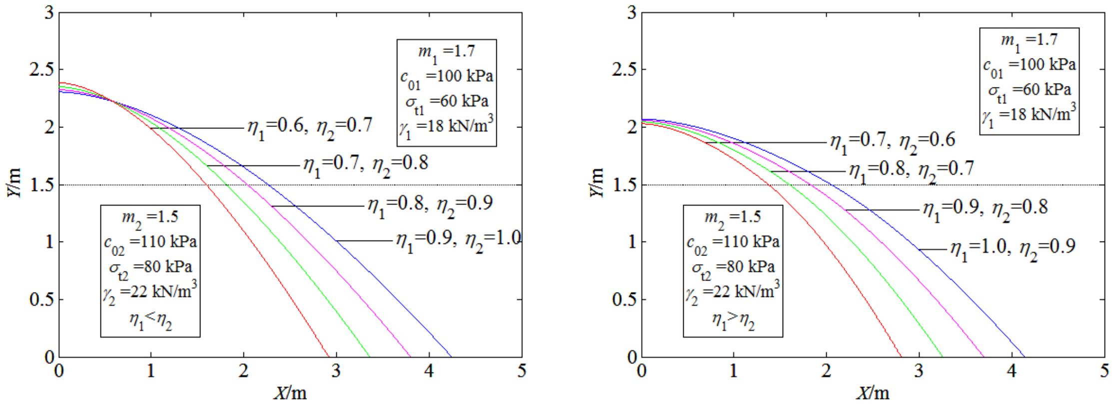


Fig. 5 The effect of dilatancy coefficient η_2 on the range of possible collapsing block



(a) $\eta_1 < \eta_2$ (b) $\eta_1 > \eta_2$

Fig. 6 The effect of dilatancy coefficients η_1 and η_2 on the range of possible collapsing block

(2) Only lower soil follows non-associated flow rule

To investigate the influence of dilatancy coefficient η when only lower geotechnical material follows non-associated flow rule, the numerical results with η_2 varying from 0.7 to 1.0 are represented in Fig. 5 corresponding to dilatancy coefficient of upper soil η_1 = 1.0, initial cohesion c_{01} = 100 kPa and c_{02} = 110 kPa, nonlinear coefficient m_1 = 1.7 and m_2 = 1.5, axial tensile stress σ_{t1} = 60 kPa and σ_{t2} = 80 kPa, unit weight γ_1 = 18 kN/m^3 and γ_2 = 22 kN/m^3, the vertical distance between the layered position and the tunnel roof h = 1.5 m, pore pressure coefficient r_u = 0.1 and supporting force q = 50 kPa. It can be seen from Fig. 5, when the other parameters remain constant, the height H_1 and the widths L_1 and L_2 of collapsing block decrease with the decrease of dilatancy coefficient η_2.

(3) Both layers follow non-associated flow rule

For investigating the influence of dilatancy coefficient η_1 when both geotechnical materials follow non-associated flow rule, the numerical results of the failure mechanism of rectangular tunnel roof are illustrated in Fig. 6 corresponding to c_{01} = 100 kPa, c_{02} = 110 kPa, m_1 = 1.7, m_2 =

1.5, σ_{t1} = 60 kPa, σ_{t2} = 80 kPa, γ_1 = 18 kN/m^3, γ_2 = 22 kN/m^3, h = 1.5 m, r_u = 0.1 and q = 50 kPa. The influence of dilatancy coefficients η_1 and η_2 on the scope of collapsing block is obtained and shown in Fig. 6 when $\eta_1 < \eta_2$ and $\eta_1 > \eta_2$. It can be seen from Fig. 6 that, when the other parameters remain constant, with both dilatancy coefficients decrease, the total range of collapsing block reduce whenever $\eta_1 < \eta_2$ or $\eta_1 > \eta_2$.

5. Conclusions

(1) Incorporating Power-Law failure criterion and non-associated flow rule, the variation principle based on limit analysis is employed to estabilsh the collapse mechanism of deep tunnel in homogeneous and layered soils considering effects of supporting force and seepage force.

(2) A failure mechanism of deep tunnel in homogeneous soil which follows non-associated flow rule is obtained by deriving the expression of velocity discontinuity equation $f(x)$. By equating the dissipation power of internal forces and the work rate of external forces, the expression of width L and height H of falling block are also obtained. By comparing with the result of Fraldi, the accuracy of this work is verified. Through analysis, the height H remains constant and the width L reduces with the decrease of dilatancy coefficient η in homogeneous soil.

(3) By assuming that the possible failure mode of deep tunnel in layered soil following non-associated flow rule is a smooth curve which is composed of two equations $f(x)$ and $g(x)$, the expressions of $f(x)$ and $g(x)$ are obtained. When the parameters of geotechnical material are the same in both layers, the results of this paper agree well with Fraldi's, so as to verify the accuracy of this study. It can also be found through analysis that dilatancy coefficients η_1 and η_2 have significant influence on the possible collapsing range of tunnel in layered soils. When only the upper soil follows non-associated flow rule, the height H_1 and the width L_1 of collapsing block increase and the width L_2 reduces with the decrease of dilatancy coefficient η_1. When only the lower layer follows non-associated flow rule, the height H_1 and the widths L_1, L_2 of collapsing block decrease with dilatancy coefficient η_2 decrease. When both layers follow non-associated flow rule and both dilatancy coefficients decrease, the potential range of collapsing block decreases in layered soils.

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