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Optimal placement and design of nonlinear dampers for building structures in the frequency domain

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Abstract. In this paper, a systematic technique is proposed for the optimal placement and design of nonlinear dampers for building structures. The concept of Output Frequency Response Function (OFRF) is applied to analytically represent the output frequency response of a building frame where nonlinear viscous dampers are fitted for suppression of vibration during earthquakes. An effective algorithm is derived using the analytical representation to optimally determine the locations and parameters of the nonlinear dampers. Various numerical examples are provided to verify the effectiveness of the optimal designs. A comparison of the vibration suppression performance with that of the frame structure under a random or uniform damping allocation is also made to demonstrate the advantages of the new designs over traditional solutions.

Keywords: optimal damper placement; nonlinear damper; frame structure; earthquake loading; vibration control

1. Introduction

Passive dampers have been widely applied for the vibration control of building structures under earthquake ground motions. These dampers include, for example, hysteretic steel dampers, viscous wall-type dampers, viscous oil dampers, viscoelastic dampers, and friction dampers etc. The optimal or effective placement and design of these dampers to achieve a desired building vibration control performance have been comprehensively studied (Takewaki 1997, 2009, Singh and Moreschi 2001, Trombetti and Silvestri 2004, Lavan and Levy 2006, Aydin *et al*. 2007, Cimellaro 2007, Fujita *et al*. 2010, Hwang *et al*. 2013). However, except a few works (Uetani *et al*. 2003, Martinez-Rodrigo and Romero 2003, Goel 2004, Lavan and Levy 2005, Attard 2007, Silvestri *et al*. 2010, Palermo *et al*. 2013, Lang *et al*. 2013), most methods assume that both the dampers and building structures behave linearly so that well-established linear system methods can be applied

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to conduct the structural analysis and damper designs.

Although linear dampers have been widely used in engineering practice, many beneficial effects and advantages of nonlinear damping have recently been revealed by researchers (Lang *et al*. 2009, Laalej *et al*. 2012). In order to exploit these beneficial effects and advantages, effective methods are needed for the optimal placement and design of nonlinear dampers for building structures. Compared with the optimal damper placement for linear systems where efficient response evaluation methods without time-history response analysis can be used, an elaborate time-history response analysis is generally required for building structures with nonlinear dampers to evaluate the values of an objective function in the optimal design procedure. In order to apply an optimal nonlinear damper placement and design in practice, a more efficient optimization algorithm is needed to resolve the problems with current approaches, in which procedures are often complicated and no explicit relationship between the design objective and design parameters can be used to facilitate the optimization process.

In addition, it has also been realised that some nonlinear characteristics of conventional dampers in building applications have to be taken into account in the damper designs. A typical example is the need to design the relief force of conventional oil dampers for building structures (Adachi *et al*. 2013a, Murakami *et al*. 2013). During earthquakes, oil dampers fitted in building structures often induce large internal forces into building frames due to intensive ground motions. To avoid possible damage that could be caused by such large forces, a so-called relief force mechanism is often introduced in oil dampers. Under this mechanism, when the internal force in oil dampers arrives at the relief force, the damping coefficient becomes small compared to the initial one so that the maximum force produced by oil dampers can be kept in a reasonable range. Therefore, there is also a real need to design the relief force of oil dampers such that, subject to practical constraints on the relief force and the associated maximum damping force, an optimal vibration control performance for building structures can be achieved.

Motivated by the needs of exploiting advantages of nonlinear dampers over conventional linear dampers, a technique for the optimal placement and design of additional nonlinear viscous dampers for vibration control of simple Multi-Degree-Of-Freedom (MDOF) systems was proposed in Lang *et al*. (2013). The effectiveness of the technique was demonstrated by the optimal design and placement of power-law type nonlinear dampers in a simplified six-story shear building model under the harmonic and earthquake loading excitations, respectively. The design issue for the relief force of oil dampers was raised in Adachi *et al*. (2013a) and further investigated in Adachi *et al*. (2013b). In Adachi *et al*. (2013a), a nonlinear time-history based sensitivity analysis method was proposed for the optimal design of oil damper relief forces to minimize the maximum interstory drift or acceleration of top-story under design earthquakes and some design constraints. In Adachi *et al*. (2013b), the effects of interstory velocity on optimal along-height allocation of oil dampers in super high-rise buildings were studied.

The present study is based on these previous works and concerned with the development of a more general frequency domain analysis based nonlinear damper design approach in order to uniformly deal with a range of design problems, including both the optimal placement and design of power law-type nonlinear dampers and the optimal design and allocation of relief forces of oil dampers in multi-story building frames. Compared to the results in Lang *et al*. (2013), in this study, a more sophisticated design algorithm is proposed that can be applied to much more complicated frame models and achieve a verified optimality. The earthquake loadings will be considered and the Vibration Power Loss Factor (VPLF) (Guo and Lang 2011) of the building top story displacement vibration will be used as the design objective function. The optimal design will be conducted subject to an equality constraint on the sum of design parameters which can, for example, be either nonlinear damping coefficients or oil damper relief forces. The detailed design procedure will be provided and numerical studies on a multi-story building frame model will be conducted to demonstrate the performance of the optimal design approach.

The present study produces a more systematic approach for the optimal design and placement of nonlinear dampers for building structures whose nonlinear characteristic can be specified by a single parameter. The method has potential to help structural engineers to derive an optimal distribution of nonlinear dampers or oil damper relief forces when commissioning damper retro-fitting projects for building structures.

2. Novelty of the study

The OFRF is a concept based on which, the output frequency response of a wide class of nonlinear systems can be represented analytically by a polynomial function of the parameters defining the system nonlinearity. In Lang *et al*. (2013), an algorithm was derived using the OFRF concept for optimal placement and design of nonlinear dampers in simple multi-degree-of-freedom systems. However, because of complexities with the more realistic multi-story frame model of building structures, the algorithm in Lang *et al* (2013) cannot be directly applied to the nonlinear damper designs for multi-story frame structures. In order to resolve this problem, in this study, new procedures are proposed to determine the step size that can be used to search for an optimal solution and, by introducing these new procedures, a new optimal nonlinear damper design algorithm for multi-story frame structures is derived to fulfil the need of the more complicated design. In addition to this novel contribution, a more general design problem is considered to produce a generic solution which can be used to uniformly deal with various design issues including, e.g., the design of power law-type nonlinear dampers and the optimal design of oil damper relief forces. Finally, the new method is applied to the optimal placement and design of nonlinear dampers in a multi-story building frame model and its performance is verified by using various criteria widely used in earthquake engineering.

3. Characteristic of nonlinear dampers

3.1 A general description of damping characteristic of nonlinear dampers

In the present study, the design of nonlinear dampers for building frame structures is considered and it is assumed that the damping characteristic of the nonlinear dampers can be represented by a general description as follows

$$f = \alpha_1 \dot{u} + \left(\alpha_2(p)\dot{u}^2 + \alpha_3(p)\dot{u}^3 +, \cdots, + \alpha_{L(p)}(p)\dot{u}^{L(p)} \right) \tag{1}$$

In Eq. (1), f represents the damping force of the nonlinear damper and $\dot{u}$ denotes the relative velocity between the two ends of the damper. α_1 and p are damper characteristic parameters, $\alpha_i(p)$ represents a continuous function of parameter *p*, and *L*(*p*) indicates that *L* is dependent on *p*.

This description basically assumes that the damping characteristic of nonlinear dampers can be approximated by a polynomial function of $\dot{u}$ with $L(p)$ and $\alpha_1, \alpha_2(p), \cdots \alpha_L(p)$ being the order and coefficients of the polynomial, respectively. In addition, it is also assumed that the dampers' nonlinear characteristic can be well determined by a single parameter p. Examples of the parameter p can be found in the next section.

3.2 Damping characteristics of power-law nonlinear dampers and oil dampers with relief mechanism

The description for the damping characteristic of nonlinear dampers given by Eq. (1) can cover a range of characteristics of commercially available nonlinear dampers. The well-known power-law nonlinear damping characteristic

$$f = c|\dot{u}|^{\alpha} \operatorname{sgn}(\dot{u}) \tag{2}$$

as given in Soong and Constantinou (1994) and Hwang (2002) can be considered as a special case of Eq. (1). In the case of α >1, this is obvious . In the case of α <1, the gradient of Eq. (2) at $\dot{u}=0$ tends to infinite, Eq. (2) can be regarded as a special case of Eq. (1) with $\alpha_1 = \infty$.

The characteristic of oil dampers with relief force mechanism as shown in Fig. 1 can be described analytically as

$$f = \begin{cases} C_1\dot{u} & \text{when} \quad |\dot{u}| < f_R / C_1 \\ \left[f_R + C_2\left(|\dot{u}| - f_R/C_1\right)\right]\operatorname{sgn}(\dot{u}) & \text{when} \quad f_R / C_1 \le |\dot{u}| \le f_R / C_1 + \left(f_{R\max} - f_R\right)/C_2 \\ f_{R\max}\operatorname{sgn}(\dot{u}) & \text{otherwise} \end{cases} \tag{3}$$

where $C_1, C_2, f_R, f_{R\max}$ are the parameters and f_R is referred to as the relief force.

In practice, C_1 is normally fixed and there is a definite relationship between C_1 and C_2 and between f_R and $f_{R\max}$. For example, $C_2 / C_1 = 0.05$ and $f_{R\max}$ =1.1 f_R.

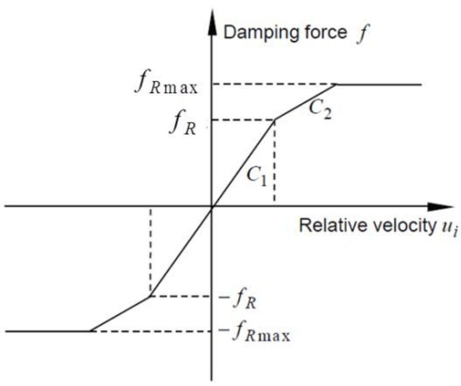


Fig. 1 Damping characteristic of oil damper with relief force mechanism

When $f_R > 0$, due to continuous property, Eq. (3) can be represented as

$$f = C_1 \dot{u} + \left(\alpha_3(f_R) \dot{u}^3 + \cdots + \alpha_{L(f_R)}(f_R) \dot{u}^{L(f_R)} \right) \tag{4}$$

where $L(f_R)$ is an odd integer. Obviously, Eq. (4) is a special case of Eq. (1) with

$$\alpha_1 = C_1 \ , \ p = f_R, \ \ \alpha_i(p) = \begin{cases} \alpha_i(f_R), & i = 3,5,\ldots,L(f_R) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

These analyses imply that Eq. (1) can be used to represent a class of characteristics of nonlinear dampers and can therefore be used to study the optimal placement and design of a class of nonlinear dampers for the vibration control of building structures during earthquakes.

4. Analytical representation of top story frequency response of building structures in term of nonlinear damping characteristic parameter

Consider an n-story multi-span plane-frame structure with $2n$ additionally fitted viscous dampers as shown in Fig. 2. The equations of motion for this model can be described by

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = -\mathbf{M}r\ddot{u}_g(t) + \mathbf{F}(t) \tag{6}$$

where **M**, **C**, **K**, **X**(t) and r denote the system mass, damping and stiffness matrices, nodal displacement vector, and influence coefficient vector of the frame, respectively. $\ddot{u}_g(t)$ denotes the base acceleration and **F**(t) represents the influence (force) of the $2n$ additionally fitted viscous dampers on the system responses.

The nodal displacement vector **X**(t) can be represented as follows.

$$\mathbf{X}(t) = \left\{ u_1 \ \cdots \ u_n \ \ v_{11} \ \ \theta_{11} \ \cdots \ v_{1s+1} \ \ \theta_{1s+1} \ \cdots \ v_{ij} \ \ \theta_{ij} \ \cdots \ v_{ns+1} \ \ \theta_{ns+1} \right\}^T \tag{7}$$

where s is the number of span of the structure, u_i $(i = 1,\cdots,n)$ represents the horizontal displacement at the ith floor, v_{ij}, θ_{ij} (i = 1,…, n j = 1…, s+1) denote the vertical and rotational displacements, respectively, at the ith story and the jth span.

In the present study, the number of the horizontal degree of freedom is assumed to be one at each floor.

Assume that the two viscous dampers at each floor are exactly the same and the damping characteristic of the dampers at ith floor can be represented by Eq. (1), that is

$$f_i = \alpha_{1i}\dot{\delta}_i + \left(\alpha_{2i}(p_i)\dot{\delta}_i^2 + \alpha_{3i}(p_i)\dot{\delta}_i^3 +,\cdots,+ \alpha_{L_i(p_i)}(p_i)\dot{\delta}_i^{L_i(p_i)} \right) \quad i = 1,\cdots,n \tag{8}$$

where α_{1i} and p_i are the linear and nonlinear characteristic parameters, respectively, of the additional dampers at the ith floor and $\dot{\delta}_i$ represents the relative velocity between the two ends of the dampers.

Obviously $\mathbf{F}(t)$ in Eq. (6) is completely determined by f_i $(i = 1,\cdots,n)$ in Eq. (8). Considering the polynomial form of Eq. (8) and the continuous property of function, $\alpha_{2i}(P_i)\, i = 1,\cdots,n$, the following expression can be introduced by following the analysis similar to that in Lang *et al*. (2013) for a simplified multi-story shear building model.

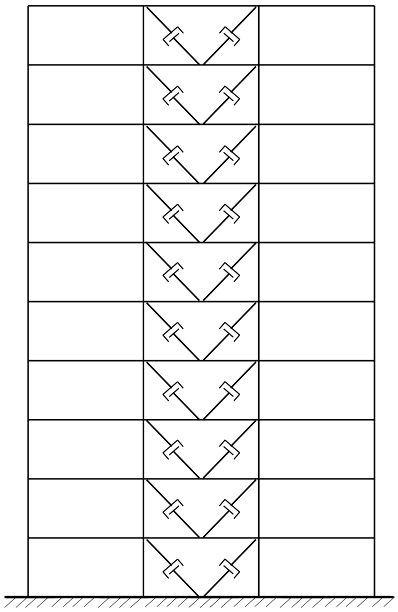


Fig. 2 *n*-story multi-span frame model of building structures

$$X_{Top}(j\omega)=\sum_{s=0}^{N}Q_{ks}(j\omega)p_k^s,\ k=1,\cdots,n \tag{9}$$

where $X_{top}(j\omega)$ is the Fourier transform of the building top story displacement, $Q_{ks}(j\omega)$, $s=1,\cdots,N$, are the functions of frequency variable ω and N is the order of this polynomial form analytical representation of $X_{top}(j\omega)$. These parameters $Q_{ks}(j\omega)$ are dependent on the base acceleration $\ddot{u}_g(t)$ and all the system parameters apart from P_k. Eq. (9) is an Output Frequency Response Function (OFRF) representation of the frequency response of the structural system defined by Eq. (6), which describes the frequency response of the top story displacement in term of the system nonlinear damping characteristic parameter p_k.

The OFRF is a concept proposed in Lang *et al*. (2007) which shows that the output frequency response of a wide class of nonlinear systems can be represented analytically by a polynomial function of the parameters which define the system nonlinearity. Eq. (9) is actually an OFRF representation of the frequency response of the structural system with a more general implication. This is because when k takes a specific value $\bar{k}$, that is, $k=\bar{k}$, Eq. (9) can be described by

$$X_{Top}(j\omega)=\sum_{s=0}^{N}Q_{\bar{k}s}(j\omega)p_{\bar{k}}^s \tag{10}$$

In Eq. (10), $Q_{\bar{k}s}(j\omega)$ $(s=1,\cdots,N)$, depend not only on the system input (base excitation) and linear characteristic parameters as defined by $\mathbf{M},\mathbf{C},\mathbf{K}$ and $\alpha_{1i}(i=1,...,n)$, but also on p_k ($k=1,...,\bar{k}-1,\bar{k}+1,...,n$), i.e. all system nonlinear damping characteristic parameters except $p_{\bar{k}}$.

This feature of the OFRF as described in Eq. (9) implies that the design of the system nonlinear damping characteristic parameters p_k $\left(k = 1,...,n\right)$ need to be conducted in an iterative way. This is one of the basic points of the algorithm that will be introduced in the next section for the optimal placement and design of nonlinear dampers represented by Eq. (1) for building structures under earthquake loadings.

5. Algorithm for optimal placement and design of nonlinear dampers in building structures under earthquake loadings

5.1 Damping characteristics

The present study considers the optimal placement and design of additional nonlinear dampers for the n -story multi-span building structure subjected to an earthquake loading. According to the second design example in Lang *et al*. (2013), the Vibration Power Loss Factor (VPLF) of the structure's top story displacement vibration is used as the design objective function. The VPLF γ was introduced in Guo and Lang (2011) and is defined as

$$\gamma = \left(1 - \frac{P}{P_0}\right) \times 100\% \tag{11}$$

where P_0 denotes the power of the original structural system response without any additional dampers and P represents the power of the structural system response when a vibration control mechanism has been introduced. Therefore, the VPLF indicates the reduction in the power of the system vibration response that can be achieved by the introduction of a vibration control mechanism.

Let us denote the time history of the top story displacement of the building during an earthquake as $x_{top}(t)$. P in Eq. (11) can then be written as

$$P = \frac{1}{T}\int_0^T \left|x_{top}(t)\right|^2 dt = \int_{-\infty}^{\infty} \left|X_{Top}(j\omega)\right|^2 \frac{d\omega}{2\pi T} \tag{12}$$

where T represents the duration of the earthquake. It is known from Eq.(9) that the following relation can be drawn.

$$\left|X_{Top}(j\omega)\right|^2 = \left|\sum_{s=0}^{N} Q_{ks}(j\omega)\, p_k^s\right|^2 = \sum_{s=0}^{2N} \tilde{Q}_{ks}(j\omega)\, p_k^s \tag{13}$$

where $\tilde{Q}_{ks}(j\omega)$ is a function of frequency of the same nature as $Q_{ks}(j\omega)$. Substituting Eq. (13) into Eq. (12) yields

$$P = P_{k0} + \sum_{s=1}^{2N} P_{ks}\, p_k^s \tag{14}$$

where $P_{ks} = \int_{-\infty}^{\infty} \tilde{Q}_{ks}(j\omega)\frac{d\omega}{2\pi T}, s = 0,...,2N$. Let $P_0 = P_{k0}$, then

$$\gamma(p_k) = \left(1 - \frac{P}{P_0}\right) \times 100 = \sum_{s=1}^{2N} \beta_{ks} p_k^s \quad (k = 1, \cdots, n) \tag{15}$$

where $\beta_{ks} = P_{ks} / (-P_{k0}) = P_{ks} / (-P_0)$.

Eq. (15) is an OFRF based representation of the design objective function which shows an analytical relationship between the VPLF of the top floor displacement and the nonlinear characteristic parameter of the additional damper inserted into the *k*th floor. Based on this representation, an algorithm will be derived in the next section for the optimal placement and design of nonlinear dampers as described by Eq. (1) for reducing vibrations of the *n*-story multi-span frame model shown in Fig. 2 during earthquakes.

5.2 Design algorithm

The basic ideas of the OFRF based algorithm for the optimal placement and design of nonlinear dampers can be summarized as follows:

Step 1: Determine the OFRF representation in terms of the damper characteristic parameter at *k*th floor with respect to a baseline of $p_1, \cdots, p_n$ and repeat this for all the *k*'s (*k*=1,…, *n*). Here a baseline is a set of initial values of $p_1, \cdots, p_n$; the OFRF with respect to a base line is a polynomial form representation of the system output frequency response in terms of an increment of $p_1, \cdots, p_n$ relative to the baseline.

Step 2: From the *n* OFRFs that have been determined in Step 1, find at which floor, an increment for the characteristic parameter of additional dampers needs to be added and how much the increment should be.

Step 3: Make the increment for the damping characteristic parameter on the floor as obtained in Step 2,

Step 4: Repeat the procedure from Step 1 to Step 3 until the design constraint

$$\sum_{s=1}^{n} p_s \le \bar{C} \tag{16}$$

is satisfied where $\bar{C}$ is the constraint on the sum of the damping characteristic parameters of added dampers.

It is worth mentioning that another well-known performance criterion for the optimal damper placement is associated with the maximum interstory drift and given by

$$\max_i \left(\delta_i\right) \le \bar{\delta} \tag{17}$$

In the present study, the evaluation of the maximum interstory drift is also conducted as an additional check on the vibration performance of building structures in the procedure of the optimal damper placement by using the above design ideas. Understanding the relationship between the maximum interstory drift and the sum of the damping characteristic parameters is helpful for structural engineers in determining the amount of the sum of the damping characteristic parameters.

The details of implementing the design and associated principles are described in the following.

Denote q as the index of baseline which also represents the step number of the iterative optimization procedure and starts from $q=0$. In order to determine the OFRF with respect to a baseline $p_1,\cdots,p_n$ step q, denote the baseline as $p_1(q),\cdots,p_n(q)$, and rewrite Eq.(15) in terms of Δp_k, an increment of p_k, as

$$\begin{aligned}\gamma_q(\Delta p_k) &= \left(1-\frac{P\,(p_1(q),...,p_k(q)+\Delta p_k,...,p_n(q))}{P\,(p_1(q),...,p_k(q),...,p_n(q))}\right) \quad k=1,\cdots,n \\ &= \sum_{s=1}^{2N}\beta_{ks}(p_1(q),...,p_k(q),...,p_n(q))\Delta p_k^s\end{aligned} \tag{18}$$

where the subscript q on the left hand side of Eq.(18) indicates that the VPLF γ is a result evaluated with respect to baseline $p_1(q),\cdots,p_n(q)$.

$\beta_{ks}(p_1(q),...,p_k(q),...,p_n(q)), s=1,...,2N,$ in Eq. (18) can be determined from at least $2N$ different increments of p_k and the corresponding values of γ. Denote these different increments of p_k as $\Delta p_k(i), i=1,...,2N$ and the corresponding values of γ as $\gamma_q(\Delta p_k(i)), i=1,...,2N$ which can be evaluated from $2N$ simulated system top floor displacement responses to a considered earthquake loading when Δp_k takes the values of $\Delta p_k(i),\ i=1,...,2N$, respectively. Then $\beta_{ks}(p_1(q),..., p_k(q),...,p_n(q))$, $s=1,...,2N$, can be found by solving the following $2N$ simultaneous equations.

$$\gamma_q(\Delta p_k(i)) = \sum_{s=1}^{2N}\beta_{ks}(p_1(q),...,p_k(q),...,p_n(q))\Delta p_k^s(i),\ i=1,\cdots,2N \tag{19}$$

Thus, by going through the procedure for each k, the OFRFs with respect to the baseline $p_1(q),\cdots,p_n(q)$ for all the floors can be determined.

In order to determine at which floor an increment for the damping characteristic parameter needs to be made, a gradient of the determined OFRF, i.e., Eq.(18) with respect to the baseline $p_1(q),\cdots,p_n(q)$ is evaluated as

$$\frac{d\gamma_q(\Delta p_k)}{d\Delta p_k}\Big|_{\Delta p_k=0} = \beta_{k1}(p_1(q),...,p_k(q),...,p_n(q)),\quad k=1,...,n \tag{20}$$

Then, the number of floor at which an increment for the damping characteristic parameter should be made can be found as $k^*(q)$ such that

$$\beta_{k^*(q)1}(p_1(q),...,p_k(q),...,p_n(q)) = \max\{\beta_{k1}(p_1(q),...,p_k(q),...,p_n(q)),\quad k=1,...,n\} \tag{21}$$

This is because Eqs. (20) and (21) imply that an increase in the damping parameter p at floor $k^*(q)$ on top of the baseline $p_1(q),...,p_k(q),...,p_n(q)$ can produce a maximum increase in the VPLF value γ.

After finding at which floor the damping characteristic parameter should be increased, the amount of the increment will be determined as follows.

First, solve equation

$$\frac{d}{d\Delta p}\left(\sum_{s=1}^{2N}\beta_{k^*(q)s}(p_1(q),\ldots,p_n(q))\Delta p^s\right)=0 \tag{22}$$

to find its minimum positive solution $\Delta\bar{p}(q)$ and take $\Delta\bar{p}(q)$ as the upper limit of the increment $\Delta p(q)$ at floor $k^*(q)$. This is because an increment of the damping characteristic parameter at floor $k^*(q)$ beyond $\Delta\bar{p}(q)$ will not be able to further reduce the value of VPLF as needed.

Second, solve the following equation for all $k\in\{1,\ldots,n\}$ except $k=k^*(q)$, respectively,

$$\sum_{s=1}^{2N}\beta_{k^*(q)s}(p_1(q),\ldots,p_n(q))\Delta p^s-\sum_{s=1}^{2N}\beta_{ks}(p_1(q),\ldots,p_n(q))\Delta p^s=0 \tag{23}$$

to find n-1 minimum positive solutions and denote them as $\Delta\bar{\bar{p}}_j(q)$, $j=1,\cdots,n-1$. Then find the increment for $p_{k^*(q)}(q)$ as

$$\Delta\tilde{p}(q)=\min\left\{\Delta\bar{p}(q),\Delta\bar{\bar{p}}(q)\right\} \tag{24}$$

where $\Delta\bar{\bar{p}}(q)=\min_j\left(\Delta\bar{\bar{p}}_j(q),\ j=1,\ldots,n-1\right)$.

The objective of using this procedure is to determine an increment for $p_{k^*(q)}(q)$ and to make sure the following conditions are satisfied when $\Delta p\in(0,\Delta\tilde{p}(q))$,

$$\frac{d\gamma_q\left(\Delta p_{k^*(q)}\right)}{d\Delta p_{k^*(q)}}>0 \tag{25}$$

$$\sum_{s=1}^{2N}\beta_{k^*(q)s}(p_1(q),\ldots,p_n(q))\Delta p^s>\sum_{s=1}^{2N}\beta_{ks}(p_1(q),\ldots,p_n(q))\Delta p^s\quad k\neq k^*(q) \tag{26}$$

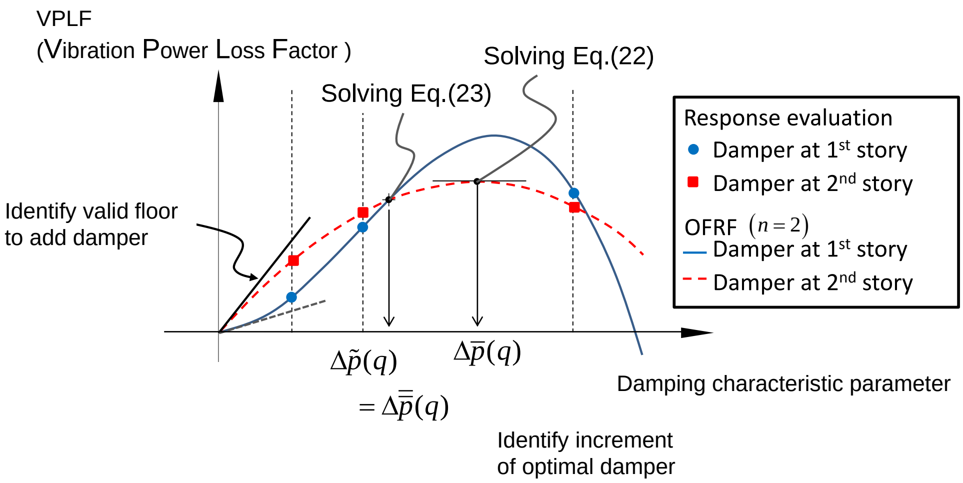


Fig. 3 Illustration of determining where (which floor) damping should be increased and how much the increment should be

Therefore, $\Delta\tilde{p}(q)$ is an optimal increment that can be made for $P_{k^*(q)}(q)$. Fig. 3 illustrates this procedure and the corresponding optimum increment for a case where n=2.

Based on the ideas and implementation details as described above, the detailed procedure of the design algorithm can be summarized as follow.

(i) Let q=0, choose a baseline for $p_1,\cdots,p_n$, and denote the baseline as $p_1(0),\cdots,p_n(0)$.

(ii) Determine coefficients $\beta_{ks}(p_1(q),...,p_k(q),...,p_n(q)),\ s=1,...,2N$ in the OFRF given by Eq.(18) with respect to baseline $p_1(q),\cdots,p_n(q)$ from $2N$ simulated system top floor displacement responses to a considered earthquake loading in the cases where Δp_k takes $2N$ specified values $\Delta p_k(i), i=1,...,2N$, respectively. Repeat this procedure for k=1,...,n to find OFRF in Eq.(18) with respect to the same baseline for all the floors.

(iii) From $\beta_{k1}(p_1(q),...,p_k(q),...,p_n(q)),\ (k=1,...,n)$, that have been determined in (ii), find $k^*(q)$, the number of floors where an increment for the damping characteristic parameter p should be made, using Eq. (21).

(iv) Determine the increment $\Delta\tilde{p}(q)$ for $p_{k^*(q)}(q)$ using Eq. (24) with $k^*(q)$ as found in (iii).

(v) Let $p_k(q+1)=p_k(q)$ for $k\in\{1,...,n\}$ but $k\neq k^*(q)$ and $p_{k^*(q)}(q+1)$ $=p_{k^*(q)}(q)+\Delta\tilde{p}(q)$.

(vi) Work out $\sum_{k=1}^{n}p_k(q+1)$

(vii) If $\sum_{k=1}^{n}p_k(q+1)<\bar{C}$, let q=q+1, and return to (ii) for the next iteration. Otherwise, let

$$p_{k^*q}(q+1)=p_{k^*(q)}(q)+\bar{C}-\sum_{k=1}^{n}p_k(q)$$

and finish the algorithm.

In the algorithm, in addition to an earthquake loading to be considered for the design, the parameters N and $\Delta p_k(i),\ i=1,...,2N$, also have to be specified. Generally speaking, the appropriate values for these parameters are problem dependent. The effects of their choices on the structural vibration control performances will be investigated using design examples.

6. Numerical examples

For the application of the proposed algorithm to the nonlinear damping designs for building structures, a 5-story 4-span plane frame structure is considered. Table 1 shows the member property of the plane frame. The possible location to fit the additional dampers is the two spans in the middle so that the issue of optimal placement of dampers on the same floor is not considered in this case study. In addition to the constraint on the sum of the damping characteristic parameters of added dampers, it is also required that the maximum interstory drift should be smaller than 0.02[m] under the specified input ground motion. This design constraint will be used to make an additional check for the building vibration performance during design earthquakes. The ground motions used as the input excitation in the optimal design are generated so as to be compatible with the target response spectrum, i.e. the safety-limit response spectrum defined in the Japanese Building Earthquake-resistant Design Code. The detailed procedure for ground motion generation can be found in Gasparini and Vanmarcke (1976). Fig. 4 shows the time histories of three different input excitations which are produced by the same target response spectrum and represented as the input A, B and C, respectively.

The optimal damper placements under different input motions (A, B, C) were derived using the design algorithm in Section 5.2 and the results are as shown in Fig. 5 when the damping exponent parameter is chosen as $\alpha = 0.3$. From these figures, it can be observed that the optimal damper placements are not similar even though the input ground motions have been generated by the same target response spectrum. In this simulation, the VPLF for the optimal design demand is set as 70%. A comparison of the damper placements and the corresponding maximum interstory drifts that have been achieved under the three different ground motions is given in Table 2. As the maximum interstory drift of the original structural frame under input B is the largest when compared with the cases under other ground motions, the input B is used as the target ground motion in the optimal design in the following simulations.

For investigating the variability of the optimal damper placement due to the difference of the input ground motion that has been used for the design, consider two cases shown in Fig. 6: (i) the input A based optimal design subjected to input A and the input B based optimal design subjected to input A, and (ii) the input C based optimal design subjected to input C and the input B based optimal design subjected to input C. For simplification in the following figures, the term “Design: A” is defined as the optimal design using input A and the term “Input: A” indicates the

Table 1 Member cross-section property

	Column		Beam
	Outer column	Inner column	
1st story	$\square\!-\!600\times600\times22$	$\square\!-\!650\times650\times22$	$\text{H-}700\times250\times14\times25$
2nd story	$\square\!-\!600\times600\times22$	$\square\!-\!650\times650\times22$	$\text{H-}700\times250\times14\times25$
3rd story	$\square\!-\!600\times600\times22$	$\square\!-\!650\times650\times22$	$\text{H-}600\times250\times12\times28$
4th story	$\square\!-\!500\times500\times19$	$\square\!-\!600\times600\times22$	$\text{H-}600\times250\times12\times28$
5th story	$\square\!-\!500\times500\times19$	$\square\!-\!600\times600\times22$	$\text{H-}600\times250\times12\times28$

Table 2 Optimal damper placement ($\alpha = 0.3$)

		Damping coefficient ([$\times10^6$ N/$(\text{m/s})^{0.3}$])					$\max_i(\delta_i)$
		1st story	2nd story	3rd story	4th story	5th story	[$\times10^{-2}$ m]
Input A	Initial	-	-	-	-	-	1.95
	Optimal	0.00	1.74	0.11	0.00	0.00	1.37
Input B	Initial	-	-	-	-	-	3.05
	Optimal	0.00	1.22	0.57	0.092	0.00	1.96
Input C	Initial	-	-	-	-	-	2.64
	Optimal	0.00	0.97	0.52	0.026	0.00	1.50

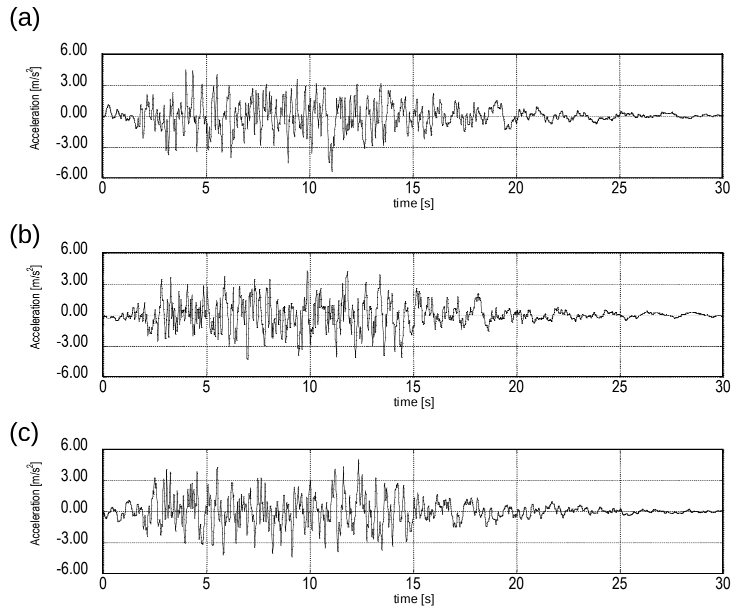


Fig. 4 Three different ground motions generated by the same target response spectrum (a) input A, (b) input B, (c) input C

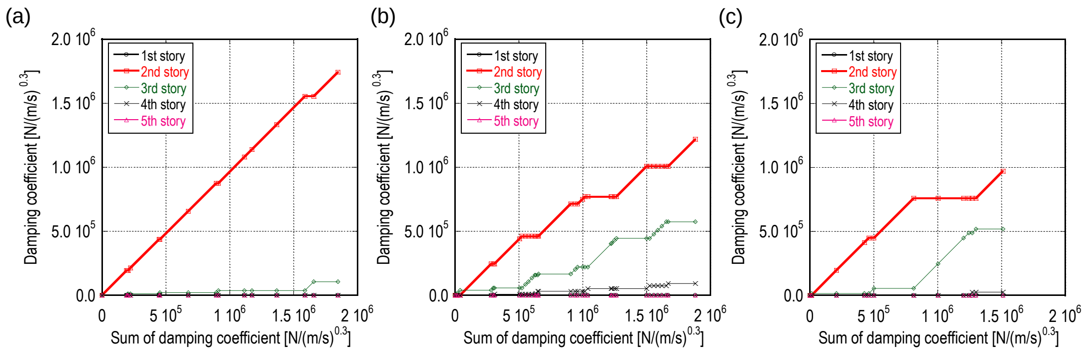


Fig. 5 Optimal damper placements ($\alpha = 0.3$) (a) input A, (b) input B, (c) input C

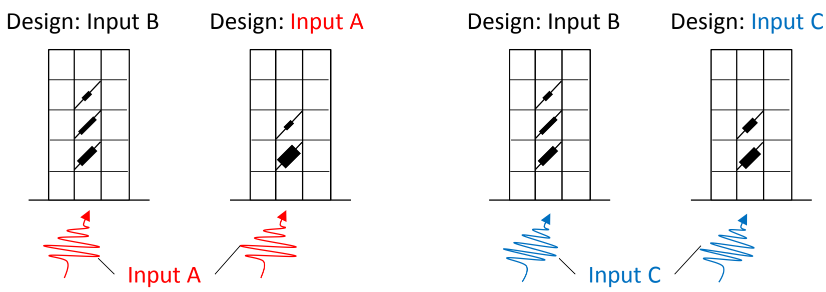


Fig. 6 Illustration of how an optimal design based on one input is verified for the case where the structure is subjected to a different input

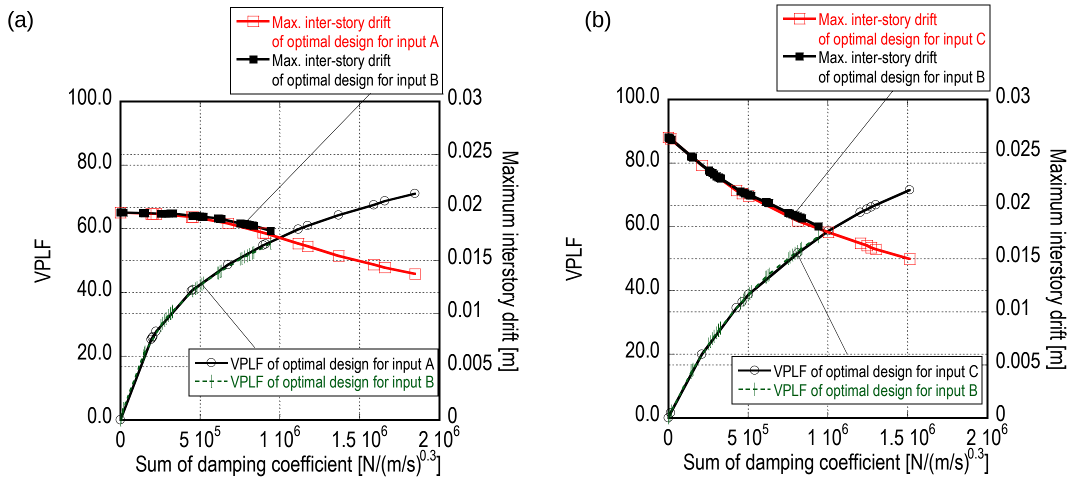


Fig. 7 Comparison of VPLF and maximum interstory drift (a) “Design: A - Input: A” VS “Design: B - Input: A”, (b) “Design: C –Input: C” VS “Design: B - Input: C”

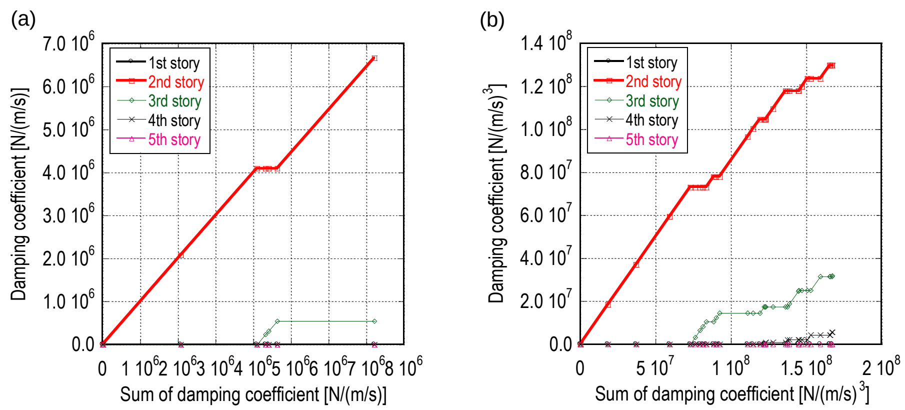


Fig. 8 Optimal damper placements under input B (a) $\alpha = 1.0$, (b) $\alpha = 3.0$

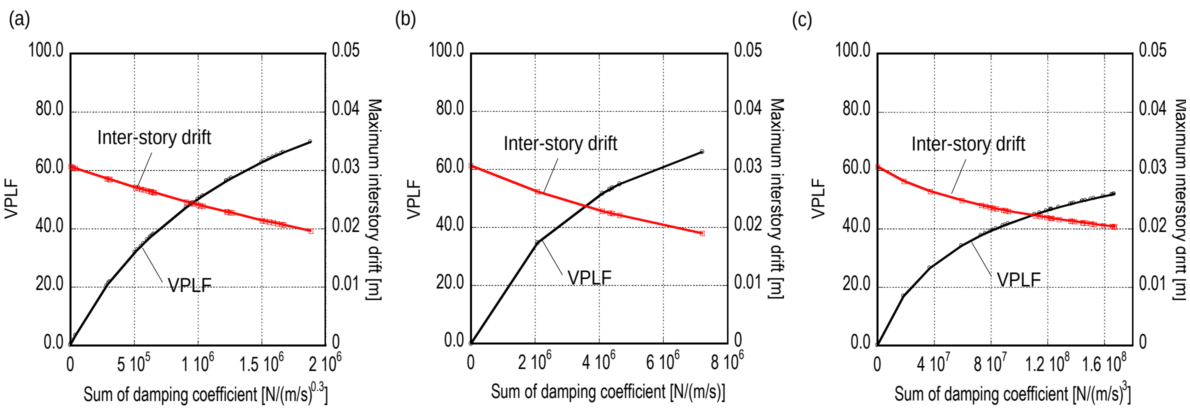


Fig. 9 Improvement of structural performance for optimal damper placements (input B) (a) $\alpha = 0.3$, (b) $\alpha = 1.0$, (c) $\alpha = 3.0$

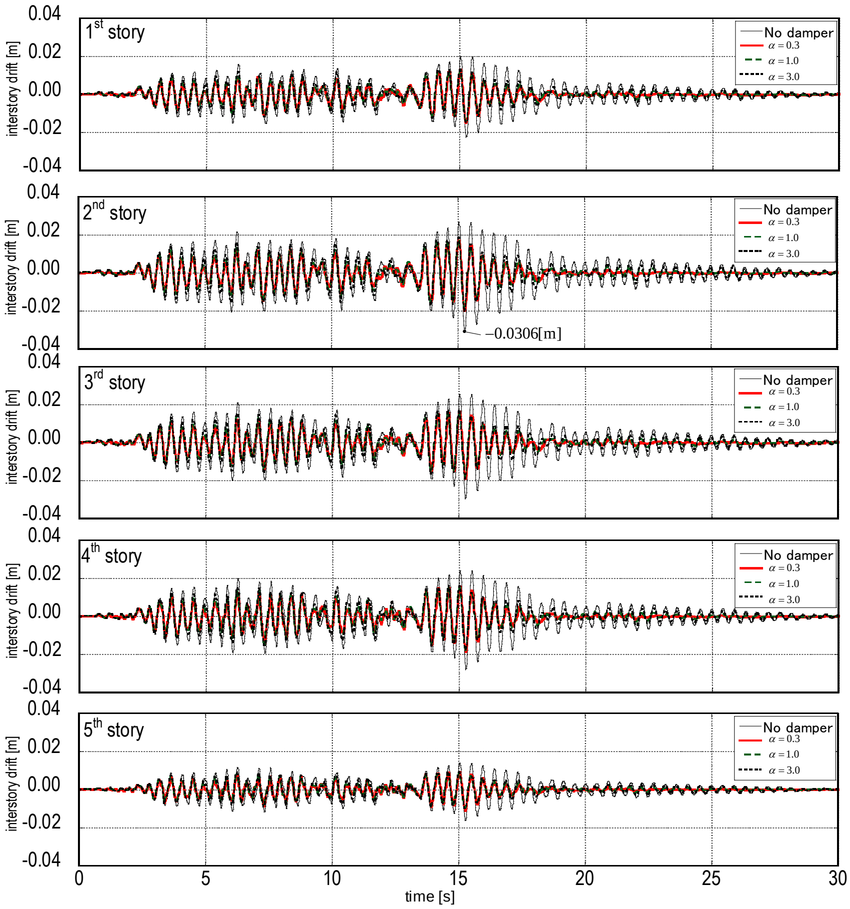
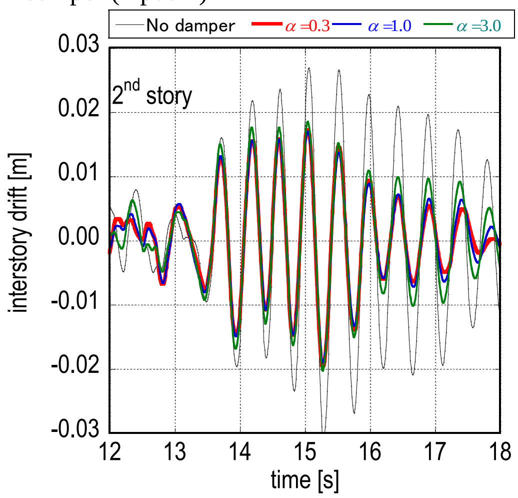


Fig. 10(a) Comparison of time-histories of interstory drift of optimal damping with that for no damper (input B)



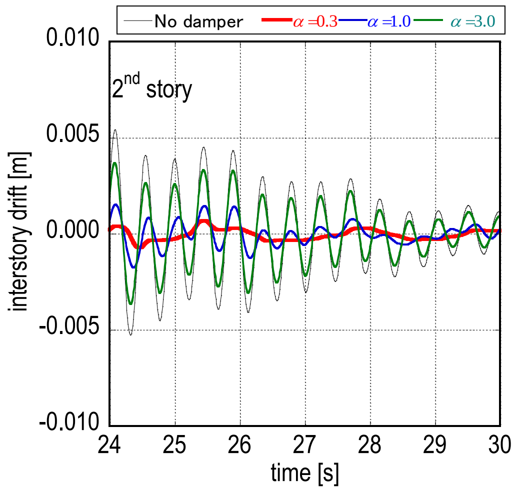


Fig. 10(b) Magnified figure (12-18 sec) Fig. 10(c) Magnified figure (24-30 sec)

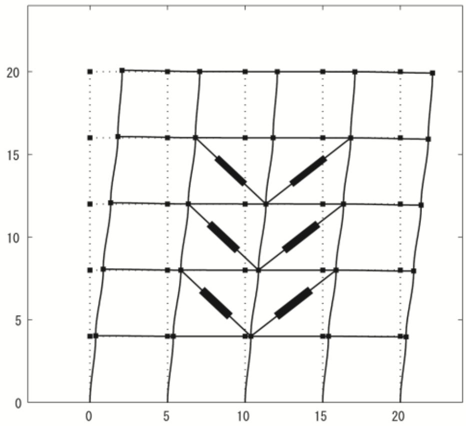


Fig. 11 Maximum deformation of the plane frame with optimal damping design ($\alpha = 0.3$)

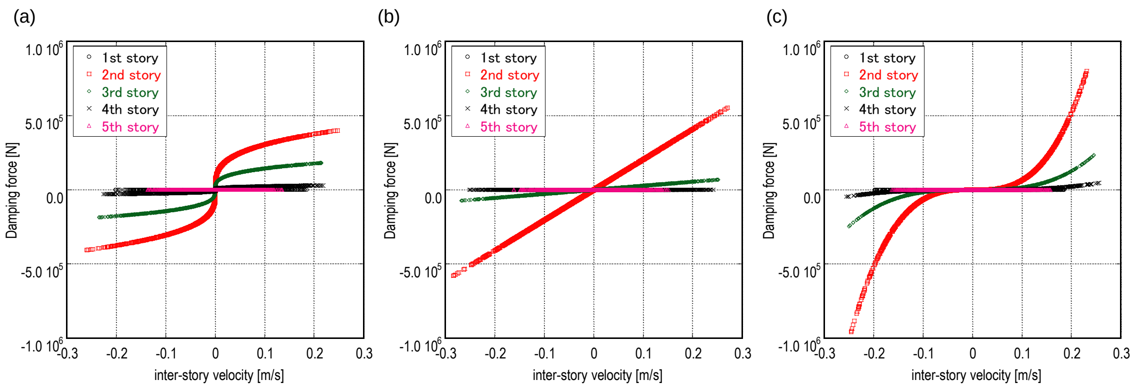


Fig. 12 Comparison of damping force (a) , (b) , (c)

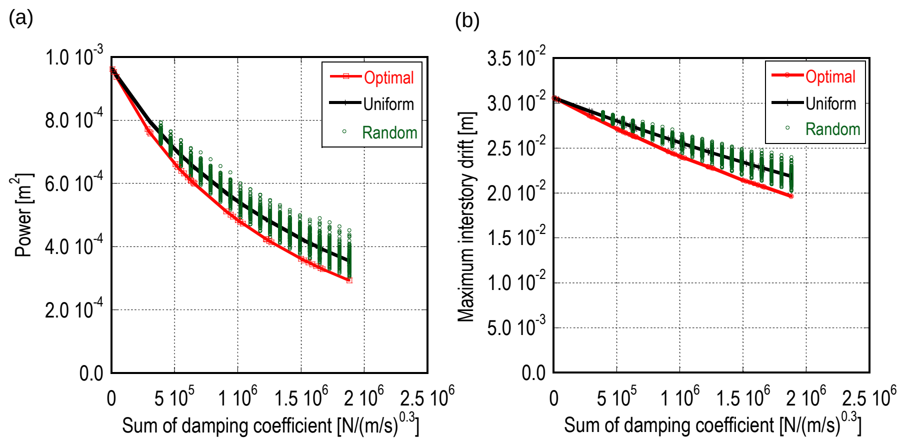


Fig. 13 Verification of the optimal damper placement ($\alpha = 0.3$) (a) Power of top horizontal displacement, (b) Maximum interstory drift

interstory drift of the building structures with added nonlinear dampers subjected to input A.

Fig. 7(a) shows a comparison of VPLF and the maximum interstory drifts of “Design: A - Input: A” and those of “Design: B - Input: A”. The added nonlinear damper is expected to be the most effective in the former case, i.e. “Design: A - Input: A”. On the other hand, Fig. 7(b) shows a comparison of VPLF and the maximum interstory drifts of “Design: C – Input: C” and those of “Design: B – Input: C”. From these figures, it can be observed that, if the input is of the same target spectrum, there is no significant difference in the structural vibration performance in terms of both VPLF and maximum interstory drift even if the designs are different. Therefore, it is concluded that, when the proposed technique is applied for the optimal nonlinear damper designs, one loading input based optimal design is also valid provided that the input loadings have the same target spectrum, i.e. different designs exhibit the same structural vibration performance in terms of both VPLF and maximum interstory drift to a common input. A reasonable value of VPLF can be determined by the structural performance demand. When the maximum interstory drift is selected as the structural performance criterion, the evaluation of the maximum interstory drift in the optimal design procedure as shown in Fig. 7 can provide useful information in the decision.

Fig. 8 shows the optimal damper placement under the input B where the damping exponent parameters are taken as $\alpha = 1.0$ and $\alpha = 3.0$ where the maximum interstory drift as the performance criteria is 0.02m. For comparing the improvement of the structural performances by the optimal damper placements for α = 0.3, 1.0 and 3.0, the variation of the VPLFs and the maximum interstory drifts are shown in Fig. 9. The maximum interstory drifts for α = 0.3, 1.0 and 3.0 at the terminal point are 0.02m. From these figures, it is found that VPLF is maximized in the optimal design with $\alpha = 0.3$. This may result from the difference of the performance of the vibration control for α = 0.3, 1.0 and 3.0 in the range of small deformation (see Fig. 10). Actually the nonlinear damper with $\alpha = 0.3$ is effective for reducing the small amplitude vibration.

Fig. 10(a) shows the comparison of the time histories of the interstory drift of the optimal damping designs with α = 0.3, 1.0 and 3.0 with those for no additional dampers subjected to the input B. The amounts of added damper are given so as to suppress the maximum interstory drift to 0.02m. It can be observed that the design constraint (the maximum interstory drift=0.02m) can be achieved in the optimal damper placement. Figs. 10 (b) and (c) show the magnified time histories during (b) 12-18s and (c) 24-30s, respectively, of the interstory drift at the 2nd story. Although the design constraints on the maximum interstory drift are satisfied in those optimal designs for α = 0.3, 1.0 and 3.0, the performance of the vibration control in the small deformation range indicates that the best performance is achieved for $\alpha = 0.3$, the second best is achieved for $\alpha = 1.0$, and the worst is in the case of $\alpha = 3.0$ (Fig. 10(c)). On the other hand, in the case of very rare earthquakes having seismic intensities larger than the design earthquakes, the nonlinear dampers with $\alpha = 3.0$ is much more effective.

Fig. 11 represents the maximum deformation of the plane frame with the optimal damping design using $\alpha = 0.3$ under input B. The deformation including nodal vertical displacements and nodal rotations can be understood.

Fig. 12 describes the comparison of the damping force-velocity relations for the optimal damper design with different damping parameters $\alpha = 0.3, 1.0, 3.0$ under input B where the maximum interstory drift is 0.02. From these figures, the difference of the non-linear and linear damper models can be observed from damping forces. The damping forces for the non-linear damper with $\alpha = 0.3$ are the smallest. Since the damping force may be related with the frame design cost, the optimal damper design using non-linear dampers with $\alpha = 0.3$ can be regarded as the best solution in this case.

For the verification of the optimality of the obtained damper placement, Fig. 13 shows the comparison of the variation of the power and the maximum interstory drift for the optimal damper placement with $\alpha = 0.3$ with those for other damper placements under input B. In Fig. 13, the uniform damper placement indicates the damper placement where the same amount of damper is added at each floor for the same sum of damping coefficients. In addition, the random damper placement denotes the damper placement where the random amount of damper is given by using Monte Carlo simulation for the same sum of damping coefficients. It can be observed from Fig. 13 that the optimal damper placement derived by the proposed OFRF based optimal design algorithm performs the most effective vibration suppression, achieving the minimum in terms of both VPLF and the maximum interstory drift.

7. Conclusions

The use of damping devices in building structures for vibration control during earthquakes has become a more important subject in structural engineering especially after 2011 Tohoku earthquake in Japan. Although the optimal design and placement of dampers in frame structures have been comprehensively investigated, most works are concerned with the design and placement of linear dampers. Considering dampers' inherent nonlinear behaviours and potential beneficial effects of nonlinear damping on structural vibration control, the analysis of structural behaviours including nonlinear dampers and the design of nonlinear dampers for structural vibration suppression during earthquake ground motions have recently been studied by some researchers. However, there is no a unified approach that can be used systematically to deal with the optimal placement and design of a wide range of nonlinear dampers to achieve a general design objective such as the reduction of top story vibration energy as required in many practical cases.

To address these problems, in the present study, a general nonlinear damper model has been proposed, which can be used to represent a range of nonlinear dampers including, for example, power-law dampers and oil dampers with relief mechanism. Then a general approach has been derived for the optimal design and placement of this class of nonlinear dampers in building frame structures. The approach is based on a formulation of the structural nonlinear response in the frequency domain using the OFRF concept that reveals a simple polynomial relationship in the frequency domain between the structural response and parameters defining structural nonlinearity. A design criterion concerning the top story vibration energy that can deal with any given loading conditions is used for the optimal design.

Comprehensive numerical studies have been conducted to apply the proposed approach to the optimal design of the nonlinear characteristic parameter of power-law dampers. The results have verified the performance of the new designs and demonstrated that the new design approach could help structural engineers to optimally retrofit or design nonlinear dampers for the vibration control of building structures subjected to a wide range of loadings including earthquake ground motions. Especially it has been demonstrated that the properties of nonlinear dampers (damping exponent parameters for power-law dampers) play a key role in the vibration suppression characteristics in a small vibration amplitude. The performance of the vibration control in the small deformation range indicates that the best performance is achieved for $\alpha = 0.3$, the second best is achieved for $\alpha = 1.0$, and the worst is in the case of $\alpha = 3.0$. On the other hand, in the case of very rare earthquakes having seismic intensities larger than the design earthquakes, the nonlinear dampers with $\alpha = 3.0$ is much more effective.

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