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# Seismic damage estimation of reinforced concrete framed structures affected by chloride-induced corrosion

# M.B. Anoop<sup>\*</sup> and K. Balaji Rao

CSIR-Structural Engineering Research Centre, CSIR Campus, Taramani, Chennai 600 113, India

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**Abstract.** A methodology for estimation of statistical properties (viz. mean and standard deviation) of the expected seismic damage to reinforced concrete framed structures subject to corrosion of reinforcement, over a specified reference time (typically the service life of the structure) is proposed in this paper. The damage to the structure under the earthquake loading is characterised by the damage index, determined using the modified Park and Ang damage model. The reduction in area, yield strength and strain at ultimate of steel reinforcement, and the reduction in compressive strength of cover concrete due to corrosion are taken into account in the estimation of damage. The proposed methodology is illustrated through an example problem. From the results obtained, it is noted that there is an increase of about 70% in the mean value of expected seismic damage to the reinforced concrete frame considered over a reference time of 30 years when effect of corrosion is taken into consideration. This indicates that there is a need to consider the effect of corrosion of reinforcement on the estimation of expected seismic damage.

**Keywords:** earthquakes; chloride-induced corrosion; reinforced concrete framed structure; expected seismic damage

# 1. Introduction

The estimation of expected damage due to earthquakes for structures located in seismically active regions is a matter of concern for design engineers and decision-making authorities in public- and private-sector organizations. This issue is more complicated for reinforced concrete (RC) structures located in coastal areas due to the degradation in strength and stiffness of the structure with time caused by the corrosion of reinforcement. Number of investigations has been reported in literature dealing with the estimation of damage to RC structures due to earthquakes. Based on experimental investigations on reinforced concrete beams subjected to different levels of reinforcement corrosion, Razak and Choi (2001) have reported that corrosion of reinforcement affect the dynamic characteristics (namely, natural frequencies and damping ratio) of the reinforced concrete beams. Thus, there is a need to consider explicitly the effect of corrosion of reinforcement while studying the seismic performance of the reinforced concrete structures.

Estimation of seismic performance of corrosion-affected reinforced concrete structures is an

<sup>\*</sup>Corresponding author, Ph.D., E-mail: balajiserc1@yahoo.com

area of active research (Choe *et al.* 2009, Kumar *et al.* 2009, Simon *et al.* 2010, Akiyama *et al.* 2011, Biondini *et al.* 2011, 2014, Fotopoulou *et al.* 2012, Ghosh and Padget 2012, Yalciner *et al.* 2012, Yüksel and Coşkan 2013, Dong *et al.* 2013, Akiyama and Frangopol 2014, Karapetrou *et al.* 2013, Chiu *et al.* 2015). However, the number of investigations on estimation of expected seismic damage to RC framed structures subject to corrosion of reinforcement is scanty. Such investigations are important for countries like India where a number of urban areas are located in the coastal region, many of which are prone to earthquake-induced ground shaking.

In the present study, the emphasis is on estimating the expected seismic damage to RC framed structures subject to corrosion of reinforcement. A methodology is proposed for determining the expected damage to corrosion affected reinforced concrete structures under earthquakes over the service life of the structure. The methodology is formulated within the framework of Monte Carlo simulation (MCS) considering both corrosion damage to the structure and earthquake loading as stochastic. The service life of the structure is divided into reference times, and the statistical properties of the expected damage to the structure at these reference times are determined. The time to corrosion initiation and rate of corrosion propagation are considered as random variables to account for the uncertainties in the material properties and variations in the exposure condition. Thus, the resistance of the building is stochastic. The occurrence of earthquakes during the reference time period is modelled as a Poisson process, and the stochastic seismic excitation is represented by an ensemble of acceleration time histories. The damage to the structure under the earthquake loading is determined from the results of an inelastic dynamic analysis. The inelastic damage analysis program IDARC 2D is used for this purpose. Determination of the statistical properties of expected damage will be useful in carrying out vulnerability analysis of the structure and in the regional risk assessment (Balaji et al. 2005).

The paper is organised as follows. The proposed methodology for expected seismic damage estimation to reinforced concrete framed structures considering the effect of corrosion is presented in the next section. An example of the estimation of expected seismic damage to a reinforced concrete frame, over different reference times ranging from 5 years to 30 years, is presented in the following section to illustrate the proposed methodology, and the results are discussed. The summary and conclusions of the study are given in the last section.

## 2. Proposed methodology for expected damage estimation

#### 2.1 Modelling chloride-induced corrosion of reinforcement

For RC structures located in marine and other aggressive environments, chloride-induced corrosion of reinforcement is identified as a major mechanism of resistance degradation. The process of chloride-induced corrosion of reinforcement in concrete can be considered as a two-stage process, namely, the corrosion initiation and the corrosion propagation (Tuutti 1982).

*Time for corrosion initiation:* The reinforcement in concrete is normally protected against corrosion by a microscopically thin oxide layer, which forms on the surface of the reinforcement due to the high alkalinity of the surrounding concrete (Kropp and Hilsdorf 1995). Chloride ions from the surroundings ingress into the concrete through the system of capillary pores and microcracks. When the chloride concentration around the reinforcement exceeds a threshold value (critical chloride concentration), the protective oxide layer dissolves and corrosion initiates. A reliable prediction model for the ingress of chloride ions into concrete should consider the

complex combination between the several transport mechanisms that include diffusion, capillary sorption and permeation (Kropp and Hilsdorf 1995). For most cases, however, diffusion mechanism is assumed to be the governing mechanism for the penetration of chlorides into concrete. Therefore simple model derived from Fick's second law of diffusion (Crank 1975) can be used to estimate the chloride penetration into concrete. If there are no chloride ions present in the concrete at the time of placing the concrete, the time to corrosion initiation ( $t_i$ ) can be determined from Fick's second law of diffusion as

$$t_i = \frac{c^2}{4D} \left[ \operatorname{erf}^{-l} \left( \frac{c_s - c_{cr}}{c_s} \right) \right]^{-2}$$
(1)

where *c* is the clear concrete cover to the reinforcement (in mm), *D* is the diffusion coefficient for chlorides in concrete (in mm<sup>2</sup>/s), *erf* is the error function,  $c_s$  is the surface chloride content (in % by weight of concrete),  $c_{cr}$  is the critical chloride concentration (in % by weight of concrete), and, erf is the error function. The time for corrosion initiation computed using Eq. 1 will be in seconds. Some of the limitations of the assumptions made in deriving the above model are: (i) concrete is neither homogeneous nor isotropic, (ii) chloride ions may be physically adsorbed onto surface of pores and chemically combined to aluminates forming chloroaluminate complexes, (iii) diffusion coefficient depends both on time and location/concentration. In spite of these limitations, the above equation is widely used in the estimation of chloride profile in concrete and the time for corrosion initiation. While more realistic models like the Mejlbro-Paulsen Model, *DuraCrete model, fib* Model Code model are available in literature (Lindvall 1999, Lindvall 2001, *fib* 2006, Moodi *et al.* 2013), the large amount of information required for applying these models prohibit their use except for very important projects. Hence, in the present study, the time for corrosion initiation is estimated using Eq. (1).

To account for variations in workmanship and exposure conditions, the diffusion coefficient at any given time, surface chloride concentration, critical chloride concentration and the clear cover to reinforcement need to be considered as random variables. Thus, the time to corrosion initiation obtained using Eq. (1) will be a random variable.

The surface chloride concentration decreases as the distance to the sea increases and also decreases with height (Di Maio *et al.* 2004). Hence, in multi-storeyed RC structures, the structural members in the ground floor will be more susceptible to corrosion than that at the uppermost floor in a given time period. Thus the time to corrosion initiation should be determined for different storey levels separately taking into account the variation of surface chloride concentration with height.

*Rate of corrosion:* Researchers have proposed different models for determining the rate of corrosion of reinforcing bar (Raupach 2006). From a brief review of these models (Balaji *et al.* 2000), it is found that the model proposed by Rodriguez *et al.* (1996) is widely accepted. Using this model, the rate of corrosion can be determined as

$$r_{corr} = 0.0116 I_{corr} \alpha \tag{2}$$

where  $r_{corr}$  is the rate of corrosion (in mm/year),  $I_{corr}$  is the average value of corrosion current density (in  $\mu$ A/cm<sup>2</sup>), 0.0116 is a factor which converts  $\mu$ A/cm<sup>2</sup> to mm/year, and  $\alpha$  is a factor for including the effect of highly localized pitting normally associated with chloride-induced corrosion (varies from 4 to 8). This model is formulated based on Faraday's law, under the assumptions that the number of electrons freed by Fe ion during the corrosion reaction is two, the atomic mass of

iron is 55.95 g/mol and the mass density of iron is 7.88 g/cm<sup>3</sup> (Vořechovská *et al.* 2009). While this model takes into consideration the localised pitting associated with chloride-induced corrosion, the reduction in corrosion current with time is not taken into account. This model gives a conservative estimate for remaining bar diameter in the case of pitting corrosion, since the corrosion pits are almost hemispherical in shape.

The remaining diameter of the reinforcing bar at any time t,  $\Phi(t)$ , can be obtained as

$$\Phi(t) = \Phi(0) - r_{orr} \left( t - t_i \right) \tag{3}$$

where  $\Phi(0)$  is the initial diameter of the reinforcing bar (in mm), *t* is the time elapsed (in years) and,  $t_i$  is the time for initiation of corrosion (in years). The rate of corrosion is influenced by the water-cement ratio (Baweja *et al.* 1998) and exposure condition (Anoop *et al.* 2003). To account for the variations in the water-cement ratio and exposure condition,  $r_{corr}$  is considered as a random variable. Since  $r_{corr}$  and  $t_i$  are random variables, the remaining diameter,  $\Phi(t)$ , will be a random variable.

Deterioration of strength of concrete due to corrosion: Cracking of cover concrete is one of the most common distressing consequences of corrosion of reinforcement in concrete. Cracking occurs due to the formation of corrosion products around the steel bars. The corrosion products occupy larger volume (6 to 8 times the original volume of steel) causing bursting stresses in the cover concrete region. In the present study, the reduced compressive strength of cover concrete due to corrosion-induced cracking is modelled as (Coronelli and Gambarova 2004)

$$f_c^* = \frac{f_c}{1 + K \varepsilon_1 / \varepsilon_{\infty}} \tag{4}$$

where K is a coefficient related to bar roughness and diameter (taken as 0.1 for medium-diameter ribbed bars) and  $\mathcal{E}_{co}$  is the strain at the peak compressive stress  $f_c$  (taken as 0.002 in the present study).  $\mathcal{E}_1$  is the average (smeared) tensile strain in the cracked concrete at right angles to the direction of the applied compression determined as

$$\varepsilon_1 = (b_f - b_o)/b_o \tag{5}$$

where  $b_o$  is the section width when there is no corrosion crack and  $b_f$  is the increased section width due to corrosion cracking.  $(b_f - b_o)$  is the increase in beam width due to corrosion cracking, given by

$$\left(b_{f} - b_{o}\right) = n_{bars} w_{cr}(t) \tag{6}$$

where  $n_{bars}$  is the number of the bars in top layer of compression region, and  $w_{cr}(t)$  is the width of crack due to corrosion at time t. Different models have been proposed by various researchers relating the level of corrosion to the formation of cracks in reinforced concrete. However, most of the models do not take into consideration the effect of loading on the corrosion-induced cracking (Anoop 2009). The following model proposed by Vidal *et al.* (2004) is based on the experimental investigations on reinforced concrete beams kept under loading and hence is more applicable to practical conditions

$$w_{cr}(t) = 0.0575 \left( \Delta A_s(t) - \Delta A_{so} \right) \tag{7}$$

where  $\Delta A_s(t)$  is the loss in area of reinforcement at time *t* (in years) due to corrosion, and  $\Delta A_{so}$  is the loss in area of steel cross-section corresponding to crack initiation due to corrosion of reinforcement, given by (Vidal *et al.* 2004)

$$\Delta A_{so} = A_{s} \left[ 1 - \left[ 1 - \frac{\alpha}{\Phi(0)} \left( 7.53 + 9.32 \frac{c}{\Phi(0)} \right) \times 10^{-3} \right]^{2} \right]$$
(8)

where  $A_s$  is the initial area of steel cross-section. It is assumed that once the width of corrosioninduced becomes 1 mm, spalling occurs (Vu and Stewart 2005) and the cover concrete is no longer effective in taking the compressive stress.

*Effect of corrosion on properties of steel reinforcement*: From the results of experimental investigations carried out by different researchers, it is found that corrosion of reinforcement reduces the yield strength, ultimate strength and ductility of the steel reinforcement (Clark *et al.* 2000, Zhu and Francois 2013). In the present study, the effect of corrosion of the properties of steel reinforcement is modelled using the following relations given by Clark *et al.* (2000)

$$f_{y}^{*} = f_{y} \left( 1 - 0.0054 A_{cor} \right)$$
(9)

$$f_u^* = f_u \left( 1 - 0.0054 A_{cor} \right) \tag{10}$$

$$\varepsilon_{su}^* = \varepsilon_{su} \left( 1 - 0.029 A_{cor} \right) \tag{11}$$

where  $f_y$ ,  $f_u$  and  $\mathcal{E}_{su}$  are the yield strength, ultimate strength and strain at ultimate stress for the reinforcing bar without corrosion, and  $A_{cor}$  is the percentage area of steel cross-section lost due to corrosion.

#### 2.2 Estimation of expected seismic damage

In the present study, the damage to the structure under the earthquake loading is characterised by the damage index. The modified Park and Ang damage model is considered for determining the damage index. The damage to the structure under earthquake excitation is determined using the nonlinear dynamic analysis program IDARC 2D. Brief descriptions of the modelling of the structure in IDARC 2D and the modified Park and Ang damage model are given in Appendix I.

Degree of Damage	Physical appearance	Damage Index	State of Building
Collapse	Partial or total collapse of building	> 1.0	Loss of building
Severe	Extensive crushing of concrete; disclosure of buckled reinforcement	0.4 - 1.0	Beyond repair
Moderate	Extensive large cracks; spalling of concrete in weaker elements	0.25 - 0.4	Repairable
Minor	Minor cracks; partial crushing of concrete in columns	0.1 - 0.25	Repairable
Slight	Sporadic occurrence of cracking	< 0.1	Repairable

Table 1 Interpretation of overall damage index using Park and Ang Model (from Park et al. 1985)

The Park and Ang damage model had already been calibrated with observed structural damage of nine reinforced concrete buildings (Park *et al.* 1985). Table 1 presents the calibrated damage index with the degree of observed damage in the structure.

#### 2.2.1 Assumptions

The following assumptions are made in the present study while estimating the expected seismic damage.

i. The effect of corrosion of reinforcement is restricted to the reduction in cross-sectional area of steel only. The loss of bond between steel and concrete and the loss in concrete cross-section due to corrosion-induced cracking are not considered. However, chloride-induced corrosion is a localized corrosion process, and the effect of loss in bond is not a major concern when assessing structures affected by local corrosion (*fib* 2000).

ii. The time-dependent effects such as change in modulus of elasticity of concrete due creep of concrete are not considered.

iii. The structure is repaired for seismic damage only, and the corrosion state of the reinforcement is not affected by the repair. The state of corrosion of the reinforcement in most of the elements of the structure is not affected by the repair, since only some of the elements would have been damaged (to an extent requiring repair) in the event of an earthquake. This is because these structures, which are engineered, would have been designed to withstand code-specified seismic forces. Also, in the repaired elements, there is no guarantee that the repair will provide adequate protection against corrosion (Wheat *et al.* 2005), which can be attributed to the inefficiency in the application of the repair measure and to the formation of corrosion macrocells (Shiessl and Raupach 1997). This assumption is also justified in view of the economic considerations. As a consequence of this assumption, the resistance deterioration of a structural element of a corrosion-affected RC framed structure subjected to earthquakes and repaired can be modelled as shown schematically in Fig. 1.

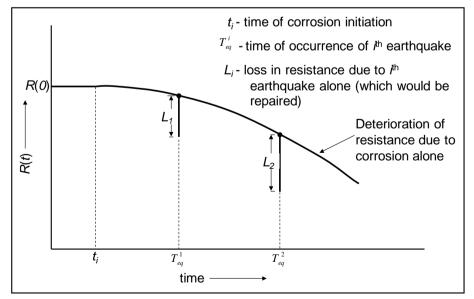


Fig. 1 Schematic representation of resistance deterioration of a RC structural element with time

iv. The time taken for repairs after each earthquake is very small compared to the reference time considered. This assumption is justified in view of the above assumption that the structure is repaired for seismic damage only, and only some of the elements would have been damaged (to an extent requiring repair) in the event of an earthquake.

v. The time for corrosion initiation and the rate of corrosion are same for all the structural elements in a given storey (thus the effect of micro-environment on corrosion hazard is ignored). This is justified since the emphasis in this study is the determination of overall seismic damage to the structure subject to chloride-induced corrosion of reinforcement.

#### 2.2.2 Procedure for estimating the expected seismic damage

The procedure evolved by Balaji *et al.* (2003, 2005) for estimating the expected seismic damage to structures in a given region is considered in the present study. The stochastic damage assessment model adopted is based on Poisson occurrence of earthquakes. This procedure is modified for including the effect of corrosion on the performance of the structure and is used in the present study. The procedure consists of the following steps.

1. Determine the mean recurrence rate (v) of the earthquake with the specified magnitude for the region in which the structure is located.

2. Determine the statistical properties (namely, mean and standard deviation) of  $t_i$  for the reinforced concrete structural elements at the different storey levels, considering the diffusion coefficient and surface chloride content as random variables, and taking into account the reduction in surface chloride concentration with height.

3. Generate one thousand values of  $t_i$  at each storey level by assuming  $t_i$  to be lognormally distributed with statistical properties as determined in Step 2.

4. Generate one thousand values of rate of corrosion propagation  $(r_{corr})$  assuming  $r_{corr}$  to be lognormally distributed

5. Generate an ensemble of hundred acceleration time-histories corresponding to the earthquake type considered (Douglas and Aochi 2014).

6. Define the reference time  $(T_{REF})$  for damage estimation.

7. Generate one thousand Poisson random variables representing the number of occurrences of earthquakes during the period  $T_{REF}$  with v as the parameter.

8. For each realisation of number of occurrences of earthquake

i. Determine the time of occurrence of each earthquake  $(T_{eq})$  by assuming the inter-arrival times of earthquakes to be exponentially distributed

ii. Compare  $T_{eq}$  with  $t_i$  at each storey level to check whether corrosion is initiated for the RC structural elements at that storey level

iii. If  $T_{eq} > t_i$  at any storey level, determine the remaining area of reinforcement for the RC structural elements at that storey level using Eq. (3) (with  $t = T_{eq}$ ). Determine the reduced compressive strength of cover concrete, and, the reduced yield strength, reduced ultimate strength and reduced strain at ultimate for the steel reinforcement.

iv. Develop the moment-curvature relation for the RC beams and RC columns at different floor levels considering appropriate remaining area of reinforcement, reduced compressive strength of cover concrete, and, the reduced yield strength, reduced ultimate strength and reduced strain at ultimate for the steel reinforcement.

v. Carry out nonlinear dynamic analysis of the structure for each acceleration time history in the ensemble and determine the damage index (in the present investigation, this has been carried

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#### out using IDARC 2D)

vi. Determine the expected overall damage index at the end of  $T_{\text{REF}}$  using Monte Carlo simulation.

9. Determine the statistical properties of expected overall damage index for one thousand realisations of number of earthquakes.

The above procedure is given in the form of a flowchart in Fig. 2. The proposed methodology is illustrated through an example problem in the next section.

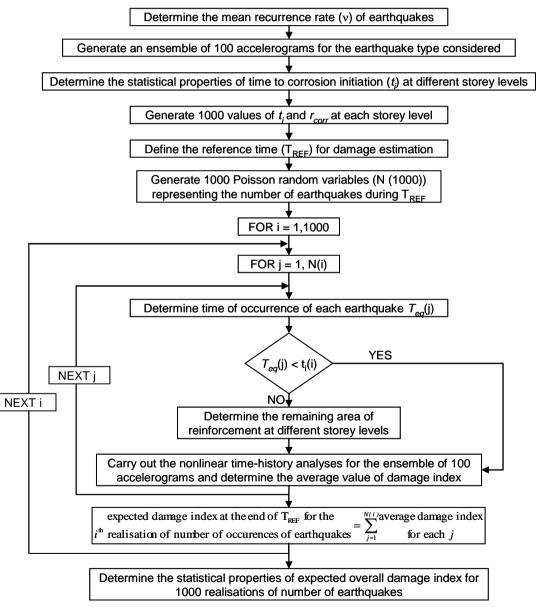


Fig. 2 Flow chart showing the procedure for estimating the expected damage to corrosion-affected RC Structures due to earthquakes

# 3. Example

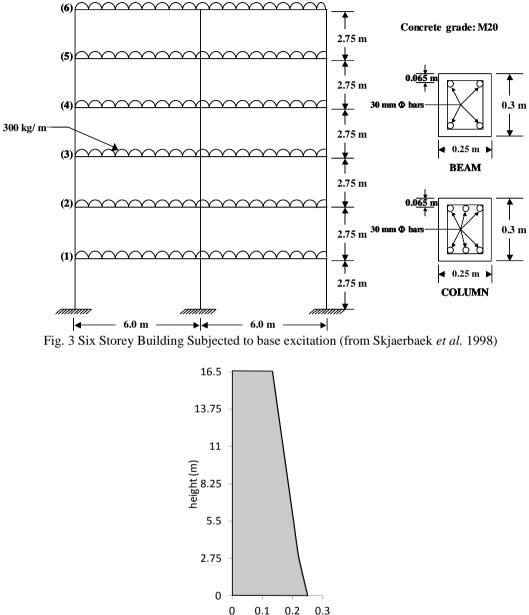
The two-bay six-storey reinforced concrete frame (Fig. 3), whose behaviour under seismic loading was already studied using model testing in laboratory by Skjærbæk et al. (1998), is considered in the present study to illustrate the proposed methodology. The cross-sectional details of the beams and columns of the frame are shown in Fig. 3. The concrete used has a compressive strength of 20 MPa and the average yield strength of steel is 600 MPa. The frame is assumed to be new at the beginning of reference time period and is assumed to be located in the coastal area in Andhra Pradesh region at a distance of about 50 m from the sea. While the coastal zone regulations specifies that a minimum distance of 500 m between the coastline and the structures, number of RC structures are being constructed even within a distance of 50 m be maintained (Sheth et al. 2006). Keeping this in view, the distance of 50 m has been considered in this study. The effect of proximity to the coastline is taken into account by considering the surface chloride content corresponding to the coastal environment. The exposure condition for the frame is characterised as severe (as per BIS (2000)). From a database of diffusion coefficient (D) values created at CSIR-SERC, Chennai, by collecting the values of D reported by various researchers based on different laboratory tests and field exposure tests (Anoop et al. 1999), the value of D for concrete with a grade of 20 MPa is taken as  $5.0 \times 10^{-6}$  mm<sup>2</sup>/s. The values for  $c_s$  at the ground level,  $c_{cr}$  and  $r_{corr}$  are selected as 0.25% by weight of concrete, 0.12% by weight of concrete and 0.18 mm/year, which are representative of the values reported in similar environments and for similar type and grade of concrete. The surface chloride content is considered to be decreasing with height (Di Maio et al. 2004), and the profile of variation of surface chloride concentration with height considered in the present study is shown in Fig. 4. While a triangular profile as suggested by Di Maio et al. (2004) would be appropriate for a high-rise building, the height of the reinforced concrete frame considered in the present study is only 16.5 m, and hence a trapezoidal profile for variation of  $c_s$  with height is considered. To account for variations in workmanship and exposure conditions, the diffusion coefficient (D), surface chloride concentration  $(c_s)$  and rate of corrosion  $(r_{corr})$  are treated as lognormally distributed random variables, with mean as the above values and an assumed coefficient of variation (cov) of 0.20. The random variables considered along with their statistical properties (namely, mean and cov) are given in Table 2.

From the records of earthquake magnitudes in Andhra Pradesh region, the mean recurrence rate (v) of earthquakes with magnitude>6.0 is determined as 0.0086 yr<sup>-1</sup> (i.e., mean return period of 116 years) (Balaji *et al.* 2004). In the present study, the reference time period has been varied as 5, 10, 15, 20, 25 and 30 years. For each reference time period, one thousand realisations of number of

Random variable	Mean <sup>*</sup>	cov**	Distribution <sup>**</sup>	
Diffusion coefficient	$5 \times 10^{-6} \text{ mm}^2/\text{s}$	0.20	Lognormal	
Surface chloride content at the ground floor level	0.25% by weight of concrete	0.20	Lognormal	
Rate of corrosion	0.18 mm/year	0.20	Lognormal	

Table 2 Random variables considered in the determination of time of corrosion initiation and in corrosion propagation

(Note: \* - representative of the values reported in similar environments and for similar type and grade of concrete, \*\* - assumed)



surface chloride content (% by wt. of concrete)

Fig. 4 Variation in surface chloride content with height assumed in the present study

earthquakes within the reference time period are generated assuming Poisson occurrence of earthquakes with v as the parameter. By considering one thousand realisations, the mean of the mean recurrence rate is obtained as 0.0085 yr<sup>-1</sup> (for reference period of 30 years), which is close to the desired value of 0.0086 yr<sup>-1</sup>. For each realisation of number of occurrences of earthquakes, the

time of occurrence of each earthquake is determined by assuming the inter-arrival times of the earthquakes to be exponentially distributed.

One of the important aspects of prediction of seismic damage in a given peninsular region is the paucity of acceleration time-histories, especially for earthquakes of larger magnitudes. In the present study, an ensemble containing 100 accelerograms has been artificially generated for the earthquake type considered using the method proposed by Deodatis and Shinozuka (1988). The envelop function for generating acceleration time-histories is taken as that of the 1940 El Centro earthquake of magnitude 6.9 (PGA=0.348 g). The 1940 El-Centro is chosen because this earthquake time-history is often used in the design of earthquake-proof structures today, particularly for the time-history analysis method. Also, the response spectrum of this earthquake has been used worldwide as a standard design spectrum (Franks et al. 1989). The values of expected damage and the statistical properties of expected damage for different reference time periods are determined using the proposed procedure. The nonlinear time-history analysis is carried out using the computer program IDARC 2D. An interface is developed to link between the generation of thousand realisations of earthquake occurrence sequences, one hundred stochastic acceleration time-histories, one thousand realisations of time of corrosion initiation for each storey and the nonlinear dynamic analysis program IDARC 2D. The time step for response analysis is taken as 0.005 seconds. Mass proportional damping is considered in the analysis. The damping value is taken as 4% of critical.

# 4. Results and discussions

It is noted that the earthquakes occur as a Poisson process. For a fixed time, the number of earthquakes in that interval follows a Poisson distribution. The thousand realisations of the Poisson random variable for a reference time period of 30 years are shown in Fig. 5. From this figure, it is noted that by considering the number of occurrences of earthquakes to follow Poisson process with mean recurrence rate as 0.0086 yr<sup>-1</sup>, the maximum number of earthquakes that can occur is 3 (considering one thousand realisations) within a reference period of 30 years. It is known that the interarryival times of earthquakes follows an exponential distribution. The times of occurrences of earthquakes corresponding to the realisations of the Poisson distribution (shown in Fig. 5) for a reference time period of 30 years are shown in Fig. 6. A typical realisation of acceleration time history is shown in Fig. 7. The average and the bounds of pseudo-acceleration response spectra (PSA) for 5% damping for the ensemble of acceleration time histories generated for 1940 El-Centro earthquake are shown in Fig. 8. The pseudo-acceleration response spectra for 5% damping for the recorded acceleration time-history for this earthquake is also shown in Fig. 8. From Fig. 8, it is noted that the simulated ensemble of acceleration time-histories encompasses the spectral acceleration features of the recorded acceleration time-history. It can also be noted from Fig. 8 that the actual spectral accelerations corresponding to the first three modes<sup>1</sup> of the reinforced concrete frame considered are well within the bounds obtained from the simulations. Since the first three modes contributes the maximum to the damage (since the energy contents corresponding to these modes are the maximum), the simulated time histories can be used for estimating the seismic damage.

<sup>&</sup>lt;sup>1</sup>The periods corresponding to the first three modes of the uncorroded reinforced concrete frame considered are 0.52 seconds, 0.16 seconds and 0.08 seconds

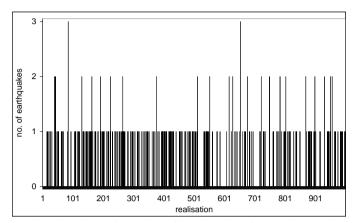


Fig. 5 One thousand realisations of number of earthquakes for Andhra Pradesh region during a reference time of 30 years

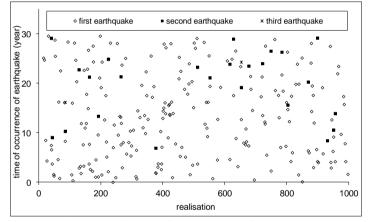


Fig. 6 One thousand realisations of time of occurrences of earthquakes corresponding to the realisations of earthquakes shown in Fig. 5 for a reference time of 30 years

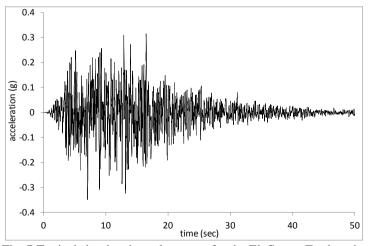


Fig. 7 Typical simulated accelerogram for the El-Centro Earthquake

The values of  $c_s$  at different floor levels are computed by assuming  $c_s$  decreases linearly with height as shown in Fig. 4. Ten thousand values of D and  $c_s$  are generated, assuming that these variables follow lognormal distribution with cov of 0.20. The values of  $t_i$  are determined at different storey levels using generated values of D and  $c_s$ . Ten thousand simulation cycles have been used to characterise the variations in  $t_i$  at a given floor level. The frequencies of corrosion initiation time for the structural elements at the ground floor level are shown in Fig. 9. Similar histograms have been obtained at other floor levels. From these histograms, it is noted that the variations in corrosion initiation time can be described by a lognormal distribution. Using the ten thousand simulated values of  $t_i$  at each floor level, the mean and standard deviation of  $t_i$  are computed. Using ten thousand simulation cycles with 95% confidence the population mean would be contained within an interval of 0.0392s (where s is the sample standard deviation) around the sample mean (Ang and Tang 1975). Similarly, 95% upper confidence limit for population variance is obtained as 0.977s<sup>2</sup>. This information is further used in the estimation of mean expected overall damage to the structure considered.

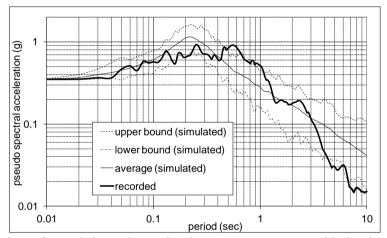


Fig. 8 Comparison of recorded pseudo-acceleration response spectrum with the simulated pseudo-acceleration response spectra (1940 El Centro earthquake, magnitude=6.9, PGA=0.348 g)

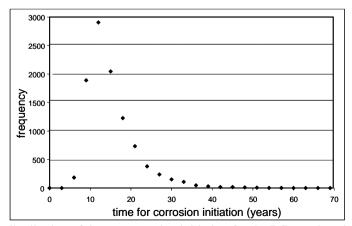


Fig. 9 Frequency distribution of time to corrosion initiation for the RC members in the ground floor

The variation in width of corrosion-induced crack and the compressive strength of cover concrete with loss in area of steel reinforcement is shown in Fig. 10. The time after corrosion initiation, considering the rate of corrosion as the mean value of  $r_{corr}$  (0.18 mm/year) is also shown in Fig. 10. From Fig. 10, it is noted that the corrosion-induced crack initiates shortly (within half an year) after corrosion initiation and spalling of cover concrete (i.e., crack width of 1 mm) occurs within three years after corrosion initiation.

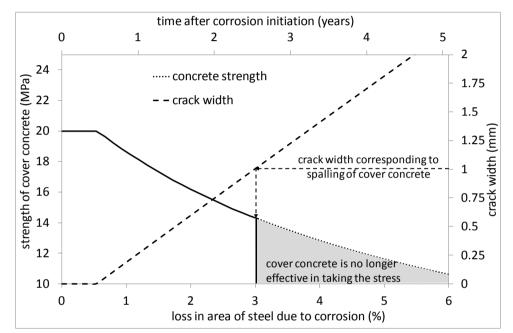


Fig. 10 Variation in width of corrosion-induced crack and the compressive strength of cover concrete with loss in area of steel reinforcement

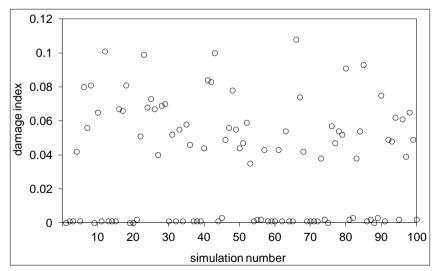


Fig. 11 Overall damage to the uncorroded RC structure subjected to stochastic seismic excitation (time t=0)

The values of overall damage index (ODI) for the RC frame with no corrosion (i.e., at time t=0) subjected to stochastic seismic excitation are shown in Fig. 11. The mean and maximum values of ODI are 0.035 and 0.108, respectively. The low values of ODI indicate that the degree of seismic damage to the structure is slight and is repairable (see Table 1). The low values of ODI also justify the assumption that the time to carry out repair is small compared to the life of the structure. It is also noted from Fig. 11 that, as expected, the seismic damage to a structure needs to be treated as random. The frequency distribution of ODI of uncorroded structure is shown in Fig. 12. From this figure, it is noted that the frequency distribution of ODI of uncorroded structure is multi-modal. This indicate the need for simulation-based approaches for studying the seismic damage to the reinforced concrete structures. The expected damage indices (E[DI]) for columns and beams at different floor levels for the RC frame with no corrosion subjected to stochastic seismic excitation are shown in Fig. 13. It is noted from Fig. 13 that the damage suffered by the beams are higher than that suffered by columns at different storeys. It is also noted that the beams at the  $2^{nd}$  and  $3^{rd}$ floor levels are subjected to the maximum damage when the effect of corrosion is not considered. This is in agreement with the observations made by Skjærbæk et al. (1998), based on shake table tests on the models of the frame, that cracking has occurred in several members of 2<sup>nd</sup> and 3<sup>rd</sup> storeys after first earthquake event while the members in the other storeys have remained undamaged. Skjærbæk et al. (1998) had also reported very large permanent deformations and severe spalling of concrete at several members of  $2^{nd}$  and  $3^{rd}$  storeys after three earthquake events.

Fig. 14 shows the variation in mean of expected value of ODI ( $\langle E[ODI] \rangle$ ) with and without considering the effect of corrosion with reference time. It is noted, as expected, that the  $\langle E[ODI] \rangle$  increases with increase in reference time in both the cases. This is because the probability of occurrence of earthquakes (one or more) within the reference time increases with increase in the reference time. It is also noted that the variation of  $\langle E[ODI] \rangle$  with time is more or less linear when effect of corrosion is not considered. This can be attributed to the assumption that the occurrences of earthquakes follow a Poisson process. The expected value of a Poisson process increases linearly with increase in time, and hence the  $\langle E[ODI] \rangle$  increases linearly with time. However, when effect of corrosion is considered, the variation in  $\langle E[ODI] \rangle$  becomes nonlinear with increase in reference time.

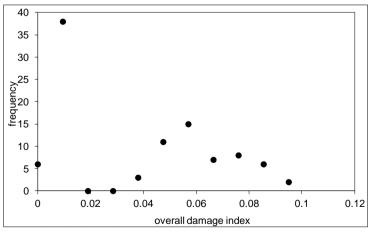


Fig. 12 Frequency distribution of overall damage index for the uncorroded RC structure subjected to stochastic seismic excitation (time t=0)

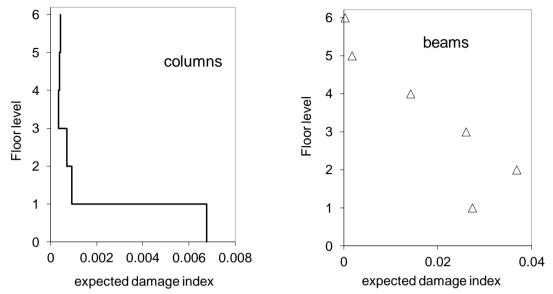
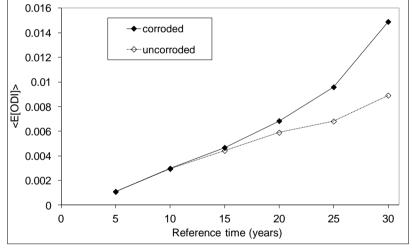
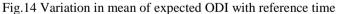


Fig. 13 Expected damage index at different floor levels for the members of the uncorroded structure under stochastic seismic excitation (time t=0)





It is also noted from Fig. 14 that the value of  $\langle E[ODI] \rangle$  is higher for a particular reference time when the effect of corrosion is considered. Fig. 15 shows the percentage increase in mean and standard deviation (SD) of E[ODI] with reference time when the corrosion effect is taken into account in the analysis. It is noted from Fig. 15 that the increase in  $\langle E[ODI] \rangle$  can be as high as about 70% with an increase in SD of about 155% for a reference time of 30 years when the effect of corrosion is considered. This shows the need to consider corrosion effects in the damage estimation. It is also noted that, as expected, the increase in mean and SD of E[ODI] increases with the increase in the reference time considered. This is because (i) the number of corroded structures increases with increase in reference time, and (ii) the effects of corrosion on the structural behaviour become more pronounced with the increase in reference time. Fig. 16 shows the percentage of occurrences when corrosion has initiated before the occurrence of earthquake in the reference time period considered together with the percentage increase in mean of E[ODI] due to corrosion. It is noted from this figure that both these quantities can be considered to be linearly varying with reference time for the frame considered.

The results indicate that the proposed procedure will be useful for the determination of statistical properties of the expected damage to the structure for different reference times. The computational costs associated are very less considering the availability of high speed computers at present. For instance, the number of nonlinear dynamic analyses required are 3100 for a reference period of 5 years and 25400 for a reference period of 30 years, and thus are computationally inexpensive. Estimation of expected damage will be useful for the decision-makers involved in earthquake disaster management.

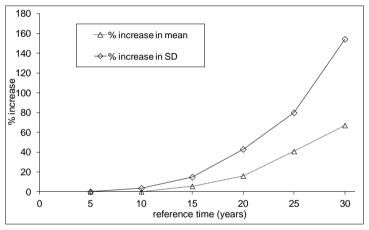


Fig. 15 Percentage increase in mean and standard deviation of E[ODI] due to corrosion

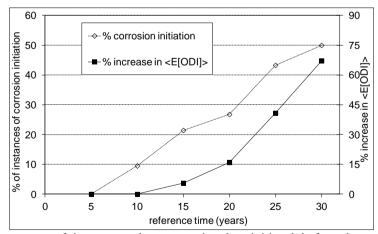


Fig. 16 Percentage of instances where corrosion has initiated before the occurrence of earthquake within the reference time period and percentage increase in  $\langle E[ODI] \rangle$  due to corrosion

## 5. Conclusions

A methodology for the estimation of statistical properties (viz. mean and standard deviation) of the expected damage to a reinforced concrete framed structure, subjected to stochastic seismic excitation, over a specified reference time (this can be typically the service life of the structure) is proposed in this paper. The proposed methodology integrates the nonlinear dynamic analysis (using IDARC 2D) of the framed structure with stochasticity in occurrences of earthquakes and the seismic excitation, in a Monte Carlo simulation framework. In addition to this, the methodology also considers the degradation in stiffness of members due to chloride-induced corrosion of reinforcement in the computation of the expected damage. The reduction in area, yield strength and strain at ultimate of steel reinforcement, and the reduction in compressive strength of cover concrete, due to corrosion are taken into account in the estimation of damage. The modified Park and Ang (MPA) model is used in the computation of expected overall damage index (ODI).

The proposed methodology is applied to estimate the statistical properties of expected damage of a typical six storied reinforced concrete framed structure (Skjærbæk et al. 1998), located in Andhra Pradesh region (for which the mean return period of earthquake is about 116 years), and exposed to severe environment (as per BIS). The earthquake type is assumed to be of El Centro type. From the results of Monte Carlo simulations carried out, it has been found that the probability distribution of expected ODI of uncorroded structure is multi-modal and the evolution of  $\langle E(ODI) \rangle$  with reference time can be approximated with a linear relation. It is also noted that there is an increase of about 15% in the mean value of expected seismic damage to the reinforced concrete frame considered for a reference time of 30 years when the effect of corrosion is taken into consideration. It is noted that the expected seismic damage to the framed structure considered is small, since the structure is designed to withstand code-specified seismic forces. These conclusions need further examination by carrying out such studies on number of framed buildings. The proposed methodology can be improved further by taking into consideration the variations in material strength (strengths of concrete and steel) and cross-sectional dimensions. The determination of statistical properties of the expected damage to the structure during different reference times will be useful in carrying out vulnerability analysis of the structure and in the regional risk assessment.

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# Appendix I

## I.1 Modelling of the structure [Valles et al. 1996]

The beam elements are modelled as flexural elements with coupled shear deformations. The hysteretic flexural behaviour is modelled using the three parameter Park model. The column elements are modelled considering flexural, shear and axial deformations. Flexural and shear components of deformations are modelled using the three parameter Park model and the axial deformation component is modelled using a linear elastic spring. For both the beam and column elements, the increase in stiffness at the joint is simulated by including a rigid length zone, the length of which can be specified depending on the dimensions of the connecting elements. The moment-curvature envelope of the section is determined from the fibre model analysis of the cross section for both the beam and column elements. The spread plasticity model, available in IDARC, is considered to include the variation of the section flexibility. The element stiffness matrix is constantly varied throughout the analysis according to the spread plasticity model and the hysteretic model.

The damage (D) to the structure under the earthquake loading is characterised by the damage index. The modified Park and Ang damage model is considered in this study for determining the damage index. A brief description of this model is given below.

#### I.2 The modified Park and Ang damage model [Valles et al. 1996]

In the Park and Ang damage model, the seismic structural damage to a reinforced concrete structural element is expressed as a linear combination of the maximum deformation and the absorbed hysteretic energy. The Park and Ang damage index for a structural element is defined as (Park *et al.* 1985)

$$DI = \frac{\delta_m}{\delta_u} + \frac{\beta}{\delta_u P_v} \int dE_h$$

where DI is the damage index (which gives an empirical measure of damage, with  $DI \ge 1.0$ indicating total damage or collapse),  $\delta_m$  is the maximum response deformation,  $\delta_u$  is the ultimate deformation of the element under static loading,  $P_y$  is the calculated yield strength of the element,  $\int dE_h$  is the hysteretic energy absorbed by the element during the response history (excluding potential energy); and  $\beta$  is a coefficient for cyclic loading effect (a function of structural parameters). A value of 0.1 for the parameter  $\beta$  has been suggested for nominal strength deterioration. The Park and Ang damage model accounts for damage due to maximum inelastic excursions, as well as damage due to the history of deformations.

- Three damage indices are computed using this damage model:
- 1. Element damage index: column, beams or shear wall elements.
- 2. Storey damage index: vertical and horizontal components and total storey damage.
- 3. Overall building damage index (ODI).

Direct application of the damage model to a structural element, a storey, or to the overall building requires the determination of the corresponding overall element, storey, or building ultimate deformations. Since the inelastic behaviour is confined to plastic zones near the ends of some members, the relation between element, storey or top storey deformations, with the local plastic rotations is difficult to establish. For the element end section damage the following modifications to the original model were introduced.

$$DI = \frac{\theta_m - \theta_r}{\theta_u - \theta_r} + \frac{\beta}{\theta_u M_v} E_h$$

where  $\theta_m$  is the maximum rotation attained during the loading history,  $\theta_u$  is the ultimate rotation capacity of the section,  $\theta_r$  is the recoverable rotation when unloading,  $M_y$  is the yield moment, and  $E_h$  is the dissipated energy in the section. The element damage is then selected as the largest damage index of the end sections.

The two additional indices: storey and overall damage indices are computed using weighting factors based on dissipated hysteretic energy at component and storey levels respectively

$$DI_{storey} = \sum (\lambda_i)_{component} (DI_i)_{component}; \quad (\lambda_i)_{component} = \left(\frac{E_i}{\sum E_i}\right)_{component}$$
$$DI_{overall} = \sum (\lambda_i)_{storey} (DI_i)_{storey}; \quad (\lambda_i)_{story} = \left(\frac{E_i}{\sum E_i}\right)_{storey}$$

where  $\lambda_i$  are the energy weighting factors, and  $E_i$  is the total absorbed energy by the component or storey "*i*".