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Plastic design of seismic resistant reinforced concrete frame

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Abstract. A new method for designing moment resisting concrete frames failing in a global mode is presented in this paper. Starting from the analysis of the typical collapse mechanisms of frames subjected to horizontal forces, the method is based on the application of the kinematic theorem of plastic collapse. The beam section properties are assumed to be known quantities, because they are designed to resist vertical loads. As a consequence, the unknowns of the design problem are the column sections. They are determined by means of design conditions expressing that the kinematically admissible multiplier of the horizontal forces corresponding to the global mechanism has to be the smallest among all kinematically admissible multipliers. In addition, the proposed design method includes the influence of second-order effects. In particular, second-order effects can play an important role in the seismic design and can be accounted for by means of the mechanism equilibrium curves of the analysed collapse mechanism. The practical application of the proposed methodology is herein presented with reference to the design of a multi-storey frame whose pattern of yielding is validated by means of push-over analysis.

Keywords: global mechanism; concrete moment resisting frames; plastic collapse theory

1. Introduction

Nowadays, the collapse mechanism control is universally recognized as one of the primary goals of the structural design process (Akiyama 1985; Bertero *et al*. 1977; Bruneau *et al*. 2011; Park 1986). The primary purpose consists in avoiding collapse mechanisms characterized by poor energy dissipation capacity, such as “soft-storey” mechanisms, assuring the development of a collapse mechanism of global type. In particular, such kind of mechanism is characterized by the location of plastic hinges at all the beam ends and at the base sections of first storey columns. Relatively to the moment resisting frames, it is obvious that the maximum number of plastic hinges is obtained when two plastic hinges develops in each bay and they are usually located at beam ends. However, for particular load conditions, plastic hinges can develop also in the mid span of the bay. In a collapse mechanism of global type the energy dissipation capacity and global ductility supply are maximized because all the dissipative zones are involved in the corresponding pattern of yielding. Conversely, all the other structural parts remains in elastic range. Therefore,

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generally speaking, dissipative zones have to be designed according to the internal actions arising from the seismic forces provided by the codes, whereas the non-dissipative zones have to be proportioned on the basis of the maximum internal actions transmitted by the dissipative zones. In a seismic resistant concrete frame, beams and columns are identified as dissipative and non-dissipative zones, respectively. These are the basic principles of capacity design approach, independently of the structural scheme and the constructional material (Lee 1996; Paulay 1977,1980). In order to avoid undesired collapse mechanisms hierarchy criterion, reported in all the modern seismic codes, suggests that at any joint, the sum of the flexural strength of the columns is greater than the sum of the flexural strength of the beams converging in the same joint (EN 1998-1 2004; NZS 3101 1982). Unfortunately, the beam-column hierarchy criterion is only able to prevent "soft-storey" mechanisms, but it does not allow the development of a collapse mechanism of global type; in fact it is a non-rigorous application of capacity design principles (Kappos 1986; Panelis *et al*. 1997). For this reason, a more sophisticated design procedure, based on the kinematic theorem of plastic collapse and on second order plastic analysis (i.e. the concept of mechanism equilibrium curve) has been presented in 1997 (Mazzolani *et al*. 1997). Starting from this first work, the "Theory of Plastic Mechanism Control" (TPMC) has been obtained as a powerful tool for the seismic design.

In particular, it consists on the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve. In fact, for any given structural typology, the design conditions to be applied in order to prevent undesired collapse mechanisms can be derived by imposing that the mechanism equilibrium curve corresponding to the global mechanism has to be located below those corresponding to all the other undesired mechanisms up to a displacement level compatible with the local ductility supply of dissipative zones. This design approach was successively extended to MRFs with RBS connections (Montuori *et al*. 2000), EB-Frames with horizontal links (i.e. split-K scheme and D-scheme) (Mastrandrea *et al*. 2003), EB-frames with inverted Y scheme (Montuori *et al*. 2014a), dissipative truss-moment frames DTMFs (Longo *et al*. 2012a, b), MRF-CBF dual systems (Giugliano *et al*. 2010) and, finally, in MRFs equipped with friction dampers (Montuori *et al*. 2014b, c).

Starting from the above background, in this paper a new application of the "Theory of Plastic Mechanism Control" is developed with reference to the reinforced concrete frames.

Furthermore, the simplicity of the proposed method will be emphasized by means of a worked example aiming to show its practical application which can be carried out even by means of hand calculations. In addition, static inelastic analyses are carried out to control the fulfilment of the desired collapse mechanism typology, i.e. a collapse mechanism of global type.

2. Theory of plastic mechanism control

TPMC allows the theoretical solution of the problem of designing a structure failing in global mode, i.e. assuring that plastic hinges develop only at beam ends while all the columns remain in elastic range with the only exception of base sections at first storey columns.

In general, three main collapse mechanism typologies that the structure is able to exhibit can be recognized: these mechanisms, depicted in Fig. 1, are to be considered undesired because they do not involve all the dissipative zones. Type-1 mechanism starts from the first storey level and

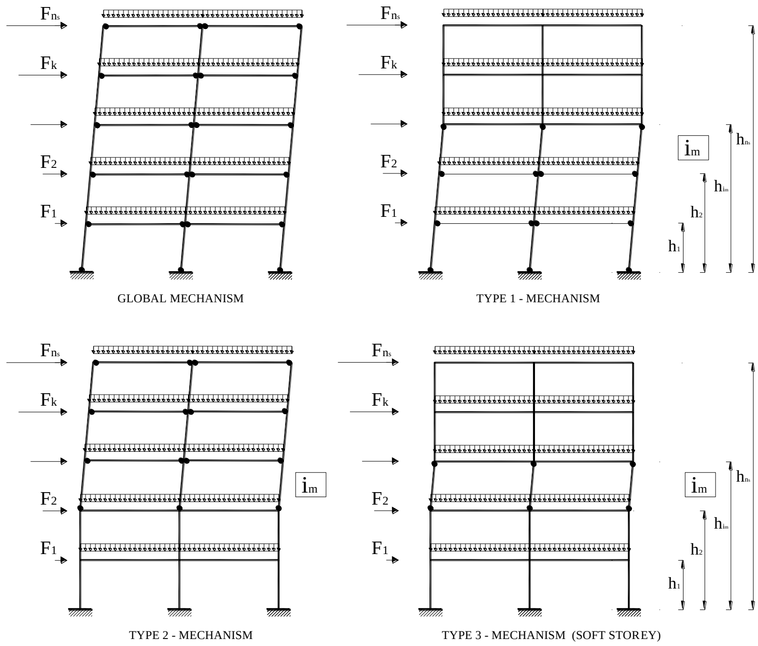

Fig. 1 Collapse mechanism typologies

Table 1 Notation

n_c	number of columns	$M_{c,im} = \sum_{i=1}^{n_c} M_{c,i,im}$	sum of plastic moments of columns at i_m-th storey
n_b	number of bays	$M_V = \sum_{k=1}^{n_s} V_k h_k$	second-order work due to vertical loads in global mechanism
n_s	number of storeys	$M_F = \sum_{k=1}^{n_s} F_k h_k$	external work due to horizontal forces in the global mechanism
i_m	index of mechanism	$M_{b.jk}$	plastic design resistance of beam at j-th bay of the k-th storey
H_o	sum of the interstorey heights of the storeys involved by the generic mechanism	$M_{b,Rd} = 2\sum_{k=1}^{n_s}\sum_{j=1}^{n_b} M_{b.jk}$	sum of the plastic design resistances of beam ends
h_k	height of the k-th storey (with k=1, 2, .., n_s)	$F = \sum_{k=1}^{n_s} F_k$	sum of the horizontal forces
$M_{c,i,im}$	plastic moment of the i-th column at i_m-th storey		

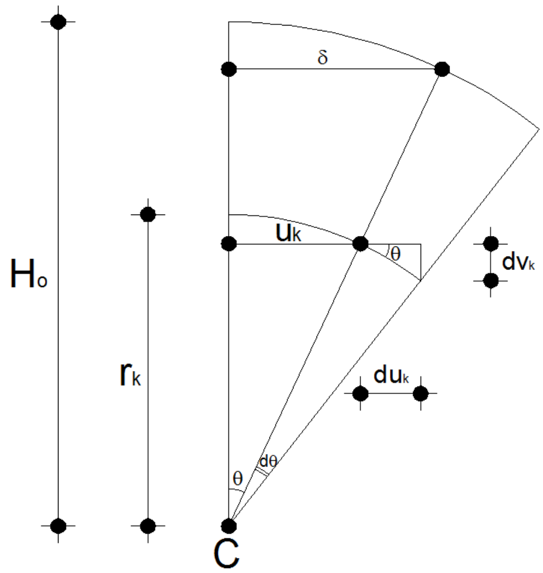

Fig. 2 Second order vertical displacements

involves an i_m -th number of storeys, for this reason plastic hinges develop at the beam ends of all the storeys involved, at the base section of the first storey columns and at the top section of the i_m -th storey columns. Type- 2 mechanism is a particular kind of mechanism which starts to the top of the structure and involves an i_m-th number of storey. In particular, all the beam ends and the base section of the i_m -th storey columns develop plastic hinges. The global mechanism, representing the design goal, is a particular case of type-2 mechanism involving all the storeys. Finally, type-3 mechanism involves only one storey, so that plastic hinges develop at the base and top section of the same storey columns. It has to be considered the worst mechanism because involves only the columns which are to be considered less dissipative than the beam sections.

In order to apply the TPMC it is of paramount importance the introduction of the concept of linearized mechanism equilibrium curve for each considered mechanism.

The mathematical expression of this curve can be written as

$$\alpha = \alpha_0 - \gamma\delta \tag{1}$$

where α_0 is the kinematically admissible multiplier of horizontal forces and γ is the slope of the mechanism equilibrium curve (Giugliano *et al*. 2010; Longo *et al*. 2012a, b; Mastrandrea *et al*. 2003; Mazzolani *et al*. 1997; Montuori *et al*. 2000; Montuori *et al*. 2014a, b, c; Montuori *et al*. 2014).

Both parameters can be derived, according to rigid-plastic theory, using the principle of virtual work. Within the framework of a kinematic approach, for any given collapse mechanism, the mechanism equilibrium curve can be easily derived by equalling the external work to the internal work. In addition, in order to account for second order effects, the external second-order work due to vertical load is also evaluated.

$$W_i = \left[\sum_{i=1}^{n_c} M_{c,i,1} + 2\sum_{k=1}^{n_s}\sum_{j=1}^{n_b} M_{b.jk}\right] d\theta = \left[M_{c,1} + M_{b,Rd}\right] d\theta \tag{2}$$

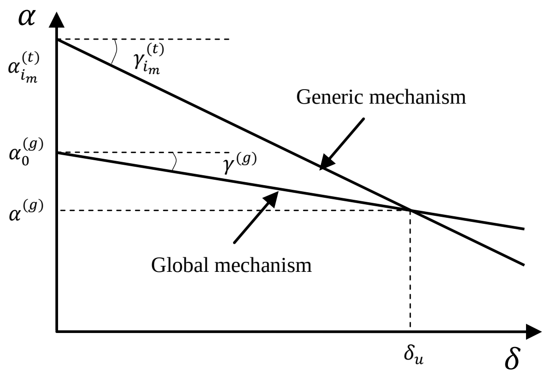

Fig. 3 Design condition

For the better comprehension of the adopted notation reference is made to Table 1. The external work due to the horizontal forces can be written as

$$W_e = \left[\alpha \sum_{k=1}^{n_s} F_k h_k\right] d\theta = [\alpha M_F] d\theta \tag{3}$$

Therefore the application of the virtual work principle provides the kinematically admissible multiplier as

$$\alpha_0^{(g)} = \frac{[M_{c,1} + M_{b,Rd}]}{M_F} \tag{4}$$

In order to compute the slope of the mechanism equilibrium curve, it is necessary to evaluate the second-order work due to vertical loads.

With reference to Fig. 2, it can be observed that the horizontal displacement of the k-th storey involved in the generic mechanism is given by $u_k = r_k sin\theta$, where r_k is the distance of the k-th storey from the centre of rotation C and θ the angle of rotation.

The top sway displacement is given by $\delta = H_o sin\theta$, where H_o is the sum of the interstorey heights of the storeys involved by the generic mechanism. In the case of global type mechanism, as shown in Fig. 1, all the storeys participate to the collapse mechanism, so that $H_o = h_{ns}$.

The relationship between vertical and horizontal virtual displacements is given by (Fig. 2)

$$dv_k = du_k tan\theta \approx du_k sin\theta = du_k \frac{\delta}{H_o} \tag{5}$$

It shows that, as the ratio dv_k / du_k is independent from the considered storey, vertical and horizontal virtual displacement vectors have the same shape. In fact, the virtual horizontal displacements are given by

$$du_k = r_k cos\theta d\theta \approx r_k d\theta \tag{6}$$

By substituting Eq. (6) in Eq. (5), the virtual vertical displacements are given by

$$dv_k = \frac{\delta}{H_o} r_k d\theta \tag{7}$$

And, therefore, they have same shape r_k of the horizontal ones. As a consequence, the second-order work due to vertical loads for the global mechanism is given by

$$W_v = \sum_{k=1}^{n_s} V_k h_k \frac{\delta}{H_o} d\theta = M_V \frac{\delta}{H_o} d\theta \tag{8}$$

By accounting for this value, the virtual work principle can be written as

$$W_i = W_e + W_v \tag{9}$$

By substituting Eqs. (2), (3) and (8) in Eq. (9) the following relation can be obtained

$$\left[M_{c,1} + M_{b,Rd}\right] d\theta = [\alpha M_F] d\theta + M_V \frac{\delta}{H_o} d\theta \tag{10}$$

By means of simple steps it is immediately recognized the form of the linearized mechanism equilibrium curve expressed by Eq. (1)

$$\alpha = \frac{M_{c,1}+M_{b,Rd}}{M_F} - \frac{\frac{1}{H_o} M_V}{M_F} \delta \tag{11}$$

Therefore, the slope of the mechanism equilibrium curve, γ, can be easily obtained. In the case of global mechanism it is given by

$$\gamma^{(g)} = \frac{\frac{1}{H_o} M_V}{M_F} = \frac{\frac{1}{h_{n_s}} M_V}{M_F} \tag{12}$$

Therefore, the linearized mechanism equilibrium curve of global mechanism $\alpha_0 = \alpha_0^{(g)} - \gamma^{(g)} \delta$ is completely defined.

It can be useful to underline that the linearization of equilibrium curve is due to the small displacement theory adopted in relation (6). In fact, due to this assumption, second-order work of to vertical loads is linear and as a consequence, also the mechanism equilibrium curve is linear.

For each considered mechanism (Fig. 1) a mechanism equilibrium curve can be obtained. In particular, for the i_m-th mechanism ($i_m = 1, 2, \ldots n_s$) of the t-th mechanism typology (t = 1,2,3) the application of kinematic theorem of plastic collapse provides

$$\alpha_{im}^{(t)} = \alpha_{0,im}^{(t)} - \gamma_{im}^{(t)} \delta \qquad t = 1,2,3 \qquad i_m = 1,2, \ldots n_s \tag{13}$$

Where $\alpha_{0,im}^{(t)}$ and $\gamma_{im}^{(t)}$ represent, respectively, the kinematically admissible multiplier and the slope of mechanism equilibrium curve of the i_m-th mechanism of the t-th mechanism typology.

In the proposed method the beam section properties are assumed to be known quantities because they are designed to resist vertical loads. As a consequence, the unknowns of the design problem are the column sections**.** They could be determined by means of design conditions expressing that the kinematically admissible multiplier corresponding to the global mechanism is the minimum among all kinematically admissible multipliers corresponding to all other mechanisms (Fig. 1).

Obviously, this design condition is able to assure the desired collapse mechanism only in case of rigid-plastic behaviour, while actual structures are characterized by elastic displacements before the development of a plastic mechanism. Due to these elastic displacements, second-order effects of vertical loads cannot be neglected. These effects can be taken into account by imposing that the mechanism equilibrium curve corresponding to the global mechanism has to lie below those corresponding to all other mechanisms i.e. the upper bound theorem of plastic design is to be satisfied for each value of the displacements δ (Fig. 3). However, the fulfilment of this requirement is necessary only up to a selected ultimate displacement δ_u, which has to be compatible with the ductility supply of structural members.

This corresponds to impose the following conditions

$$\alpha_0^{(g)} - \gamma^{(g)}\delta_u \leq \alpha_{0,im}^{(t)} - \gamma_{im}^{(t)}\delta_u \tag{14}$$

for $i_m = 1,2,3,\ldots,n_s$ and $t = 1,2,3$.

It is important to underline that, for any given geometry of the structural system, the slope of mechanism equilibrium curve attains its minimum value when the global type mechanism is developed. This issue assumes a paramount importance in TPMC allowing the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve by simply checking the equation for the value $\delta = \delta_u$, as depicted in Fig. 3. Therefore, there are $3n_s$ design conditions to be satisfied for a structural scheme having n_s storeys.

With reference to i_m-th mechanism of type-1, the kinematically admissible multiplier of seismic horizontal forces is given by

$$\alpha_{0,i_m}^{(1)} = \frac{M_{c,1} + 2\sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk} + M_{c,im}}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k} \tag{15}$$

while the slope of the mechanism equilibrium curve is

$$\gamma_{i_m}^{(1)} = \frac{1}{h_{i_m}} \frac{\sum_{k=1}^{i_m} V_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} V_k}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k} \tag{16}$$

With reference to i_m-th mechanism of type-2 the kinematically admissible multiplier of seismic horizontal forces is given by

$$\alpha_{0,i_m}^{(2)} = \frac{M_{c,im} + 2\sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk}}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m-1})} \tag{17}$$

while the slope of the mechanism equilibrium curve is

$$\gamma_{i_m}^{(2)} = \frac{1}{h_{n_s} - h_{i_m-1}} \frac{\sum_{k=i_m}^{n_s} V_k (h_k - h_{i_m-1})}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m-1})} \tag{18}$$

It is useful to note that, for i_m=1 Eq. (17) and Eq. (18) are coincident with Eq. (4) and Eq. (12) respectively, because in such case the mechanism is coincident with the global one.

In addition, these relations for $i_m = 1$ include the term $h_{i_m-1} = h_0$ which is to be assumed equal to zero.

Finally, with reference to i_m-th mechanism of type-3, the kinematically admissible multiplier of horizontal forces, is given by

$$\alpha_{0,i_m}^{(3)} = \frac{2M_{c,im}}{(h_{i_m} - h_{i_m-1})\sum_{k=i_m}^{n_s} F_k} \tag{19}$$

In addition, the corresponding slope of the mechanism equilibrium curve is given by

$$\gamma_{i_m}^{(3)} = \frac{\sum_{k=i_m}^{n_s} V_k}{(h_{i_m} - h_{i_m-1})\sum_{k=i_m}^{n_s} F_k} \tag{20}$$

3. Design algorithm

The above mentioned relations can be used to design concrete frames failing in global mode and, therefore, having a mechanism equilibrium curve given by Eq. (1)), with the kinematically admissible multiplier of horizontal forces given by Eq. (4) and the slope given by relation (12). The design algorithm to solve this problem is constituted by the following steps

a) Selection of a design top sway displacement δ_u compatible with the ductility supply of structural members. To this scope the plastic rotation capacity of beams can be assumed equal to 0.04 rad so that $\delta_u = 0.04 \cdot h_{ns}$ where h_{n_s} is the height of the structure.

b) Design of beam sections to withstand vertical loads acting in the non-seismic load combination. The preliminary design of beam can be made by considering a bending moment belonging to the range $qL^2/8 \div qL^2/10$ being q the load acting on the beam in the vertical load combination (see worked example). It is important to underline that the presented procedure, considers only symmetric structures characterized by symmetrical beam sections with symmetrical reinforcement. This limitation allows to consider for each beam just one plastic moment.

c) Computation, by means of Eqs. (16), (18) and (20), of the slopes of mechanism equilibrium curves $\gamma_{i_m}^{(t)}$ which are known quantities because they depend on loads (vertical and horizontal) and frame geometry.

d) For each considered i_m value, Eq. (14) provides the following relations where the unknown quantities are represented by $M_{c,im}$ and $M_{c,1}$, which are the sum of plastic moments of columns at i_m-th storey and at first storey, respectively. It is important to note that for $t = 2$ Eq. (14) is an identity because global mechanism is obtained by type 2 mechanism for $i_m = 1$. Furthermore, for $i_m = 1$, type 1 and type 3 mechanisms are coincident. This observation can be immediately derived from Fig. 1 and, in addition, it is easy to check that $\alpha_{0,1}^{(1)} = \alpha_{0,1}^{(3)}$ and $\gamma_1^{(1)} = \gamma_1^{(3)}$.

As a consequence, for $i_m = 1$ there is only a design condition where the only unknown is represented by $M_{c,1}$. This value can be found by substituting the values of $\alpha_0^{(g)}, \gamma^{(g)}, \alpha_{0,1}^{(1)} \left(or\ \alpha_{0,1}^{(3)}\right)$ and $\gamma_1^{(1)} \left(or\ \gamma_1^{(3)}\right)$ in Eq. (14) that gives

$$M_{c,1} \geq \frac{M_{b,Rd} + \left(\gamma_1^{(3)} - \gamma^{(g)}\right) \cdot \delta_u \cdot M_F}{2\frac{M_F}{h_1 F} - 1} \tag{21}$$

The above relation is of paramount importance from the practical point of view, because it allows to design first storey columns by means of a closed form solution easy to be applied by hand calculations.

e) The sum of the required plastic moments of columns can be distributed among the columns in different ways which are at the discretion of the designer. In this case, the following simple rule can be adopted

$$M_{c,i,1} = \frac{M_{c,1}}{n_c} \tag{22}$$

for $i = 1, \ldots, n_c$.

It is important to underline that the way of distributing the sum of required plastic moments expressed by Eq. (22) is not mandatory, in fact, any other distribution among the columns of storey 1 having as sum the value $M_{c,i,1}$ is perfectly equivalent. The choice has been made according to several analyses carried out on different structures in order to provide a cheaper solution.

f) Design of the columns at first storey. It starts by considering a section able to resist to vertical loads at ultimate limit state ($N_{V,SLU}$). The base of the section is, initially, assumed equal to b = 30 cm and the height is calculated with the following equation

$$h = \frac{N_{V,SLU}}{\nu \cdot b \cdot f_{cd}} = \frac{N_{V,SLU}}{0.5 \cdot b \cdot f_{cd}} \geq 30cm \tag{23}$$

For a given value of b and h, the reinforcements of the section are to be designed. At this aim it is very important to consider the shape of the M-N interaction domain. In fact, for a concrete frame, the maximum axial force does not necessarily implicate the worst condition, as happens in steel members, because it depends on the zone of the domain where the design point is located. If the design condition falls at the left of the M-N interaction diagram peak, there is a resistant moment increasing with the increase of axial load (Fig. 4 (a)). Conversely, at the right of the peak the dual condition is noted (Fig. 4 (b)).

The main problem affecting the M-N interaction domain configuration is that it is impossible to immediately obtain the value of the design axial force. The problem can be solved by considering two values of axial force

- a maximum value N_{max}, given by the sum of the axial forces due to vertical loads, in the seismic load combination ($N_{q,E}$) and the maximum of the axial force related to the shear actions due to the plastic hinges developed at the beam ends ($N_{M,E}$)

$$N_{max} = N_{q,E} + N_{M,E} \tag{24}$$

- a minimum value N_{min}, obtained by subtracting the same terms mentioned above

$$N_{min} = N_{q,E} - N_{M,E} \tag{25}$$

For a generic column and for a fixed direction of the earthquake, if the axial load is given by Eq. (24), then, for the opposite direction of horizontal forces, the axial load contribution is given by Eq. (25). Consequently, a design moment must be associated with those axial forces.

In particular the proposed design procedure considers, for both values of the axial forces, the value of the design moment obtained with the Eq. (22), namely M_{PR}. In conclusion the design points are

$$A\ (N_{min}, M_{PR}) \qquad\qquad B\ (N_{max}, M_{PR}) \tag{26}$$

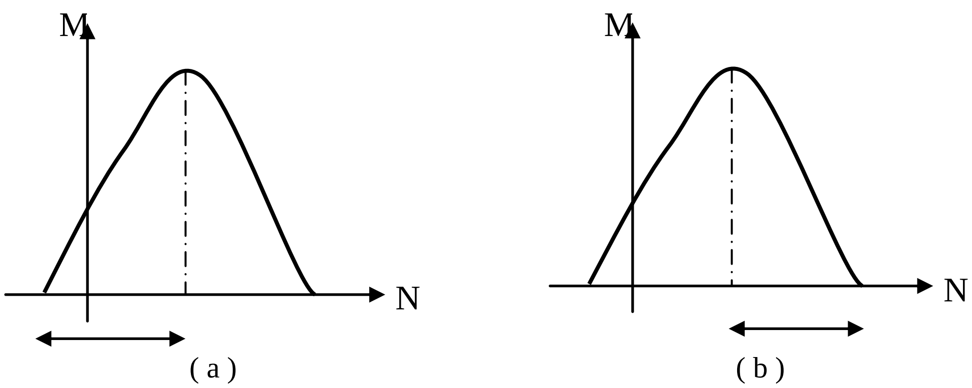

Fig. 4 M-N interaction domain

Anyway, this aspect will be better clarified in the worked example (Fig. 6).

g) Design of the reinforcement of columns at first storey. The process consists, essentially, in assuming an initial quantity of reinforcement (corresponding to the minimum value allowed by the adopted code) which is increased until the design points fall into the internal side of the M-N performance domain. If the maximum percentage of reinforcement prescribed by the regulations is reached, then the procedure is repeated by increasing the section dimensions. Once the columns are designed the obtained value of $M_{c.1}$, namely $M_{c,Rd,1}$, is generally greater than the required minimum value provided by Eq. (21). Therefore, the kinematically admissible multiplier $\alpha_0^{(g)}$ corresponding to the global mechanism is to be evaluated accordingly, i.e. by means of Eq. (4) by replacing the term $M_{c.1}$, with the value $M_{c,Rd,1}$ resulting from designed sections.

h) Computation of the required sum of plastic moments of columns $M_{c,im}$ for $i_m > 1$ imposing that the i_m-th mechanism equilibrium curves of type 1, 2 and 3 have to be located above the curve of global one, i.e. by applying relation (14). In fact, for a fixed value of i_m, relation (14) provides three values of $M_{c,im}$, namely $M_{c,im}^{(t)}$ for t = 1,2,3. In particular, in order to avoid the i_m-th mechanism of type 1, the minimum required value of $M_{c,im}$ is

$$M_{c,im}^{(1)} \geq \left(\alpha_0^{(g)} + \gamma_{i_m}^{(1)}\delta_u\right)\left(\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k\right) - M_{c,Rd,1} - 2\sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk} \quad (27)$$

In addition, in order to avoid the i_m-th mechanism of type-2, the minimum required value of $M_{c,im}$ is

$$M_{c,im}^{(2)} \geq \left(\alpha_0^{(g)} + \gamma_{i_m}^{(2)}\delta_u\right)\sum_{k=i_m}^{n_s} F_k\left(h_k - h_{i_m-1}\right) - 2\sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk} \quad (28)$$

Finally to avoid the i_m-th mechanism of type-3, the minimum required value of $M_{c,im}$ is

$$M_{c,im}^{(3)} \geq \left(\alpha_0^{(g)} + \gamma_{i_m}^{(3)}\delta_u\right)\frac{\left(h_{i_m} - h_{i_m-1}\right)}{2}\sum_{k=i_m}^{n_s} F_k \quad (29)$$

Eqs. (27), (28) and (29) have been derived from Eq. (14) for $i_m > 1$ and $t = 1$, $t = 2$ and $t = 3$, respectively.

i) Computation of the required sum of the plastic moments of columns for each storey as the maximum value among those coming from the above design conditions:

$$M_{c,im} = max\left\{M_{c,im}^{(1)}, M_{c,im}^{(2)}, M_{c,im}^{(3)}\right\} \qquad \text{for } i_m > 1 \quad (30)$$

j) The sum of the required plastic moments of columns at each storey, is distributed among all the storey columns with the same procedure as for the columns on the first storey i.e. according to the following relation

$$M_{c,i,im} = \frac{M_{c,im}}{n_c} \qquad \text{for } i = 1, \ldots, n_c. \quad (31)$$

k) Design of columns at each storey. The procedure is the same as that explained in the points f) and g).

l) If necessary, a technological condition is imposed by requiring, starting from the base, that the column sections cannot increase along the building height. If this condition requires the change of sections at first storey then the procedure needs to be repeated from point f). In fact, in this case, a new value of $M_{c,Rd,1}$ is obtained and, as a consequence, also a new value of $\alpha_0^{(g)}$ is to be evaluated. As a consequence the values $M_{c,im}^{(1)}$, $M_{c,im}^{(2)}$, $M_{c,im}^{(3)}$ obtained by relations (27), (28), (29) also change. On the contrary, if the condition only requires the change of sections at the upper storeys, i.e. without the involvement of first storey columns, then the design step k) is to be repeated in order to consider the new section dimensions.

The check of technological condition could look redundant because it is common for both axial force and shear demand to increase gradually in lower stories but ,when the proposed procedure is applied, as reported in the worked example, the required sum of plastic moments of j-th storey can be bigger than the required sum of plastic moments at (j – 1)-th storey.

4. Worked example

In order to show the practical application of the proposed design procedure, the seismic design of a four-bay five-storey moment resisting frame is presented in this section. The inelastic behaviour of the designed structure is successively examined by means of a push-over static inelastic analysis, confirming the fulfilment of the design goal, i.e. the location of the yielding zones at the beam ends with the only exception of the base section of first-storey columns.

The structural scheme of the frame to be designed is shown in Fig. 5. The structural scheme is symmetric: the external bay span is equal to 7 m and the internal bay span is equal to 4 m.

The interstorey height is equal to 3 m. The characteristic values of the vertical loads acting on the beams are equal to 19.5 kN/m and 12 kN/m for permanent (G_k) and live (Q_k) actions, respectively. The structural materials adopted are concrete C25/30 and reinforcement of steel grade B450C. According to Eurocode 8, the value of the period of vibration to be used for preliminary design is

$$T = 0.075\, H^{3/4} = 0.075 \cdot 15^{3/4} \approx 0.57\, s \tag{32}$$

where H is the total height of the frame. With reference to the design spectrum for stiff soil conditions (soil class A of Eurocode 8) and by assuming a behaviour factor q equal to 3.9, the horizontal seismic forces are those depicted in Fig. 5. In the following, the numerical development of the design steps for the structural scheme described above is provided.

a) Selection of the design top sway displacement

The selection of the maximum top sway displacement up to which the global mechanism has to be assured is a very important design issue, because the value of this displacement governs the magnitude of second order effects accounted for in the design procedure. A good criterion to choose the design ultimate displacement δ_u is to relate it to the plastic rotation supply of beams or beam-to-column connections by assuming $\delta_u = \theta_u \cdot h_{ns}$ (where θ_u can be assumed equal to 0.04 rad). As a consequence, the design value of the top sway displacement has been assumed equal to

$$\delta_u = 0.04 \cdot h_{ns} = 0.04 \cdot 15 = 0.60\, m \tag{33}$$

b) Design of beam sections to withstand vertical loads.

The load acting on the frame in the vertical load combination is

$$Q_{SLU} = 1.3\, G_k + 1.5\, Q_k = 43.35\, kN/m \tag{34}$$

For the design of the beams has been considered a bending moment equal to

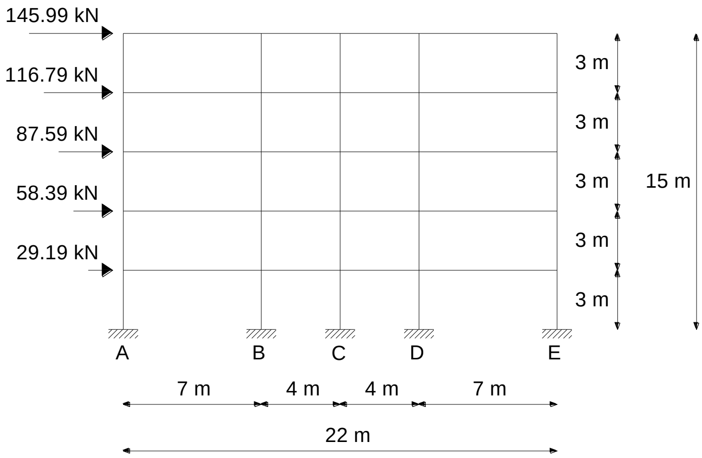

Fig. 5 Structural scheme of the designed frame

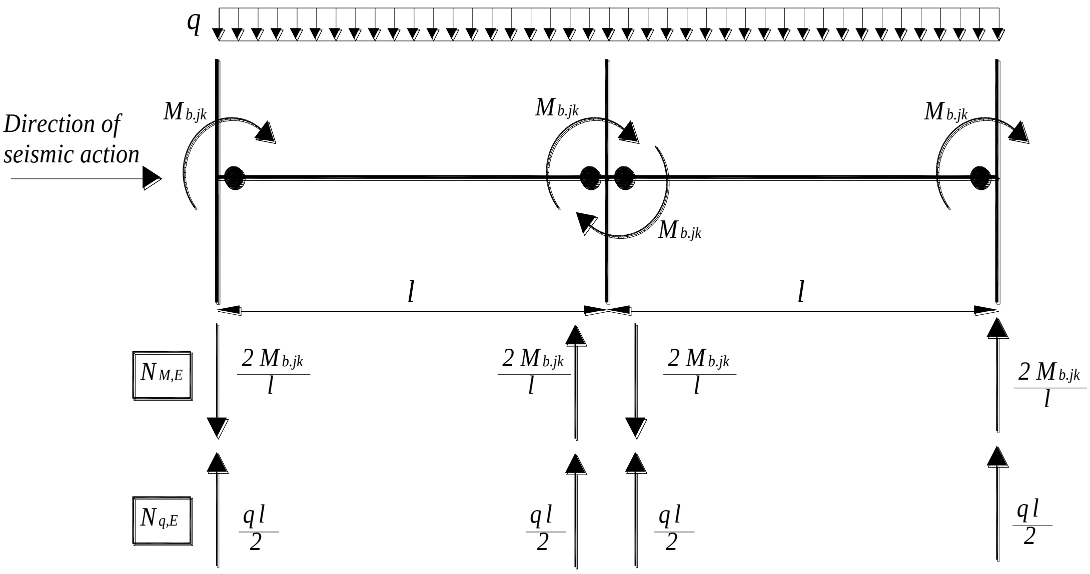

Fig. 6 Loads transmitted by the beams to the columns at collapse state

Table 2Axial forces acting at collapse state in the columns

STOREY	Columns A and E		Columns B and D		Column C	
i_m	$N_{q,E}$ [kN]	$N_{M,E}$ [kN]	$N_{q,E}$ [kN]	$N_{M,E}$ [kN]	$N_{q,E}$ [kN]	$N_{M,E}$ [kN]
1	404.25	476.63	635.25	54.2	462.00	0
2	323.40	381.3	508.20	43.36	369.60	0
3	242.55	285.98	381.15	32.52	277.20	0
4	161.70	190.65	254.10	21.68	184.80	0
5	80.85	95.33	127.05	10.84	92.40	0

Table 3 Slopes of mechanism equilibrium curves (cm^{-1})

STOREY i_m	$\gamma_{i_m}^{(1)}$	$\gamma_{i_m}^{(2)}$	$\gamma_{i_m}^{(3)}$
1	0.0193	0.0032	0.0193
2	0.0090	0.0036	0.0166
3	0.0057	0.0045	0.0145
4	0.0041	0.0062	0.0129
5	0.0032	0.0116	0.0116

Table 4 Design of the column sections at first storey

STOREY	COLUMN	$M_{c,i,1}$ [kNm]	$b\ x\ h$	$A_s = A_s'$	N_{min} [kN]	N_{max} [kN]
1°	A	465.08	30x60	6 Φ 24	- 72.38	880.88
	B		30x60	5 Φ 20	581.05	689.45
	C		30x50	7 Φ 20	462.00	462.00
	D		30x60	5 Φ 20	581.05	689.45
	E		30x60	6 Φ 24	- 72.38	880.88

Table 5 Sum of plastic moments of column required at each storey to avoid undesired mechanism

STOREY i_m	$M_{c,im}^{(1)}$ [kNm]	$M_{c,im}^{(2)}$ [kNm]	$M_{c,im}^{(3)}$ [kNm]
1	<u>2720.48</u>	-	2720.48
2	<u>2893.59</u>	1344.47	2119.03
3	<u>3317.87</u>	184.07	1750.97
4	<u>3095.36</u>	-545.12	1275.11
5	<u>2010.44</u>	-627.53	691.45

Table 6 Design of column sections at each storey

STOREY	COLUMN	$M_{c,im}$ [kNm]	$b\ x\ h$	$A_s = A_s'$	N_{min} [kN]	N_{max} [kN]
2°	A	578.71	30x60	5 Φ 28	- 57.90	704.70
	B		30x60	5 Φ 24	581.05	551.56
	C		30x60	6 Φ 24	369.60	369.60
	D		30x60	5 Φ 24	581.05	551.56
	E		30x60	5 Φ 28	- 57.90	704.70
3°	A	663.57	30x60	4 Φ 32	- 43.43	528.53
	B		30x60	6 Φ 24	348.63	413.67
	C		30x60	5 Φ 28	277.20	277.20
	D		30x60	6 Φ 24	348.63	413.67
	E		30x60	4 Φ 32	- 43.43	528.53
4°	A	619.07	30x60	5 Φ 28	- 28.95	352.35
	B		30x60	6 Φ 24	232.42	275.78
	C		30x60	6 Φ 24	184.80	184.80
	D		30x60	6 Φ 24	232.42	275.78
	E		30x60	5 Φ 28	- 28.95	352.35
5°	A	402.08	30x50	6 Φ 24	- 14.48	176.18
	B		30x50	7 Φ 20	116.21	137.89
	C		30x50	5 Φ 24	92.40	92.40
	D		30x50	7 Φ 20	116.21	137.89
	E		30x50	6 Φ 24	- 14.48	176.18

Table 7 Sum of plastic moments of column required at each storey to avoid undesired mechanism

STOREY i_m	$M_{c,im}^{(1)}$ [kNm]	$M_{c,im}^{(2)}$ [kNm]	$M_{c,im}^{(3)}$ [kNm]
1	<u>2763.19</u>	-	2763.19
2	<u>2873.40</u>	1375.54	2124.47
3	<u>3307.00</u>	204.26	1755.63
4	<u>3091.47</u>	-534.25	1278.61
5	<u>2010.44</u>	-623.64	693.40

Table 8 Design of column sections at each storey for earthquake from left to right

STOREY	COLUMN	$M_{c,i,im}$ [kNm]	$b\ x\ h$	$A_s = A'_s$	$N_{LR,i,im}$ [kN]	$M_{c,Rd,i,im}$ [kNm]
1°	A	465.08	30x60	6 Φ 24	-72.38	555.67
	B		30x60	5 Φ 20	689.45	482.83
	C		30x60	6 Φ 20	462.00	511.71
	D		30x60	5 Φ 20	581.05	466.49
	E		30x60	6 Φ 24	880.88	746.46
2°	A	574.68	30x60	5 Φ 28	-57.89	636.52
	B		30x60	7 Φ 20	551.56	594.28
	C		30x60	6 Φ 24	369.60	668.32
	D		30x60	7 Φ 20	464.84	578.64
	E		30x60	5 Φ 28	704.70	803.48
3°	A	661.40	30x60	4 Φ 32	-43.42	669.60
	B		30x60	6 Φ 24	413.67	677.45
	C		30x60	5 Φ 28	277.20	724.63
	D		30x60	6 Φ 24	348.63	663.75
	E		30x60	4 Φ 32	528.53	805.43
4°	A	618.29	30x60	5 Φ 28	-28.95	644.29
	B		30x60	6 Φ 24	275.78	647.30
	C		30x60	6 Φ 24	184.80	624.51
	D		30x60	6 Φ 24	232.42	636.88
	E		30x60	5 Φ 28	352.35	741.57
5°	A	402.08	30x50	6 Φ 24	-14.47	465.73
	B		30x50	7 Φ 20	137.89	410.23
	C		30x50	5 Φ 24	92.40	411.17
	D		30x50	7 Φ 20	116.21	405.53
	E		30x50	6 Φ 24	176.18	507.19

$$M_{max} = \frac{Q_{SLU} \cdot L^2}{8} \tag{35}$$

Therefore, by imposing the base of the section equal to b=30 cm, is possible to calculate the height of the beam through the following design relation

$$d = r\sqrt{\frac{M_{Sd}}{b}} \tag{36}$$

Assuming $\xi = 0.25$ and $\rho = 0.25$ a value of $r = 0.19$ is obtained. As a consequence the amount of reinforcement is given by

$$A_s = A'_s = \frac{M_{Sd}}{0.85 \cdot h \cdot f_{sd}} \tag{37}$$

Obviously the number of steel bars in the beam is such that

$$M_{Rd} > M_{Sd} \tag{38}$$

c) Computation of the axial load acting at collapse state in the columns.

According to the global mechanism, axial forces in the columns at collapse state depend both from the distributed loads acting on the beams and from the shear action due to the development of plastic hinges at the beam ends, as depicted in Fig. 6 (with reference to the earthquake from left to right).

So that, the total load transmitted by the beams to the columns is the sum of two contributions. The first one, $N_{q,E}$, is related to the vertical loads acting in the seismic load combination (i.e. the sum of $ql/2$ type contributions). The second one, $N_{M,E}$, is related to the shear actions due to the plastic hinges developed at the beam ends (i.e. the sum of $2M_{b,jk}/l$ type contributions).

In Table 2, the two contributions $N_{q,E}$, $N_{M,E}$ are reported for each storey both for internal columns and for external columns.

Table 9 Design of column sections at each storey for earthquake from right to left

STOREY	COLUMN	$M_{c,i,im}$ [kNm]	$b\ x\ h$	$A_s = A'_s$	$N_{RL,i,im}$ [kN]	$M_{c,Rd,i,im}$ [kNm]
1°	A	465.08	30x60	6 Φ 24	880.88	746.46
	B		30x60	5 Φ 20	581.05	466.49
	C		30x60	6 Φ 20	462.00	511.71
	D		30x60	5 Φ 20	689.45	482.83
	E		30x60	6 Φ 24	-72.38	555.67
2°	A	574.68	30x60	5 Φ 28	707.70	803.48
	B		30x60	7 Φ 20	464.84	578.64
	C		30x60	6 Φ 24	369.60	668.32
	D		30x60	7 Φ 20	551.56	594.28
	E		30x60	5 Φ 28	-57.89	636.52
3°	A	661.40	30x60	4 Φ 32	528.53	805.43
	B		30x60	6 Φ 24	348.63	663.75
	C		30x60	5 Φ 28	277.20	724.63
	D		30x60	6 Φ 24	413.67	677.45
	E		30x60	4 Φ 32	-43.42	669.60
4°	A	618.29	30x60	5 Φ 28	352.35	741.57
	B		30x60	6 Φ 24	232.42	636.88
	C		30x60	6 Φ 24	184.80	624.51
	D		30x60	6 Φ 24	275.78	647.30
	E		30x60	5 Φ 28	-28.95	644.29
5°	A	402.08	30x50	6 Φ 24	176.18	507.19
	B		30x50	7 Φ 20	116.21	405.53
	C		30x50	5 Φ 24	92.40	411.17
	D		30x50	7 Φ 20	137.89	410.23
	E		30x50	6 Φ 24	-14.47	465.73

d) Computation of the slopes of mechanism equilibrium curve $\gamma_{i_m}^{(t)}$.

By means of Eqs. (16), (18) and (20) the slopes of mechanism equilibrium curves are computed. These values are reported in Table 3

In particular it is important to underline that the slope value corresponding to the global mechanism $\gamma^{(g)} = \gamma_1^{(2)}$, is the minimum among all the $\gamma_{i_m}^{(t)}$ values

$$\gamma^{(g)} = 0.0032\ cm^{-1} \tag{39}$$

e) Computation of the required sum of plastic moments of columns at first storey $M_{c,1}$.

As previously pointed out, the required sum of plastic moments of columns at first storey is provided by Eq. (21).

In the examined case, this sum is equal to $M_{c,1} = 2325.424\ kNm$ and has to be distributed among the columns proportionally to their number. Therefore the required bending moment for each column $M_{c,i,1}$, the section, the upper and lower reinforcement, the axial force for both directions of the earthquake are reported in Table 4. The sum of obtained column plastic moments at first storey is: $M_{c,Rd,1} = 2720.482\ kNm$ which is greater than the required one.

f) Computation of seismic horizontal forces corresponding to the ultimate design displacement.

The value of $\alpha_0^{(g)}$ obtained from Eq. (4) is equal to $\alpha_0^{(g)} = 2.6599$

g) Computation of the required sum of plastic moments of columns $M_{c,im}^{(t)}$ *at any storey, to avoid undesired mechanism by means of equations (27) (28) and (29).*

h) Computation of the maximum value of $M_{c,im}$.

The sum of the plastic moments of columns governing the column design at each storey is given in Table 5 by the underlined values. It can be recognized that, in the examined case, the need to avoid type-1 mechanism always governs the design of columns

i) Design of column sections at each storey.

The required sum of column plastic moments $M_{c,i,im}$, the section, the upper and lower reinforcement, the axial force for both directions of the earthquake are reported in Table 6.

j) Checking of technological condition

By observing Table 4 and Table 6 it can be noted that there is a column section at the first storey which is smaller than the corresponding one required at the second storey, therefore, this condition generates a technological condition at the first storey. As a consequence, the value of $M_{c,Rd,1}$ needs to be updated and the procedure needs to be repeated from the step e). In Table 7 the new value of required sum of plastic moments of columns $M_{c,im}^{(t)}$ at any storey are reported

With reference to Fig. 6, that is, to the earthquake from left to right, the axial force corresponding $N_{LR,i,im}$ and the obtained bending resistance $M_{c,Rd,i,im}$ are reported in Table 8. With reference to the earthquake from right to left, the axial force corresponding $N_{RL,i,im}$ and the obtained bending resistance $M_{c,Rd,i,im}$ are reported in Table 9

5. Validation of the design procedure

In order to validate the design procedure, a static non-linear analysis (push-over) has been carried out to investigate the actual seismic response of the designed frame by means SAP2000 computer program (CSI 2007). This analysis has the primary aim to confirm the development of the desired collapse mechanism typology and to evaluate the obtained energy dissipation capacity, testing the accuracy of the proposed design methodology.

Regarding the structural modelling, the mechanical non-linearities, have been concentrated at beam and column ends by means of plastic hinge elements. The constitutive law of such plastic hinge elements is provided by a rigid plastic moment-rotation curve. The type of hinge depends on the element considered i.e. by its internal action. In fact, for the beams and the columns M3 and P-M3 hinge type have been considered, respectively. In case of P-M3 hinge type, the interaction domain P-M has been evaluated for each column and used in SAP2000 computer program.

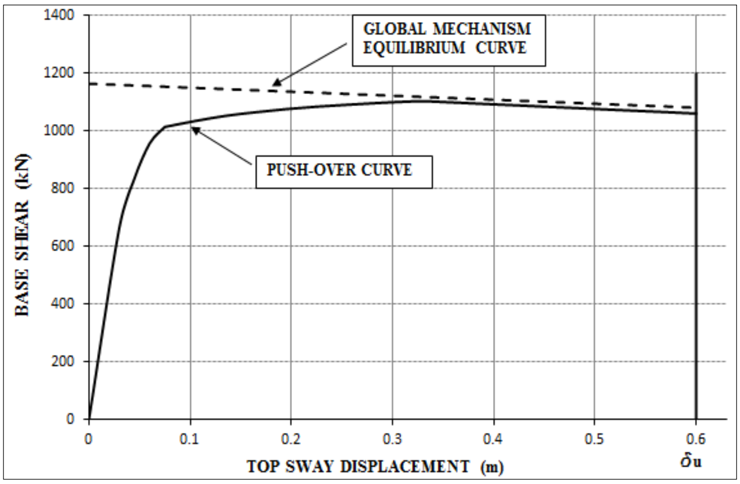

Fig. 7 Push-over curve with the global mechanism equilibrium curve

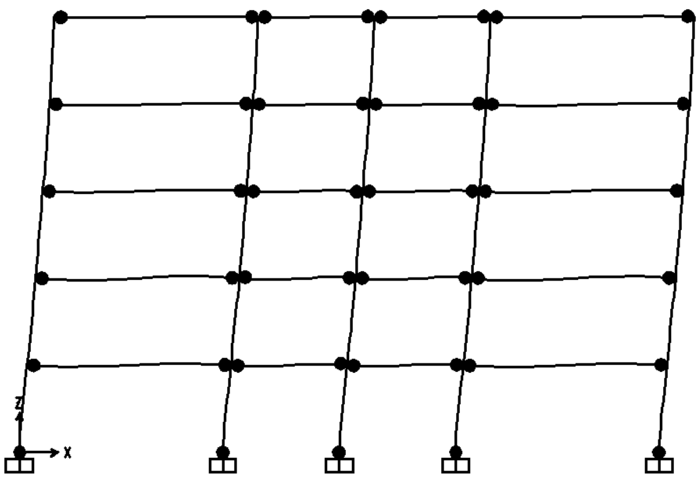

Fig. 8 Pattern of yielding of the designed frame at $\boldsymbol{\delta = \delta_u}$

Table 10 Limitation of interstorey drift

STOREY	d_s [mm]	d_r [mm]	υ	$d_r\ \upsilon$	0.005 h
1°	7.3204	0.9663		0.4831	1.5
2°	6.3540	1.4082		0.7041	1.5
3°	4.9457	1.8060	0.5	0.9030	1.5
4°	3.1397	1.9290		0.9645	1.5
5°	1.2107	1.2107		0.6053	1.5

The results of the push-over analysis are mainly constituted by base shear – top sway displacement curve which is depicted in Fig. 7. In the same figure also a straight line is given, i.e. the one corresponding to the linearized mechanism equilibrium curve of global mechanism whose expression, for the designed frame, is

$$\alpha = 2.6599 - 0.0032\ \delta \tag{40}$$

Obviously, the base shear depicted in Fig. 7 is, in this case, obtained by multiplying the value of α, given by Eq. (40), for the design base shear corresponding to $\alpha = 1$.

The comparison between the capacity curve and the above straight line provides a first confirmation of the accuracy of the proposed design procedure.

A further confirmation, even the most important, of the fulfilment of the design objective is represented by the pattern of yielding developed at the occurrence of the design ultimate displacement. In fact, developed plastic hinges are shown in Fig. 8 and their pattern is in perfect agreement with the global mechanism.

Finally, in order to fulfill the serviceability requirements the interstorey drift have been checked with reference to the limit reported in the Eurocode 8. In particular the considered limit refers to buildings having non structural elements of brittle materials attached to the structure

$$d_r\ \nu \leq 0.005\ \mathrm{h} \tag{41}$$

If this serviceability requirement is not verified the structural stiffness can be improved by increasing the beam sections or the ultimate design displacement. In fact, in both cases the final results will be a more rigid structure with respect to the one obtained in the worked example herein presented. In Table 10 the final results are reported.

6. Conclusions

In this paper a methodology called "Theory of Plastic Mechanism Control" for the design of reinforced concrete moment resisting frames has been presented. On the base of the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve, the Theory of Plastic Mechanism Control allows to evaluate the sum of plastic moments of the columns required at each storey in order to develope a collapse mechanism of global type. The closed form solution of the design conditions makes the design procedure very easy to be applied even by means of hand calculations and, therefore, it could also be suggested for code purpose by definitely solving the problem of collapse mechanism control whose importance in seismic design is universally recognised. Beam-column hierarchy criterion, commonly suggested by seismic codes, appears only as a very rough approximation when compared to TPMC and its theoretical background. The reliability of the proposed design procedure has been also demonstrated through its application to a four-bays, five-storeys frame, leading to the fulfilment of the design objective, i.e. the development of a collapse mechanism of global type, as it has been confirmed by the results of the push-over static inelastic analysis. The proposed methodology can be considered as belonging to the Performance Based Seismic Design philosophy (SEAOC. 1995. Vision 2000). In fact, in order to satisfy the limit states of “Life Safe” or “Near Collapse” the designer has to promote a dissipative collapse mechanism avoiding the so called “soft storey mechanism”. In addition, it is useful to underline that the proposed procedure constitutes a rigorous application of the capacity design principles. In fact, beams are designed in order to bear external loads, while columns are designed according to the maximum internal actions transmitted by the dissipative zones.

As already stated in point b) of the design algorithm, the limit of the procedure herein presented is constituted by the simmetry considered both for the structural scheme and for the beam sections. In fact, only symmetric structures characterized by symmetrical beam sections with symmetrical reinforcement have been considered. This represents the main limit of the procedure and the its overcoming is the main objective of the future developments of the work.

Even though, more design examples need to be developed and structural behaviour should be further checked by means of incremental dynamic analyses, the results obtained in this work with reference to reinforced concrete are decisively encouraging and in perfect agreement with the ones obtained in the case of steel moment resisting frames.

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