Earthquakes and Structures, *Vol. 8, No. 6 (2015) 1299-1323*
DOI: http://dx.doi.org/10.12989/eas.2015.8.6.1299

Modelling of magneto-thermoelastic plane waves at the interface of two prestressed solid half-spaces without energy dissipation

Rajneesh Kakar*[1] and Shikha Kakar[2a]

[1]*Faculty of Engineering & Technology, GNA University, Hargobindgarh, Phagwara, Chotti Baradari, 163/1, Jalandhar-144022, India*
[2]*Department of Electronics, SBBS University, Padhiana, Chotti Baradari, 163/1, Jalandhar-144022, India*

(Received November 9, 2014, Revised December 7, 2014, Accepted December 22, 2014)

Abstract. A model for reflection and refraction of magneto-thermoelastic SV-waves at the interface of two transversely isotropic and homogeneous solid half spaces under initial stress by applying classical dynamical theory of thermoelasticity is purposed. The reflection and refraction coefficients of SV-waves are obtained with ideal boundary conditions for SV-wave incident on the solid-solid interface. The effects of magnetic field, temperature and initial stress on the amplitude ratios after numerical computations are shown graphically with MATLAB software for the particular model.

Keywords: initial stress; reflection; refraction; relaxation time; temperature; magnetic field

1. Introduction

The Earth is under high initial stress, the main cause of these initial stresses in the medium is slow process of creep and quenching, difference of temperature and gravity changes. The coupled field of magneto-thermoelasticity arises due to mutual interactions between thermal field, an externally applied magnetic field and the elastic deformation present in the solid body. That is why; it is of large interest to study the influence of these initial stresses and magnetic field on the propagation of thermoelastic waves in the Earth. Inspite of the fact that the Maxwell equations governing the electro-magnetic field have been known for quite a long time (Landsu and Lifshitz 1960), the interest in the coupled fields of magneto-thermoelasticity is of recent origin. This is due to the fact only recently has been recognized the possibility of applying these coupled theories in such practical situations as optics, acoustics, geophysics, plasma physics and earthquake science. Duhamel (1937) purposed coupling of strain and temperature fields by introducing dilation term in the equation of thermal conductivity, but this equation was inconsistent to laws of thermodynamics. Duhamel's theory was further modified by several others scientists and solved number of interesting problems. It was Biot (1965) who gave a satisfactory solution of the

*Corresponding author, Professor, E-mail: rajneesh.kakar@gmail.com
[a]HOD, E-mail: shikha_kakar@rediffmail.com

http://www.techno-press.org/journals/eas&subpage=7 ISSN: 2092-7614 (Print), 2092-7622 (Online)

equations of thermal conductivity with the theory of coupled thermoelasticity. Keeping this drawback in view some researchers such Lord and Shulman (1967), Fox (1969), Gurtin and Pipkin (1969), Meixner (1970) modified the Fourier law so as to obtain an equation of heat conduction in hyberbolic differential form. These works include the time needed for acceleration of the heat flow in the heat conduction equation and takes into account the coupling between the temperature and strain fields. The equations thus obtained are hyberbolic. This paradox in the existing coupled theory of thermoelasticity has also been discussed by Boley (1999). Effects have been made to modify the classical Fourier constitutive law (subjected to themodynamical constraints), connecting the heat flux vector and temperature gradient. This is done with a view to obtain a wave-type equation of heat conduction incorporating the so-called "second sound effect" or the "thermal relaxation effect". Montanaro (1964) introduced hydrostatic initial stress to study the isotropic linear thermo elasticity by using Biot's equations. Chattopadhyay and Kumar (2006) investigated reflection and refraction of waves for isotropic and anisotropic medium without initial stress. Chattopadhyay *et al.* (2007) considered the problem of reflection and refraction of quasi P and SV waves at the interface of fibre-reinforced media without initial stress and temperature. Singh (2009) discussed SV-wave for homogeneous and inhomogeneous elastic half-spaces without initial stress. Deswal *et al*. (2011) reviewed reflection and refraction of waves in thermally conducting viscous liquid half-spaces. Kumar and Kumar (2012) discussed reflection and refraction of thermoelastic waves at the interface of a prestressed elastic half-space with voids half-space. Shekhar and Parvez (2013) investigated plane waves in transversely isotropic dissipative half space under rotation and magnetic field. Chakraborty (2011) took wrong parameter in the basic equation for study elastic waves under initial stress and temperature field. Chakraborty and Singh (2013) presented a model for reflection and refraction of elastic SV-waves at the interface of two solid half spaces under initial stress and temperature without magnetic field. Therefore, the authors have taken magnetic field, temperature and stress to study reflection and refraction of elastic waves in an elastic medium without energy dissipation because magnetic field of the medium also an important factor along with initial stress and temperature in the reflection and refraction of elastic waves. MATLAB software (Version 7.6.0.324 (R2008a), Trademark of Mathworks. Inc. U.S. Patent) is used to represent the various graphs associated with the reflection and refraction amplitude ratios.

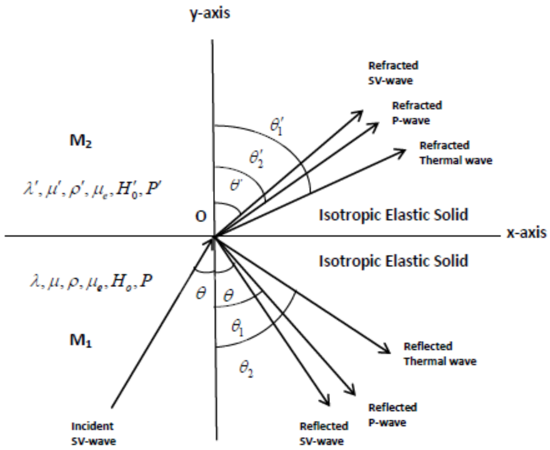

Fig. 1 Reflection and refraction of magneto-thermoelastic plane waves

2. Formulation of the problem

We consider two transversely isotropic, homogeneous elastic half spaces under constant magnetic field (along *z*-axis) and initial tensile stress *P* along *x*-axis at absolute temperature *T* (Fig. 1). An elastic plane SV-wave (rotational wave) is incident in medium M_1 at the plane interface such that it is partially reflected as SV-wave (rotational wave) in medium M_1, partially refracted SV-wave (rotational wave) in medium M_2, one reflected P-wave (dilatational wave) in medium M_1, one refracted P-wave (compressional wave) in medium M_2, one reflected thermal wave (compressional wave) in medium M_1 and one refracted P-wave (dilatational wave) in medium M_2 as shown (Fig. 1). Therefore, on striking the SV-wave at y=0 making an angle θ in the solid half space M_1, it will have one reflected rotational wave making an angle θ and two reflected compressional waves at an angle θ_1 and θ_2 (Fig. 1). The incident SV-wave has refracted components in the upper half space at angles of θ', θ_2' and θ_1' for refractive SV-wave, refractive P-wave and refracted thermal wave respectively.

3. Governing equations

The magnetic field is taken parallel to the *Z*-axis, therefore third *Z*-component '*w*'of the displacement vector and all other quantities are independent of *Z*. Following Lotfy (2011) the governing equations in the absence of all body forces are given by

Strain-displacement relations

$$e_{ij} = \frac{1}{2}(\mathrm{u}_{i.j} + \mathrm{u}_{j,\mathrm{i}}) \tag{1}$$

where, $\mathrm{u}_i = (u, v, 0)$ is the components of displacement vector.

Stress displacement relation

$$s_{ij} = \lambda\, \mathrm{e}\,\delta_{ij} + 2\mu e_{ij} - \gamma\left(1 + \tau\frac{\partial}{\partial t}\right)\mathrm{T}\delta_{ij}, \tag{2}$$

The modified heat conduction equation

$$\mathrm{K}\nabla^2\mathrm{T} = \rho c_p\left(\frac{\partial \mathrm{T}}{\partial t} + \tau_0\frac{\partial^2\mathrm{T}}{\partial t^2}\right) + \gamma\,\mathrm{T}_0\left(\frac{\partial^2 u}{\partial x\partial t} + \frac{\partial^2 v}{\partial y\partial t} + \tau_0\delta_{ij}\left[\frac{\partial^3 u}{\partial x\partial t^2} + \frac{\partial^3 v}{\partial y\partial t^2}\right]\right) \tag{3}$$

The components of Lorentz force

$$F_x = \mu_e(\mathrm{J}\times\mathrm{H})_x, \quad F_y = \mu_e(\mathrm{J}\times\mathrm{H})_y, \quad F_z = 0 \tag{4}$$

The uniform magnetic field intensity $\mathrm{H}(0,0,\mathrm{H}_0)$ is parallel to *Z*-axis; it induces electric field *E* and magnetic field *h*. These variations in magnetic and electric fields are given by Maxwell's equations

$$\mathrm{curl\ h} = \mathrm{J} + \frac{\partial \mathrm{D}}{\partial t} \tag{5}$$

$$\text{curl E} = -\frac{\partial \mathrm{B}}{\partial t} \tag{6}$$

$$\text{div B} = 0,\ \ \text{div B} = 0,\ \ \mathrm{B} = \mu_e \mathrm{H},\ \ \mathrm{D} = \varepsilon_e \mathrm{E} \tag{7}$$

$$\mathrm{E} = -\mu_e \left[\frac{\partial u}{\partial t} \times \mathrm{H} \right] \tag{8}$$

where $\partial u/\partial t$ is the particle velocity of the medium, and the influence of temperature gradient on J is also neglected. The steady-state deformed position is measured from dynamic displacement vector, which is assumed to be ignored.

The components of magnetic intensity vector in the medium

$$\mathrm{H}_x = \mathrm{H}_y = 0,\ \mathrm{H}_z = \mu_e[\mathrm{H}_0 + \mathrm{h(x,z,t)}] \tag{9}$$

$$\mathrm{J}_x = -\mathrm{H}_0 \frac{\partial e}{\partial y} + \mu_e \mathrm{H}_0 \varepsilon_e \frac{\partial^2 v}{\partial t^2},\ \mathrm{J}_y = \mathrm{H}_0 \frac{\partial e}{\partial x} - \mu_e \mathrm{H}_0 \varepsilon_e \frac{\partial^2 u}{\partial t^2},\ \mathrm{J}_z = 0 \tag{10}$$

From Eqs. (5)-(10) into Eq. (4) we obtain

$$F_x = \mu_e \mathrm{H}_0^2 \frac{\partial e}{\partial x} - \mu_e^2 \mathrm{H}_0^2 \varepsilon_e \frac{\partial^2 u}{\partial t^2},\ \ F_y = \mu_e \mathrm{H}_0^2 \frac{\partial e}{\partial y} - \mu_e^2 \mathrm{H}_0^2 \varepsilon_e \frac{\partial^2 v}{\partial t^2},\ \ F_z = 0 \tag{11}$$

where h=-H_0(0,0,e)

Following Biot (1965), the dynamical equations of motion for the propagation of wave in two dimensions

$$\frac{\partial s_{xx}}{\partial x} + \frac{\partial s_{xy}}{\partial y} - P\frac{\partial \Omega}{\partial y} + F_x = \rho \frac{\partial^2 u}{\partial t^2} \tag{12}$$

$$\frac{\partial s_{xy}}{\partial x} + \frac{\partial s_{yy}}{\partial y} - P\frac{\partial \Omega}{\partial x} + F_y = \rho \frac{\partial^2 v}{\partial t^2} \tag{13}$$

where s_{xx}, s_{yy} and s_{xy} are incremental thermal stress components. The first two are principal stress components along *X*- and *Y*-axes, respectively and last one is shear stress component in the *X*-*Y* plane and *u*, *v* are the displacement components along *X* and *Y* directions respectively.

Following Biot (1965), the stress-strain relations with incremental isotropy

$$s_{xx} = (\lambda + 2\mu + P)e_{xx} + (\lambda + P)e_{yy} + 2\mu e_{xx} - \gamma\left(\mathrm{T} + \tau \frac{\partial \mathrm{T}}{\partial x} \right) \tag{14}$$

$$s_{yy} = \lambda e_{xx} + (\lambda + 2\mu)e_{yy} - \gamma\left(\mathrm{T} + \tau \frac{\partial \mathrm{T}}{\partial x} \right) \tag{15}$$

$$s_{xy} = 2\mu e_{xy} \tag{16}$$

where

$$e_{xx}=\frac{\partial u}{\partial x},\qquad e_{yy}=\frac{\partial v}{\partial x},\qquad e_{xy}=\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) \tag{17}$$

4. Solution of the problem

From Eq. (12), Eq. (13), Eq. (14), Eq. (15), Eq. (16) and Eq. (17), we get

$$(\lambda+2\mu+P)\frac{\partial^2 u}{\partial x^2}+\left(\lambda+\mu+\frac{P}{2}\right)\frac{\partial^2 v}{\partial x\partial y}+\left(\mu+\frac{P}{2}\right)\frac{\partial^2 u}{\partial^2 y}+\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}\right)=\rho\frac{\partial^2 u}{\partial t^2}+\gamma\left(\frac{\partial \mathrm{T}}{\partial x}+\tau\frac{\partial^2 \mathrm{T}}{\partial t\partial x}\right) \tag{18}$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+\left(\lambda+\mu+\frac{P}{2}\right)\frac{\partial^2 u}{\partial x\partial y}+\left(\mu-\frac{P}{2}\right)\frac{\partial^2 v}{\partial^2 x}+\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}\right)=\rho\frac{\partial^2 v}{\partial t^2}+\gamma\left(\frac{\partial \mathrm{T}}{\partial y}+\tau\frac{\partial^2 \mathrm{T}}{\partial t\partial y}\right) \tag{19}$$

From Eq. (18) and (19) by using classical dynamical theory: $\tau=\tau_0=0, \delta_{ij}=0,$ we get

$$(\lambda+2\mu+P)\frac{\partial^2 u}{\partial x^2}+\left(\lambda+\mu+\frac{P}{2}\right)\frac{\partial^2 v}{\partial x\partial y}+\left(\mu+\frac{P}{2}\right)\frac{\partial^2 u}{\partial^2 y}+\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}\right)=\rho\frac{\partial^2 u}{\partial t^2}+\gamma\frac{\partial \mathrm{T}}{\partial x} \tag{20}$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+\left(\lambda+\mu+\frac{P}{2}\right)\frac{\partial^2 u}{\partial x\partial y}+\left(\mu-\frac{P}{2}\right)\frac{\partial^2 v}{\partial^2 x}+\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}\right)=\rho\frac{\partial^2 v}{\partial t^2}+\gamma\frac{\partial \mathrm{T}}{\partial y} \tag{21}$$

Eq. (3) can be modified by using classical dynamical theory: $\tau=\tau_0=0, \delta_{ij}=0,$ as

$$K\nabla^2\mathrm{T}=\rho c_p\left(\frac{\partial \mathrm{T}}{\partial t}+\tau\frac{\partial^2 \mathrm{T}}{\partial t^2}\right)+\gamma\mathrm{T}_0\left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right] \tag{22}$$

Eq. (20) and Eq. (21) can be solved by choosing potential functions ϕ and ψ as

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial y}\ \text{ and }\ v=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y} \tag{23}$$

From Eq. (20) and (23), we get

$$\nabla^2\phi=\frac{\rho}{(\lambda+2\mu+\mu_e H_0^2+P)}\frac{\partial^2\phi}{\partial t^2}+\frac{\gamma}{(\lambda+2\mu+\mu_e H_0^2+P)}\left(\mathrm{T}+\tau\frac{\partial \mathrm{T}}{\partial t}\right) \tag{24}$$

$$\nabla^2\psi=\frac{\rho}{\left(\mu+\frac{P}{2}\right)}\frac{\partial^2\psi}{\partial t^2} \tag{25}$$

From Eq. (21) and (23), we get

$$\nabla^2\phi=\frac{\rho}{(\lambda+2\mu+\mu_e H_0^2)}\frac{\partial^2\phi}{\partial t^2}+\frac{\gamma}{(\lambda+2\mu+\mu_e H_0^2)}\left(\mathrm{T}+\tau\frac{\partial \mathrm{T}}{\partial t}\right) \tag{26}$$

$$\nabla^2\psi = \frac{\rho}{\left(\mu - \frac{P}{2}\right)}\frac{\partial^2\psi}{\partial t^2} \tag{27}$$

Eq. (24) and Eq. (26) represent magneto- thermo compression waves along *x*- axis and *y*- axis respectively, whereas Eq. (25) and Eq. (27) represent magneto- thermo distortional waves along *x*-axis and *y*- axis respectively. For initial stress along *x*- axis, the four Eqs. (24)-(27) reduced to two equations as

$$\nabla^2\phi = \frac{1}{c_1^2}\frac{\partial^2\phi}{\partial t^2} + \frac{\gamma}{(\lambda + 2\mu + \mu_e H_0^2 + P)}\left(T + \tau\frac{\partial T}{\partial t}\right) \tag{28}$$

$$\nabla^2\psi = \frac{1}{c_2^2}\frac{\partial^2\psi}{\partial t^2} \tag{29}$$

where,

$$c_1^2 = \frac{(\lambda + 2\mu + \mu_e H_0^2 + P)}{\rho} \text{ and } c_2^2 = \frac{\left(\mu - \frac{P}{2}\right)}{\rho} \tag{30}$$

c_1 is known as P-wave velocity and c_2 is called SV-wave velocity. Also, for P-wave v=0 and for SV-wave u=0.

Now, from Eqs. (21) and (23), we get

$$K\nabla^2\mathrm{T} = \rho c_p\left(\frac{\partial \mathrm{T}}{\partial t}\right) + \gamma\,\mathrm{T}_0\left[\frac{\partial}{\partial t}\left(\nabla^2\phi\right)\right] \tag{31}$$

where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

The solution of Eq. (28), Eq. (29) and Eq. (31) is plane harmonic waves travelling perpendicular to the x-y plane, which is given as

$$\phi = \phi_1 \exp[i\{k(x\sin\theta + y\cos\theta) - \omega t\}] \tag{32}$$

$$\psi = \psi_1 \exp[i\{l(x\sin\theta + y\cos\theta) - \omega t\}] \tag{33}$$

$$\mathrm{T} = \mathrm{T}_1 \exp[i\{k(x\sin\theta + y\cos\theta) - \omega t\}] \tag{34}$$

where, k and l are compression and rotational wave numbers, ω is angular frequency.

From Eq. (28), Eq. (32) and Eq. (34), we get

$$c_1^2\left(\frac{\omega^2}{c_1^2} - k^2\right)\phi_1 - \frac{\gamma}{\rho}\mathrm{T}_1 = 0 \tag{35}$$

From Eq. (31), Eq. (32) and Eq. (34), we get

$$-i\mathrm{T}_0\gamma\omega k^2\phi_1+\left(i\rho c_p\omega-\mathrm{K}k^2\right)\mathrm{T}_1=0 \tag{36}$$

In order to satisfy Eq. (35) and Eq. (36), the determinant of the coefficients of both Eq. (35) and Eq. (36) will be zero, therefore

$$\begin{vmatrix} c_1^2\left(\dfrac{\omega^2}{c_1^2}-k^2\right) & -\dfrac{\gamma}{\rho} \\ -i\mathrm{T}_0\gamma\omega k^2 & \left(i\rho c_p\omega-\mathrm{K}k^2\right) \end{vmatrix}=0 \tag{37}$$

Expanding Eq. (37), we get

$$\Delta^4-\left(1+\overline{\tau}_{\mathrm{T}}-i\overline{\varphi}\right)\Delta^2-i\overline{\varphi}=0 \tag{38}$$

where,

$$\Delta=\frac{\omega}{kc_1}=\left(\frac{c_2}{c_1}\right)^2,\quad \varphi=\frac{\omega\mathrm{K}}{\rho c_p c_1^2},\quad \tau_{\mathrm{T}}=\frac{\gamma^2\,\mathrm{T}_0}{\rho^2 c_p c_1^2} \tag{39}$$

The roots of Eq. (38) and are given as

$$\Delta_1=\frac{\omega}{k_1c_1},\quad \Delta_2=\frac{\omega}{k_2c_2} \tag{40}$$

Eq. (38) is biquadratic in Δ, it means that there are two compessional waves travelling with two different velocities. Therefore, on striking the rotational wave at y=0 making an angle θ in the solid half space it will have one reflected rotational wave making an angle θ and two reflected compressional waves at an angle θ_1 and θ_2 (Fig. 1). Therefore from the above discussion we can take displacement potential and perturbation temperature in the following form

$$\phi=\alpha_1\exp[i\{k_1(x\sin\theta_1+y\cos\theta_1)-\omega t\}]+\alpha_2\exp[i\{k_2(x\sin\theta_2+y\cos\theta_2)-\omega t\}] \tag{41}$$

$$\psi=\beta_0\exp[i\{l(x\sin\theta+y\cos\theta)-\omega t\}]+\beta_1\exp[i\{l(x\sin\theta-y\cos\theta)-\omega t\}] \tag{42}$$

$$\mathrm{T}=\delta_1\exp[i\{k_1(x\sin\theta+y\cos\theta)-\omega t\}]+\delta_2\exp[i\{k_2(x\sin\theta_2+y\cos\theta_2)-\omega t\}] \tag{43}$$

where, α_1,α_2 represent amplitudes of the reflected P-wave, thermal P-wave and β_0 represents amplitude of incident SV wave and β_1 is the amplitude of reflected SV-waves respectively.

Since incident wave strikes the interface at y=0 making an angle θ with x-axis as shown (Fig. 1), the wave gets reflected in medium M_1 and refracted in the second medium M_2 giving three waves one refracted SV-wave at an angle θ', one refracted P-wave at an angle θ'_1 and one refracted thermal wave at an angle θ'_2. Therefore, for medium M_2, the quantities $c_1,c_2,\varphi,\tau_{\mathrm{T}},\Delta_1,\Delta_2,\Delta$ in medium M_1 changed to-as given below

$$\begin{gathered} c_1'^2=\frac{(\lambda'+2\mu'+\mu_e'H_0'^2+P')}{\rho'},\quad c_2'^2=\frac{\left(\mu'-\dfrac{P'}{2}\right)}{\rho'},\quad \varphi'=\frac{\omega\mathrm{K}'}{\rho' c_p c_1'^2}, \\ \tau_{\mathrm{T}}'=\frac{\gamma'^2\,\mathrm{T}_0}{\rho'^2 c_p' c_1'^2},\quad \Delta_1'=\frac{\omega}{k_1'c_1'},\quad \Delta_2'=\frac{\omega}{k_2'c_2'},\quad \Delta=\frac{\omega}{k'c_1'}=\frac{c_2'}{c_1'}. \end{gathered} \tag{44}$$

The angles $\theta, \theta_1, \theta_2, \theta', \theta'_1, \theta'_2$ are related to respective wave numbers $k_1, k_2, l, k'_1, k'_2, l'$ as Also, and are related to respective wave numbers as

$$k_1 \sin\theta_1 = k_2 \sin\theta_2 = l \sin\theta = k'_1 \sin\theta'_1 = k'_2 \sin\theta'_2 = l' \sin\theta' \tag{45}$$

Eq. (45) can be written in terms to Snell's law as

$$\frac{\sin\theta_1}{c_1\Delta_1} = \frac{\sin\theta_2}{c_1\Delta_2} = \frac{\sin\theta}{c_1\Delta} = \frac{\sin\theta'_1}{c'_1\Delta'_1} = \frac{\sin\theta'_2}{c'_1\Delta'_2} = \frac{\sin\theta'}{c'_1\Delta'} \tag{46}$$

We can take displacement potential and perturbation temperature for medium M_2 in the following form

$$\phi' = \alpha'_1 \exp[i\{k'_1(x\sin\theta'_1 + y\cos\theta'_1) - \omega t\}] + \alpha'_2 \exp[i\{k'_2(x\sin\theta'_2 + y\cos\theta'_2) - \omega t\}] \tag{47}$$

$$\psi' = \beta'_1 \exp[i\{l'(x\sin\theta' + y\cos\theta') - \omega t\}] \tag{48}$$

$$\mathrm{T}' = \delta'_1 \exp[i\{k'_1(x\sin\theta' + y\cos\theta') - \omega t\}] + \delta'_2 \exp[i\{k'_2(x\sin\theta'_2 + y\cos\theta'_2) - \omega t\}] \tag{49}$$

Introducing Eq. (41) and Eq. (43) in into Eq. (31), we get

$$\delta_1 = \frac{\omega^2\rho}{\gamma}\left(\frac{\tau_\mathrm{T}}{\left(\Delta_1^2 + i\varphi\right)}\right) \text{ and } \delta_2 = \frac{\omega^2\rho}{\gamma}\left(\frac{\tau_\mathrm{T}}{\left(\Delta_2^2 + i\varphi\right)}\right) \tag{50}$$

Now substituting Eq. (50) in Eq. (49), we get

$$\mathrm{T} = \frac{\omega^2\rho}{\gamma}\left(\frac{\tau_\mathrm{T}}{\left(\Delta_1^2 + i\varphi\right)}\right)\alpha_1 \exp[i\{k_1(x\sin\theta + y\cos\theta) - \omega t\}] + \frac{\omega^2\rho}{\gamma}\left(\frac{\tau_\mathrm{T}}{\left(\Delta_2^2 + i\varphi\right)}\right)\alpha_2 \exp[i\{k_2(x\sin\theta_2 + y\cos\theta_2) - \omega t\}] \tag{51}$$

Similarly for medium M_2, introducing Eq. (47) and Eq. (49) in into Eq. (31), we get

$$\delta'_1 = \frac{\omega^2\rho'}{\gamma'}\left(\frac{\tau'_\mathrm{T}}{\left(\Delta_1'^2 + i\varphi'\right)}\right) \text{ and } \delta'_2 = \frac{\omega^2\rho'}{\gamma'}\left(\frac{\tau'_\mathrm{T}}{\left(\Delta_2'^2 + i\varphi'\right)}\right) \tag{52}$$

Now substituting Eq. (52) in Eq. (49), we get

$$\mathrm{T}' = \frac{\omega^2\rho'}{\gamma'}\left(\frac{\tau'_\mathrm{T}}{\left(\Delta_1'^2 + i\varphi'\right)}\right)\alpha'_1 \exp[i\{k'_1(x\sin\theta' + y\cos\theta') - \omega t\}] + \frac{\omega^2\rho'}{\gamma'}\left(\frac{\tau'_\mathrm{T}}{\left(\Delta_2'^2 + i\varphi'\right)}\right)\alpha'_2 \exp[i\{k'_2(x\sin\theta'_2 + y\cos\theta'_2) - \omega t\}] \tag{53}$$

5. Boundary conditions and Reflection and refraction coefficients

The initial conditions are supplemented by the following boundary conditions:
i. Continuity condition for displacement at Y=0

$$\begin{aligned} u &= u' \\ v &= v' \\ w = w' &= 0 \end{aligned} \tag{54a}$$

ii. Continuity condition for initial stress at Y=0

$$\begin{gathered}\nabla f_x = \nabla f'_x \text{ i.e., } s_{xy} + Pe_{xy} = s'_{xy} + P'e'_{xy} \\ \nabla f_y = \nabla f'_y \text{ i.e., } s_{yy} = s'_{yy}\end{gathered} \tag{54b}$$

iii. Continuity condition for temperature at Y=0

$$\frac{\partial \mathrm{T}}{\partial y} = \frac{\partial \mathrm{T}'}{\partial y} \tag{54c}$$

iv. Continuity condition for temperature at Y=0

$$\mathrm{T} = \mathrm{T}' \tag{54d}$$

Introducing Eq. (14), Eq. (15), Eq. (16) and Eq. (23) in the first boundary condition (i) for medium M_1 and corresponding equations for the medium M_2, we get

$$\left(\mu + \frac{P}{2}\right)\left[2\frac{\partial^2\phi}{\partial x\partial y} + \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial^2 y}\right] = \left(\mu' + \frac{P'}{2}\right)\left[2\frac{\partial^2\phi'}{\partial x\partial y} + \frac{\partial^2\psi'}{\partial x^2} + \frac{\partial^2\psi'}{\partial^2 y}\right] \tag{55}$$

Introducing Eq. (14), Eq. (15), Eq. (16) and Eq. (23) in the second boundary condition (ii) for medium M_1 and corresponding equations for the medium M_2, we get

$$\begin{aligned}&\left(\lambda + \mu_e H_0^2\right)\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + 2\mu\frac{\partial^2\phi}{\partial y^2} + 2\mu\frac{\partial^2\psi}{\partial x\partial y} - \gamma\mathrm{T} \\ &\qquad = \left(\lambda' + \mu'_e H_0'^2\right)\left(\frac{\partial^2\phi'}{\partial x^2} + \frac{\partial^2\phi'}{\partial y^2}\right) + 2\mu'\frac{\partial^2\phi'}{\partial y^2} + 2\mu'\frac{\partial^2\psi'}{\partial x\partial y} - \gamma'\mathrm{T}'\end{aligned} \tag{56}$$

Using Eq. (41), Eq. (42), Eq. (47), Eq. (48) Eq. (51) and Eq. (53) in Eq. (54), Eq. (55) and Eq. (56), we get

$$\sin\theta\left(\frac{\alpha_1}{\beta_0}\right) + \sin\theta\left(\frac{\alpha_2}{\beta_0}\right) - \sin\theta\left(\frac{\alpha'_1}{\beta_0}\right) - \sin\theta\left(\frac{\alpha'_2}{\beta_0}\right) + \cos\theta\left(\frac{\beta_1}{\beta_0}\right) + \frac{c_1}{c'_1}\left(\frac{\Delta}{\Delta'}\right)\cos\theta'\left(\frac{\beta'_1}{\beta_0}\right) = \cos\theta \tag{57}$$

$$\left(\frac{\Delta}{\Delta_1}\right)\cos\theta_1\left(\frac{\alpha_1}{\beta_0}\right) + \left(\frac{\Delta}{\Delta_2}\right)\cos\theta_2\left(\frac{\alpha_2}{\beta_0}\right) + \frac{c_1}{c'_1}\left(\frac{\Delta}{\Delta'_1}\right)\cos\theta'_1\left(\frac{\alpha'_1}{\beta_0}\right) + \frac{c_1}{c'_1}\left(\frac{\Delta}{\Delta'_2}\right)\cos\theta'_2\left(\frac{\alpha'_2}{\beta_0}\right) - \sin\theta\left(\frac{\beta_1}{\beta_0}\right) + \sin\theta\left(\frac{\beta'_1}{\beta_0}\right) = \sin\theta \tag{58}$$

$$\begin{aligned}&\left(\frac{\Delta^2}{\Delta_1^2}\right)\sin 2\theta_1\left(\frac{\alpha_1}{\beta_0}\right) + \left(\frac{\Delta^2}{\Delta_2^2}\right)\sin 2\theta_2\left(\frac{\alpha_2}{\beta_0}\right) + \bar{\rho}\xi\left(\frac{\Delta'^2}{\Delta_1'^2}\right)\sin 2\theta'_1\left(\frac{\alpha'_1}{\beta_0}\right) + \bar{\rho}\xi\left(\frac{\Delta'^2}{\Delta_2'^2}\right)\sin 2\theta'_2\left(\frac{\alpha'_2}{\beta_0}\right) \\ &\qquad + \cos 2\theta\left(\frac{\beta_1}{\beta_0}\right) - \bar{\rho}\xi\cos 2\theta'\left(\frac{\beta'_1}{\beta_0}\right) = -\cos 2\theta\end{aligned} \tag{59}$$

$$\begin{aligned}&\left[1 - 2\Delta^2\sin^2\theta_1 + \left(1 + \sin^2\theta_1\right)\eta - \frac{\Delta_1^2\tau_{\mathrm{T}}}{\left(\Delta_1^2 + i\varphi\right)}\right]\left(\frac{1}{\Delta_1^2}\right)\left(\frac{\alpha_1}{\beta_0}\right) \\ &\qquad + \left[1 - 2\Delta^2\sin^2\theta_2 + \left(1 + \sin^2\theta_2\right)\eta - \frac{\Delta_2^2\tau_{\mathrm{T}}}{\left(\Delta_2^2 + i\varphi\right)}\right]\left(\frac{1}{\Delta_2^2}\right)\left(\frac{\alpha_2}{\beta_0}\right)\end{aligned} \tag{60}$$

$$-\bar{\rho}\left[1-2\Delta'^2\sin^2\theta_1'+\left(1+\sin^2\theta_1'\right)\eta'-\frac{\Delta_1'^2\tau_T'}{\left(\Delta_1'^2+i\varphi'\right)}\right]\left(\frac{1}{\Delta_1'^2}\right)\left(\frac{\alpha_1'}{\beta_0}\right)$$
$$-\bar{\rho}\left[1-2\Delta'^2\sin^2\theta_2'+\left(1+\sin^2\theta_2'\right)\eta'-\frac{\Delta_2'^2\tau_T'}{\left(\Delta_2'^2+i\varphi'\right)}\right]\left(\frac{1}{\Delta_2'^2}\right)\left(\frac{\alpha_2'}{\beta_0}\right)$$
$$-\bar{\rho}\left(1+\eta_1\right)\sin 2\theta\left(\frac{\beta_1}{\beta_0}\right)-\bar{\rho}\left(1+\eta_1'\right)\sin 2\theta'\left(\frac{\beta_1'}{\beta_0}\right)=-\left(1+\eta_1\right)\sin 2\theta$$

$$\frac{\tau_T}{\left(\Delta_1^2+i\varphi\right)}\frac{\cos\theta_1}{\Delta_1}\left(\frac{\alpha_1}{\beta_0}\right)+\frac{\tau_T}{\left(\Delta_2^2+i\varphi\right)}\frac{\cos\theta_2}{\Delta_2}\left(\frac{\alpha_2}{\beta_0}\right)+\frac{\tau_T'}{\left(\Delta_1'^2+i\varphi'\right)}\frac{\bar{\rho}\Delta_1}{\bar{\gamma}\Delta_1'}\frac{\cos\theta_1'}{\Delta_1'}\left(\frac{\alpha_1'}{\beta_0}\right)+\frac{\tau_T'}{\left(\Delta_2'^2+i\varphi'\right)}\frac{\bar{\rho}\Delta_2}{\bar{\gamma}\Delta_2'}\frac{\cos\theta_2'}{\Delta_2'}\left(\frac{\alpha_2'}{\beta_0}\right)=0 \tag{61}$$

$$\frac{\tau_T}{\left(\Delta_1^2+i\varphi\right)}\left(\frac{\alpha_1}{\beta_0}\right)+\frac{\tau_T}{\left(\Delta_2^2+i\varphi\right)}\left(\frac{\alpha_2}{\beta_0}\right)-\frac{\tau_T'}{\left(\Delta_1'^2+i\varphi'\right)}\frac{\bar{\rho}}{\bar{\gamma}}\left(\frac{\alpha_1'}{\beta_0}\right)-\frac{\tau_T'}{\left(\Delta_2'^2+i\varphi'\right)}\frac{\bar{\rho}}{\bar{\gamma}}\left(\frac{\alpha_2'}{\beta_0}\right)=0 \tag{62}$$

where

$$\bar{\gamma}=\frac{\gamma'}{\gamma},\ \ \bar{\rho}=\frac{\rho'}{\rho},\ \ \eta=\frac{P}{c_1^2\rho},\ \ \eta'=\frac{P'}{c_1'^2\rho'},\ \eta_1=\frac{P}{2c_2^2\rho},\ \ \eta_1'=\frac{P'}{2c_2'^2\rho'}\ \text{ and }\ \xi=\frac{1+2\eta_1'}{1+2\eta_1}. \tag{63}$$

Now eliminating $\left(\frac{\alpha_1}{\beta_0}\right),\left(\frac{\alpha_2}{\beta_0}\right),\left(\frac{\alpha_1'}{\beta_0}\right),\left(\frac{\alpha_2'}{\beta_0}\right),\left(\frac{\beta_1}{\beta_0}\right)$ and $\left(\frac{\beta_1'}{\beta_0}\right)$ from Eq. (57), Eq. (58), Eq. (59), Eq. (60), Eq. (61) and Eq. (62), we can write 6 non-homogeneous equations into matrix form as

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix}\times\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{bmatrix}=\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} \tag{64}$$

where,

$$c_{11}=\sin\theta, c_{12}=\sin\theta, c_{13}=-\sin\theta, c_{14}=-\sin\theta, c_{15}=\cos\theta, c_{16}=\left(\frac{\Delta}{\Delta'}\right)\cos\theta',$$

$$c_{21}=\left(\frac{\Delta}{\Delta_1}\right)\cos\theta_1, c_{22}=\left(\frac{\Delta}{\Delta_2}\right)\cos\theta_2, c_{23}=\frac{c_1}{c_1'}\left(\frac{\Delta}{\Delta_1'}\right)\cos\theta_1', c_{24}=\frac{c_1}{c_1'}\left(\frac{\Delta}{\Delta_2'}\right)\cos\theta_2', c_{25}=-\sin\theta,$$

$$c_{26}=-\sin\theta, c_{31}=\left(\frac{\Delta^2}{\Delta_1^2}\right)\sin 2\theta, c_{32}=\left(\frac{\Delta^2}{\Delta_2^2}\right)\sin 2\theta_2, c_{33}=\bar{\rho}\xi\left(\frac{\Delta'^2}{\Delta_1'^2}\right)\sin 2\theta_1', c_{34}=\bar{\rho}\xi\left(\frac{\Delta'^2}{\Delta_2'^2}\right)\sin 2\theta_2',$$

$$c_{35}=\cos 2\theta, c_{36}=-\bar{\rho}\xi\cos 2\theta', c_{41}=\left[1-2\Delta^2\sin^2\theta_1+\left(1+\sin^2\theta_1\right)\eta-\frac{\Delta_1^2\tau_T}{\left(\Delta_1^2+i\varphi\right)}\right]\left(\frac{1}{\Delta_1^2}\right),$$

$$c_{42}=\left[1-2\Delta^2\sin^2\theta_2+\left(1+\sin^2\theta_2\right)\eta-\frac{\Delta_2^2\tau_T}{\left(\Delta_2^2+i\varphi\right)}\right]\left(\frac{1}{\Delta_2^2}\right),$$

$$c_{43}=-\bar{\rho}\left[1-2\Delta'^2\sin^2\theta_1'+\left(1+\sin^2\theta_1'\right)\eta'-\frac{\Delta_1'^2\tau_T'}{\left(\Delta_1'^2+i\varphi'\right)}\right]\left(\frac{1}{\Delta_1'^2}\right),$$

$$c_{44}=-\bar{\rho}\left[1-2\Delta'^2\sin^2\theta_2'+\left(1+\sin^2\theta_2'\right)\eta'-\frac{\Delta_2'^2\tau_T'}{\left(\Delta_2'^2+i\varphi'\right)}\right]\left(\frac{1}{\Delta_2'^2}\right),\ c_{45}=-\bar{\rho}\left(1+\eta_1\right)\sin 2\theta,$$

$$c_{46}=-\bar{\rho}\left(1+\eta_1'\right)\sin 2\theta',\ c_{51}=\frac{\tau_T}{\left(\Delta_1^2+i\varphi\right)}\frac{\cos\theta_1}{\Delta_1},\ c_{52}=\frac{\tau_T}{\left(\Delta_2^2+i\varphi\right)}\frac{\cos\theta_2}{\Delta_2},$$

$$c_{53}=\frac{\tau_T'}{\left(\Delta_1'^2+i\varphi'\right)}\frac{\bar{\rho}\Delta_1}{\bar{\gamma}\Delta_1'}\frac{\cos\theta_1'}{\Delta_1'}\left(\frac{1}{\Delta_1'^2}\right),\ c_{54}=\frac{\tau_T'}{\left(\Delta_2'^2+i\varphi'\right)}\frac{\bar{\rho}\Delta_2}{\bar{\gamma}\Delta_2'}\frac{\cos\theta_2'}{\Delta_2'},\ c_{55}=0,\ c_{56}=0,$$

$$c_{61}=\frac{\tau_T}{\left(\Delta_1^2+i\varphi\right)},\ c_{62}=\frac{\tau_T}{\left(\Delta_2^2+i\varphi\right)},\ c_{63}=-\frac{\tau_T'}{\left(\Delta_1'^2+i\varphi'\right)}\frac{\bar{\rho}}{\bar{\gamma}},\ c_{64}=-\frac{\tau_T'}{\left(\Delta_2'^2+i\varphi'\right)}\frac{\bar{\rho}}{\bar{\gamma}},\ c_{65}=0,\ c_{66}=0,$$

$$q_1=\cos\theta, q_2=\sin\theta, q_3=-\cos 2\theta, q_4=-\left(1+\eta_1\right)\sin 2\theta, q_5=0, q_6=0.$$

$r_1=\left(\dfrac{\alpha_1}{\beta_0}\right)=$ Reflection coefficients of P-wave (R_1)

$r_2=\left(\dfrac{\alpha_2}{\beta_0}\right)=$ Reflection coefficients of thermal wave (R_2)

$r_3=\left(\dfrac{\alpha_1'}{\beta_0}\right)=$ Refraction coefficient of P-wave (R_3)

$r_4=\left(\dfrac{\alpha_2'}{\beta_0}\right)=$ Refraction coefficient of thermal wave (R_4)

$r_5=\left(\dfrac{\beta_1}{\beta_0}\right)=$ Reflection coefficient of SV-wave (R_5)

$r_6=\left(\dfrac{\beta_1'}{\beta_0}\right)=$ Refraction coefficient of SV-wave (R_6)

6. Numerical analysis and discussion

Above theory clearly indicates that the amplitude ratios $|R_i|$, (i=1,2……6) depend on the angle of incidence of the incident wave. In order to study in greater detail, the dependence of these amplitude coefficients on temperature, stress and magnetic parameter together with the angle of incidence, we compute the amplitude ratios. Following material constants are taken for solid-solid interface (Table 1). For the values of relevant physical constants (Table 1), the system of Eq. (64) is solved for reflection and refraction coefficients by the application of the MATLAB software at different angles of incidence varying from 0° to 90°. The matrix elements are calculated for thermo parameters, magnetic parameters, initial stress parameters and particular angle of incidence (see Appendix). The variations of the amplitude coefficients of various reflected and transmitted waves are shown graphically with the angle of incidence of the incident SV (rotational) wave for the various values of coupling parameters, stress parameters, temperature parameters and magnetic parameters.

6.1 The incident wave is SV wave and both the surfaces are under tensile initial stress a=0.2, 0.3, 0.4 and b=0.233, 0.333, 0.433

Fig. 2 shows the variation of refraction amplitude ratio of SV wave with angle of incidence when both the half-spaces are under initial tensile stress, refraction coefficient of the corresponding refracted SV-wave shows three maxima at angles of incidence 50°, 20° and 60° respectively for values of initial stress *a*=0.2, 0.3, 0.4 and *b*=0.233, 0.333, 0.433. Beyond the value of incident angle 70° there is sharp decrease in refraction coefficient of SV wave. Fig. 3 shows the variation of reflection amplitude ratio of SV wave with angle of incidence when both the half-spaces are under initial tensile stress *a*=0.2, 0.3, 0.4 and *b*=0.233, 0.333, 0.433. Reflection coefficient of SV-wave shows three maxima at angles of incidence 50°, 20° and 70° respectively for values of initial stress *a*=0.2, 0.3, 0.4 and *b*=0.233, 0.333, 0.433. Fig. 4 shows the variation of refraction amplitude ratio of thermal wave with angle of incidence when both the half-spaces are under initial tensile stress *a*=0.2, 0.3, 0.4 and *b*=0.233, 0.333, 0.433, magnetic parameter $\mu_e H_0^2 = 1\times 10^9\,\text{N/m}^2$ for medium M_1 and $\mu_e^{'} H_0^{'2} = 1.1\times 10^9\,\text{N/m}^2$ for medium M_2 and coupling parameters $\bar{\varphi} = 0.005$ for medium M_1 and $\bar{\varphi}' = 0.003$ for medium M_2 respectively. Fig. 5 represents the variation of refraction amplitude ratio of P wave with angle of incidence when both the half-spaces are under initial tensile stress *a*=0.2, 0.3, 0.4 and *b*=0.233, 0.333, 0.433, magnetic parameter $\mu_e H_0^2 = 1\times 10^9\,\text{N/m}^2$ for medium M_1 and $\mu_e H_0^2 = 1\times 10^9\,\text{N/m}^2$ for medium M_2 and coupling parameters $\bar{\varphi} = 0.005$ for medium M_1 and $\bar{\varphi}' = 0.003$ for medium M_2 respectively. Refraction coefficient of P wave is almost constant for incidence angle 50° and beyond angle 50°, sharp decrease in amplitude ratio of the refracted P wave. Fig. 6 shows the variation of refraction amplitude ratio of thermal wave with angle of incidence when both the half-spaces are under initial tensile stress *a*=0.2, 0.3, 0.4 and *b*=0.233, 0.333, 0.433. Refraction coefficient of thermal wave is almost constant for incidence angle 40°. The reflection coefficient of P-wave with angle of incidence when both the half-spaces are under initial tensile stress *a*=0.2, 0.3, 0.4 and *b*=0.233, 0.333, 0.433 are shown in Fig. 7.

6.2 The incident wave is SV wave and upper half space is stress free b=0

Fig. 8 shows the variation of refraction amplitude ratio of SV wave with angle of incidence when upper half-spaces is stress free but magnetic parameter $\mu_e H_0^2 = 1.1\times 10^9\,\text{N/m}^2$,

Table 1(Medium constants)

Medium M_1		Medium M_2	
$\bar{\tau}_T$	0.005	$\bar{\tau}'_T$	0.003
ρ	4×10^3 kg/m^3	ρ'	2.2×10^3 kg/m^3
$\mu_e H_0^2$	1×10^9 N/m^2	$\mu'_e H_0'^2$	1.1×10^9 N/m^2
λ	8×10^{10} N/m^2	λ'	4×10^{10} N/m^2
$\bar{\varphi}$	0.005	$\bar{\varphi}'$	0.003
α_t	16.6×10^{-6} K^{-1}	α'_t	15.6×10^{-6} K^{-1}
K	401 W/(m.K)	K'	301 W/(m.K)
c_p	0.39 KJ/Kg K	c'_p	0.33 KJ/Kg K
μ	4×10^{10} N/m^2	μ'	2×10^{10} N/m^2
$\bar{\gamma}$	0.8	$\bar{\gamma}$	0.8
$\bar{\rho}$	0.9	$\bar{\rho}$	0.9
Δ_1	≈1	Δ'_1	≈1
Δ_2	0.07	Δ'_2	0.05

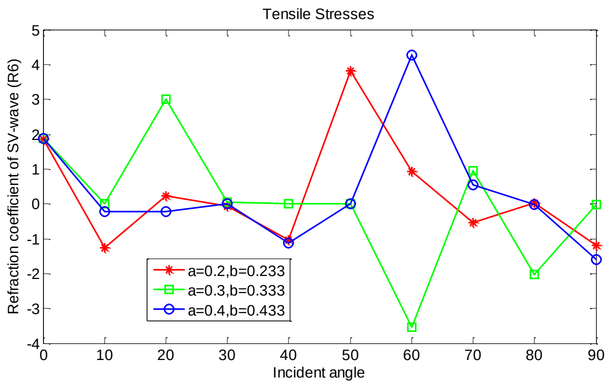

Fig. 2 Variation of refraction amplitude ratio of SV wave with angle of incidence when both the half-spaces are under initial tensile stress

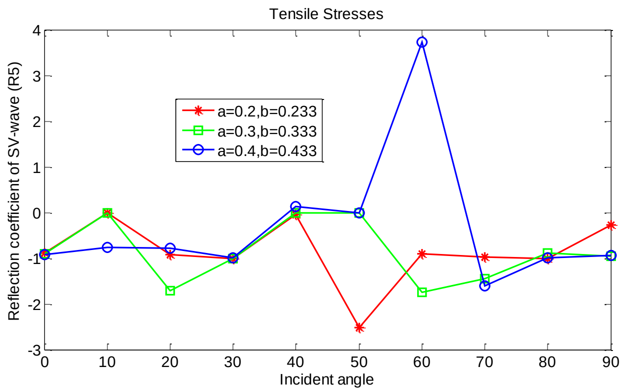

Fig. 3 Variation of reflection amplitude ratio of SV wave with angle of incidence when both the half-spaces are under initial tensile stress

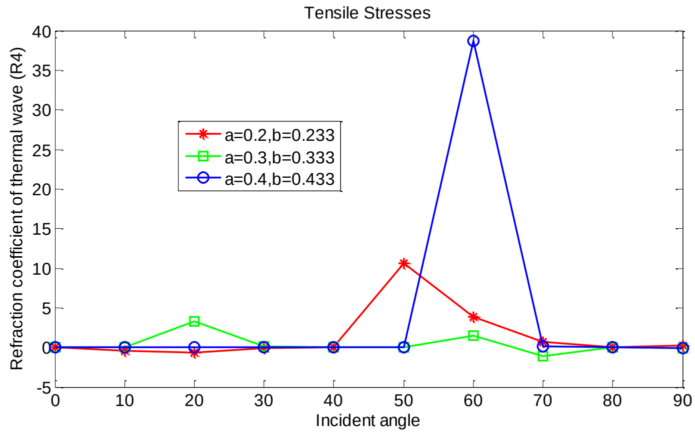

Fig. 4 Variation of refraction amplitude ratio of thermal wave with angle of incidence when both the half-spaces are under initial tensile stress

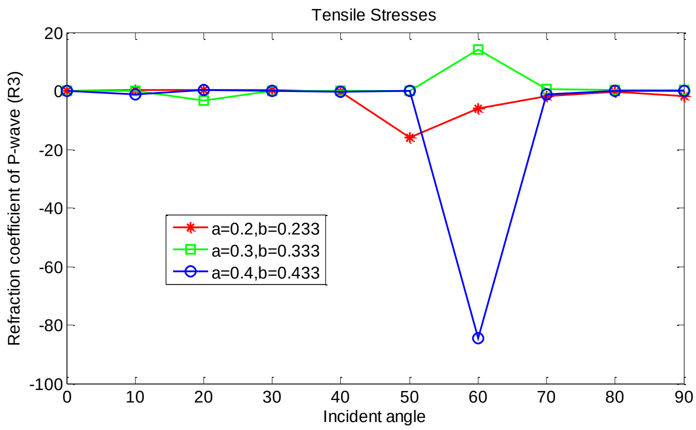

Fig. 5 Variation of refraction amplitude ratio of P wave with angle of incidence when both the half-spaces are under initial tensile stress

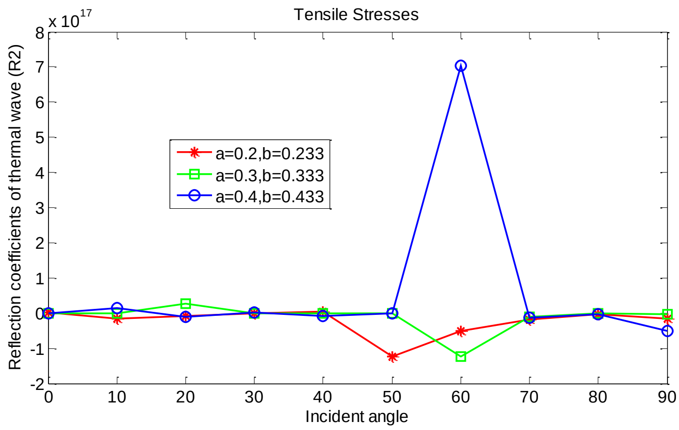

Fig. 6 Variation of refraction amplitude ratio of thermal wave with angle of incidence when both the half-spaces are under initial tensile stress

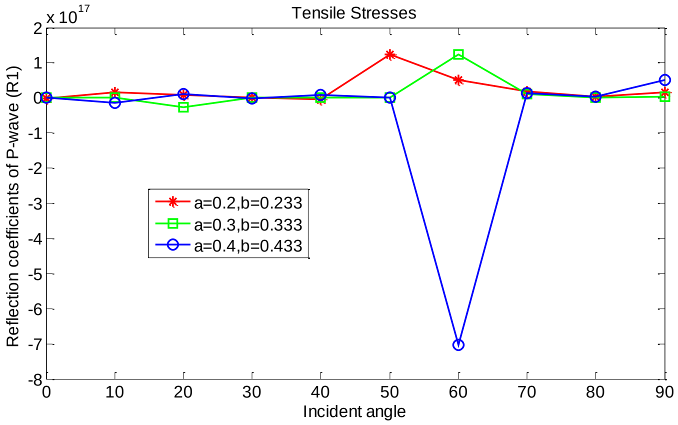

Fig. 7 Variation of reflection amplitude ratio of P-wave with angle of incidence when both the half-spaces are under initial tensile stress

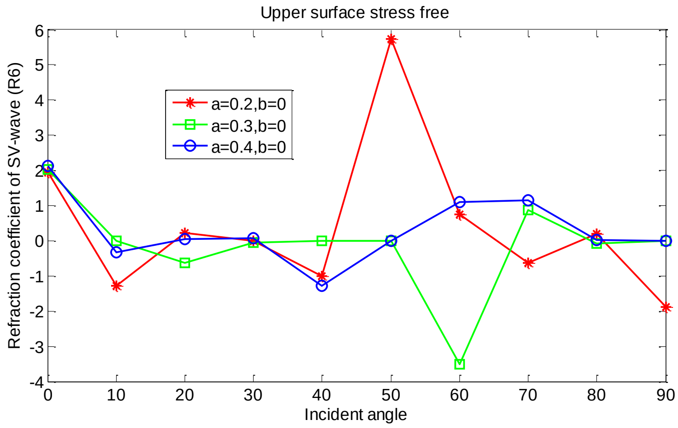

Fig. 8 Variation of refraction amplitude ratio of SV wave with angle of incidence when upper half-spaces is stress free

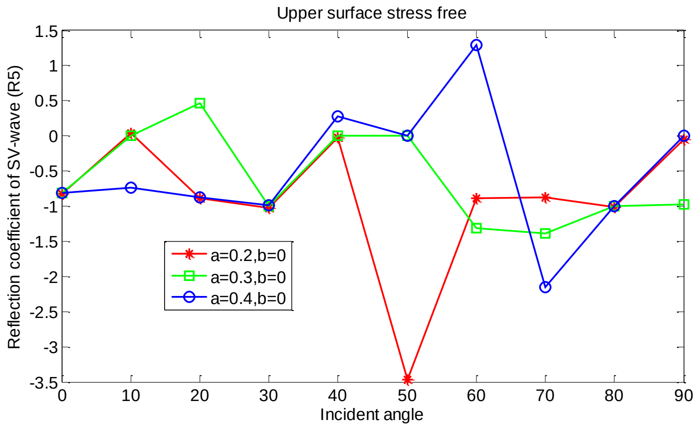

Fig. 9 Variation of refraction amplitude ratio of P-wave with angle of incidence when upper half-spaces is stress free

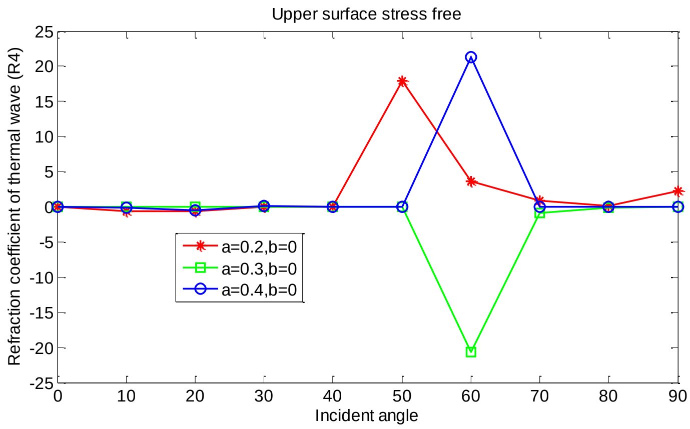

Fig. 10 Variation of reflection amplitude ratio of SV wave with angle of incidence when upper half-spaces is stress free

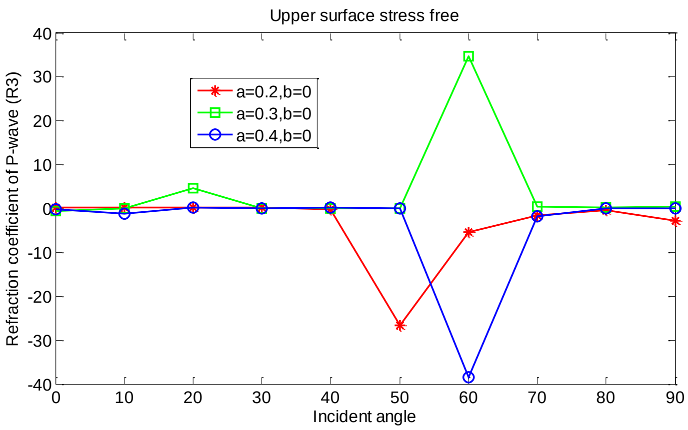

Fig. 11 Variation of refraction amplitude ratio of thermal wave with angle of incidence when upper half-spaces is stress free

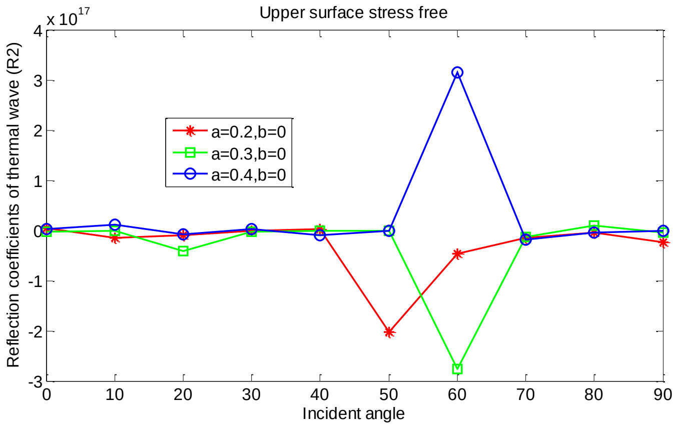

Fig. 12 Variation of reflection amplitude ratio of P-wave with angle of incidence when upper half-spaces is stress free

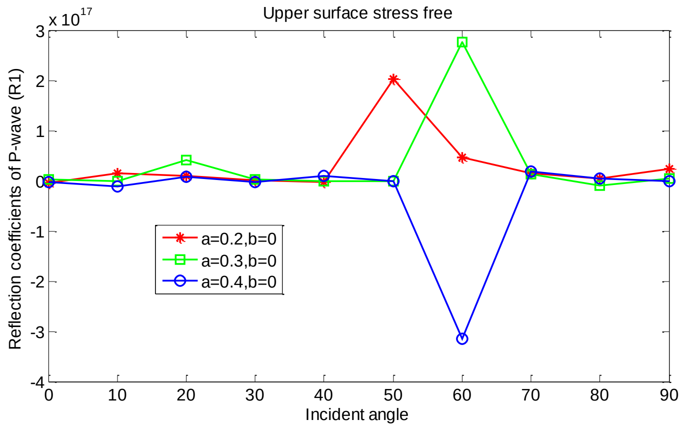

Fig. 13 Variation of reflection amplitude ratio of thermal wave with angle of incidence when upper half-spaces is stress free

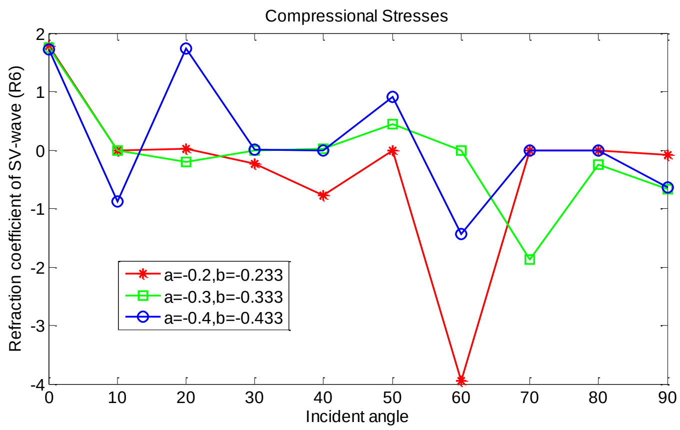

Fig. 14 Variation of refraction amplitude ratio of SV wave with angle of incidence when both the half-spaces are under initial compressional stress

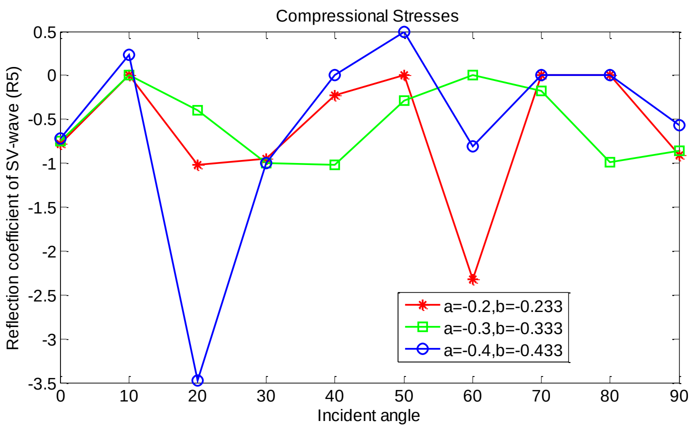

Fig. 15 Variation of reflection amplitude ratio of SV wave with angle of incidence when both the half-spaces are under initial compressional stress

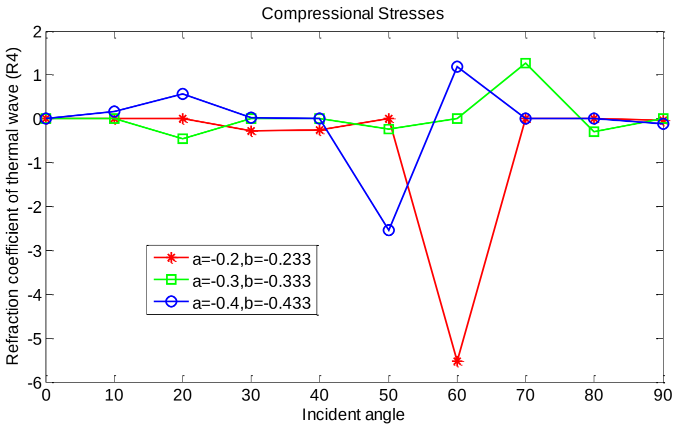

Fig. 16 Variation of refraction amplitude ratio of thermal wave with angle of incidence when both the half-spaces are under initial compressional stress

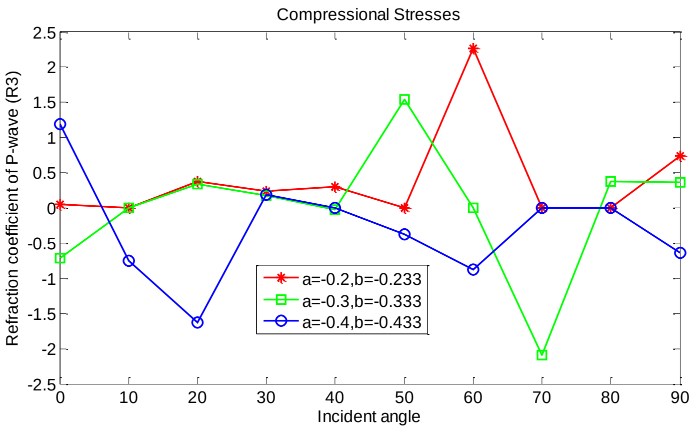

Fig. 17 Variation of refraction amplitude ratio of P-wave with angle of incidence when both the half-spaces are under initial compressional stress

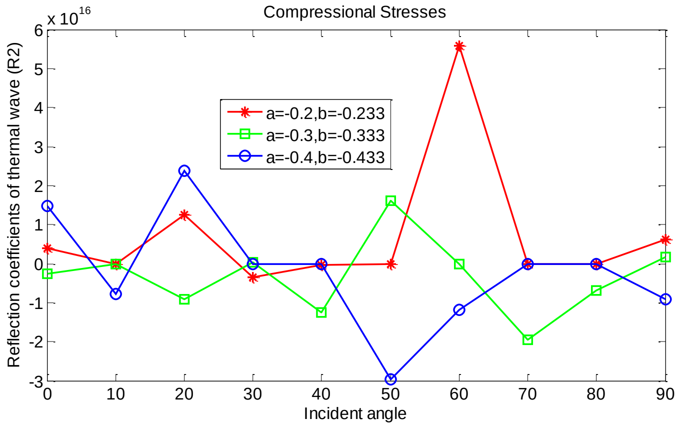

Fig. 18 Variation of reflection amplitude ratio of thermal wave with angle of incidence when both the half-spaces are under initial compressional stress

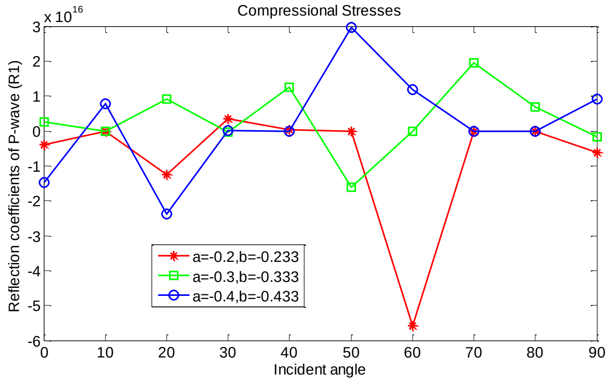

Fig. 19 Variation of reflection amplitude ratio of P-wave with angle of incidence when both the half-spaces are under initial compressional stress

$\mu_e' H_0'^2 = 1.1\times 10^9\,\mathrm{N/m^2}$ and coupling parameters $\bar{\varphi} = 0.005$ and $\bar{\varphi}' = 0.003$ for medium M_1 and M_2 respectively are present. Fig. 9 shows the variation of refraction amplitude ratio of P-wave with angle of incidence when upper half-spaces is stress free but magnetic parameter $\mu_e H_0^2 = 1.1\times 10^9\,\mathrm{N/m^2}$, $\mu_e' H_0'^2 = 1.1\times 10^9\,\mathrm{N/m^2}$ and coupling parameters $\bar{\varphi} = 0.005$ and $\bar{\varphi}' = 0.003$ for medium M_1 and M_2 respectively are present.

Fig. 10 shows the variation of reflection amplitude ratio of SV wave with angle of incidence when upper half-spaces is stress free but magnetic parameter $\mu_e H_0^2 = 1.1\times 10^9\,\mathrm{N/m^2}$, $\mu_e' H_0'^2 = 1.1\times 10^9\,\mathrm{N/m^2}$ and coupling parameters $\bar{\varphi} = 0.005$ and $\bar{\varphi}' = 0.003$ for medium M_1 and M_2 respectively are present. Fig. 11 shows the variation of refraction amplitude ratio of thermal wave with angle of incidence when upper half-spaces is stress free but magnetic parameter $\mu_e H_0^2 = 1.1\times 10^9\,\mathrm{N/m^2}$, $\mu_e' H_0'^2 = 1.1\times 10^9\,\mathrm{N/m^2}$ and coupling parameters $\bar{\varphi} = 0.005$ and $\bar{\varphi}' = 0.003$ for medium M_1 and M_2 respectively are present. Fig. 12 shows the variation of reflection amplitude ratio of P-wave with angle of incidence when upper half-spaces is stress free but magnetic parameter $\mu_e H_0^2 = 1.1\times 10^9\,\mathrm{N/m^2}$, $\mu_e' H_0'^2 = 1.1\times 10^9\,\mathrm{N/m^2}$ and coupling parameters $\bar{\varphi} = 0.005$ and $\bar{\varphi}' = 0.003$ for medium M_1 and M_2 respectively are present. Fig. 13 shows the variation of reflection amplitude ratio of thermal wave with angle of incidence when upper half-spaces is stress free but magnetic parameter $\mu_e H_0^2 = 1.1\times 10^9\,\mathrm{N/m^2}$, $\mu_e' H_0'^2 = 1.1\times 10^9\,\mathrm{N/m^2}$ and coupling parameters $\bar{\varphi} = 0.005$ and $\bar{\varphi}' = 0.003$ for medium M_1 and M_2 respectively are present.

6.3 The incident wave is SV wave and both the surfaces are under initial compressive stress a=-0.2, -0.3, -0.4 and b=-0.233, -0.333, -0.433.

Fig. 14 is plotted for refraction amplitude ratio of SV wave with angle of incidence when both the half-spaces are under initial compressional stress a=-0.2, -0.3, -0.4 and b=-0.233, -0.333, -0.433. Refracted waves are greatly influenced by the stresses, temperature and magnetic field of the medium. For stresses a=-0.2, -0.4 and b=-0.233, -0.433, the minima occurs at 60°, but for stress a=-0.3, b=-0.333, the minima for the wave occurs at 70°. Fig. 15 shows the variation of reflection amplitude ratio of SV wave with angle of incidence when both the half-spaces are under initial compressional stress a=-0.2, -0.3, -0.4 and b=-0.233, -0.333, -0.433. For stresses a=-0.2,

-0.4 and *b*=-0.233, -0.433, the two minima occurs at 20° and 60°, but for stress *a*=-0.3, *b*=-0.333, the maxima for the wave occurs at 70°. Fig. 16 shows the variation of refraction amplitude ratio of thermal wave with angle of incidence when both the half-spaces are under initial compressional stress *a*=-0.2, -0.3, -0.4 and *b*=-0.233, -0.333, -0.433. For stresses *a*=-0.2, -0.4 and *b*=-0.233, -0.433, the two minima occurs at 50° and 60°, but for stress *a*=-0.3, *b*=-0.333, the maxima for the wave occurs at 70°. Fig. 17 is plotted for refraction amplitude ratio of P-wave with angle of incidence when both the half-spaces are under initial compressional stress *a*=-0.2, -0.3, -0.4 and *b*=-0.233, -0.333, -0.433. For stresses *a*=-0.2 and *b*=-0.233 the maxima occurs at 60°, but for stress *a*=-0.3, *b*=-0.333, the minima for the wave occurs at 70°. Fig. 18 shows the variation of reflection amplitude ratio of thermal wave with angle of incidence when both the half-spaces are under initial compressional stress *a*=-0.2, -0.3, -0.4 and *b*=-0.233, -0.333, -0.433. It is observed that maxima and minima changes for reflection and refraction of the wave in the medium. Fig. 19 is plotted for reflection amplitude ratio of P-wave with angle of incidence when both the half-spaces are under initial compressional stress *a*=-0.2, -0.3, -0.4 and *b*=-0.233, -0.333, -0.433. Figs. 14-19 show the continuous amplitude ratio for reflection and refraction of SV, P and thermal waves for incident angle 60° and beyond that angle the pattern is discontinuous. This rate of change of the amplitude ratios is not uniform. This is due to presence of magnetic field in the mediums.

7. Conclusions

Numerical calculations in detail are presented for the cases of magneto-thermoelastic SV waves incident at the solid-solid interface of the model considered. It has been observed from the above numerical values of the reflection and refraction coefficients of SV-wave that negligibly amount of energy is reflected or refracted for thermal wave and small amount of energy is reflected and refracted for P-wave and SV-wave. These observations are same in both the case of tensile initial stresses and compressional stresses. For the both cases of incidence, it is observed that the amplitude ratios change with the change in thermal and magnetic parameter. However, the rate of change of the amplitude ratios is not uniform. The effect of elasticity is also observed on various reflected and refracted waves. The SV-waves, P-waves and thermal waves are greatly influenced by magnetic field, temperature field and initial stress present in the mediums.

Acknowledgments

The authors are thankful to unknown reviewers for their valuable comments.

Funding

This research received no specific grant from any funding agency in the public, commercial or not-for-profit sectors.

Conflict of interest

None declared.

References

Biot, M.A. (1965), *Mechanics of Incremental Deformations*, John Wiley and Sons, Inc., New York.

Biot, M. (1956), "Thermoelasticity and irreversible thermodynamics", *J. Appl. Phys*., **27**(3), 240-253.

Boley, B.A. (1964), *Theory of thermal stresses*, John Wiley.

Chakraborty, N. and Singh, M.C. (2011), "Reflection and refraction of a plane thermoelastic wave at a solid-solid interface under perfect boundary condition, in presence of normal initial stress", *Appl. Math. Model.*, **35**(11), 5286-5301.

Chakraborty, N. (2013), "Reflection of plane elastic waves at a free surface under initial stress and temperature field", *TEPE*, **2**(2), 47-54.

Chattopadhyay, A. and Kumar, R. (2006), "Reflection and refraction of waves at the interface of an isotropic medium over a highly anisotropic medium", *Acta Geophys.*, **54**(3), 239-249.

Chattopadhyay, A., Venkateswarlu, R.L.K. and Chattopadhyay, A. (2007), "Reflection and refraction of quasi P and SV waves at the interface of fibre-reinforced media", *Adv. Stud. Theor. Phys*., **1**(2), 57-73.

Duhamel, J.M.C. (1937), "Second mémorie sur les phénomenes thermomecaniques", *J. 1' Ecole Polytech. Paris*, **15**(25), 1-57.

Deswal, S., Singh, L. and Singh, B. (2011), "Reflection and refraction at an interface between two dissimilar thermally conducting viscous liquid half-spaces", *Int. J. Pure Appl. Math.*, **70**(6), 807-824.

Fox, N. (1969), "Generalised thermoelaticity", *Int. J. Eng. Sci*., **7**(4), 437.

Gurtin, M.E. and Pipkin, A.C. (1969), "A general theory of heat conduction with finite wave speeds", *Arch. Rat. Mech. Anal.*, **31**(2), 113-126.

Kumar, R. and Kumar, R. (2012), "Reflection and refraction of elastic waves at the interface of an elastic half-space and initially stressed thermoelastic with voids half-space", *Multidiscip. Model. Mater. Struct.*, **8**(3), 355-379.

Landau, L.D. and Lifshitz, E.M. (1960), *Elastrodynamics of Continuous Media (English Translation)*, Pergamon Press, London, New York.

Lord, H.W. and Shulman, Y. (1967), "A generalized dynamical theory of thermoelasticity", *J. Mech. Phys. Solid.*, **15**(5), 299-309.

Lotfy, K. (2011), "Transient disturbance in a half-space under generalized magneto-thermoelasticity with a stable internal heat source under three theories", *Multidiscip. Model. Mater. Struct.*, **7**(1), 73-90.

Meixner, J. (1970), "On the linear theory of heat conduction", *Archives Rat. Mech. Anal.*, **39**(2), 108-119.

Montanaro, A. (1999), "On singular surface in isotropic linear thermoelasticity with initial stress", *J. Acoust. Soc. Am.*, **106**(3), 1586-1588.

Neumann, F. (1885), *Vorlesungen uber die Theorie der Elastizität der festoon Körpern*, Leipzig.

Nowacki, W. (1958), *Thermoelasticity*, Addison-Wesley Pub. Com. Inc., London.

Nowinski, J.L. (1978), *Theory of thermoelasticity with application*, Sijthoff and Noord-hoof International, The Netherlands.

Shekhar, S. and Parvez, I.A. (2013), "Effect of rotation, magnetic field and initial stresses on propagation of plane waves in transversely isotropic dissipative half space", *Appl. Math.*, **4**(08), 107-113.

Singh, S.S. (2009), “Reflection and transmission of SV-wave at a plane interface between homogeneous and inhomogeneous elastic half-spaces”, *Sci. Vision*, **9**(2), 73-78.

CC

Appendix

Solving Eq. (38), we get the solution as

$$\Delta^2 = \frac{1}{2}\left[\left(1+\bar{\tau}_{\mathrm{T}} - i\bar{\varphi}\right) \pm \sqrt{\left(1+\bar{\tau}_{\mathrm{T}} - i\bar{\varphi}\right)^2 + 4i\bar{\varphi}}\,\right]$$

Approximate expression for reflection coefficients is obtained by assuming practical values of $\bar{\tau}_{\mathrm{T}} \ll 1$ and $\bar{\varphi} \ll 1$ for elastic materials. Solving Eq. (38) and retaining only first degree terms of $\bar{\tau}_{\mathrm{T}}$ and $\bar{\varphi}$, we get

$$\Delta_1 = 1 + \frac{1}{2}\bar{\tau}_T(1 - i\bar{\varphi}) \text{ and } \Delta_2 = -i^{\frac{1}{2}}\bar{\varphi}^{\frac{1}{2}}\Delta_1 = \frac{1}{\sqrt{2}}(i-1)\bar{\varphi}^{\frac{1}{2}}\Delta_1$$

Similarly, for medium M_2

$$\Delta_1' = 1 + \frac{1}{2}\bar{\tau}_{\mathrm{T}}'(1 - i\bar{\varphi}') \text{ and } \Delta_2' = -i^{\frac{1}{2}}\bar{\varphi}'^{\frac{1}{2}}\Delta_1' = \frac{1}{\sqrt{2}}(i-1)\bar{\varphi}'^{\frac{1}{2}}\Delta_1'$$

For sake of convenience, we consider λ=2μ and λ'=2μ', therefore the stress parameters for both the mediums are $a = \frac{P}{2\mu}$ and $b = \frac{P'}{2\mu'}$ respectively. Therefore $c_1^2, c_2^2, c_1'^2, c_2'^2, \Delta^2, \Delta'^2, \eta, \eta', \eta_1, \eta_1'$ and ξ can be written in terms of '*a*' and '*b*' as

$$c_1^2 = \frac{(\lambda + 2\mu + \mu_e H_0^2 + P)}{\rho} = \frac{\mu_e H_0^2}{\rho} + \frac{4\mu}{\rho}\left(1 + \frac{a}{2}\right),\quad c_1'^2 = \frac{(\lambda' + 2\mu' + \mu_e' H_0'^2 + P')}{\rho'} = \frac{\mu_e' H_0'^2}{\rho'} + \frac{4\mu'}{\rho'}\left(1 + \frac{b}{2}\right),$$

$$c_2^2 = \frac{\left(\mu - \frac{P}{2}\right)}{\rho} = \frac{\mu(1-a)}{\rho},$$

$$c_2'^2 = \frac{\left(\mu' - \frac{P'}{2}\right)}{\rho'} = \frac{\mu'(1-b)}{\rho'},$$

$$\Delta^2 = \frac{c_2^2}{c_1^2} = \frac{(1-a)}{\left(\mu_e H_0^2/\mu\right) + 4(1 + a/2)},$$

$$\Delta'^2 = \frac{c_2'^2}{c_1'^2} = \frac{(1-b)}{\left(\mu_e' H_0'^2/\mu'\right) + 4(1 + b/2)},$$

$$\eta = \frac{P}{c_1^2\rho} = \frac{P}{(\lambda + 2\mu + \mu_e H_0^2 + P)} = \frac{P}{\mu_e H_0^2 + (4\mu + P)} = \frac{1}{\left(\mu_e H_0^2/\mu a\right) + \left((2/a) + 1\right)},$$

$$\eta' = \frac{P'}{c_1'^2\rho'} = \frac{P'}{(\lambda' + 2\mu' + \mu_e' H_0'^2 + P')} = \frac{P'}{\mu_e' H_0'^2 + (4\mu' + P')} = \frac{1}{\left(\mu_e' H_0'^2/\mu' b\right) + \left((2/b) + 1\right)},$$

$$\eta_1 = \frac{P}{2c_2^2\rho} = \frac{P}{2\left(\mu - \frac{P}{2}\right)} = \frac{1}{\left((1/a)-1\right)},$$

$$\eta_1' = \frac{P'}{2c_2'^2\rho'} = \frac{P'}{2\left(\mu' - \frac{P'}{2}\right)} = \frac{1}{\left((1/b)-1\right)},$$

$$\xi = \frac{1+\dfrac{2}{\left((1/b)-1\right)}}{1+\dfrac{2}{\left((1/a)-1\right)}},$$

$$\sin\theta_1 = \frac{\Delta_1}{\Delta}\sin\theta, \sin\theta_2 = \frac{\Delta_2}{\Delta}\sin\theta, \sin\theta_1' = \frac{\Delta_1'}{\Delta}\frac{c_1'}{c_1}\sin\theta, \sin\theta_2' = \frac{\Delta_2'}{\Delta}\frac{c_1'}{c_1}\sin\theta,$$

$$\sin\theta' = \frac{\Delta'}{\Delta}\frac{c_1'}{c_1}\sin\theta, \cos\theta_1 = \left[1-\frac{\Delta_1^2}{\Delta^2}\sin^2\theta\right]^{\frac{1}{2}}, \cos\theta_2 = \left[1-\frac{\Delta_2^2}{\Delta^2}\sin^2\theta\right]^{\frac{1}{2}},$$

$$\cos\theta_1' = \left[1-\frac{c_1'^2}{c_1^2}\frac{\Delta_1'^2}{\Delta^2}\sin^2\theta\right]^{\frac{1}{2}}, \cos\theta_2' = \left[1-\frac{c_1'^2}{c_1^2}\frac{\Delta_2'^2}{\Delta^2}\sin^2\theta\right]^{\frac{1}{2}}, \cos\theta' = \left[1-\frac{c_1'^2}{c_1^2}\frac{\Delta'^2}{\Delta^2}\sin^2\theta\right]^{\frac{1}{2}}.$$

$$\begin{vmatrix}
\sin\theta & \sin\theta & -\sin\theta & -\sin\theta & \cos\theta & \left(\frac{(1-a)^{\frac{1}{2}}}{(1-b)^{\frac{1}{2}}}\right)\left(\frac{(2+b)^{\frac{1}{2}}}{(2+a)^{\frac{1}{2}}}\right)\left[1-\frac{(1-b)}{(1-a)}\sin^2\theta\right]^{\frac{1}{2}} \\
\left(\frac{(1-a)^{\frac{1}{2}}}{2(1+a)^{\frac{1}{2}}}\right)\left[1-\frac{2(2+a)}{(1-a)}\sin^2\theta\right]^{\frac{1}{2}} & \left(\frac{(1-a)^{\frac{1}{2}}}{2(1+a)^{\frac{1}{2}}}\right)\left[1-\frac{2(2+a)}{(1-a)}\sin^2\theta\right]^{\frac{1}{2}} & \left(\frac{0.01(1-a)^{\frac{1}{2}}}{\left[1+\frac{b}{2}\right]^{\frac{1}{2}}}\right)\left[1-\frac{2(2+b)}{(1-a)}\sin^2\theta\right]^{\frac{1}{2}} & \left(\frac{0.01(1-a)^{\frac{1}{2}}}{\left[1+\frac{b}{2}\right]^{\frac{1}{2}}}\right)\left[1-\frac{2(2+b)}{(1-a)}\sin^2\theta\right]^{\frac{1}{2}} & -\sin\theta & -\sin\theta \\
\left(\frac{(1-a)}{4(1+a)}\right)\sin 2\theta & \left(\frac{(1-a)}{4(1+a)}\right)\sin 2\theta & .9\left(\frac{0.2(1-a)}{(2+b)}\right)\left(\frac{2(1+\frac{b}{2})^{\frac{1}{2}}}{(1-a)^{\frac{1}{2}}}\right)\sin 2\theta & .9\left(\frac{0.2(1-a)}{(2+b)}\right)\left(\frac{.1(1+\frac{b}{2})^{\frac{1}{2}}}{(1-a)^{\frac{1}{2}}}\right)\sin 2\theta & \cos 2\theta & -.9\left(\frac{0.1(1-a)}{(1+\frac{b}{2})}\right)\left[1-\frac{(1-b)}{(1-a)}\sin^2 2\theta\right]^{\frac{1}{2}} \\
\left(.95-2\sin^2\theta+(1+\left[\frac{(4+2a)}{(1-a)}\right]\sin^2\theta\right) & \left(.95-2\sin^2\theta+(1+\left[\frac{(4+2a)}{(1-a)}\right]\sin^2\theta\right) & -.9\left(.95-2\sin^2\theta+(1+\left[\frac{(4+2b)}{(1-b)}\right]\sin^2\theta\right) & -.9\left(.95-2\sin^2\theta+(1+\left[\frac{(4+2b)}{(1-b)}\right]\sin^2\theta\right) & -.9\left(1-\frac{a}{(1-a)}\right)\sin 2\theta & -.9\left(1-\frac{b}{(1-b)}\right)\left[1-\frac{(1-b)^{\frac{1}{2}}}{(1-a)^{\frac{1}{2}}}\sin 2\theta\right] \\
0.005\left[1-\frac{(4+2b)}{(1-a)}\sin^2\theta\right]^{\frac{1}{2}} & 0.005\left[1-\frac{(4+2b)}{(1-a)}\sin^2\theta\right]^{\frac{1}{2}} & 0.0033\left[1-\frac{(4+2b)}{(1-a)}\sin^2\theta\right]^{\frac{1}{2}} & 0.0033\left[1-\frac{(4+2b)}{(1-a)}\sin^2\theta\right]^{\frac{1}{2}} & 0 & 0 \\
.005 & .005 & 0.0033 & -0.0033 & 0 & 0
\end{vmatrix}
\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{bmatrix}
=
\begin{bmatrix} \cos\theta \\ \sin\theta \\ -\cos 2\theta \\ -\left(\frac{1}{(1-b)}\right)\sin 2\theta \\ 0 \\ 0 \end{bmatrix}$$

Nomenclature

λ, μ	Lame's constants
ρ	density
σ	Poisson's ratio
c_p	specific heat at constant strain
s_{ij}	components of stress tensor
T	absolute temperature
T_0	reference temperature chosen so that $\lvert \mathrm{T}-\mathrm{T}_0/\mathrm{T}_0 \rvert \ll 1$
P	Initial pressure ($s_{yy}-s_{xx}$)
e_{ij}	components of strain tensor
K	thermal conductivity
J	current density vector
μ_e	$1/[2(1+\sigma)]$ magnetic permeability
ε_e	electric permittivity
H	initial uniform magnetic intensity vector
h	induced magnetic field
H_0	magnetic field component
E	induced electric field vector
D	electric displacement vector
B	magnetic displacement vector
F	Lorentz force
δ_{ij}	Kronecker delta
τ_{T}	thermoelastic coupling constant
u_i	components of displacement vector
τ_0, τ	relaxation times
t	time
e	($\partial u/\partial x+\partial v/\partial y+\partial w/\partial z$) cubical dilatation
Ω	$1/2(\partial v/\partial x-\partial u/\partial y)$ rotational component
α_t	coefficient of linear thermal expansion
γ	$(3\lambda+2\mu)\alpha_t$
k	wave number
ω	angular frequency