Earthquakes and Structures, *Vol. 8, No. 6 (2015) 1481-1497*
DOI: http://dx.doi.org/10.12989/eas.2015.8.6.1481

Vertical seismic response analysis of straight girder bridges considering effects of support structures

Tong Wang[*1,3], Hongjing Li[2a] and Yaojun Ge[3b]

[1]*College of Civil Engineering, Shanghai Normal University, Shanghai 201418, China*
[2]*College of Civil Engineering, Nanjing Tech University, Nanjing 211800, China*
[3]*Department of Bridge Engineering, Tongji University, Shanghai 200092, China*

(Received September 13, 2014, Revised November 4, 2014, Accepted December 5, 2014)

Abstract. Vertical earthquake ground motion may magnify vertical dynamic responses of structures, and thus cause serious damage to bridges. As main support structures, piers and bearings play an important role in vertical seismic response analysis of girder bridges. In this study, the pier and bearing are simplified as a vertical series spring system without mass. Then, based on the assumption of small displacement, the equation of motion governing the simply-supported straight girder bridge under vertical ground motion is established including effects of vertical deformation of support structures. Considering boundary conditions, the differential quadrature method (DQM) is applied to discretize the above equation of motion into a MDOF (multi-degree-of-freedom) system. Then seismic responses of this MDOF system are calculated by a step-by-step integration method. Effects of support structures on vertical dynamic responses of girder bridges are studied under different vertical strong earthquake motions. Results indicate that support structures may remarkably increase or decrease vertical seismic responses of girder bridges. So it is of great importance to consider effects of support structures in structural seismic design of girder bridges in near-fault region. Finally, optimization of support structures to resist vertical strong earthquake motions is discussed.

Keywords: vertical seismic response; girder bridge; support structure; pier; bearing; earthquake ground motion; differential quadrature method

1. Introduction

In the past several decades, or even since the origin of structural seismic design, the vertical component of earthquake ground motion has almost always been neglected in design and analysis of aseismic structures, though the earthquake ground motion is actually three-dimensional. This routine is mainly driven by the common perception, established based on far-fault earthquake recordings, that vertical earthquake ground motion is too small to amplify vertical responses of structures, and that structures have sufficient overstrength against the vertical ground motion

*Corresponding author, Ph.D., E-mail: tongwang@shnu.edu.cn
[a]Professor, E-mail: harbiner@163.com
[b]Professor, E-mail: yaojunge@tongji.edu.cn

http://www.techno-press.org/journals/eas&subpage=7 ISSN: 2092-7614 (Print), 2092-7622 (Online)

because they have already been designed for the gravitational acceleration. However, in recent years, more and more near-fault recordings with prominent vertical ground motions were acquired, for example, a remarkable vertical peak ground acceleration (PGA) of more than 2 g was recorded during the aftershock of the 1985 Nahanni earthquake in Canada, with a ratio of at least 2 to horizontal peak ground acceleration (Weichert *et al*. 1986), and more recently a breath-taking vertical PGA of 38.66 m/s^2 was captured in the 2008 Iwate-Miyagi Nairiku earthquake in Japan (Takabatake and Matsuoka 2012). Similar phenomenon with intense vertical ground motions was also observed in other earthquakes, such as the 1979 Imperial Valley, 1989 Loma Prieta, 1994 Northridge, 1995 Kobe, 1999 Chi-Chi, 2011 Tohoku and 2011 Christchurch earthquake (Shrestha 2014). Such near-fault data have eventually changed the misleading assumption that the vertical ground motion can be taken to be two thirds of the horizontal motion, as postulated by Newmark *et al*. (1973). At short periods and near-source distances, vertical component of the ground motion may be noticeably more severe than the horizontal component and cause serious damage to structures by magnifying vertical dynamic responses, such as bending moment, shear and axial force. Papazoglou and Elnashai (1996) collated a body of field evidence on damaging effects of vertical earthquake motion, including many compressive failures of bridge piers. In fact, as early as nearly half a century ago, Chopra (1966) addressed the importance of vertical earthquake ground motion in design and performance assessment of structures. Yet it didn’t receive more attention until after several earthquakes around 1990s mentioned ahead (Kalkan and Graizer 2007).

Vertical ground motion may magnify midspan bending moment and end shear of the girder, as well as compressive axial force of the pier, thus resulting in severe structural failures of girder bridges (Kunnath *et al*. 2008). Past earthquakes have provided plenty of field evidence on it (Han *et al*. 2009, Papazoglou and Elnashai 1996). Recently, especially in the years of this century, many studies have been devoted to investigate the effects of vertical ground motion on dynamic responses of girder bridges. Button *et al*. (2002) analyzed the responses of typical highway bridges subjected to a suite of vertical motions with different magnitudes, sites and fault distances, and compared results excluding and including vertical motions. Kunnath *et al*. (2008) numerically studied the responses of typical highway overcrossings subjected to combined effects of vertical and horizontal components of near-fault ground motions, and found that the vertical components of ground motions cause significant amplification in the axial force demand in the columns and moment demands in the girder both at the midspan and at the face of the bent cap. Meanwhile, Warn and Whittaker (2008) investigated the influence of vertical earthquake excitation on the response of a bridge isolated with low-damping rubber and lead-rubber bearings through earthquake simulation testing, and found significant amplification of vertical responses by comparison. Kim *et al*. (2011) analytically assessed the effects of vertical earthquake ground motions with different vertical-to-horizontal peak acceleration ratios on RC bridge piers, and compared the results with the case of horizontal-only excitation. They found that inclusion of the vertical components of ground motions has an important effect on the response at all levels and components. Gulerce *et al.* (2012) established seismic demand models for typical highway overcrossings by incorporating critical EDPs (Engineering Demand Parameters) and combined effects of horizontal and vertical ground motion IMs (Intensity Measures) depending on the type of the parameter and the period of the structure. Wang *et al*. (2013) explored the effects of vertical ground motions on the component fragility of a coupled bridge-soil-foundation system, and found that presence of vertical ground motions has a minor effect on the failure probabilities of piles and expansion bearings, while it has a great influence on fixed bearings. Fardis and Tsionis (2013) developed close-form solutions for distributed-mass symmetric multi-span bridges with restrained ends, and studied systematically the effects of stiffness and number of piers on natural vibration of structures. More recently, Lee and Mosalam (2014) found by shaking table tests that vertical excitation has the potential to degrade the shear capacity of bridge columns. However, influence of vertical ground motion on seismic responses of girder bridges is still not understood very well, though so many efforts have been done. Thus there is still need to carry out more thorough studies.

The majority of the above studies were carried out generally through shake table-based model tests or FEM-based numerical simulations. This paper tries another way. The equation of motion governing vertical seismic responses of the straight girder bridge is firstly established based on the principle of small displacement and considering axial deformation of support structures. Then the differential quadrature method (DQM) is applied to discretize the above equation of motion into a MDOF (multi-degree-of-freedom) system, considering all boundary conditions. Finally, vertical seismic responses of this MDOF system are calculated by a step-by-step integration method. In constructing the governing equation, the pier and bearing are simplified as a vertical series spring system without mass. Details on derivation of the governing equation are presented in Section 2. Section 3 shows the fundamental of DQM. Discretization of the governing equation using DQM is given in Section 4. Section 5 analyzes effects of support structures on dynamic responses of the girder bridge under different vertical strong ground motions, and discusses optimization of support structures to resist vertical strong motions. Finally, some conclusions are summarized in Section 6.

2. Formulation of the governing equation

In practice, the girder is usually supported on the ground via piers and bearings. As main support structures, piers and bearings play an important role in vertical seismic response analysis of girder bridges. Fig. 1 shows a single-span straight girder bridge to be considered in this study. Both support conditions are supposed to be simple supports for the purpose of only considering vertical motions, and the girder cannot jump up from the bearings. Generally, the girder is much heavier than the pier and bearing, so here the pier and bearing are simplified as a massless series spring system as shown in Fig. 2 for not considering the axil vibration of themselves. The slenderness ratio of the girder is usually large enough to avoid considering shear deformation and rotational inertia. And for simplicity, except for the above assumptions, we also suppose that the ground is rigid for not considering soil-structure interaction, and that all displacements are elastic and linear to satisfy the superposition principle, i.e., the assumption of small displacement.

The coordinate system adopted is shown in Fig. 2, in which the x axis is defined as the central axis of the girder at its original (or static) position. $y(x,t)$ is the absolute displacement relative to x

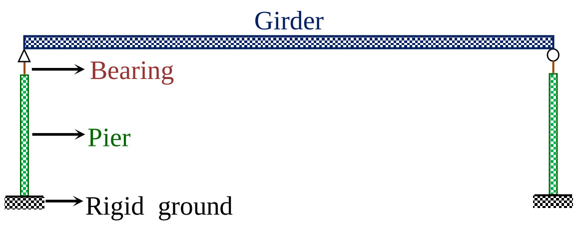


Fig. 1 Sketch of a single-span simplly-supported straight girder bridge

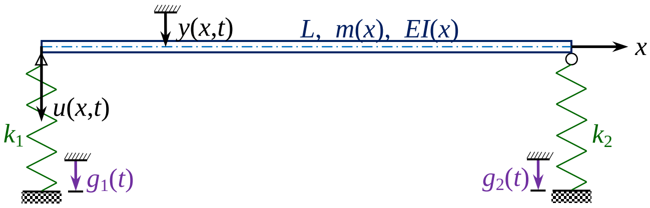


Fig. 2 Simplified analytical model of the girder bridge under vertical ground motion

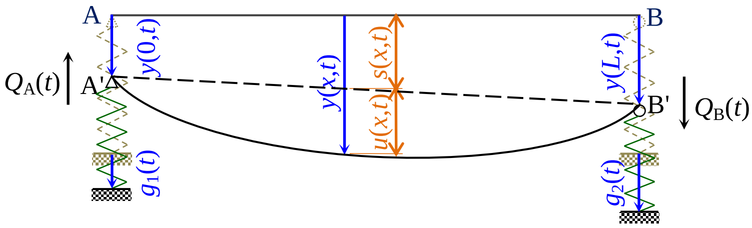


Fig. 3 Vertical displacement of the girder bridge

axis, and $u(x,t)$ is the displacement relative to the rigid central axis, i.e., the deformation from the broken line A'B' to the curve A'B' as shown in Fig. 3. Based on the principle of small displacement, $y(x,t)$ and $u(x,t)$ are both vertical. L, $m(x)$ and $EI(x)$ are the span, the mass per unit length and the flexural stiffness, respectively. $g_i(t)$ (i=1,2) is the vertical ground motion exerted at the ith pier, and k_i (i=1,2) is the vertical stiffness of the ith series spring, which can be determined by

$$k_i = \frac{k_{\mathrm{p}i} k_{\mathrm{b}i}}{k_{\mathrm{p}i} + k_{\mathrm{b}i}}, \text{ for } i=1,2 \tag{1}$$

where $k_{\mathrm{p}i}$ and $k_{\mathrm{b}i}$ are the vertical stiffness of the ith pier and bearing, respectively.

Fig. 3 illustrates the vertical displacement of the girder. The solid line AB is the original (static) position of the girder, while the curve A'B' is the final position. The broken line A'B' is the position of the rigid girder due to spring distortion. The total displacement $y(x,t)$ is equal to the rigid body displacement $s(x,t)$ plus the flexure displacement $u(x,t)$, i.e.

$$y(x,t) = s(x,t) + u(x,t) \tag{2}$$

in which $s(x,t)$ can be determined by linear interpolation, i.e.

$$s(x,t) = \left(1 - \frac{x}{L}\right) y(0,t) + \frac{x}{L} y(L,t) \tag{3}$$

According to the action and reaction principle, the end shear of the girder, i.e., $Q_{\mathrm{A}}(t)$ and $Q_{\mathrm{B}}(t)$ shown in Fig. 3, should equal the spring force, that is

$$k_1[y(0,t) - g_1(t)] = Q_{\mathrm{A}}(t) \tag{4}$$

$$k_2[y(L,t) - g_2(t)] = -Q_{\mathrm{B}}(t) \tag{5}$$

where

$$Q_A(t) = Q(0,t) = M'(0,t) = -[EI(x)u''(x,t)]'_{x=0} \tag{6}$$

$$Q_B(t) = Q(L,t) = M'(L,t) = -[EI(x)u''(x,t)]'_{x=L} \tag{7}$$

in which the prime "$'$" represents differentiation with respect to x.

Substitute Eqs. (4)-(7) into Eq. (3) and the resulting equation into Eq. (2), then one gets

$$\begin{aligned} y(x,t) = u(x,t) + \left(1-\frac{x}{L}\right)g_1(t) - \frac{1}{k_1}\left(1-\frac{x}{L}\right)[EI(x)u''(x,t)]'_{x=0} \\ +\frac{x}{L}g_2(t) + \frac{x}{k_2 L}[EI(x)u''(x,t)]'_{x=L} \end{aligned} \tag{8}$$

Excluding the internal damping and assuming the external damping force proportional to the deformation velocity, the equation of motion governing the beam shown in Fig. 3 can be written as (Clough and Penzien 1995)

$$m(x)\ddot{y}(x,t) + c_E(x)\dot{u}(x,t) + [EI(x)u''(x,t)]'' = P(x,t) \tag{9}$$

where the over dot "$\dot{}$" represents differentiation with respect to the time t, $c_E(x)$ is the external damping factor, and $P(x,t)$ is the external force.

Substituting Eq. (8) into Eq. (9) yields

$$\begin{aligned} m(x)\ddot{u}(x,t) - \frac{m(x)}{k_1}\left(1-\frac{x}{L}\right)\left[EI(x)\ddot{u}''(x,t)\right]'_{x=0} + \frac{xm(x)}{k_2 L}\left[EI(x)\ddot{u}''(x,t)\right]'_{x=L} \\ +c_E(x)\dot{u}(x,t) + \left[EI(x)u''(x,t)\right]'' = P(x,t) - m(x)\left[\left(1-\frac{x}{L}\right)\ddot{g}_1(t) + \frac{x}{L}\ddot{g}_2(t)\right] \end{aligned} \tag{10}$$

Eq. (10) is just the equation of motion governing vertical seismic responses of straight girder bridges considering axil deformation of support structures. And this equation can be reduced into the one without effects of support structures by setting $1/k_1=1/k_2=0$.

For convenience, $m(x)$, $EI(x)$ and $c_E(x)$ are all supposed to be constant in this study, i.e., $m(x)=m$, $EI(x)=EI$ and $c_E(x)=c$. Moreover, the span L is generally very small relative to the propagation velocity of the vertical earthquake motion, so the traveling wave effect of the earthquake motion is negligible, i.e., $g_1(t)=g_2(t)=g(t)$. Then Eq. (10) becomes

$$m\ddot{u}(x,t) - \frac{mEI}{k_1}\left(1-\frac{x}{L}\right)\ddot{u}'''(0,t) + \frac{mEIx}{k_2 L}\ddot{u}'''(L,t) + c\dot{u}(x,t) + EIu^{\mathrm{IV}}(x,t) = P(x,t) - m\ddot{g}(t) \tag{11}$$

For simple supports, the boundary conditions of Eq. (11) or (10) are

$$u(0,t)=0 \tag{12}$$

$$u(L,t)=0 \tag{13}$$

$$u''(0,t)=0 \tag{14}$$

$$u''(L,t)=0 \tag{15}$$

And the initial conditions can be denoted as

$$u(x,0)=u_0(x) \tag{16}$$

$$\dot{u}(x,0)=\dot{u}_0(x) \tag{17}$$

In Eq. (11), there are two terms with variable coefficient and mixed partial derivative at one point, which make it difficult (or impossible) to solve this equation analytically though it is linear. Numerical methods are better choice at present, such as the time-domain method (Law *et al*. 1997) and frequency-domain method (Wang *et al*. 2012). The former is relatively easier to be implemented for this problem than the latter. Besides, the earthquake data applied in the paper are essentially discrete or numerical, which makes it straightforward to use a step-by-step method.

3. Differential quadrature method

The DQM, proposed by Bellman and Casti (1971), is a high-efficiency numerical solution technique for the initial and/or boundary value problems of physical and engineering sciences. It is conceptually simple and the implementation is straightforward. Unlike conventional methods such as finite difference method (FDM) and finite element method (FEM), DQM can produce highly accurate solutions with minimal computational efforts. Many successful applications of DQM in mechanics have been reported (Bert *et al*. 1988, Shu and Richards 1992, Bert and Malic 1996, Liew *et al.* 2001, Malekzadeh and Farid 2007, Kaya 2010, Bozdogan 2012, Rajasekaran 2013).

The idea of DQM is essentially an extension of the integral quadrature method (Shu 2000). It approximates the derivative of a function at any sampling grid point as the weighted linear sum of all the functional values along a mesh line within domain under consideration. Without losing generality, one can consider a one-dimensional function $f(x)$ defined in $[a,b]$. Using DQM, the rth-order derivative of $f(x)$ with respect to x at a point x_i can be expressed as

$$f^{(r)}(x_i)=\sum_{k=1}^{n} w^{(r)}(i,k)\,f(x_k)\text{, for } i=1,2,\ldots,n \text{ and } r=1,2,\ldots,n\text{-}1 \tag{18}$$

where $a=x_1<x_2<\ldots<x_{n-1}<x_n=b$ are the discrete grid coordinates that can be arbitrarily distributed in space, n is the total grid number, and $w^{(r)}(i,k)$ is the corresponding weighting coefficient which can be calculated from various interpolation base functions. In this paper, the Lagrange interpolation polynomials are used to calculate these weighting coefficients. The weighting coefficients of the first-order derivative can be obtained by (Shu 2000)

$$w^{(1)}(i,j)=\prod_{\substack{k=1\\k\neq i,j}}^{n}(x_i-x_k)\Bigg/\prod_{\substack{k=1\\k\neq j}}^{n}(x_j-x_k)\text{, for } i\neq j \tag{19}$$

$$w^{(1)}(i,i)=\sum_{\substack{k=1\\k\neq i}}^{n} w^{(1)}(i,k) \tag{20}$$

where "Π" is the continuous multiply symbol, and i, j=1,2,…,n.

Then, the weighting coefficients for higher-order derivatives can be obtained directly by the matrix multiplication approach, i.e.

$$[w^{(r)}]=[w^{(1)}]^r \tag{21}$$

where $[w^{(r)}]$ is the weighting coefficient matrix for the rth-order derivative.

Choice of sampling points can notably affect the differential quadrature (DQ) solution. It has been demonstrated that non-uniform grids can enhance the accuracy of the DQ solution and can tackle the problem of deterioration of the quadrature solution (Bert and Malik 1996). So this study will adopt a non-uniform mesh defined in particular by

$$x_i=\frac{(a+b)}{2}-\frac{(b-a)}{2}\cos[(i-1)\pi/(n-1)]\text{, for } i=1,2,\ldots,n \tag{22}$$

4. Discretization of the governing equation

The key process of using DQM to discretize the governing Eq. (11) is to appropriately apply the boundary conditions of Eqs. (12)-(15). Obviously, Eq. (11) is a fourth-order partial differential equation with respect to x, and accordingly has four boundary conditions, whereas there are only two boundary points, i.e., two boundary conditions (a Dirichlet and a Neumann) for one boundary point. How to apply two boundary conditions at one boundary point? Many efforts have been devoted to this problem (Jang *et al.* 1989, Wu and Liu 2000, Wang *et al.* 2005). In this study, a built-in method proposed by Wang *et al.* (2005) is adopted. It is to consider boundary conditions during formulation of the weighting coefficients for higher-order derivatives. Details are as follows.

Firstly, x is divided in the domain $[0,L]$ into n grid points defined by Eq. (22), i.e., $0=x_1<x_2<\ldots<x_{n-1}<x_n=L$. Then, using DQM, one gets

$$\{u'\}=[w^{(1)}]\{u\} \tag{23}$$

where

$$\{u\}=\{u(x_1,t),\ u(x_2,t),\cdots,\ u(x_n,t)\}^{\mathrm{T}} \tag{24}$$

$\{u'\}$ is the first-order derivative of $\{u\}$ with respect to x, and $[w^{(1)}]$ is the corresponding weighting coefficient matrix which can be expanded as

$$[w^{(1)}]=\begin{bmatrix} w^{(1)}(1,1) & \cdots & w^{(1)}(1,n) \\ \vdots & \ddots & \vdots \\ w^{(1)}(n,1) & \cdots & w^{(1)}(n,n) \end{bmatrix} \tag{25}$$

Considering the Dirichlet boundary conditions of Eqs. (12)-(13), Eq. (23) becomes

$$\{u'\}=[\bar{w}^{(1)}]\{U\} \tag{26}$$

where

$$[\bar{w}^{(1)}]=\begin{bmatrix} w^{(1)}(1,2) & \cdots & w^{(1)}(1,n-1) \\ \vdots & \ddots & \vdots \\ w^{(1)}(n,2) & \cdots & w^{(1)}(n,n-1) \end{bmatrix} \tag{27}$$

$$\{U\}=\{u(x_2,t),\cdots,u(x_{n-1},t)\}^{\mathrm{T}} \tag{28}$$

By canceling $u'(x_1,t)$ and $u'(x_n,t)$ from $\{u'\}$, Eq. (26) becomes

$$\{U'\}=[W^{(1)}]\{U\} \tag{29}$$

where $\{U'\}$ is the first-order derivative of $\{U\}$ with respect to x, and

$$[W^{(1)}]=\begin{bmatrix} w^{(1)}(2,2) & \cdots & w^{(1)}(2,n-1) \\ \vdots & \ddots & \vdots \\ w^{(1)}(n-1,2) & \cdots & w^{(1)}(n-1,n-1) \end{bmatrix} \tag{30}$$

Similarly, one gets

$$\{u''\}=[w^{(1)}]\{u'\}=[w^{(1)}][\bar{w}^{(1)}]\{U\}=[\bar{w}^{(2)}]\{U\} \tag{31}$$

$$\{U''\}=[\tilde{w}^{(1)}][\bar{w}^{(1)}]\{U\}=[W^{(2)}]\{U\} \tag{32}$$

where

$$[\tilde{w}^{(1)}]=\begin{bmatrix} w^{(1)}(2,1) & \cdots & w^{(1)}(2,n) \\ \vdots & \ddots & \vdots \\ w^{(1)}(n-1,1) & \cdots & w^{(1)}(n-1,n) \end{bmatrix} \tag{33}$$

The Dirichlet boundary conditions of Eqs. (12)-(13) have been built in $[W^{(1)}]$ and $[W^{(2)}]$. In the same way, consider the Neumann boundary conditions of Eqs. (14)-(15), then one obtains

$$\{u'''\}=[w^{(1)}]\{u''\}=[\bar{w}^{(1)}]\{U''\}=[\bar{w}^{(1)}][W^{(2)}]\{U\}=[\bar{w}^{(3)}]\{U\} \tag{34}$$

$$\{U'''\}=[W^{(1)}][W^{(2)}]\{U\}=[W^{(3)}]\{U\} \tag{35}$$

and

$$\{u^{\mathrm{IV}}\}=[w^{(1)}]\{u'''\}=[w^{(1)}][\bar{w}^{(3)}]\{U\}=[\bar{w}^{(4)}]\{U\} \tag{36}$$

$$\{U^{\mathrm{IV}}\}=[\tilde{w}^{(1)}][\bar{w}^{(3)}]\{U\}=[W^{(2)}][W^{(2)}]\{U\}=[W^{(4)}]\{U\} \tag{37}$$

From Eq. (34), one obtains

$$u'''(0,t)=u'''(x_1,t)=\{W^{(5)}\}\{U\} \tag{38}$$

$$u'''(L,t)=u'''(x_n,t)=\{W^{(6)}\}\{U\} \tag{39}$$

in which

$$\{W^{(5)}\}=\{w^{(1)}(1,2),w^{(1)}(1,3),\cdots,w^{(1)}(1,n-1)\}[W^{(2)}] \tag{40}$$

$$\{W^{(6)}\}=\{w^{(1)}(n,2),w^{(1)}(n,3),\cdots,w^{(1)}(n,n-1)\}[W^{(2)}] \tag{41}$$

Clearly, the boundary conditions of Eqs. (12)-(15) have been built in $[W^{(3)}]$, $[W^{(4)}]$, $\{W^{(5)}\}$ and $\{W^{(6)}\}$. Using Eqs. (37)-(39) to discretize Eq. (11) yields

$$[M]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}=\{F\} \tag{42}$$

where

$$[M]=m[I_{\mathrm{m}}]-\frac{mEI}{k_1}([I_{\mathrm{m}}]-[X]/L)\{I_{\mathrm{v}}\}\{W_5\}+\frac{mEI}{Lk_2}[X]\{I_{\mathrm{v}}\}\{W_6\} \tag{43}$$

$$[C]=c[I_{\mathrm{m}}] \tag{44}$$

$$[K]=EI[W_4] \tag{45}$$

$$\{F\}=\{P\}-m\{I_{\mathrm{v}}\}\ddot{g}(t) \tag{46}$$

in which $[I_{\mathrm{m}}]$ is an $(n\text{-}2)\times(n\text{-}2)$ identity matrix, $\{I_{\mathrm{v}}\}$ is an $(n\text{-}2)$-dimensional column vector with unit elements, $\{P\}$ is the external force vector, i.e.

$$\{P\}=\{P(x_1,t),P(x_2,t),\cdots,P(x_n,t)\}^{\mathrm{T}} \tag{47}$$

and $[X]$ is an $(n\text{-}2)\times(n\text{-}2)$ diagonal matrix, i.e.

$$[X]=\mathrm{diag}(x_2,x_3,\cdots,x_{n\text{-}1}) \tag{48}$$

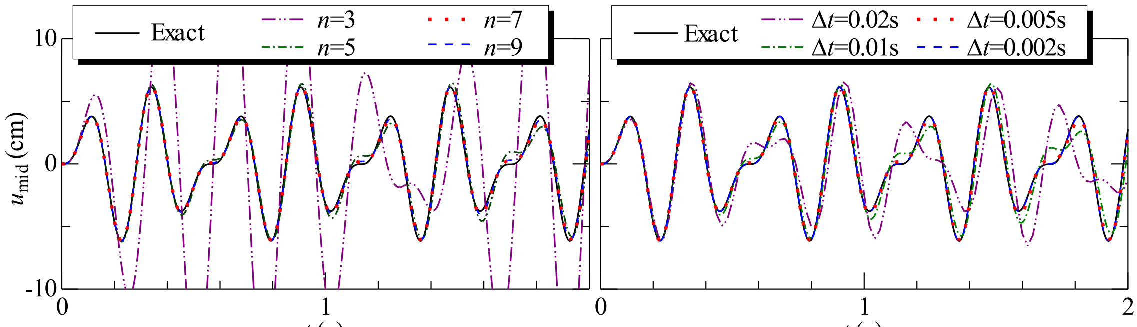
Till now, the governing Eq. (11) has been discretized into a MDOF system of Eq. (42). The vertical bending deformation $u(x,t)$ of the girder bridge can be obtained directly by using some step-by-step method to solve this MDOF system. Then other responses such as shear and bending moment can be calculated indirectly from the obtained $u(x,t)$. Using the assumption of uniformity and substituting Eqs. (38)-(39) into Eqs. (6)-(7) yield calculation formulas for $Q_{\mathrm{A}}(t)$ and $Q_{\mathrm{B}}(t)$, i.e.

$$Q_{\mathrm{A}}(t)=-EI\{W^{(5)}\}\{U\} \tag{49}$$

$$Q_{\mathrm{B}}(t)=-EI\{W^{(6)}\}\{U\} \tag{50}$$

5. Calculations and discussions

To investigate vertical seismic responses of girder bridges, Eq. (11) is discretized according to the process shown in Section 4, then the resulting Eq. (42) is solved by the Newmark-β method with β=1/4, i.e., the constant average acceleration method. A computer program is accomplished in MATLAB. While before starting the study, the compiled computer program is firstly verified by a relatively simple example.



(a) Δt=0.005s (b) n=7

Fig. 4 Comparison of numerical and analytical results

Table 1 Errors of the calculated u_{mid} relative to the analytical result at t=1.82s with changing Δt and n

Δt (s)	Error (%)							
	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10
0.020	211.03	-64.12	-123.09	-118.90	-118.59	-118.60	-118.61	-118.61
0.010	252.50	-108.85	-48.18	-31.47	-30.48	-30.50	-30.51	-30.51
0.005	237.41	-95.28	-20.96	-7.30	-6.49	-6.50	-6.51	-6.51
0.002	231.37	-89.80	-14.01	-1.70	-0.96	-0.97	-0.98	-0.98

5.1 Verification of compiled computer program

Consider the forced vibration of a simply-supported Euler beam governed by

$$m\ddot{u}(x,t) + EIu^{\mathrm{IV}}(x,t) = P(x,t) \tag{51}$$

where all symbols are the same as those in Eq. (11). The above equation can be directly obtained from Eq. (11) by setting $1/k_1=1/k_2=0$, c=0 and $g(t)$=0. And the boundary conditions are still Eqs. (12)-(15). Suppose the external force $P(x,t)=Q\sin(\pi x/L)\sin(pt)$, in which Q=the maximum force, L=span, and p=the frequency of the dynamic force, and the initial state of the beam is static. Then the analytical solution for the displacement are obtained as (Rao 2011)

$$u(x,t) = \frac{Q}{m}\sin(\pi x/L)\frac{\sin pt - (m/\omega)\sin \omega t}{\omega^2 - p^2} \tag{52}$$

in which $\omega=\pi^2(EI/mL^4)^{1/2}$.

The verification is carried out when L=10 m, m=420 kg/m, $EI=4.7726\times10^7$ N·m^2, $Q=1\times10^4$ N/m and $p=2\pi/0.28335$ rad/s. Fig. 4(a) shows 2-second time histories of the midspan displacement u_{mid} obtained at Δt=0.005s and n=3,5,7,9, and Fig. 4(b) shows 2-second time histories of u_{mid} obtained at n=7 and Δt=0.02s, 0.01s, 0.005s, 0.002s. Table 1 shows the errors of calculated u_{mid} relative to the analytical one at t=1.82s with changing Δt and n. It should be noted here that the results for even n are not the direct numerical results. They are obtained by using the Lagrange interpolation which will not change the accuracy of solution because the DQM used here is also based on the Lagrange interpolation. From Fig. 4, the numerical solutions converge quickly to the analytical one with increasing n or with decreasing Δt, and similar conclusions can be obtained from Table 1. It can also be found from Table 1 that the numerical results will not change or only change a little when $n>7$, and that if Δt is too large, the calculated results cannot converge to the analytical solution with increasing n. Obviously, when $\Delta t=0.002$s, $n=6$ can yield sufficiently accurate results, which demonstrates the high efficiency of DQM. Validity of the compiled computer program in this study is efficiently verified by this example.

5.2 Effects of support structures

In the following study of the vertical seismic responses of the girder bridge, the external force $P(x,t)$ in Eq. (11) is omitted according to the convention in seismic analysis of structures, and the initial state of the bridge is static. The basic parameters of the bridge are set as shown in Table 2, in which all parameters are determined according to a practical engineering except the factor c. It should be noted that the damping factor c is difficult to be determined, so an empirical valued is adopted here. Moreover, k_1 and k_2 are assumed to be identical in this study in order to facilitate analysis. Detailed analysis about k_1 and k_2 can be found subsequently.

Vertical strong ground motion data considered in this study are chosen from the PEER Strong Motion Database, and are shown in Table 3, in which PGA and NPTS represent the peak ground acceleration and the total number of time samplings, respectively. All the seismic data are obtained from near-fault sites with remarkable PGAs.

Mesh-dependence study is firstly conducted for input M1 with and without considering effects of support structures. Table 4 shows variation of the absolute maximum value of u_{mid}, noted as $\max|u_{\mathrm{mid}}|$, with changing n, in which "ES" means "Elastic Support", i.e., $k_1=k_2=4\times10^9$ N/m, and "RS" means "Rigid Support", i.e., $1/k_1=1/k_2=0$. Similar to Table 1, results for even n are obtained indirectly by using the Lagrange interpolation. One can see from Table 4 that numerical results converge quickly to a steady state with increasing mesh number n, while the results will not change or only change a little when $n>9$. Similar conclusions can be obtained for other inputs which are omitted. So $n=9$ is set for all cases in the following study.

Table 2 Basic parameters of the girder bridge

L (m)	m (kg/m)	c (N·s/m^2)	EI (N·m^2)	k_1 (N/m)	k_2 (N/m)
30	3×10^4	1.2×10^4	1×10^{11}	4×10^9	4×10^9

Table 3 Input of vertical earthquake motion

No.	Time	Name	Station	PGA (g)	Δt (s)	NPTS
M1	1979/10/15 23:16	Imperial Valley	942 El Centro Array #6	1.655	0.005	7807
M2	1987/10/01 14:42	Whittier Narrows	289 Whittier N. Dam upstream	0.505	0.005	5884
M3	1989/10/18 00:05	Loma Prieta	16LGPC	0.890	0.005	4993
M4	1992/04/25 18:06	Cape Mendocino	89005 Cape Mendocino	0.754	0.020	1500
M5	1995/01/16 20:46	Kobe	0 Takarazuka	0.433	0.010	4096
M6	1999/09/20	Chi-Chi, Taiwan	CHY080	0.724	0.005	18000

Table 4 Mesh dependence of max|u_{mid}| for input M1 with and without considering support structures

	max|u_{mid}| (cm)								
	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=11
RS	4.68	3.64	5.09	5.03	5.00	5.06	5.04	5.04	5.04
ES	4.30	3.90	5.48	5.43	5.35	5.37	5.35	5.35	5.35

Fig. 5 shows time histories of u_{mid} with and without considering effects of support structures under six vertical strong excitations shown in Table 3. In order to give a clear show, only 20s of the time histories are presented in the figure. It can be found by comparison that support structures have distinct effects on u_{mid}, especially for the cases of M3 and M4. Considering support structures, vertical seismic responses may be increased such as Figs. 5(a) and 5(e)-(f), or decreased such as Figs. 5(b)-(d). Effects of support structures depend on characteristics of the vertical earthquake motion exerted on the structures.

Corresponding to Fig. 5, Table 5 presents the absolute maximum value max|u_{mid}| with and without considering effects of support structures under different vertical excitations. Meanwhile, errors of case ES relative to case RS are also given for comparison. Considering support structures, max|u_{mid}| decreases by around 15% for the cases of M3 and M4, while increases by nearly 10% for the case of M5. Effects of support structures for the case of M6 are relatively small.

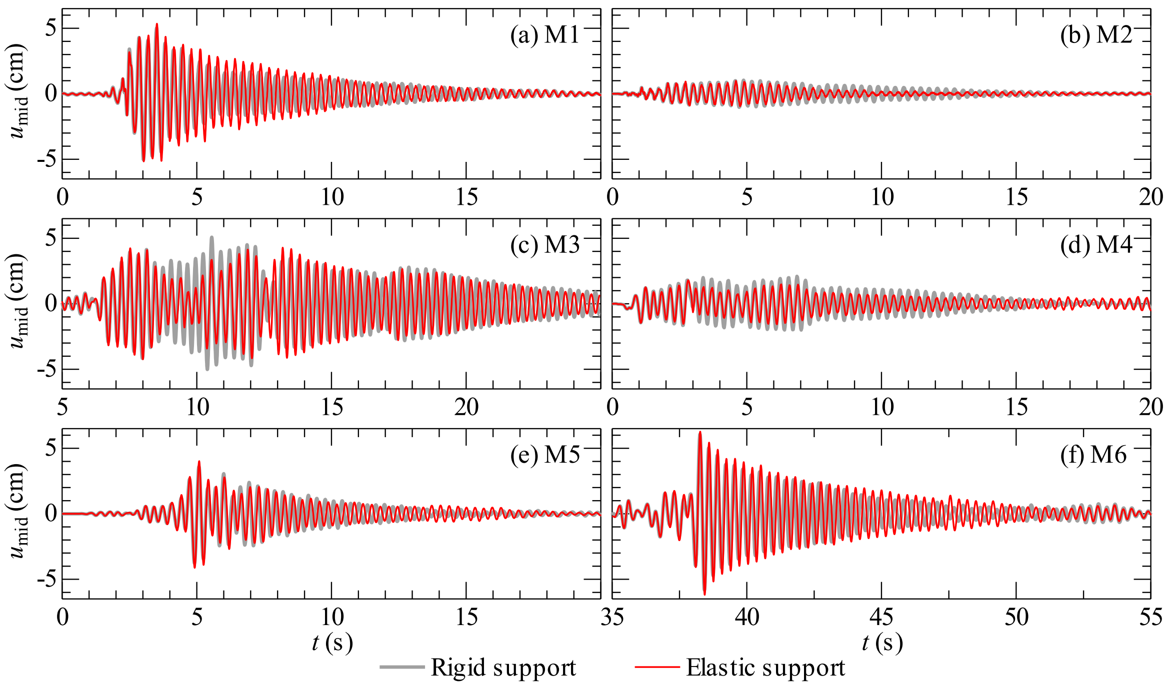


Fig. 5 Time histories of u_{mid} with and without considering effects of support structures under different vertical excitations

Table 5 Comparison of max|u_{mid}| under different vertical excitations

		M1	M2	M3	M4	M5	M6
max|u_{mid}| (cm)	RS	5.04	1.03	5.08	2.10	3.77	6.08
	ES	5.35	0.98	4.29	1.80	4.13	6.27
Error (%)		6.17	-4.89	-15.68	-14.27	9.61	3.27

Similar analysis can be made for end shears $Q_A(t)$ and $Q_B(t)$. In this study, Q_A and Q_B are identical according to the assumptions of uniformity, $g_1(t)=g_2(t)$ and $k_1=k_2$. So, only results of Q_A are given in the following. Similar to Fig. 5, Fig. 6 compares time histories of Q_A with and without considering support structures under different vertical excitations, and correspondingly Table 6 presents the absolute maximum value max|Q_A| with and without considering support structures under different vertical excitations. Compared with Fig. 5, similar conclusions can be obtained from Fig. 6 for the cases of M3, M5 and M6, while different ones for the cases of M1, M2 and M4, in which there are obviously high-frequency components. Compared with Table 5, similar conclusions are obtained from Table 6 for the cases of M4-M6, while different ones for the cases of M1-M2. For the case of M1, support structures decrease max|Q_A| by around 8%, while increase max|u_{mid}| by around 6%. And for the case of M2, support structures increase max|Q_A| by more than 25%, while decrease max|u_{mid}| by around 5%. So influence of support structures on two different responses may be opposite.

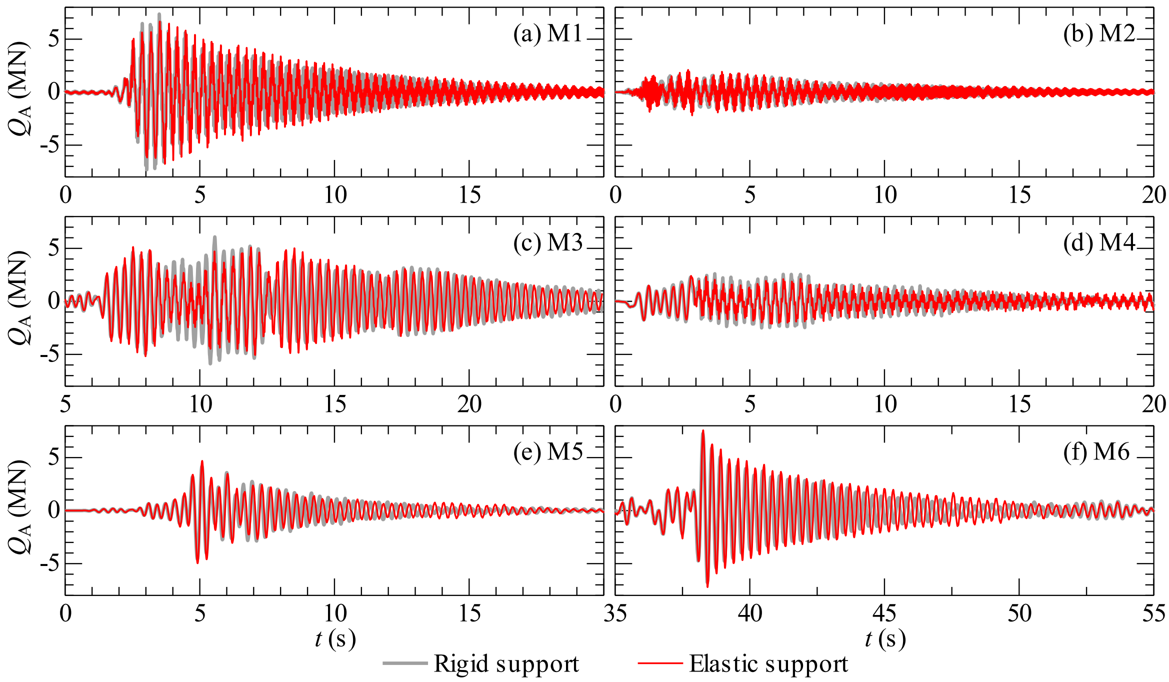


Fig. 6 Time histories of Q_A with and without considering effects of support structures under different vertical excitations

Table 6 Comparison of max|Q_A| under different vertical excitations

		M1	M2	M3	M4	M5	M6
max|Q_A| (MN)	RS	7.37	1.75	6.08	2.60	4.59	7.26
	ES	6.78	2.20	5.19	2.40	4.98	7.58
Error (%)		-8.01	25.46	-14.70	-7.72	8.52	4.31

From the above, near-fault vertical strong earthquake motion may induce severe dynamic responses for girder bridges, and support structures play an important role in this process. So it is of great importance to consider effects of support structures in structural seismic design of girder bridges in near-fault region.

5.3 Optimization of support structures

It has been found in Section 5.2 that support structures can remarkably change vertical seismic responses of girder bridges in near-fault region. In this subsection, how to optimize support structures to resist vertical strong earthquake motions will be discussed. The example of the girder bridge shown in Section 5.2 is still used here, while the difference is that the values of k_1 and k_2 can be changed with keeping k_1=k_2 unchanged. In order to conveniently express the effects of support structures, a non-dimensional amplification factor is defined,

$$\alpha(R)=\frac{\max|R|_{\mathrm{ES}}}{\max|R|_{\mathrm{RS}}} \tag{53}$$

in which R can be any concerned response such as u_{mid} and Q_A, and the subscriptions “ES” and “RS” have the same meanings as those in Table 4.

Fig. 7 presents variations of $\alpha(u_{\mathrm{mid}})$ and $\alpha(Q_A)$ with k_1 and k_2 under six vertical strong ground excitations shown in Table 2. Besides, the mean value of these lines is also given. Clearly, the six variation lines of $\alpha(u_{\mathrm{mid}})$ or $\alpha(Q_A)$ are all fairly uneven, especially in the region of k_1, k_2=1×10^8-1×10^{10} N/m, while have a similar trend that $\alpha(u_{\mathrm{mid}})$ or $\alpha(Q_A)$ rises concussively with increasing k_1 and k_2, and arrives at a maximum value in the region of 1×10^8-1×10^{10} N/m, and finally approaches the horizontal line of the value 1. It is easy to understand from the dynamic perspective that characteristics of the variation lines of $\alpha(u_{\mathrm{mid}})$ or $\alpha(Q_A)$ depend on the spectral characteristics of the corresponding earthquake motion. It can also be found by comparison that the variation lines of $\alpha(u_{\mathrm{mid}})$ are almost the same as those of $\alpha(Q_A)$ except for the case of M2. In Fig. 7(b), there is a peak of more than 2.6 at k_1, k_2=5×10^9 N/m for the case of M2, while in Fig. 7(a) the corresponding value is less than 1.1. However, the two mean variation lines are relatively smooth. Generally it can be concluded from the mean variation lines that effects of support structures can be neglected for the considered girder bridge when k_1, k_2>7×10^{10} N/m, and benefit structures to resist earthquakes when k_1, k_2<9×10^7 N/m. Whereas in the region of 9×10^7 N/m<k_1, k_2<7×10^{10} N/m, support structures mostly increase and occasionally decrease responses of structures.

From the above, one can optimize stiffness of support structures to resist vertical strong ground motions. However, the stiffness of support structures should obviously be determined according to engineering practice. The support structures must firstly meet the requirements of the vertical bearing capacity, so their stiffness cannot be too small in practice.

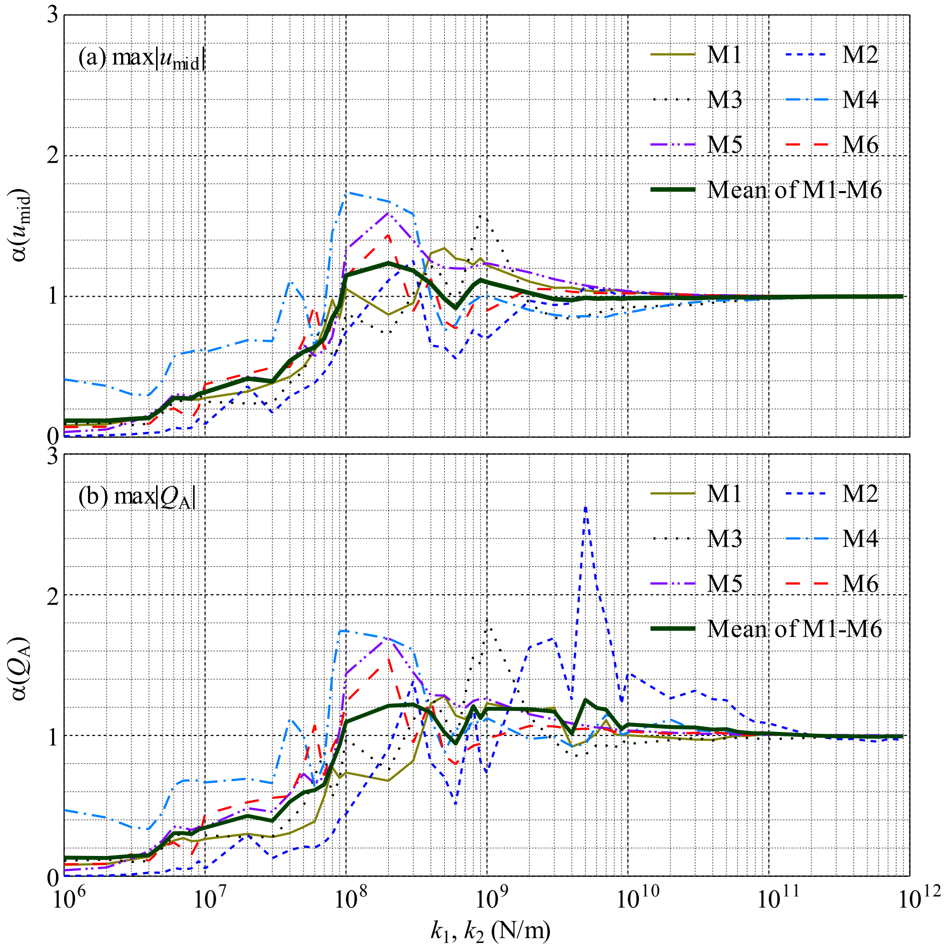


Fig. 7 Variations of max|u_{mid}| and max|Q_A| with k_1 and k_2 under different vertical excitations

6. Conclusions

Based on the assumption of small displacement, the equation of motion governing vertical seismic responses of simply-supported straight girder bridges is established considering vertical deformation of support structures which are simplified as massless springs. DQM is applied to discretize the governing equation into a MDOF system. Then, the Newmark-β method with β=1/4 is adopted to solve this MDOF system to obtain vertical seismic responses of the girder bridge. Six near-fault vertical ground motions with remarkable PGAs chosen from the PEER Strong Motion Database are considered. Numerical results indicate that support structures play an important role in vertical seismic response analysis of girder bridges. Considering effects of support structures, the absolute maximum midspan vertical deformation and end shear of the girder may increase or decrease remarkably, perhaps by more than 25%. Besides, support structures may have opposite effects on different responses of structures, for example, they may decrease the absolute maximum midspan vertical deformation and increase the absolute maximum end shear at the same time. Finally optimization of support structures to reduce vertical seismic responses of the bridge is studied through changing stiffness of support structures. It is found that support structures can be neglected when their stiffness exceeding a maximum value (7×10^{10} N/m for the considered bridge in this paper), and benefit structures to resist earthquakes when their stiffness under a minimum value (9×10^{7} N/m for the considered bridge in this paper), while may increase or decrease responses between the above two values. However, the stiffness of support structures should be determined according to engineering practice.

This paper presents a simple method to investigate the vertical seismic responses of girder bridges. It can also be applied to continuous girder bridges though only single-span ones are considered here. Whereas, with increasing length of considered girder bridges, the traveling wave effect of earthquake motion may need to be considered. Further study on vertical seismic response analysis of girder bridges will be continued.

Acknowledgments

The research described in this paper was financially supported by the National Key Basic Research Program of China (973 Program, Grant No. 2013CB036300) and the National Natural Science Foundation of China (Grant No. 51278245).

References

Bellman, R. and Casti, J. (1971), "Differential quadrature and long-term integration", *J. Math. Anal. Appl.*, **34**(2), 235-238.

Bert, C.W., Jang, S.K. and Striz, A.G. (1988), "Two new approximate methods for analyzing free vibration of structural components", *AIAA J.*, **26**(5), 612-618.

Bert, C.W. and Malik, M. (1996), "Differential quadrature method in computational mechanics: A review", *Appl. Mech. Rev.*, **49**(1), 1-28.

Bozdogan, K.B. (2012), "Differential quadrature method for free vibration analysis of coupled shear walls", *Struct. Eng. Mech.*, **41**(1), 67-81.

Button, M.R., Cronin, C.J. and Mayes, R.L. (2002), "Effect of vertical motions on seismic response of highway bridges", *J. Struct. Eng*., ASCE, **128**(12), 1551-1564.

Chopra, A.K. (1966), "The importance of the vertical component of earthquake motions", *Bull. Seismol. Soc. Am.*, **56**(5), 1163-1175.

Clough, R.W. and Penzien, J. (1995), *Dynamics of Structures* (3rd Edition), Computers & Structures, Inc., Berkeley, CA, USA.

Fardis, M.N. and Tsionis, G. (2013), "Eigenvalues and modes of distributed-mass symmetric multispan bridges with restrained ends for seismic response analysis", *Eng. Struct.*, **51**, 141-149.

Gulerce, Z., Erduran, E., Kunnath, S.K. and Abrahamson, N.A. (2012), "Seismic demand models for probabilistic risk analysis of near fault vertical ground motion effects on ordinary highway bridges", *Earthq. Eng. Struct. Dyn.*, **41**(2), 159-175.

Han, Q., Du, X., Liu, J., Li, Z., Li, L. and Zhao, J. (2009), "Seismic damage of highway bridges during the 2008 Wenchuan earthquake", *Earthq. Eng. Eng. Vib.*, **8**(2), 263-273.

Jang, S.K., Bert, C.W. and Striz, A.G. (1989), "Application of differential quadrature to static analysis of structural components", *Int. J. Numer. Meth. Eng.*, **28**(3), 561-577.

Kalkan, E. and Graizer, V. (2007), "Coupled tilt and translational ground motion response spectra", *J. Struct. Eng*., ASCE, **133**(5), 609-619.

Kaya, B. (2010), "Solution of the advection-diffusion equation using the differential quadrature method",

KSCE J. Civ. Eng., **14**(1), 69-75.
Kim, S.J., Holub, C.J. and Elnashai, A.S. (2011), "Analytical assessment of the effect of vertical earthquake motion on RC bridge piers", *J. Struct. Eng.*, ASCE, **137**(2), 252-260.
Kunnath, S.K., Erduran, E., Chai, Y.H. and Yashinsky, M. (2008), "Effect of near-fault vertical ground motions on seismic response of highway overcrossings", *J. Bridge Eng.*, **13**(3), 282-290.
Law, S.S., Chan, T.H.T. and Zeng, Q.H. (1997), "Moving force identification: A time domain method", *J. Sound Vib.*, **201**(1), 1-22.
Lee, H. and Mosalam, K.M. (2014), "Seismic evaluation of the shear behavior in reinforced concrete bridge columns including effect of vertical accelerations", *Earthq. Eng. Struct. Dyn.*, **43**(3), 317-337.
Liew, K.M., Teo, T.M. and Han, J.B. (2001), "Three-dimensional static solutions of rectangular plates by variant differential quadrature method", *Int. J. Mech. Sci.*, **43**(7), 1611-1628.
Malekzadeh, P. and Farid, M. (2007), "A DQ large deformation analysis of composite plates on nonlinear elastic foundations", *Compos. Struct.*, **79**(2), 251-260.
Newmark, N.M., Blume, J.A. and Kapur, K.K. (1973), "Seismic design spectra for nuclear power plants", *J. Power Div*., ASCE, **99**(2), 287-303.
Papazoglou, A.J. and Elnashai, A.S. (1996), "Analytical and field evidence of the damaging effect of vertical earthquake ground motion", *Earthq. Eng. Struct. Dyn.*, **25**(10), 1109-1137.
Rajasekaran, S. (2013), "Free vibration of tapered arches made of axially functionally graded materials", *Struct. Eng. Mech.*, **45**(4), 569-594.
Rao, S.S. (2011), *Mechanical Vibration*, Prentice Hall, Upper Saddle River, NJ, USA.
Shrestha, B. (2014), "Seismic response of long span cable-stayed bridge to near-fault vertical ground motions", *KSCE J. Civ. Eng.*, **19**(1), 180-187.
Shu, C. (2000), *Differential Quadrature and Its Applications in Engineering*, Springer-Verlag, London, UK.
Shu, C. and Richards, B.E. (1992), "Application of generalized differential quadrature to solve two-dimensional incompressible Navier-Stokes equations", *Int. J. Numer. Meth. Fluid.*, **15**(7), 791-798.
Takabatake, H. and Matsuoka, M. (2012), "Origin of the anomalously large upward acceleration associated with the 2008 Iwate-Miyagi Nairiku earthquake", *Earthq. Struct.*, **3**(5), 675-694.
Wang, X., Liu, F., Wang, X. and Gan, L. (2005), "New approaches in application of differential quadrature method to fourth-order differential equations", *Commun. Numer. Meth. Eng.*, **21**(2), 61-71.
Wang, Y.J., Shi, J. and Xia, Y. (2012), "Dynamic responses of an elastic beam moving over a simple beam using modal superposition method", *J. Vibroeng.*, **14**(4), 1824-1832.
Wang, Z., Padgett, J.E. and Dueñas-Osorio, L. (2013), "Influence of vertical ground motions on the seismic fragility modeling of a bridge-soil-foundation system", *Earthq. Spectra*, **29**(3), 937-962.
Warn, G.P. and Whittaker, A.S. (2008), "Vertical earthquake loads on seismic isolation systems in bridges", *J. Struct. Eng.*, ASCE, **134**(11), 1696-1704.
Weichert, D.H., Wetmiller, R.J. and Munro, P. (1986), "Vertical earthquake acceleration exceeding 2 g? The case of the missing peak", *Bull. Seismol. Soc. Am.*, **76**(5), 1473-1478.
Wu, T.Y. and Liu, G.R. (2000), "The generalized differential quadrature rule for initial-value differential equations", *J. Sound Vib.*, **233**(2), 195-213.

IT