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Modified seismic analysis of multistory asymmetric elastic buildings and suggestions for minimizing the rotational response

George K. Georgoussis*

Department of Civil Engineering, School of Pedagogical and Technological Education (ASPETE), N. Heraklion 14121, Attica, Greece

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Abstract. A modified procedure is presented for assessing the seismic response of elastic non-proportionate multistory buildings. This procedure retains the simplicity of the methodology presented by the author in earlier papers, but it presents higher accuracy in buildings composed by very dissimilar types of bents. As a result, not only frequencies and peak values of base resultant forces are determined with higher accuracy, but also the location of the first mode center of rigidity (m1-CR). The closeness of m1-CR with the axis passing through the centers of floor masses (mass axis) implies a reduced rotational response and it is demonstrated that in elastic systemsa practically translational response is obtained when this point lies on the mass axis.Besides, when common types of buildings are detailed as planar structures under a code load, this response is maintained in the inelastic phase of their response as a result of the almost concurrent yielding of all the resisting bents. This property of m1-CR can be used by the practicing engineer as a guideline to form a structural configuration which will sustain minimum rotational response, simply by allocating the resisting elements in such a way that this point lies close to the mass axis. Inelastic multistory building structures, detailed as above, may be regarded as torsionally balanced multistory systems and this is demonstrated in eight story buildings, composed by dissimilar bents, under the ground motions of Kobe 1995 (component KJM000) and Friuli 1976 (component Tolmezzo E-W).

Keywords: modal analysis; eccentric structures; modal centre of rigidity; minimum torsion

1. Introduction

The structural behavior of asymmetric multistory buildings having resisting bents with stiffness matrices which are proportional to each other (proportionate buildings) and the centers of floor masses on the same vertical line, can be obtained by determining (i) the response of the corresponding uncoupled multi-story structure and, (ii) for each mode of vibration of the latter structure, by analyzing an equivalent torsionally coupled single story system (Kan and Chopra 1977a, Hejal and Chopra 1989, Athanatopoulou *et al*. 2006). The same analysis can also be applied in shear type buildings with different static eccentricities at the various floor levels (Kan and Chopra 1977b). Recently, this analysis was extended by the author to non-proportionate buildings, by introducing the concept of the modal stiffness of the bents which provide the lateral

*Corresponding author, Professor, E-mail: ggeorgo@tee.gr

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stiffness of a given structure. It is shown (Georgoussis 2009, 2010, 2012) that the peak elastic response of medium height buildings can be derived by analyzing two equivalent single-story modal systems, each of which has a mass equal to the *k*-mode effective mass, M_k^* (k = 1, 2), of the uncoupled multi-story structure, and is supported by elements with a stiffness equal to the product of M_k^* with the first mode (when k = 1) or second mode (when k = 2) squared frequencies of the corresponding real bents (element frequencies)of the assumed multi-story structure.In medium height buildings the first mode equivalent single-story modal system has a dominant role on the overall response and its stiffness centre constitutes the first mode centre of rigidity (m_1-CR). The closeness of this point with the mass axis implies a reduced rotational response and in uniform systems, the vertical axis which passes through m_1-CR may be regarded as a stiffness axis (or 'elastic axis') in the sense that, when this axis coincides with the mass axis, an almost translational response is obtained under a ground motion.

Reviewing the literature it can be seen that the absence of a definition of the 'elastic axis' has led to many investigations about the issue of establishing a set of points located at the floor levels of a multi-story building with properties similar to those of the stiffness center of single-story systems. Since the oscillatory response of single story systems is due to the distance between the centers of mass and rigidity (CM and CR), usually referred to as static eccentricity, early studies on elastic multi-story systems (e.g. Poole 1977; Humar 1984; Smith and Vezina 1985; Jiang *et al*. 1986) have led to different definitions about the magnitude of this eccentricity at the various floor levels. Cheung and Tso (1986) proposed the 'rigidity centers (CRs)' as a reference axis for structural applications. These are the points that when a given distribution of lateral loading passes through them only translational movement of the floors will occur. However, apart from the proportionate structures these points are load dependant and their space distribution is very irregular, even in uniform structures composed of different types of bents.More recently, Makarios and Anastassiadis (1998a,b) introduced the 'axis of optimum torsion' as a reference axis with promising results (Makarios 2005, 2008; Makarios *et al*. 2006). This axis can be determined by means of an indirect static analysis by applying a set of floor torques equal in magnitude to the lateral forces at the same floors. An alternative mathematical procedure was proposed by Marino and Rossi (2004).

In recent years the rotational response of multistory asymmetric structures, composed by inelastic bents, has received major attention (qualitative overviews are presented by De Stefano and Pintucchi, 2008 and Anagnostopoulos *et al*. 2013) and an alternative strategy for controlling this response in multistory structures designed to withstand ground motions into the inelastic region is presented by Aziminejad *et al.* (2008) and Aziminejad and Moghadam (2009). In these studies the problem of element strength distribution on the rotational response of the structure is studied by using a proper configuration of the centers of mass, strength and stiffness according to the findings obtained from single story systems with elements having strength dependant stiffness (Myslimaj and Tso 2002, 2004). Interesting results are also highlighted by Lucchini *et al*. (2008) concerning the response of shear type 3-story inelastic buildings under strong ground motions and, also, by Stathopoulos and Anagnostopoulos (2005).

The first objective of this study is to improve the aforementioned methodology (Georgoussis 2009, 2010, 2012) in analyzing elastic systems, by introducing the concept of the element effective frequency. The concept of this frequency is outlined in section 2, by means of the approximate continuum method, because of the advantage of this method to provide the response of building structural systems in a parametric form. The higher accuracy of using the element 'effective' frequencies (instead of the element frequencies), in calculating the frequency data of symmetrical buildings composed by dissimilar bents is demonstrated in section 3 and a brief description of the formulation of the aforesaid methodology, in systems with simple asymmetry,is given in section 4.

The second objective of the paper is to show that when the mass axis passes through m_1-CR, implying that the response in the elastic phase is practically translational, an almost concurrent yielding of all elements may preserve this type of response into the inelastic phase. The concept of this approach originates from the response of eccentric single story systems. Such systems, with coincident the centres of mass and rigidity and elasto-plastic elements having a strength distribution proportional to the stiffness distribution (usually called torsionally balanced (TB) models) present a purely translational inelastic response under strong ground excitations. For this reason they are used as 'reference' models in relevant studies (e.g. Correnza *et al*. 1994; Chandler *et al*. 1996; Wong and Tso 1994). This behaviour is attained because yielding is initiated at the same instant for all elements and the element force balance about CM is preserved into the inelastic phase, leading to a translational response throughout the ground shaking. In the case of a multi-story building, where the mass axis passes through m_1-CR, such a response into the inelastic phase may be obtained when the strength assignment of the various bents is based on a planar static analysis under a set of lateral forces simulating a 'seismic loading' (e.g. a set of floor forces having the shape of the 'inverted triangle' and summing to the code base shear).

The accuracy of the proposed modified methodology in elastic systems is demonstrated, in section 5,in buildings composed by dissimilar bents. Data of basic dynamic quantities (periods, top rotations and resultant base forces) of 8-story building models are presented and comparisons are made with the old procedure and with the results derived from the accurate SAP2000 computer program. Particular attention is paid on the accuracy of assessing the location of m_1-CR. The same building models are used to demonstrate that a practically translational response may be expected in inelastic systems when the mass axis is passing through m_1-CR and the bent strength assignment has been derived from a planar static analysis under a code lateral loading. Two characteristic ground motions (Kobe 1995, component KJM000 and Friuli 1976, component Tolmezzo E-W*)*, selected from the strong ground motion database of the Pacific Earthquake Engineering Research (PEER) Center (hppt://peer.berkely.edu) and scaled to a PGA=0.5g, are used to compare rotations and base torques of the assumed 8-story models.

2. Approximating the modal stiffness of building structures- element effective frequencies

Consider the symmetrical plan of the uniform multistory building of Fig. 1. Let's assume that the resisting elements are pairs of bents aligned in two orthogonal directions (*i*-bents are aligned along the x-direction and *j*-bents along the y-direction) symmetrically to the center of mass (CM), and therefore at each floor the center of mass coincides with the center of resistance of the structural system.

The assumed bents may be of different type (e.g.: walls, frames, coupled walls, wall frame assemblies) with quite dissimilar stiffness matrices, but the lateral stiffness of the building, in any of the directions, is given by the sum of the stiffnesses of the bents in the same direction. For example, along the y-direction:

$$\mathbf{K}_{\mathrm{y}} = \Sigma \mathbf{K}_{\mathrm{j}} \tag{1}$$

where $\mathbf{K}_{\mathrm{j}}$ is the stiffness matrix of the *j*-bent and $\mathbf{K}_{\mathrm{y}}$ the stiffness matrix of the complete structure.

In the earthquake analysis of building systems, basic dynamic properties (frequencies, effective modal masses, etc) are required and, nowadays, they can be easily derived by standard engineering software applicable to discrete member models. An analysis with such a model (using stiffness and mass matrices and displacement vectors)is favored by practicing engineers, but it is model specific and it does not providea deep insight in the response of building systems. Such an overview may be obtained by the approximate continuum method (differential equation formulation), which has the advantage of providing the behavior in a parametric form.

Using the latter method (Heidebrecht and Stafford Smith 1973; Heidebrecht 1975), the undamped equation of free motion of a common cantilever type planar bent (say the *j*-bent of Fig. 1(b), which may be a uniform over the height wall, frame or a wall frame assembly) is given by

$$EI_j u^{iv} - GA_j u'' = \omega_j^2 mu \tag{2}$$

where I_j,GA_j are the sum of column inertias and the shear rigidity of the *j*-bent respectively, *m* is the mass of the structure per unit height, *E* the modulus of elasticity, *u* the horizontal deflection of the bent and ω_j its frequency. In case of a symmetrical building structure with rigid floor diaphragms, as that of Fig. 1, the corresponding equation of the complete structure along the y-direction is

$$EIu^{iv} - GAu'' = \omega^2 mu \tag{3}$$

where $I = \Sigma I_j$ and $GA = \Sigma\, GA_j$

The frequencies of the system expressed by Eq. (3) can be evaluated from the formula:

$$\omega = \lambda_1 \lambda_2 \sqrt{\frac{EI}{mH^4}} \tag{4}$$

where $\lambda_1 = \beta\lambda_o$, $\lambda_2 = \sqrt{\left(\lambda_1^2 + (\alpha H)^2\right)}$

$$\alpha H = \sqrt{GA/EI}\,H \tag{5}$$

λ_o =1.875, 4.694, 7.855 for the first three modes of vibration.

The corresponding non-dimensional shapes of vibration are

$$\Psi = \cos\lambda_1 s - \cosh\lambda_2 s - Q(\sin\lambda_1 s - (\lambda_1/\lambda_2)\sinh\lambda_2 s), \tag{6}$$

where s=z/H, $Q = \dfrac{\cos\lambda_1 + (\lambda_2/\lambda_1)^2\cosh\lambda_2}{\sin\lambda_1 + (\lambda_2/\lambda_1)^2\sinh\lambda_2}$

In the expressions above, *H* is the height of the structure and *β* is a coefficient that may be assumed equal to unity when *αH* is less than 6. For higher values of *αH*, the accurate value of *β* is

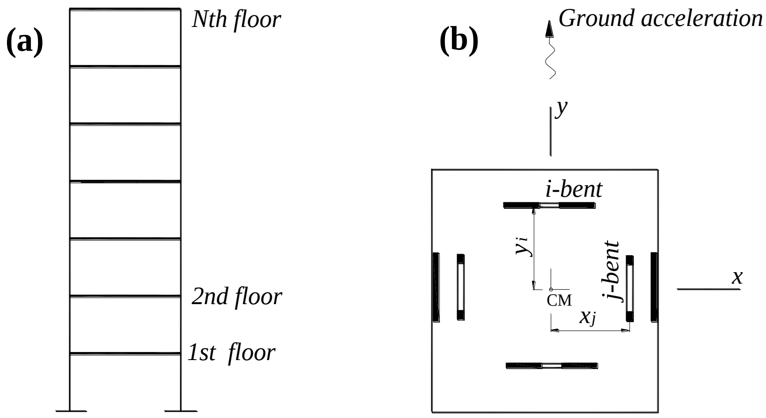

Fig. 1(a) Elevation of a uniform multi-story building with ;(b) symmetrical floor plan

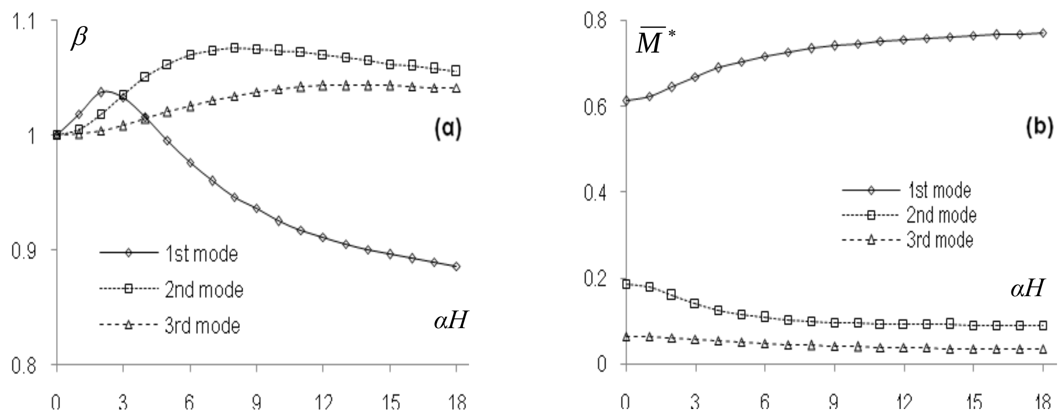

Fig. 2 (a) Parameter β and; (b) effective mass ratios the first three modes of vibration

required, especially when the fundamental frequency is to be estimated (Heidebrecht 1975). Exact values of β for the first three modes of vibration (Georgoussis 2006) in relation to the parameter αH, are shown in Fig. 2(a). Similarly, the effective modal masses (Georgoussis 2009), given by formula:

$$M^* = \frac{\left(m\int_0^1 \Psi ds\right)^2}{m\int_0^1 \Psi^2 ds} \tag{7}$$

are shown in Fig. 2(b), as ratios of the total mass (mH), i.e.: $\overline{M}^* = M^* / mH$

In the earthquake analysis of linear systems, the aforesaid dynamic properties (frequency, effective modal mass) of the symmetrical building of Fig. 1, for any k-mode of vibration (k=1,2,..), are adequate to provide the peak modal base shear through a design acceleration spectrum. For the case of a ground excitation along the y-direction, this peak shear represents the response of an

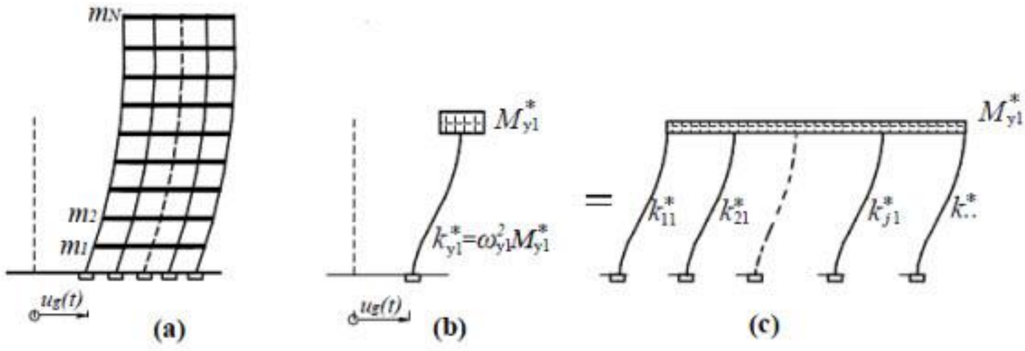

Fig. 3 (a) Typical first mode deformation profile of a multi-story building; (b) the first mode single-story system with the modal stiffness; (c) the contribution of each bent to the modal stiffness

equivalent single-degree-of-freedom (SDOF) system which has a mass equal to M^*_{yk} and stiffness equal to

$$k^*_{yk} = \omega^2_{yk} M^*_{yk} \tag{8}$$

In Eq. (8) ω_{yk} represents the k-mode frequency of the building along the y-direction and M^*_{yk} is the corresponding modal mass. The aforesaid SDOF, for the first mode of vibration of the multi-story building of Fig. (3a), is shown in Fig. 3(b)).

It is of particular interest to note that:

(i) Making the hypothesis that the y-direction lateral stiffness of the assumed building is provided exclusively by the j-bent (j=1,2..), which possess a stiffness ratio

$$\alpha_j H = \sqrt{GA_j / EI_j}\, H \tag{9}$$

that is assumed to be less than 6, the corresponding frequencies, for the first three modes of vibration, can be derived from Eq.(4) by replacing αH with $\alpha_j H$ and assuming that β is equal to unity. In fact, this element frequency, ω_{jk}, denotes the frequency of a subsystem which has the same mass as the actual structure but its lateral stiffness depends entirely on the j-bent.

(ii) Simple mathematic calculations show that the sum of the squares of the aforementioned ω_{jk} (j=1, 2, ..) element frequencies is equal to the square k-mode frequency of the building, i.e.:

$$\omega^2_{yk} = \Sigma \omega^2_{jk} \tag{10}$$

Eq. (10) demonstrates the contribution of the individual j-bent to the total modal stiffness of the complete structure. That is, each j-bent (subsystem) contributes to the modal stiffness of Eq. (8) in proportion to its square frequency ω_{jk}. For example, considering the first mode of vibration (k=1) of the building shown in Fig. 3(a) and its modal oscillator of Fig. 3(b), the j-bent represents an element of this oscillator (Fig. 3(c)) with a stiffness equal to

$$k^*_{jk} = \omega^2_{jk} M^*_{yk} \tag{11}$$

This technique has been used in the past by the author (Georgoussis 2009, 2010) to extend the analysis introduced by Kan and Chopra (1977a) on proportionate buildings to mixed-bent-type structures. At present, a modified expression is proposed for the evaluation of the above *k*-mode stiffness of the *j*-bent (subsystem), to cover a wider range of combinations of dissimilar bents having stiffness ratios $\alpha_j H$ much higher than 6. The objective is to increase the accuracy of the aforementioned methodology to practically any combination of dissimilar bents. This modification is based on the following considerations:

1. The equality shown by Eq. (10) is based on the assumption that the coefficient β in the first of Eqs. (5) is equal to unity. For stiffness ratios $\alpha_j H$ much higher than 6, the coefficient β may receive different values (Fig. 2(a)) and this makes the sum $\Sigma\omega^2_{jk}$ an approximation of ω^2_{yk}. In fact, for the first mode of vibration this sum is a lower bound of ω^2_{yk} as it represents Southwell's formula (Newmark and Rosenblueth, 1971) which provides a lower estimate of the fundamental frequency of the structure.
2. What is really needed is a better estimate of the modal stiffness of the *j*-subsystem (bent). According to its description above, the modal stiffness of such a subsystem, for the *k*-mode of vibration, is equal to

$$k^*_{jk} = \omega^2_{jk} M^*_{jk} \tag{12}$$

where M^*_{jk} is now the *k*-mode effective mass of the particular *j*-subsystem, depending on the $\alpha_j H$ stiffness ratio. In the general case, M^*_{jk} is different from M^*_{yk} and this can be seen in Fig. 2(b), where it is shown that different values of αH provide different effective modal mass ratios.

3. As the total lateral stiffness of the structure is equal to sum of the stiffnesses of the various *j*-bents which constitute its resisting system (Eq. (1)), it is appropriate to approximate the *k*-mode stiffness of this structure, given by Eq. (8), by the sum of the modal stiffnesses of the corresponding subsystems (Eq. (12)). Expressing Eq. (12) as

$$k^*_{jk} = \omega^2_{jk} M^*_{jk} = \omega^2_{jk} \frac{M^*_{jk}}{M^*_{yk}} M^*_{yk} = \overline{\omega}^2_{jk} M^*_{yk} \tag{13}$$

a modified formula is obtained to assess the squared *k*-mode frequency of the complete system, as

$$\omega^2_{yk} = \Sigma\overline{\omega}^2_{jk} \tag{14}$$

and further, this formula indicates that the contribution of the *j*-bent to the modal stiffness of Eq. (8) is proportional to its squared effective frequency $\overline{\omega}^2_{jk}$

In conclusion, Eq. (13) is similar to Eq. (11), but it takes into account the effect of the mode shape of a particular bent on the corresponding effective modal mass. The sum of the modal stiffnesses of the various bents, given by Eq. (13), is approximating the modal stiffness of the complete structure k^*_{yk} more accurately than the sum of the Eq. (11), and this approach makes the approximate method presented by the author in earlier papers more accurate. As shown in the next sections, dynamic data of symmetric and asymmetric building structures (periods and base resultant forces) of higher accuracy are derived when the element effective frequencies, $\overline{\omega}_{jk} = \omega_{jk}\sqrt{M^*_{jk}/M^*_{yk}}$, instead of ω_{jk} (j = 1,2,..), are used in the approximate method.

3. Frequencies of symmetrical buildings by the modified procedure

Consider the wall-frame assembly shown in Fig. 4. The two individual subsystems (wall and frame), although they belong to the same family of shear-flexure cantilevers, have different vibration shapes (shear walls deflect in a flexural mode in the lower part -concavity downwind-, while rigid frames deflect in a shear mode in the upper part -concavity upwind). The effect of these profiles can be seen on the effective modal mass ratios of Fig. 2(b). The shear wall (W) has a stiffness ratio $\alpha_w H$ equal to zero, while the corresponding ratio $\alpha_f H$ of common frames(F) may receive values higher than 15. The dual system shown in Fig. 4 represents a combination of dissimilar bents which produces an assembly (W+F) of different mode shapes from those of the individual bents.

The wall of Fig. 4 is assumed to have a cross section of 30 × 400 cm and the frame is consisting of two columns 60 × 60 cm, at a distance of 5m, connected by beams 25x60cm at floor heights equal to 3.5 m. The frame shear rigidity was calculated by the formula provided by Heidebrecht and Stafford Smith (1973). Five different heights, H, of the example structure are considered: 14 m (4 stories), 24.5 m (7 stories), 35 m (10 stories), 45.5 m (13 stories) and 56 m (16 stories). In all cases $\alpha_w H$=0, while for the frame the stiffness ratio $\alpha_f H$, for the different building heights, receives respectively the values 4.94, 8.64, 12.34, 16.05 and 19.75. For the wall-frame assembly the corresponding αH ratios are respectively equal to 0.57, 1.00, 1,42, 1.85 and 2.28. In Fig. 5 are shown the first mode frequency ratios $(\omega_m/\omega)_{app}$ and $(\omega_e/\omega)_{app}$ derived by the approximate method of the continuous medium. Frequency ω is determined by means of Eq. (4) on the grounds of the αH ratios, taking into account the exact value of coefficient β (Fig. 2(a)). Frequencyω_e is given from Eq. (10) on the grounds of the individual $\alpha_w H$ and $\alpha_f H$ ratios and using again the exact values of the correspondingβ coefficients (procedure suggested in earlier papers).

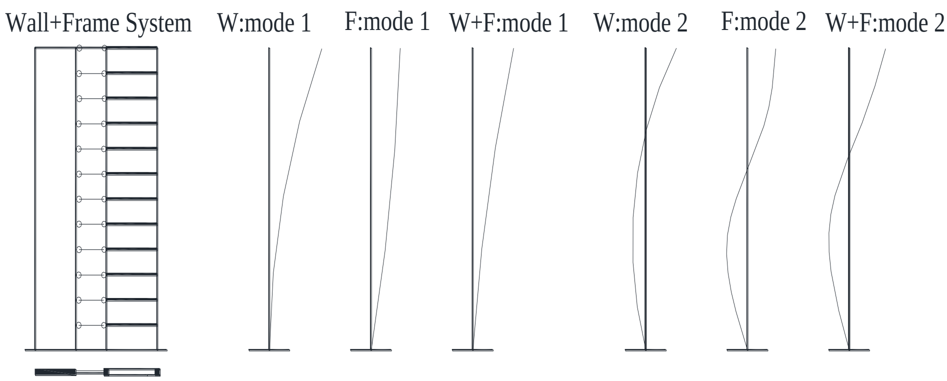

Fig. 4Wall-frame assembly; mode shapes of individual bents and the dual systemstiffness

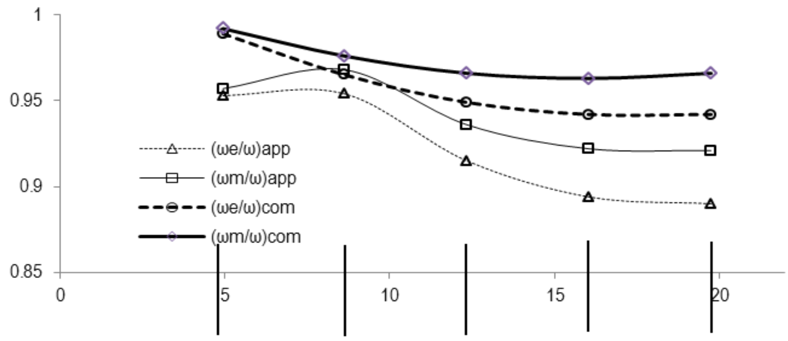

Fig. 5First mode frequency ratios

Frequencyω_m is obtained through Eq. (14) as suggested by the present modified procedure. It is evident that when the dissimilarity of the combined bents is increasing (denoted by higher values of $\alpha_f H$) the difference between the two procedures is enlarged: the thin solid line $(\omega_m/\omega)_{app}$ deviates from the thin dotted line $(\omega_e/\omega)_{app}$ coming closer to unity. This demonstrates the superior accuracy of the proposed modified method.

In this figure are also shown the aforementioned frequency ratios ($(\omega_m/\omega)_{com}$ and $(\omega_e/\omega)_{com}$) derived by analyzing the assumed structural models by the accurate SAP2000-V11 computer program. The analysis was performed by assuming a typical floor mass equal to 50 kNs^2/m and a modulus of elasticity equal to 25 × 10^{6}kN/m^2. These ratios are closer to unity, but the proposed modified procedure (thick solid line) presents again data of superior accuracy compared with that of earlier papers (thick dotted line). The frequency ratios $(\omega_m/\omega)_{app}$ and $(\omega_e/\omega)_{app}$ for the next two modes of vibration (second and third) are very close to unity with an error of less than 3.2% for the approximate procedure and 1.5% for the discrete models (SAP2000 results).

4. Analysis of mono-symmetric buildings by the modified procedure

Having introduced the concept of the element effective frequency as above, the modified procedure for estimating periods and peak values of base resultant forces of uniform buildings with simple asymmetry (Georgoussis 2009), is implemented in brief by the following steps:

(i) Expressing the modal stiffness of any *i*-bent in the x-direction in a similar manner, e.g.:

$$k_{ik}^{*} = \omega_{ik}^{2} M_{ik}^{*} = \omega_{ik}^{2} \frac{M_{ik}^{*}}{M_{yk}^{*}} M_{yk}^{*} = \overline{\omega}_{ik}^{2} M_{yk}^{*} \tag{15}$$

where ω_{ik}, M_{ik}^{*} are the *k*-mode frequency and effective mass respectively of the subsystem which has the same mass as the actual structure but its stiffness depends entirely on the *i*-bent, and $\overline{\omega}_{ik}$ is its corresponding effective frequency.

(ii) Constructing the undamped equation of motion of the *k*-mode single-story system. For a ground motion along the y-direction, this system has a mass equal to M_{yk}^{*} and it is supported by elements having stiffnesses given from Eqs. (13) and (15). Its equation of motion, in a coordinate system with the origin at the center of mass(Fig. 6a), is as follows:

$$\mathbf{M}_k^* \ddot{\mathbf{U}}_k + \mathbf{K}_k^* \mathbf{U}_k = -\mathbf{M}_k^* \mathbf{1} \ddot{u}_g \tag{16}$$

where

$\mathbf{M}_k^* = M_{yk}^* \begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}$ the effective k-mode mass matrix(r: the radius of gyration of the typical floor)

$\mathbf{U}_k = \langle u_k \quad \theta_k \rangle^{\mathrm{T}}$ is the corresponding modal displacement vector at CM

$\mathbf{K}_k^* = \begin{bmatrix} k_y^* & k_{yw}^* \\ k_{wy}^* & k_w^* \end{bmatrix}_k$ is the effective k-mode stiffness matrix

$\mathbf{1}^{\mathrm{T}} = \langle 1 \quad 0 \rangle^{\mathrm{T}}$ is the unit matrix, and (17)

$$k_{yk}^* = \Sigma k_{jk}^* = \Sigma(\omega_{jk}^2 M_{jk}^*) = M_{yk}^* \Sigma \overline{\omega}_{jk}^2$$

$$k_{wk}^* = \Sigma x_j^2 k_{jk}^* + \Sigma y_i^2 k_{ik}^* = M_{yk}^* \Sigma(x_j^2 \overline{\omega}_{jk}^2 + y_i^2 \overline{\omega}_{ik}^2)$$

$$k_{ywk}^* = k_{wyk}^* = \Sigma x_j k_{jk}^* = M_{yk}^* \Sigma x_j \overline{\omega}_{jk}^2$$

Full description of the modal quantities derived by this analysis is given in Georgoussis, 2009. It is worth noticing here that the coupled Eqs. (16), for the first mode (k=1) single-story system, provide the response quantities of the first two modes of vibration. Therefore, when the stiffness matrix of Eqs. (16), for k=1,is decoupled,the first two modes of vibration (translational and rotational) are decoupled and in the case of a ground motion along the y-direction the response for a low height building will be practically translational. The stiffness matrix of Eq. (16) is decoupled when the term $k_{ywk}^* (= k_{wyk}^*)$ is equal to zero. In fact, this condition specifies that the first mode center of rigidity (m_1-CR) of the corresponding single-story system coincides with CM. Therefore, in medium or low height structures, where the first two modes of vibration determine their response, a structural arrangement in which the location of m_1-CR coincides with CM, is expected to sustain a practically translational response. Generally, the x-coordinate of m_1-CR is given as

$$x_{m1-CR} = \frac{\Sigma(x_j \overline{\omega}_{j1}^2)}{\Sigma(\overline{\omega}_{j1}^2)} \tag{18}$$

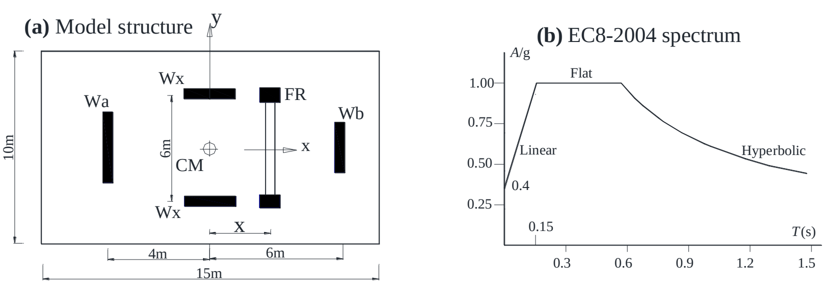

Fig.6 (a) Example structure; (b) Code-recommended design spectrum

5. Systems analyzed

To illustrate the application and accuracy of the proposed method, the example structure shown in Fig. 6(a) was analyzed. This is an 8-story monosymmetric uniform building structure,with a floor plan 15x10m,composed by dissimilar bents:two structural walls (Wa and Wb) and a moment resisting frame (FR) are aligned along the y-direction and a pair of wall bents (Wx) is oriented in the axis of symmetry. The structural walls Wa and Wb are of cross sections 30 × 500 cm and 30 × 400 cm respectively, while the moment resisting frame FR consists of two 80x80cm columns, 6m apart, connected by beams of a cross section 35 × 70 cm. The x-direction wall bents Wx are of the same dimensions as Wb and they are located symmetrically to CM at distances of 3m. The total mass per floor is m=120kNs^2/m, the radius of gyration about CM is r = 5.204 m, the story height is 3.5 m and the modulus of elasticity (E) is assumed equal to 25 × 10^6 KN/m^2. The center of mass at each floor lies on a vertical line passing through the centroid of the orthogonal floor plan at each level. To investigate the accuracy of the proposed method to a broader range of building structures, different structural configurations of the example structure are examined as follows: walls Wa and Wb are assumed to be located at a fixed positions, the first on the left of CM in a distance equal to 4m and the second on the right of CM at a distance of 6 m, while the frame FR is taking all the possible locations along the x-axis.

At first the elastic response of the assumed models is examined. The accuracy of the proposed modified approximate procedure to predict periods of vibrations and base resultant forces, in the case of a dynamic excitation (along the y-direction) characterized the EC8-2004 recommended response spectrum (Fig. 6(b)) is investigated by comparison with the results derived by means of the computer program SAP2000-V11. The resultsare also compared with those obtained by the approximate methodology presented by the author in earlier papers (Georgoussis 2009, 2012). In practical terms, the difference between the two approximate procedures is based on the grounds that at present the formulation of Eq. (16) is based on the element effective frequencies $\overline{\omega}_{jk}$ and $\overline{\omega}_{ik}$, while in the older version this formulation is based on the real frequencies of the various bents ω_{jk} and ω_{ik} .To apply the proposed method, the first pair of frequencies of the various bent-subsystems is required, and also their effective modal masses. Denoting with M the total mass of the structure (M = 8m = 960kNs^2/m), these quantities for wall Wa are as follows:

ω_{wa1}=5.922/s, ω_{wa2}=34.278/s and $\overline{M}^*_{wa1} = M^*_{wa1}/M = 0.66$, $\overline{M}^*_{wa2} = 0.212$.

For wall Wb (and Wx): ω_{wb1} = 4.261/s, ω_{wb2} = 25.397/s and $\overline{M}^*_{wb1} = 0.658$, $\overline{M}^*_{wb2} = 0.208$.

For frame FR: ω_{f1}=3.529/s, ω_{f2}=11.771/s and $\overline{M}^*_{wb1} = 0.774$, $\overline{M}^*_{wb2} = 0.116$

The first two effective modal masses of the uncoupled structure, in the y-direction, normalized with respect to the total mass, are respectively equal to $\overline{M}^*_{y1} = M^*_{y1}/M = 0.668$ and $\overline{M}^*_{y2} = 0.202$. From these data, the element effective frequencies of all the bents shown in Fig. (6a)can be determined. For wall Wa: $\overline{\omega}_{wa1} = 5.886/s$, $\overline{\omega}_{wa2} = 35.116/s$ .For wall Wb/Wx: $\overline{\omega}_{wb1} = 4.229/s$, $\overline{\omega}_{wb2} = 25.771/s$ and for frame FR: $\overline{\omega}_{f1} = 3.798/s$, $\overline{\omega}_{f2} = 8.920/s$.

The inelastic response of the assumed model structures was investigated under two characteristic ground motions (Kobe 1995, component KJM000 and Friuli 1976, component Tolmezzo E-W*)*, selected from the strong ground motion database of the Pacific Earthquake Engineering Research (PEER) Center (hppt://peer.berkely.edu) and scaled to a PGA=0.5g (unidirectional excitations along the y-axis). For all the possible locations of FR, inelastic analyses, by means of the computer program SAP2000-V11, were performed to evaluate top rotations and base shears and torques. All resisting elements (bents) are assumed to have only in-plane stiffness and their strength assignment is based on a planar static analysis under an external lateral loading with floor forces having the shape of the 'inverted triangle' and summing to a base (design) shear equal to V_d = 2400kN (approximately equal to 25% of the total weight of the structure).

More specifically, allowing for plastic hinges at the bases of walls Wa and Wb and detailing frame FR according to the strong column-weak beam philosophy (that is, allowing plastic hinges at the ends of the beams and at the foot of the ground floor columns), this static analysis leads to the following results: (i) the bending (yield) capacity at the plastic hinges at the base of walls Wa and Wb are respectively equal to 25025 and13475kNm and, (ii) the bending (yield) capacity of the plastic hinges at the ends of the beams of FR (from the top downwards) is equal to 519, 607, 594, 583, 544, 475, 368, and 222kNm respectively, while the corresponding capacity at the plastic hinges at the base of the ground columns of FR equals 383kNm. The strength of walls Wx, aligned in the y-directon, is the same as Wb. All the nonlinear response history analyses were performed by means of the program SAP2000-V11, using inelastic link elements at the assumed locations of plastic hinges. The moment-rotation relationships of these elements were assumed bilinear with a post-yielding stiffness ratio of the generalized load-deformation curve, equal to 4%. The aforesaid analyses were performed using the numerical implicit Wilson-θ time integration method, with the parameter θ taken equal to 1.4.

6. Discussion of results

The elastic response of the assumed models is shown in Figs. 7 and 8.The first four periods of vibration of the example structures of Fig. 6(a), computed by the proposed modified method (T_m: green lines) for different locations of the frame system FR (indicated by the normalized coordinate $\bar{x} = x/r$), are shown in Fig. 7, together with the accurate computer values (T_{com}: black lines) and those obtained by the methodology presented by the author in earlier papers (T_{ea}: red lines). For the first (thick solid lines), the third (thin solid lines) and the fourth (thin dotted lines) mode of vibration, both the approximate procedures are very close to the accurate computer ones. For the second mode of vibration (thick dotted lines) the proposed modified procedure presents values closer to the accurate ones with an error less than 3.2%, while the error of the older procedure is higher reaching a value 6%. It is worth noting here that the first pair of the approximate periods (first and second) is derived from the first mode (k = 1) equivalent single story system, while the second pair of the approximate periods (third and fourth) is obtained from the second mode (k = 2) equivalent single story system.

Base shears (in the y-direction) and torques, for the case of the spectrum of Fig. 6(b), are shown in Fig. 8. Normalized shears and torques $\overline{V}_m$ and $\overline{T}_m$ (green lines) represent the approximate peak results obtained by the proposed modified procedure, through the CQC rule, on the grounds of the peak modal data derived from the analysis of the first (k = 1) and second mode (k = 2) equivalent single-story systems. The aforesaid base shears are normalized in respect to the total shear, V_o, of the uncoupled structure and the mentioned torques are normalized in respect to the product: rV_o. In

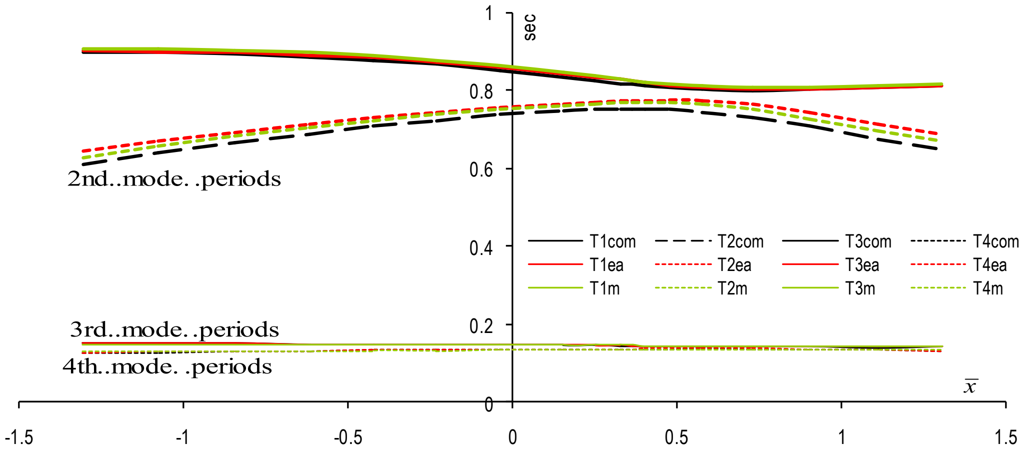

Fig. 7 Periods of vibration of the example structure

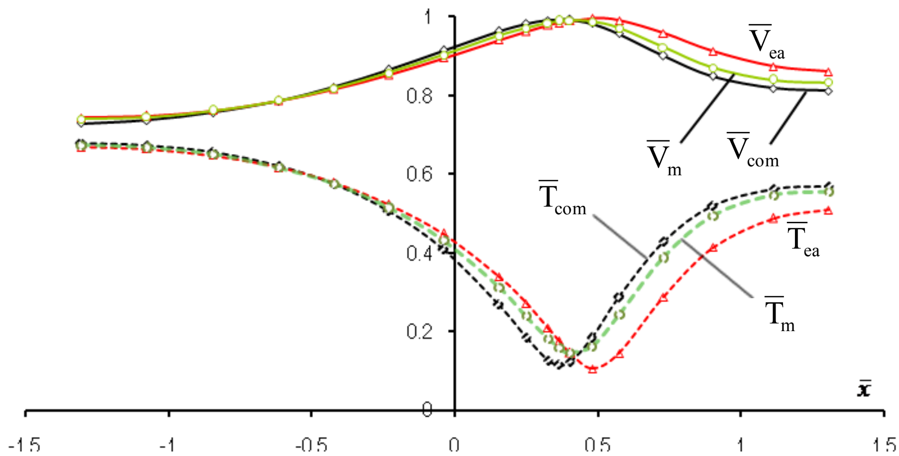

Fig. 8 Peak base shears and torques for the case of EC8-2004 spectrum

a similar manner the data $\overline{V}_{ea}$ and $\overline{T}_{ea}$ (red lines) are derived on the grounds of the methodology presented in author's earlier papers.

In Fig. 8 are also shown the accurate data $\overline{V}_{com}$ and $\overline{T}_{com}$ (black lines) given by the computer program SAP2000-V11 on the basis of the first 12 peak modal values combined according to the CQC rule (the damping ratio in each mode of vibration was taken equal to 5%).Envisaging this figure it can be seen that the proposed modified procedure provides data (green lines) closer to those of the computer analysis (black lines) than the data of the approach presented by the author in earlier papers (red lines). The location of frame FR, which predicts anm$_1$-CR point coincident with CM, is equal to $\bar{x} = x/r = 0.41$. The older procedure provides a value equal to $\bar{x} = 0.48$.The

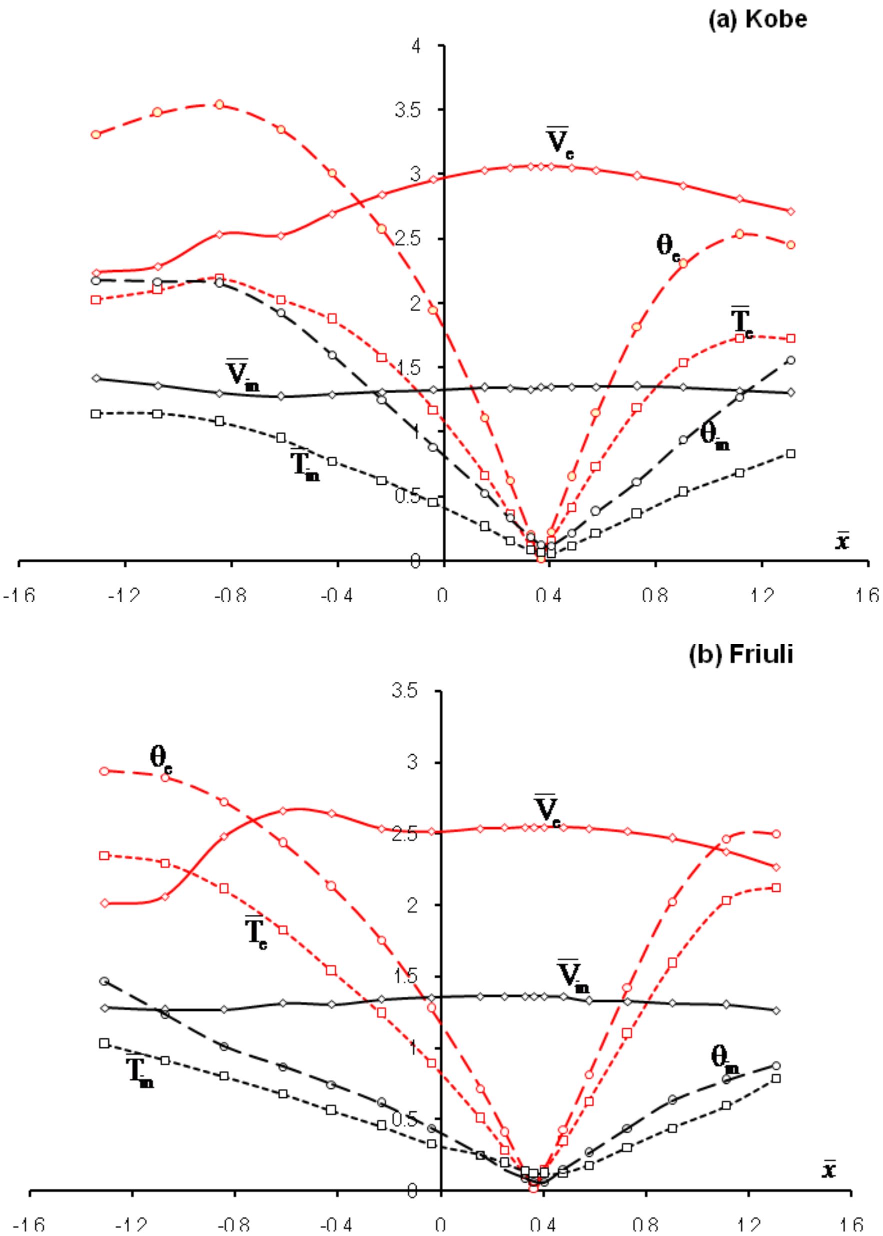

Fig. 9 Top rotations (x10^{-2}, rads) and normalized base shears and torques under ground motions

aforesaid numerical results, shown in Figs. 7 and 8, are in agreement with those of different structural configurations presented by the author and his collaborators in a recent conference paper (Georgoussis *et al*. 2013a).

The response of the inelastic model structures, as described in the previous section, under the

Kobe 1995 (component KJM000) and Friuli 1976 (component Tolmezzo E-W) excitations, are shown in Fig. 9. In order to compare elastic and inelastic behaviors, the elastic responses of the assumed models under the same excitations are also presented in this figure. Three response parameters, obtained by time history analyses assuming a 5% damping ratio, are shown: top rotations, θ, normalized base shears and normalized base torques. The red lines represent the peak elastic response (top rotations: θ_e, are shown by dashed lines, normalized base shears: $\overline{V}_e = V_e/V_d$ by solid lines and normalized base torques: $\overline{T}_e = T_e/rV_d$ by dotted lines) and the corresponding black lines represent the peak inelastic behavior (θ_{in}, $\overline{V}_{in} = V_{in}/V_d$, $\overline{T}_{in} = T_{in}/rV_d$).The response of the inelastic systems is smoother and the overall rotational behavior is smaller than that obtained by the elastic behavior. This finding confirms observations on single story systems that after yielding asymmetric systems have the tendency to deform further in a translational mode (e.g. Kan and Chopra, 1981; Ghersi and Rossi, 2001). Minimum elastic rotational response is obtained when the frame FR is located at $\bar{x} = 0.37$, while for the inelastic systems such a response is observed when FR is located at $\bar{x} = 0.40$.This finding is in agreement with that observed by the author and his collaborators in a recent paper on different structural configurations (Georgoussis *et al*. 2013b). Note, that at such locations of FR, the mass axis is practically passing through the first mode center of rigidity, implying that the elastic response of the system along the y-direction is virtually translational. As the strength distribution has been determined by a planar static analysis, this response results in an almost in-phase yielding of the bents aligned in the y-direction, leading to a minimum rotational response.

It is worth reminding here a comment by Lucchini *et al*. (2009) concerning the behavior of single story buildings: their nonlinear response depends on how the building enters the nonlinear range, which in turn depends on its elastic properties (i.e., the stiffness and mass distributions), and on the capacities of its resisting elements (i.e., the strength distribution). The numerical results, shown in Fig. 9, are in agreement with Lucchini’s comment.

7. Conclusions

Frequencies and basic earthquake response (resultant base shears and torque) of eccentric, medium height uniform buildings, composed by dissimilar bents, can be estimated from the analysis of two equivalent, single-story modal systems, the masses of which are determined from the first two vibration modes of the uncoupled multi-story structure and the stiffnesses of the resisting elements are determined from the corresponding individual bents when they are assumed to have, as planar frames, the mass of the complete structure. The proposed elastic analysis, as demonstrated by a limited amount of numerical results, improves the accuracy of the methodology developed in the past and provides the location of the first mode center of rigidity with superior accuracy. The main property of this point is that when it lies on the mass axis, the response of elastic building structures is basically translational. Therefore, as it is quite easy to determine this point with simple hand calculations, the proposed procedure can be used as a guideline in the preliminary stage of a structural application to determine the optimum structural arrangement in terms of minimum torsional response.

Another important feature of the seismic response of common building structures, when the first mode center of rigidity lies on the mass axis, is that their virtually translational elastic behavior is preserved in the inelastic phase when the strength assignment of the resisting bents is derived from a planar static analysis under a code lateral loading. This is demonstrated in common 8-story buildings under two characteristic ground motions.

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