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Applications of artificial intelligence and data mining techniques in soil modeling

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Abstract. In recent years, several computer-aided pattern recognition and data mining techniques have been developed for modeling of soil behavior. The main idea behind a pattern recognition system is that it learns adaptively from experience and is able to provide predictions for new cases. Artificial neural networks are the most widely used pattern recognition methods that have been utilized to model soil behavior. Recently, the authors have pioneered the application of genetic programming (GP) and evolutionary polynomial regression (EPR) techniques for modeling of soils and a number of other geotechnical applications. The paper reviews applications of pattern recognition and data mining systems in geotechnical engineering with particular reference to constitutive modeling of soils. It covers applications of artificial neural network, genetic programming and evolutionary programming approaches for soil modeling. It is suggested that these systems could be developed as efficient tools for modeling of soils and analysis of geotechnical engineering problems, especially for cases where the behavior is too complex and conventional models are unable to effectively describe various aspects of the behavior. It is also recognized that these techniques are complementary to conventional soil models rather than a substitute to them.

Keywords: artificial intelligence; data mining; neural network; genetic programming; evolutionary computation; soil modeling; geotechnical engineering.

1. Introduction

In the past few decades the finite element method (FEM) has been used successfully to predict the response of systems across a whole range of industries including geotechnical engineering. In this numerical analysis the behavior of the actual material is approximated with that of an idealized material that deforms in accordance with some constitutive relationships. Therefore the choice of an appropriate constitutive model which adequately describes the behavior of the material plays a significant role in the accuracy and reliability of the numerical predictions.

During the past few decades several constitutive models have been developed for different geomaterials based on mechanics (e.g., Desai *et al.* 1986, Duncan and Chang 1970, Einstein and Hirschfeld 1973, Kawamoto *et al.* 1988, Lade and Duncan 1975, Roscoe and Schofield 1963). Most of these models involve determination of material parameters, many of which have no physical meaning (Shin and Pande 2000). The engineering properties of geomaterials exhibit varied and

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uncertain behavior due to the complex and imprecise physical processes associated with the formation of these materials. Recognition of nonlinear behavior of soils and rocks becomes increasingly important in design, stability analysis, prediction and control of failure for geotechnical engineering projects. In spite of considerable complexities of constitutive theories, due to the erratic nature of soils and rocks, none of the existing constitutive models can completely describe the real behavior of various types of these materials under various stress paths and loading conditions.

In conventional constitutive material modeling, an appropriate mathematical model is initially selected and the parameters of the model (material parameters) are then identified from appropriate physical tests on representative samples to capture the material behavior. When these constitutive models are used in numerical analysis (e.g., FEA), the accuracy with which the selected material model represents the various aspects of the actual material behavior and also the accuracy of the identified material parameters affect the accuracy of the numerical predictions.

In recent years, some researchers have attempted to build nonlinear constitutive models based on computer-aided pattern recognition methods. Artificial neural network (ANN) has been the most widely used pattern recognition technique to model the constitutive material behavior. This paper presents a review and evaluation of different AI and data mining techniques that have been proposed for modeling of soils. In particular, it covers the applications artificial neural networks, genetic programming and evolutionary polynomial regression techniques in soil modeling and some geotechnical engineering problems.

2. Data driven techniques

In recent years, by pervasive developments in computational software and hardware, several computer aided pattern recognition and data mining techniques have been emerged and developed. The main idea behind a pattern recognition system is that it learns adaptively from experience and extracts various discriminants, each appropriate for its purpose. Although there are other general purpose data-driven techniques, artificial neural network (ANN) and genetic programming (GP) are the most widely used pattern recognition methods that have been utilized to model complex engineering problems and capture nonlinear interactions between various parameters in a system. In this approach model construction is usually divided into three stages: (i) function identification, (ii) parameter estimation and (iii) validation. For model construction, a physical system with an output y, dependent on a set of inputs X and parameters θ, can be mathematically formulated as

$$y = F(\boldsymbol{X}, \theta) \tag{1}$$

where F is a function in an m-dimensional space where m is the number of inputs. Data-driven techniques tend to reconstruct F from input-output data. GP generates a population of expressions for F, coded in tree structures of variable size, and performs a global search of the best fit expression for F. ANN goal, on the other hand, is to map F rather than to find a feasible structure for it.

3. Artificial neural network

Artificial neural networks (ANNs) are computational models broadly inspired by the organization of the human brain. The most important features of a neural network are its abilities to learn and to

be error tolerant. In other words an artificial neural network is able to acquire, represent, and compute a mapping from a multivariate space of information to another given a set of data representing that mapping (Garrett 1994). ANN models are adaptive and capable of generalization. They can handle imperfect or incomplete data, and can capture nonlinear and complex interactions among variables of a system. Because of these features, the artificial neural network is emerging as a powerful tool for modeling.

At the most abstract description, a neural network can be considered of as a black box, where data is fed from one side, and processed by the neural network which then produces an output according to the supplied input (Caudill 1991). Although a neural network can usually process any kind of data (e.g., qualitative or quantitative information), the data fed into the neural network should be pre-processed (e.g., filtered, transformed) to enable faster training and better performance. In fact, the selection, pre-processing, and coding of information is one of the main issues to deal with when working with neural networks.

A neural network generally consists of an input layer, one or more hidden layers and an output layer of neurons. The neurons are the processing units within the neural network and are usually arranged in layers. Each layer is composed of several processing units. The processing units are fully connected to processing units of the succeeding layer. The information is propagated through the neural network layer by layer. Connections are the paths between neurons where all the information flows within a neural network. A neuron collects information from all preceding neurons relative to the flow of the information and propagates its output to the neurons in the following layer. The output of each preceding neuron is modulated by a corresponding weight and a bias. This output is then modified by transfer function and becomes the final output of the neuron (Dayhoff 1990). This signal is then propagated to the neurons of the next layer. The most frequently used and efficient learning procedure for multi-layer neural networks is the back-propagation learning algorithm based on the generalized delta rule (Rumelhart *et al.* 1994). The back-propagation learning rule can be used to adjust the weights and biases of a network in order to minimize the sum-squared error of the network. This is done by continually changing the values of the network weights and biases in the direction of steepest descent with respect to error. Derivatives of the error vector are calculated for the network's output layer and then back-propagated through the network until derivatives of error are available for each hidden layer.

Training refers to the process that repeatedly applies input vectors to the network and calculates errors with respect to the target vectors and then finds new weights and biases with the learning rule. It repeats this cycle until the sum-squared error falls beneath an error goal, or a maximum number of epochs is reached. Training a feed-forward network with the back-propagation learning rule is most frequently used in function approximation and pattern recognition. More detailed description of ANNs is out of the scope of this paper. Texts describing aspects and features of ANN models and architectures in greater detail can be found in the literature (e.g., Lippmann 1987, Flood and Kartam 1994).

3.1 Application of artificial neural network for soil modeling

Modeling of soil behavior plays an important role in dealing with issues related to soil mechanics and foundation engineering. The application of ANN offers an alternative means for the modeling of soil behavior. A neural network based constitutive model (NNCM) is fundamentally different from a conventional constitutive model (Zhu *et al.* 1998a). One of its distinctive features is that it is

based on experimental data rather than on assumptions made in developing the constitutive model. Furthermore, NNCM requires no material parameters to be identified. These features ascertain the NNCM to be an objective model that can truly represent the natural connections among variables, rather than a subjective model which assumes the variables obey a set of predefined relations.

The NNCM learns from experimental data and forms neural connections stimuli from the learning process. Because of its unique learning, training and prediction characteristics, ANN has great potential in soil engineering applications, particularly for the situations where good experimental data are available and where conventional constitutive modeling may be difficult and time consuming.

A significant number of NNCMs have been developed for modeling of geomaterials. The application of neural network (NN) for constitutive modeling was first proposed by Ghaboussi et al (1990, 1991) for concrete. Later on Ellis *et al.* (1992) and Ghaboussi *et al.* (1994) applied the concept of neural network-based constitutive modeling to model the behavior of geomaterials. These works indicated that NNCMs can effectively capture nonlinear material behavior.

The common procedure of using ANN for constitutive modeling involves training a neural network using laboratory (or in-situ) data to learn the material behavior. The trained network is then used to predict the behavior of the material under new loading conditions. The advantages of using ANN when it is trained directly from some experimental (or *in-situ*) data is obvious. If the training data contains enough and relevant information, the trained network should be able to generalize the material behavior to new loading conditions.

Among various types of neural networks, multi-layer feed-forward back-propagation network is known to be the most suitable architecture to describe the nonlinear relationships, and so far, has been the main type of neural network used to describe material constitutive behavior (Hashash *et al.* 2004). The role of the ANN is to attribute a given set of output vectors to a given set of input vectors. When applied to the constitutive description, the physical nature of these input-output data is determined by the measured quantities like stresses, strains, pore pressures, temperatures, etc. A typical NNCM is shown schematically in Fig. 1.

In the simple example shown in Fig. 1 one input layer, two hidden layers and one output layer are considered for the network. Three principal strain components (ε_1, ε_2 and ε_3) for an assumed medium are input and a forward pass through the network including simple computations results in

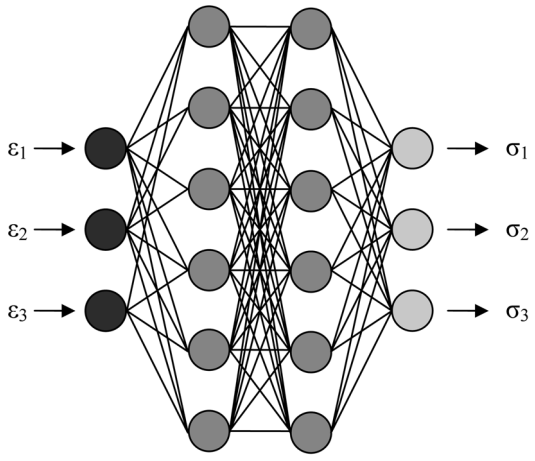

Fig. 1 A simple NNCM

the prediction of three corresponding principal stresses (σ_1, σ_2 and σ_3) in the output layer. Every neuron in each layer is connected to every neuron in the next layer and each such connection has associated with it a "connection weight". The knowledge stored in the developed network is represented by the set of connection weights. The neural network is trained by appropriately modifying its connection weights through the set of "training cases" until the predicted output variables agree satisfactory with the desired variables. The "back-propagation" term (Rumelhart *et al.* 1986) refers to the algorithm by which the observed error in the predicted output variables is used to modify the connection weights.

Encouraged by the attractive features of neural networks, after exploration of the potential of ANN for constitutive modeling during early 90's; a number of NNCMs for different materials were developed. Millar and Clarici (1994) showed the capability of ANN for modeling of behavior of rocks in rock mechanics applications. They used laboratory test results of axial stress-axial strain measurements for training and testing of ANN. Four different ANN models, in terms of number of hidden neurons, were developed and it was shown that ANNs are able to predict the stress-strain relationship with good accuracy. In this work, a multilayer perceptron architecture with back-propagation training algorithm was used. The input-output set for training the model was:

Inputs: σ_3, ε_1, ε_3 and (sign$[d\sigma_1/d\varepsilon_1]$)

Output: σ_1

where σ_1 is the major principal stress, σ_3 is the confining pressure, ε_1 is the major principal strain, ε_3 is the minor principal strain and sign$[d\sigma_1/d\varepsilon_1]$ is the sign of the gradient of the stress-strain curve. The fact that the latter input parameter was required for the ANN training implied that some indication of the history of stress state was necessary as input in the training process.

Ellis *et al.* (1995) modeled the stress-strain relation of sands using ANN and showed good agreement between laboratory data and modeling results. A series of undrained triaxial tests on mortar sand was used to develop the models. Two different types of architecture were used to evaluate the ability of ANN for modeling sand behavior. They were the conventional neural network without feedback and the sequential NN with feedback.

In a sequential network (Fig. 2), at the initial phase of the training a pattern is input to the plan units. Feed forward process occurs as in the standard backpropagation algorithm, producing the first output pattern. This output is then copied back to the current state units for the next feed forward process. The sequential NN has the potential in incorporating the path dependency of mechanical behavior into the model. In order to accommodate this aspect, the input-output parameters for the model should be variables of time. Based on the results it was found that the sequential NN worked better than the conventional backpropagation NN. Thus the authors proposed a sequential network with three layers which had 10 neurons in the intermediate layer and its input-output parameters

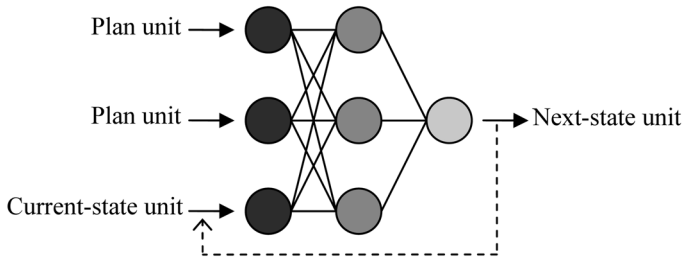

Fig. 2 Architecture of a typical sequential NN

were:

Inputs: ${}^{i}\sigma_1$, σ_3, ${}^{i}\varepsilon_1$, ${}^{i}u$, OCR, D_r and C_u

Outputs: ${}^{i+1}\sigma_1$ and ${}^{i+1}u$

where u is the pore water pressure, OCR is the over consolidation ratio which reflects the previous stress history, D_r is the initial relative density and C_u is the coefficient of uniformity which characterizes the grain size distribution of sand. A constant value of 0.0405% was used for the axial strain increment, $\Delta\varepsilon_1$. Based on the reported results, the NN predictions, in particular the values of pore water pressure, were not very accurate. Later, it was argued that a prescribed strain rate (0.0405% per minute) has to be defined in order to make predictions with this model (Najjar and Basheer 1996). This issue limits the developed network as applicable only to a specific case with a strain rate of 0.0405% per minute.

Millar and Calderbank (1995) showed that a single multilayer feedforward neural network is able to predict the deformability behavior of rock. Data used to train the neural network model was derived from the results of a series of simulations of triaxial tests using commercial explicit finite difference software, FLAC. The authors made some modifications to their ANN training approach in order to resolve the deficiencies associated with the earlier work (Millar and Clarici 1994) and make their model worthy for immediate use as a stand alone constitutive relationship in a numerical modeling code. For this purpose the authors used the same ANN architecture as their earlier work, but they revised the way the input-output parameters were introduced to ANN in the training procedure. The input-output parameter sets used for the training of their revised ANN based model were:

Inputs: ${}^{i}\sigma_1$, ${}^{i}\sigma_3$, ${}^{i}(\Delta\varepsilon_1)$ and ${}^{i}(\Delta\varepsilon_3)$

Outputs: ${}^{i+1}\sigma_1$ and ${}^{i+1}\sigma_3$

where $(\Delta\varepsilon_1)$ and $(\Delta\varepsilon_3)$ are the increments of major and minor principal strains, respectively. The data, which were produced by triaxial test simulation in FLAC using strain softening model available within this software, had to be scaled within the interval between –0.5 and 0.5 for the training process. Also the value of minor principal stress, σ_3, was considered not to be identical for a single test. This was done through the superposition of a component of noise to the input values on each presentation of the data to the NN. The optimum NN structure obtained for the constitutive relationship was then used to develop a user defined constitutive model, called NN UDM, back in FLAC. Although the accuracy of the NN model over the training data was good however its prediction ability was so poor and the actual behavior of the NN UDM was far from desired behavior when it was used in place of the standard strain softening constitutive model within FLAC.

Amorosi *et al.* (1996) also adopted a neural network based representation for constitutive behavior of geomaterials. The data obtained from undrained triaxial tests on a particular clay (Vallericca clay) was used to develop the NN model. The input-output parameter sets used in this work were

Inputs: ${}^{i}\sigma_1$, σ_3, ${}^{i}u$, ${}^{i}(\Delta\varepsilon_1)$ and OCR

Outputs: ${}^{i+1}\sigma_1$ and ${}^{i+1}u$

The constitutive behavior of Vallericca clay was shown to be adequately represented with the trained NN model. The developed model had a back propagation multilayered perceptron architecture

with three layers. The input and output layers used 5 and 2 nodes respectively while the hidden layer contained four nodes.

Logar and Turk (1997) presented a constitutive model for soft soils using a neural network. The results from oedometer loading tests on a silty soil were used to train a feed forward neural network. Based on the source of the available data the following input-output set was used to develop the model

Inputs: σ', Z, w_0, w_l and w_p

Output: e

where σ' is effective stress, Z is the depth from which the sample was taken, w_0 is natural water content, w_l is liquid limit, w_p is plastic limit and e is void ratio. The optimum NN structure was obtained by a single hidden layer consisting of 35 hidden neurons. The results for approximation of oedometer curves by NN were relatively accurate compared to the experimental measurements with average error of around 10% for the training phase. The trained neural network was used to determine the tangential oedometer modulus as

$$E_{oed} = -(1 + {}^{i}e)\frac{\Delta\sigma'}{\Delta e} = -(1 + {}^{i}e)\frac{{}^{i+1}\sigma' - {}^{i}\sigma'}{{}^{i+1}e - {}^{i}e} \tag{2}$$

The above equation was then used, instead of the elastic parameter, in a finite element code to model the amount of settlement in an embankment. The results were reported to be comparable with those obtained using a cap model for deformation.

Penumadu and Chameau (1997) presented a model for soil behavior within a unified environment based on NN. The same triaxial data as used by Ellis *et al.* (1995) was used for training and testing of the NN sand model. Also stress-strain data obtained from a series of strain controlled undrained triaxial tests on clay was used for training and testing the NN clay model. The same type of NN as the one used in Ellis *et al.* (1995) (feed back sequential NN) was again used in this work. The NN architecture and results for Mortar sand were identical to those presented in Ellis *et al.* (1995), however for clay a different NN architecture including one hidden layer with 10 nodes was selected. The input-output set for the clay model was

Inputs: ${}^{i}\tau$, $\dot{\varepsilon}$, ${}^{i}\varepsilon$, ${}^{i}(\Delta\varepsilon)$

Output: ${}^{i+1}\tau$

where ${}^{i}\tau$ is shear stress and $\dot{\varepsilon}$ is the rate of strain increment.

Zhu *et al.* (1998a) presented a recurrent neural network (RNN) model for simulating and predicting shear behavior of two different soils. A recurrent neural network is a network where the connections between the units form a directed cycle. Recurrent neural networks must be approached differently from feed forward neural networks, both when analyzing their behavior and training them. Hidden nodes in an RNN can transmit their outputs to both input layer and output layer simultaneously (Elman 1990). A typical architecture of an RNN with one hidden layer is shown in Fig. 3.

Laboratory based experimental data were used for modeling including a set of strain controlled undrained tests and a set of stress controlled drained tests performed on a residual Hawaiian volcanic soil. The choice of input-output variables was different due to different sources of data. For

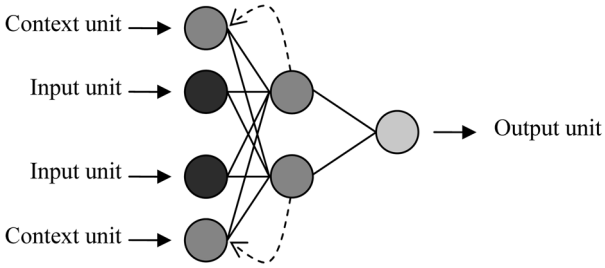

Fig. 3 A typical recurrent neural network

the strain controlled test the goal was to measure stress response of the specimen to a given strain value, therefore the selected input-output variables for RNN training were:

Inputs: ${}^{i}q$, ${}^{i}\sigma_3'$, ${}^{i}u$, ${}^{i}\varepsilon_1$, ${}^{i}(\Delta\varepsilon_1)$ and ${}^{i}e$

Outputs: ${}^{i+1}q$ and ${}^{i+1}u$

where $q = \sigma_1 - \sigma_3$ is deviatoric stress.

In contrast, as for a stress controlled test the shear stress and stress increment were known in advance, the goal was to predict the strain response of the specimen due to a stress increase. Thus for such test results, the selected input-output parameters for the RNN were:

Inputs: ${}^{i}\sigma_1'$, ${}^{i}\sigma_3'$, ${}^{i}(\Delta\sigma_1')$, ${}^{i}(\Delta\sigma_3')$, ${}^{i}u$, ${}^{i}\varepsilon_1$, ${}^{i}\varepsilon_v$ and ${}^{i}e$

Outputs: ${}^{i+1}\varepsilon_1$ and ${}^{i+1}\varepsilon_v$

In both models, RNN structure with one hidden layer containing 20 nodes was found to generate the minimal sum squared error. Good agreement between the modeling results and the observed experimental data showed the efficiency of the RNN approach in modeling complex soil behavior. The authors suggested that such an RNN model could be applicable to other soils if appropriate input and output parameters are chosen.

Zhu *et al.* (1998b) published a similar work in which the same NNCM (in terms of network type, architecture and input-output set) was used to model soil behavior, using generally the same data as in Zhu *et al.* (1998a). However in this work the authors proposed that in the network structure one hidden layer with 20 and 35 nodes is suitable for the modeling of the strain controlled undrained tests and stress controlled drained tests respectively.

Ghaboussi *et al.* (1998) described a new indirect method, called autoprogressive training, for training neural network material models from structural tests to learn complex stress-strain behavior of materials. The global data measured form a structural load-deflection test was used to train the network. The main premise of the work was that the structural tests usually generate a large number of spatial patterns of stresses and strains that can be used for training. The term "autoprogressive training" referred to a process in which the neural network is itself an integral part of the iterative algorithm that is used to create the stress-strain training cases from the global response data. This method differs from common applications of NN models in the sense that there is not a known set of data to train the network, but the material model is extracted iteratively from global measurements using nonlinear finite element analysis (Haj-Ali *et al.* 2001). The applications discussed in this paper show a procedure that can be used to create the stress-strain training data for the neural network material model, having knowledge of the global load vs. deflection response of the structure. In

contrast to previous applications of neural networks in constitutive modeling, in this method there was not "a priori" a set of directly measured information that accurately represents the material behavior but this information must be extracted from the recorded structural response. Based on the results of two simple examples presented in this paper, the predictions of the neural network trained in this way were consistent. However, the minimum number of measured structural responses, and their type and locations on the structure, that are required in order to uniquely determine a neural network material model is an important theoretical issue that remains to be addressed.

Sidarta and Ghaboussi (1998) modified the earlier Ghaboussi *et al.* (1998) work in order to develop a neural network based constitutive model for geomaterials using autoprogressive training. They used a non-uniform material test which had a non-uniform distribution of stresses and strains within the specimen. Then the measured boundary forces and displacements were applied in a finite element model of the test to generate the input and output data for training the neural network material model. Using the data generated in that way, the autoprogressive method was used to train the neural network material model. Three drained triaxial tests on Sacramento River sand were considered in this work. The tests were performed with end friction condition, and the relative densities of the samples ranged from loose to medium dense to dense. The measured axial forces and confining pressures were directly from the test data. The radial displacements of the outer surface of each sample were determined by assuming a parabolic distribution. These measured force and displacement boundary conditions were used in the autoprogressive method. The components of stress and strain, which were required to train the neural network material model, were constructed artificially in the finite element model of the test. In the model, the components of current strain and void ratio, (together with stresses and strains of the previous history points where necessary) were used as input to predict the components of current stress as output. The results indicated that the material behavior becomes increasingly more complex (requiring more history point modules) with increasing the soil density.

The trained neural networks were used in finite element analysis of actual triaxial test with end friction as well as finite element analyses of hypothetical tests with no end friction. The results of the analysis with end friction matched well with those of the actual experiment. However the results of the forward analysis of the hypothetical tests with no end friction showed significant differences with the actual experimental results. The work presented in this paper, introduced an improvement over conventionally trained neural network based constitutive models for geomaterials. The attraction of the non-uniform test, used in this study, is that a range of stress levels and a variety of stress paths may be represented in a single test, therefore the test results contain information on material behavior for different stress levels and stress paths. If that information could be extracted, then the results of a single non-uniform material test may be sufficient for training a neural network constitutive model and there is no need for a large number of conventional triaxial tests with different stress paths to produce the training data.

Ghaboussi and Sidarta (1998) and Sidarta and Ghaboussi (1998) presented a nested adaptive neural network (NANN) for constitutive modeling. The idea behind this approach is that the material data has an inherent structure and one type of such inherent internal structure in data is the nested structure. Basically nested adaptive neural networks take advantage of the nested structure of the material test data, and reflect it in the architecture of the neural network. A nested neural network consists of several modules. The starting point of building a NANN is to develop a base module to represent the material behavior in the lowest function space in the data structure. This base module is a standard multi-layer feed-forward neural network. The base module is then augmented by

attaching added modules to form a higher level NANN. The process is theoretically open ended and more and more modules can be added. The added modules themselves are also standard multi-layer feed-forward neural networks. The developed NANNs were applied to modeling of drained and undrained behavior of Sacramento River sand in triaxial compression tests. The objective was to model a material behavior in both drained and undrained conditions for a range of initial void ratios and initial confining pressures. First a base module was developed and then the history point modules were added. The results indicated that the effect of history points on the material behavior becomes increasingly more complex and difficult for the neural network to learn. With increasing the number of history points, the number of inputs can increase significantly, which after even a few steps this can make the network massively complex and result in much higher computational time and cost.

Another neural network based constitutive relationship was presented by Penumadu and Zhao (1999) to model stress-strain and volume change behavior of sand and gravel under drained triaxial compression test conditions. The NNCMs presented in this paper were developed based on a large database comprised of nearly 250 triaxial test results collected from literature. Two neural network sand models (Sand-Low and Sand-High) were developed to model the test results on sand in the low confining pressure (less than 700 kPa) and high confining pressure (higher than 700 kPa) range. The division at 700 kPa was chosen arbitrarily by the authors. Also a single model was developed for test results on gravel.

A sequential neural network structure (Fig. 3) was used and like other NNCMs, back-propagation algorithm was employed to train the neural networks. The final optimum network architecture had three layers with eleven neurons in input layer, fifteen neurons in the hidden layer and two neurons in output layer. The number of hidden units was determined using a trial and error procedure. The selected input-output variables for NN training were:

Inputs: ${}^{i}\sigma_d$, ${}^{i}\varepsilon_v$, σ_3', ${}^{i}\varepsilon_1$, ${}^{i}(\Delta\varepsilon_1)$, e, n_s, h, D_{50}, C_u, C_c

Outputs: ${}^{i+1}\sigma_d$, ${}^{i+1}\varepsilon_v$

Seven of the eleven inputs were used to describe the hardness of the mineral (h), shape factor (n_s), equivalent particle size and the particle size distribution (D_{50}, C_u, C_c), void ratio (e) and effective confining pressure (σ_3'). The current state units of stress and strain were represented with three inputs using deviator stress (${}^{i}\sigma_d$), axial strain (${}^{i}\varepsilon_1$) and volumetric strain (${}^{i}\varepsilon_v$). For a given specimen conditions and current state units the objective of neural network was to predict deviator stress (${}^{i+1}\sigma_d$) and volumetric strain (${}^{i+1}\varepsilon_v$) of the next state of an input axial strain increment (${}^{i}(\Delta\varepsilon_1)$). An interesting feature for training the network in this research was that a fixed set of axial strain increments were chosen consistently for all the test data. This means that the value of strain increment was chosen to increase at a constant magnitude (e.g., 0.1%). The original experimental data (deviator stress-axial strain and volumetric strain-axial strain) were not recorded at a specific strain increment. The authors obtained the training pattern corresponding to the considered strain increment by digitalizing the data and using cubic spline interpolation (Press *et al.* 1992). It was observed that the neural network material models obtained in this research were able to represent the constitutive behavior of cohesionless soil with reasonable accuracy. This NNCM was later used in Penumadu *et al.* (2000) to simulate triaxial tests.

Habibagahi and Bamdad (2003) used neural network to describe the mechanical behavior of unsaturated soils. A multilayer perceptron, sequential architecture with feed back capability was

chosen in this network. Triaxial test results on Lateritic gravel, reported by Toll (1998), were used as database. The final network which was obtained through a trial and error procedure had three layers with 9 neurons in the input layer and three neurons in the output layer. The optimal number of nodes in the hidden layer was worked out to be five. The input-output parameters set for this NNCM for unsaturated soils were:

Inputs: ${}^{i}q$, ${}^{i}\varepsilon_v$, ${}^{i}E(U_a - U_w)$, ${}^{i}\varepsilon_1$, ${}^{i}(P - U_a)$, ${}^{i}(P - U_w)$, $(U_a - U_w)$, S_r, ρ_d and θ

Outputs: ${}^{i+1}q$, ${}^{i+1}\varepsilon_v$ and ${}^{i+1}E(U_a - U_w)$

In the input parameters set, four neurons, namely, soil water content θ, dry density ρ_d, degree of saturation S_r, and soil suction $(U_a - U_w)$, represent the initial condition of the specimen before shearing. The other six neurons, namely, axial strain ${}^{i}\varepsilon_1$, change in suction ${}^{i}E(U_a - U_w)$, mean effective stresses with respect to pore air and water pressures (${}^{i}(P - U_a)$ and ${}^{i}(P - U_w)$), volumetric strain ${}^{i}\varepsilon_v$, and deviatoric stress ${}^{i}q$ are the input variables that must be updated incrementally during training based on the outputs received from the previous increment of training. It was shown that the trained network was able to model the mechanical behavior (stress-strain, volume change and change in suction) of unsaturated soils with reasonable accuracy. The authors also proposed that the model may be used to simulate triaxial tests (artificial tests) under similar conditions.

In addition to the works mentioned above, some other researchers have also applied NN for constitutive modeling of geomaterials using different datasets (e.g., Banimahd *et al.* 2005, Najjar *et al*. 1999, Wu *et al.* 2001). The results of these works also show the capability of NN in stress-strain prediction of different soils.

3.2 Implementation of NNCM in finite element method

As has been described in the previous section, to date, many researchers have attempted to model the various aspects of the constitutive behavior of geomaterials with neural networks. Although these works are different in terms of their details and terminology; however most of their results have indicated that NNs have the ability to represent materials responses to different load paths with reasonable accuracy. From this, in theory, it is seen that in a numerical analysis tool such as FEM, it is possible to replace a conventional (analytical) constitutive model with a suitably trained NNCM. However the focus of most of the investigations has been on the description of the constitutive behavior itself. As a result little is known about the performance of NNCMs in engineering analyses. The main reason for this appears to be the fact that there are considerable difficulties in incorporating a general NNCM in finite element codes (Shin and Pande 2002).

Shin and Pande (2000) presented a self learning FE code in which a NNCM was used instead of conventional constitutive models and showed that the application of a constitutive law in the form of a neural operator leads to some qualitative improvement in the application of FEM in engineering practice. They presented a procedure where data for training neural network based constitutive model were acquired from planned monitoring of structural tests. Unlike conventional procedures where generally material testing is performed to extract the stress-strain relationship and identify material parameters, in this work, inverse analysis was carried out to identify material parameters from monitored global structural response. In this way the self learning capability of the software was expected; however for this purpose the results of structure behavior needed to be available in advance. It is obvious that depending on the mesh size of the problem under consideration, large

amount of data may be accumulated with increasing the number of self learning cycles which can results in severe computer storage and CPU time problems during training. To address this problem a limited number of monitoring points were selected in the structure and the data corresponding to these points were used to train the NNCM. Selection of the number and location of monitoring points is therefore of considerable importance in identifying a reliable NNCM. It was stated that such trained NNCM will need to be treated with caution for modeling the behavior of other structures; as it is apparent that an NNCM may predict the correct response at a few points, yet may be completely inadequate to predict the response at others.

Shin and Pande (2001) showed that in their self learning finite element code the tangential constitutive matrix of the material can be computed as it is possible to obtain partial derivatives of the neural network model which has been trained though total stress and stress data. The capabilities of the developed FE code were illustrated by analyzing a rock specimen under uniaxial cylindrical compression (with fixed ends). Shin and Pande (2002) proposed a strategy to generate additional data from general homogeneous material tests in order to train NNCM. This was done by taking advantage of isotropy when it is applicable to the material under consideration. A boundary value problem of a circular cavity in a plane stress plate was modeled with the self-learning FE code using NNCM trained with the enhanced dataset. The self-learning FE analyses showed comparable results with FE analyses using conventional constitutive models.

Drakos *et al.* (2006) presented a NNCM and stated that the model is equivalent of the hardening soil model. Synthetic data for training the NNCM was generated using the Hardening Soil Model (HSM) available in the commercial software PLAXIS and choosing a set of arbitrary parameters, typical of sands, for the HSM. The performance of the trained NNCM was then validated by using this model for numerical analysis of two simple foundation and excavation problems.

Lefik and Schrefler (2003) used a neural network for constitutive modeling of nonlinear material behavior and highlighted some of the difficulties in the constitutive description in incremental form.

Hashash *et al.* (2004) described some of the issues related to the numerical implementation of NNCM in finite element analysis and derived a closed-form solution for material stiffness matrix for the neural network constitutive model.

Javadi *et al.* (2002, 2003, 2004a, 2004b, 2005) carried out extensive research on application of neural networks in constitutive modeling of complex materials in general and soils in particular. They developed an intelligent finite element method (NeuroFE code) based on the incorporation of a back-propagation neural network in finite element analysis. The intelligent finite element model was applied to a wide range of boundary value problems including several geotechnical engineering applications and it was shown that ANNs can be very efficient in learning and generalizing the constitutive behavior of complex materials such as soils, rocks and others.

3.3 Other applications of ANN in geomechanics

ANNs have been applied to a wide range of geotechnical engineering problems such as pile bearing capacity (e.g., Abu-Kiefa 1998, Goh 1996), site characterization (e.g., Juang *et al.* 2001), soil behavior (e.g., Zhu *et al.* 1998), liquefaction potential (e.g., Juang and Chen 1999), slope stability (e.g., Lu and Rosenbaum 2003), underground openings (e.g., Benardos and Kaliampakos 2004, Javadi 2006) and many others. Toll (1996) presents a review of engineering applications of AI techniques.

3.4 Advantages and shortcomings of neural networks

A neural network based constitutive model has several advantages including:

(i) It provides a unified approach to constitutive modeling of all materials;

(ii) It does not require any arbitrary choice of the constitutive (mathematical) model. The incorporation of an ANN in FE procedure avoids the need for complex yielding/plastic potential/failure functions, flow rules, etc. There is no need to check yielding, to compute the gradients of the plastic potential curve or to update the yield surface;

(iii) There are no material parameters to be identified;

(iv) As a neural network learns the material behavior directly from raw experimental data the ANN based constitutive model is the shortest route from experimental research (data) to numerical modeling;

(v) The numerical parameters of the neural network-based constitutive models are easily and automatically defined and NNCM can be incorporated in a FE code in a very natural manner. A trained network can be incorporated in a FE code/procedure in the same way as a conventional constitutive model. It can be incorporated either as incremental or total stress-strain strategies. An intelligent FE method can be used for solving boundary value problems in the same way as a conventional FEM;

(vi) An additional advantage of NNCM is that as more data becomes available, the material model can be improved by re-training the ANN.

Although it has been shown by various researchers that ANNs offer great advantages in the analysis of many geotechnical engineering problems, but in general, they suffer from a number of drawbacks. One of the main disadvantages of the ANN (and NNCM) is that the optimum structure of the network (such as number of inputs, hidden layers, transfer functions, etc.) must be identified a priori, which is usually done through a time consuming trial and error procedure. In this respect, some attempts have been made to address optimal design of ANN structure based on a multi-objective strategy to find trade-off between model simplicity and accuracy (Giustolisi and Simeone 2006). Another major disadvantage of neural network based models is the large complexity of the network structure, as it represents the knowledge in terms of a weight matrix and biases which are not accessible to user understanding. In other words NN models provide no insight into the way inputs affect the output and are therefore considered as a black box class of model. The lack of interpretability of NN models has inhibited them from achieving their full potential in real world problems (Lu *et al.* 2001). In addition, as ANNs perform function approximation through large parameterization and the use of simple functional structures (transfer functions), parameter estimation and overfitting problems represent other major disadvantages of a model constructed by ANN (Giustolisi 2002).

4. Genetic programming

Genetic programming which was introduced in the early 90s by Koza (1992), is an evolutionary computing method that generates a transparent and structured representation of the data provided. Evolutionary algorithms (EAs) are search techniques based on computer implementation of some of the evolutionary mechanisms found in nature (such as selection, crossover and mutation) in order to solve a function identification problem. The function identification problem is to search for a function

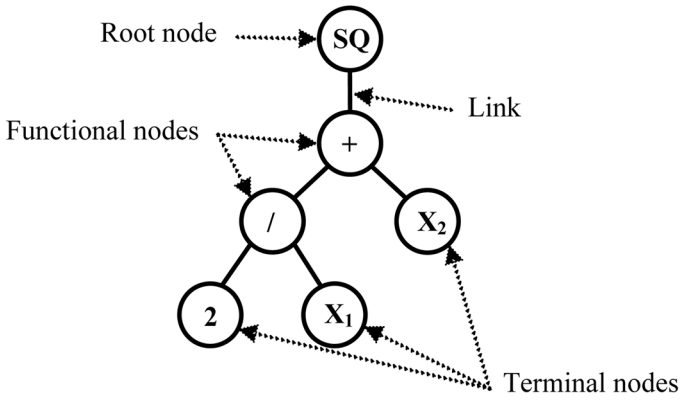

Fig. 4 Typical GP tree representing function $(2/x_1+x_2)^2$

in a symbolic form that fits a set of experimental data.

Genetic algorithm (GA) and genetic programming (GP) are the major types of evolutionary algorithms. GP is a generalization and an extension of GA. GAs are generally used in parameter optimization to evolve the best values for a given set of model parameters, whereas GPs give the basic structure of the approximation model together with the values of its parameters. While a GA uses a string of numbers to represent the solution, the GP combines a high level symbolic representation with the search efficiency of the GA to form the best possible model for the system.

Representation schemes in genetic programming are composed of nodes which are elements from a terminal set (constants e.g., 2 and/or variables e.g., x_1, x_2, etc.) and a functional set (mathematical operators that generate the model e.g., ± and x^y, etc.). A typical genetic programming tree, representing the simple algebraic expression $(2/x_1 + x_2)^2$ is shown in Fig. 4.

The result of the GP process is a set of random trees of different sizes and shapes, each exhibiting a different fitness with respect to the objective function. If the set of applied functions is sufficiently rich, tree structures are capable of representing hierarchical programs of any complexity.

The nature of genetic programming (GP) allows the user to gain additional information on how the system performs, i.e., gives an insight into the relationship between input and output data. Once a population of computer programs has been randomly created, the process of evolving the population proceeds using the simple principles as for GAs, with the minor difference that, strings of functions and terminals are reproduced, crossed over and mutated rather than strings of binary codes. Evolutionary algorithms maintain a population of structures that evolve according to the rules of natural selection and some operators inspired from natural genetics such as reproduction or crossover. Each individual in the population receives a measure of its fitness in the current environment. The fitness criteria are calculated by the objective function i.e., how good the individual is at competing with the rest of the population. At each generation a new population is created by the process of selecting individuals according to their fitness and breeding them together using the genetic operators (crossover and mutation). The existing population will then be replaced with the new population. The procedure continues until the termination criterion, which can be either the maximum number of generations or a particular allowable error, is satisfied. After the termination criterion is met, the single best program in the final population is designated as the result of the GP process.

4.2 Application of GP in geomechanics

Application of genetic programming in the field of civil engineering is quite new and original, and it has just started to be used in the field of geotechnical engineering. Indeed the very pioneering works relating to investigation of the capability of genetic programming in the field of geotechnics have been published recently by the authors (e.g., Javadi and Rezania 2006, Javadi *et al.* 2006, Rezania and Javadi 2007). Javadi *et al.* (2006) introduced GP as a new approach for determination of liquefaction induced lateral spreading. This is a complex geotechnical problem because of the large number of parameters (i.e., parameters describing the earthquake strength, geology of the site and the soil characteristics) involved. In this work GP models were trained and validated using a database of SPT-based case histories. Separate models were presented to estimate lateral displacements for free face and for gently sloping ground conditions. It was shown that the GP models are able to learn, with a very high accuracy, the complex relationship between lateral spreading and its contributing factors in the form of a function. It was also shown that the attained function can be used to generalize the learning to predict liquefaction induced lateral spreading for new cases not used in the construction of the model. The results of the developed GP models were compared with those of a commonly used model and the advantages of the proposed GP model were highlighted. It was shown that the GP based models for lateral spreading determination, offer an improvement over the most commonly used, multi linear regression (MLR), model (Youd *et al.* 2002) for this problem. Rezania and Javadi (2007) utilized genetic programming for prediction of settlement of shallow foundations on cohesionless soils. It was shown that the application of the traditional methods for prediction of settlement of shallow foundations could lead to very large errors. A new GP based model was developed and presented in this paper. Comparison of the results showed that the predictions by the proposed GP model provide significant improvements over the traditional methods and also outperforms the ANN based models.

5. Evolutionary polynomial regression

Evolutionary polynomial regression (EPR) is a data-driven method based on evolutionary computing, aimed to search for polynomial structures representing a system. Genetic programming and neural network are both very powerful non-linear modeling techniques, but they have their own drawbacks. GP tends to search for mathematical expressions for F in Eq. (1) using an evolutionary approach, but the parameter values (vector θ) are generated as non-adjustable constants, referred to as ephemeral random constants. Therefore the constants do not necessarily represent optimal values as in numerical regression methods and good structures of F can be missed in the process (Giustolisi and Savic 2006). Furthermore the number of terms in GP based expressions can greatly exceed and the evolutionary search within GP can be quite slow. Some of the disadvantages of ANN approach have been highlighted in section 3.4.

EPR is classified as a symbolic grey box technique which can construct clearly structured model expressions for a given set of data. To avoid the problem of mathematical expressions growing rapidly in length with time associated with GP, in EPR the evolutionary procedure is conducted in the way that it searches for the exponents of a polynomial function with a fixed maximum number of terms, rather than performing a general evolutionary search as used in conventional GP. Furthermore, during one execution it returns a number of expressions with increasing numbers of terms up to a

limit set by the user, to allow the optimum number of terms to be selected.

In general, EPR is a two-stage technique for constructing symbolic models; (*i*) initially, using standard genetic algorithm (GA), it searches for the best form of the function structure, i.e., a combination of vectors of independent inputs, X (Eq. (1)) and (*ii*) secondly it performs a least squares regression to find the adjustable parameters, θ, for each combination of inputs. In this way a global search algorithm is implemented for both the best set of input combinations and related exponents simultaneously, according to the user-defined cost function.

The global search for the best form of function is performed by means of a standard GA over the values in the user defined vector of exponents. The GA operates based on Darwinian evolution which begins with random creation of an initial population of solutions. Each parameter set in the population represents the individual's chromosomes. Each individual is assigned a fitness based on how well it performs in its environment. Through crossover and mutation operations, with the probabilities P_c and P_m respectively, the next generation is created. Fit individuals are selected for mating, whereas weak individuals die off. The mated parents create a child (offspring) with a chromosome set which is a mix of parents' chromosomes. It is also possible that one parent chromosome undergoes mutation operation to form the offspring. The EPR process continues over generations and stops when the termination criterion, which can be either the maximum number of generations, the maximum number of terms in the target mathematical expression or a particular allowable error, is satisfied. Description of the mathematical formulation and details of the EPR procedure is outside the scope of the current paper and can be found in, e.g., Giustolisi and Savic (2006).

5.1 Application of EPR in geomechanics

EPR is a recently developed methodology that was originally used for environmental modeling by its developers (Giustolisi and Savic 2006, Giustolisi *et al.* 2007, Doglioni *et al.* 2008). However the capability and outstanding performance of EPR approach in dealing with problems related to other disciplines of civil engineering including geotechnical, structural and earthquake engineering were investigated by the authors of this paper (e.g., Javadi *et al.* 2007, Rezania and Javadi 2006, Rezania and Javadi 2008b).

Javadi and Rezania (2008a) introduced the EPR as a new approach for analysis of a number of geotechnical engineering problems. They investigated the feasibility of using this method for capturing nonlinear interaction between input and output variables in geotechnical systems. The efficiency of the EPR methodology was illustrated by application to a number of complex practical geotechnical engineering problems which are difficult to solve or interpret using conventional approaches. The merits and limitations of the proposed method were discussed.

Rezania *et al.* (2008a) highlighted some of the complexities involved in the analysis of many civil engineering phenomena and the shortcoming of traditional methods in describing such complexities. They presented EPR as a means of for capturing nonlinear interactions between various parameters of civil engineering systems. They illustrated the capabilities of the EPR methodology by application to two complex civil engineering problems including evaluation of uplift capacity of suction caissons and shear strength of reinforced concrete deep beams. The results showed that the proposed EPR models provide significant improvement over the existing models. They also indicated that, for design purposes, the EPR models are easy to use and provide results that are more accurate than the existing methods. It was concluded that the new approach overcomes the shortcomings of the

traditional and ANN-based methods in analysis of civil engineering systems.

Rezania and Javadi (2008a) presented a new EPR-based approach for prediction of settlement of shallow foundations. The EPR model was developed and verified using a large database of SPT (standard penetration test) based case histories involving measured settlements of shallow foundations. The results of the EPR model were compared with those of a number of commonly used traditional methods and an ANN based model. It was shown that the EPR model is able to learn, with a very high accuracy, the complex relationship between foundation settlement and its contributing factors in the form of a function and generalize the learning to predict settlement of foundations for new cases not used in the development of the model. They highlighted the advantages of the proposed EPR model over the conventional methods and the ANN based model.

Rezania *et al.* (2008b) used EPR for determination of liquefaction potential of sands. EPR models were developed and validated using a database of 170 liquefaction and non-liquefaction field case histories for sandy soils based on CPT (cone penetration test) results. Three models were presented to relate liquefaction potential to soil geometric and geotechnical parameters as well as earthquake characteristics. The results of the developed EPR models were compared with a conventional model and a number of neural network based models. It was shown that the proposed EPR model provides more accurate results than the conventional model and the accuracy of the EPR results is better than or at least comparable to that of the neural network based models proposed in the literature.

Javadi and Rezania (2008b) presented an innovative approach to constitutive modeling of materials in finite element analysis using EPR. The proposed approach provides a unified framework for modeling of complex materials, using evolutionary polynomial regression-based constitutive model (EPRCM), integrated in finite element analysis. The advantages of EPRCM over conventional constitutive models and NNCMs were highlighted. The proposed algorithm provides a transparent relationship for the constitutive material model that can be easily incorporated in a finite element model. The application of the EPRCM for material modeling in finite element analysis was illustrated through a number of examples.

The main advantage of EPR over ANN appears to be that it provides the optimum structure for the material constitutive model representation as well as its parameters, directly from raw experimental (or field) data. The advantage compared with genetic programming is in producing compact and well-structured mathematical expressions.

6. Discussion

In conventional constitutive modeling of materials, an appropriate mathematical model is initially selected and the parameters of this model (material parameters) are then identified from appropriate physical tests on representative samples to capture the material behavior. When these constitutive models are used in numerical analysis, the accuracy with which the selected material model represents the various aspects of the actual material behavior affects the accuracy of the numerical predictions. In the past two decades, the use of artificial neural networks has been introduced as an alternative approach to constitutive modeling of materials. These studies indicated that neural network-based constitutive models can be very efficient in learning and generalizing the constitutive behavior of complex materials. It has also been shown that the neural network based constitutive model (NNCM) can be incorporated in a finite element (or finite difference) code as a material model. Although it has been shown by various researchers that ANNs offer great advantages in

constitutive modeling of materials, the application of NNCM in finite element analysis of engineering problems is still in its infancy and the majority of the applications so far have been limited to simple boundary value problems and relatively straight forward aspects of material behavior. The main shortcomings of the NNCMs, which have prevented them from achieving their full potential, are the back box nature of ANN and the fact that the optimum structure of the ANN must be identified a priori.

To address the shortcomings of the neural network-based approach, in recent years *new* approaches have been proposed for modeling of soils and other geomaterials using Genetic Programming (GP) and Evolutionary Polynomial regression (EPR). GP and EPR are evolutionary computing techniques that generate a transparent and structured representation of the system being studied. The main advantage of GP and EPR over ANN is that they provide the optimum structure for the material constitutive model representation as well as its parameters, directly from raw experimental (or field) data. The advantage of EPR compared with GP is in producing compact and well-structured mathematical expressions. GP and EPR provide a structured representation for the constitutive material model that can be readily incorporated in the finite element method. It is envisaged that the establishment of a unified framework for modeling of materials with complex behavior using ANN, GP or EPR will be valuable across the field in various disciplines of engineering. However, in the authors' opinion, the development of these new modeling techniques should be done in parallel with developments in conventional constitutive modeling rather than a replacement for them.

7. Conclusions

A considerable number of neural network-based models have been developed for constitutive modeling of soils. Many of these models have been developed as simple prototypes to show the applicability of these techniques in modeling of specific soils. Only few models have been integrated in numerical (e.g., FE) models of engineering systems. The majority of these systems use neural network as a unified approach to constitutive modeling of complex materials. The results of these works have collectively shown the potentials of NNCM for modeling of soil behavior. More recently, a number of other data mining systems have been proposed that appear to be able to address few shortcomings of the neural network based models. It is envisaged that, while the establishment of the new unified frameworks for modeling of materials will be valuable across the board in various disciplines of engineering, however the authors believe that the development of these new techniques should be done in parallel with developments in conventional constitutive modeling rather than a replacement for them. Development of numerical models that include a range of conventional constitutive models besides the new AI and data mining-based models will increase the range of options for modeling of complex materials. In this way, for materials whose behavior is understood and sufficiently described by one of conventional constitutive models, an appropriate model can be selected by the user. However, for cases where the behavior is too complicated to be described by a conventional model but a sufficient amount of experimental data is available, the new modeling tools offer great advantages in numerical analysis of engineering systems. In any case, it should be noted that all these models should be used by engineers and the importance of engineering judgment in interpretation of numerical results should not be underestimated.

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