Progressive collapse vulnerability in 6-Story RC symmetric and asymmetric buildings under earthquake loads

Somayyeh Karimiyan^{*1}, Ali Husseinzadeh Kashan² and Morteza Karimiyan³

¹Department of Earthquake Engineering, Taha University, Tehran, Iran

²Department of Industrial Engineering, Faculty of Engineering, Tarbiat Modares University, Tehran, Iran. ³Applied Science and Technology University, Tehran, Iran.

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Abstract. Progressive collapse, which is referred to as the collapse of the entire building under local damages, is a common failure mode happened by earthquakes. The collapse process highly depends on the whole structural system. Since, asymmetry of the building plan leads to the local damage concentration; it may intensify the progressive collapse mechanism of asymmetric buildings. In this research the progressive collapse of regular and irregular 6-story RC ordinary moment resisting frame buildings are studied in the presence of the earthquake loads. Collapse process and collapse propagation are investigated using nonlinear time history analyses (NLTHA) in buildings with 5%, 15% and 25% mass asymmetry with respect to the number of collapsed hinges and story drifts criteria. Results show that increasing the value of mass eccentricity makes the asymmetric buildings become unstable earlier and in the early stages with lower number of the collapsed hinges. So, with increasing the mass eccentricity in building, instability and collapse of the entire building occurs earlier, with lower potential of the progressive collapse. It is also demonstrated that with increasing the mass asymmetry the decreasing trend of the number of collapsed beam and column hinges is approximately similar to the decreasing trend in the average story drifts of the mass centers and stiff edges. So, as an alternative to a much difficult-to-calculate local response parameter of the number of collapsed hinges, the story drift, as a global response parameter, measures the potential of progressive collapse more easily.

Keywords: progressive collapse mechanism; asymmetric reinforced concrete buildings; earthquake load

1. Introduction

Progressive collapse mechanism in a structure means the collapse of a large portion or the build ing which is initiated by the propagation of local damages in such a way that the structural system cannot bear the main structural loads (Ellingwood 2006). Vehicular collision, accidental overload, aircraft impact, design/construction error, fire, gas explosions, bomb explosions, hazardous materi als, etc are recognized as a number of abnormal loads which can potentially be the trigger of progr essive collapse in the various buildings (Somes 1973), (Burnett 1975). Column removal can be cau

^aAssistant Professor., E-mail: a.kashan@modares.ac.ir

^{*}Corresponding author, Ph.D., E-mail: s.karimiyan@iiees.ac.ir

^bM.Sc., E-mail: info@karimiyan.com

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sed by the earthquake load, too (Ettouney *et al.* 2012, Pekau and Cui 2005, Lu *et al.* 2008a and 20 08b, Wibowo and Lau 2009, Kim *et al.* 2011, Yi *et al.* 2008 and 2011, Li *et al.* 2012, Nateghi and Parsaeifard 2013).

According to the current design specifications, 2D macro models of ten-story steel moment resi sting frames which seismically designed to bear high and moderate seismic risks have been develo ped to compare the progressive collapse mechanism using beam and column finite-element models. Results of the simulation demonstrated that the progressive collapse potential in the frames which have been designed for moderate seismic risk, is more than those designed for high seismic risk. T o have a better resistance to progressive collapse, system strength is more effective than the improved ductile detailing in the structural systems and also the alternate path method was recognized as a valuable approach against the progressive collapse potential (Khandelwal *et al.* 2012).

Macro model-based simulation method is a 2D practical approach to evaluate the progressive c ollapse potential of the RC moment resisting frame buildings in different seismic zones (Bao *et al.* 2012). This procedure was compared in RC frames which had been designed based on low, moder ate and high seismicity zone provisions. According to the results, using special reinforced concrete moment resisting frame in the structures and designing according to the high seismic risk provisio ns are more effective ingredients than the RC moment frames which were designed based on low o r moderate seismicity zone provisions.

Gurley (2012) investigated the earthquake resistance to progressive collapse and the collapse me echanisms through comparing double span mechanisms in GSA guidelines and sway collapse mec hanisms in the earthquake engineering, subjected to the columns removal resultant by the explosio n loads. According to the results, earthquake damages can eliminate the load bearing elements fro m the structural system almost similar to the explosion loads. Therefore, it is justifiable to evaluate the progressive collapse mechanism in the presence of earthquake loads. Assessment of the ductil e detailing for the removed columns demonstrated that designing against the earthquake loads and including the double span mechanisms at the removed columns, can improve the structural system resistance to the progressive collapse potential of the buildings.

Alternate path method was used to evaluate the progressive collapse potential of the buildings (Ettouney *et al.* 2012). Assessment of the general stability in sway and non sway frames which are equipped with the lateral load resisting elements demonstrated that taking the global response of the damaged building in to account is essential to investigate the progressive collapse mechanism. Besides, the progressive collapse evaluations should be applied to the explosion and seismic loads.

The alternate path method was investigated in two typical integrated cast-in-situ slab and non-integrated slab with various levels of the seismic fortification to evaluate the potential of the progressive collapse (Yi *et al.* 2011). The results of the NLTHA showed that framed beams contribute to resist against the progressive collapse due to the local damages in different locations of the buildings. Inadequacy of the lateral stiffness in columns causes the collapse horizontally distributed through the stories. Investigation of the slab effects and the various levels of the seismic design demonst rated that the catenary action provides resistance to progressive collapse potential of RC buildings in the continuous beams with an adequate horizontal constraint at both ends.

Tsai and Lin (2008) studied the relationship between the earthquake-resistant reinforced concre te structures and the progressive collapse potential of RC buildings based on GSA guidelines. Res ults of the linear static, nonlinear static and nonlinear dynamic analyses under columns removal in various locations illustrated that GSA criteria should be different for two various nonlinear analyse s. A dynamic amplification factor equal to 2 causes the nonlinear static approach to be conservativ e. According to the results, it is necessary to consider the dynamic amplification factor in the inela

stic dynamic effects of the GSA linear method and it is possible to predict the progressive collapse mechanism of RC buildings subjected to the columns removal.

Pekau and Cui (2005) simulated the progressive collapse mechanism of the 12-story, 3-bay precast panel shear wall subjected to the earthquakes loads with distinct element method (DEM) program. Results of shear ductility demand assessments in the mechanical connectors and integrity analyses showed that the precast panel shear wall can automatically provide the demands of shear slip in horizontal joints and shear ductility in vertical joints when it satisfies the seismic requirements.

To evaluate collapse of the RC buildings, a database of experimental test results in reinforced c oncrete beams, was developed by Lignos and Krawinkler (2012) to simulate the dynamic response of the RC elements. This database was used to quantify the main parameters of the cyclic moment -rotation relationship at plastic hinge regions subjected to the earthquake. The application of datab ase in the field of performance-based earthquake engineering has been successfully evaluated in co llapse assessment of the RC buildings.

Concentrated plasticity method in reinforced concrete buildings was compared in lumped plastic hinges, which were defined based on FEMA-356 and those defined according to the properties of reinforced concrete members in an eight-story and two four-story buildings under different ductilities (Eslami and Ronagh 2012). Results demonstrated that FEMA-356 hinges underestimate the str ength and the displacement capacity of the buildings with low ductility frames.

A number of models were developed by Biskinis and Fardis (2009) to calculate the moment-rot ation, secant stiffness at flexural yielding and the ultimate deformation in beams and columns according to the database of the experimental tests in RC members. Obvious and straightforward relations were presented which are independent from analyzing the moment-curvature. According to the results, these models are useful and valuable to evaluate the RC buildings.

Panagiotakos and Fardis (2009) developed relationships to specify the deformations of the RC s tructural elements at yielding or failure, based on the properties of the beam and column elements according to the results of experimental data on reinforced concrete beams and columns. Results s howed that the curvature formulations provide the results which are in accordance with the experimental tests results, but with a large and considerable scatter.

Kyakula and Wilkinson (2004) proposed an advanced spread plasticity model which determine s the start point of yielding, actual length of the yield zones and can transfer the points of contra fle xure throughout the beams with the assumption that columns and joints being remained elastic. Ac cording to the results, the advanced model enhances the precision of the global displacements, join t rotations and the inter story drift ratios up to 25%.

Many research papers in the field of progressive collapse mechanism have studied column rem oval subjected to the collisions or explosion loads (Kaewkulchai and Williamson 2003, Lew 2003, Bažant and Verdure 2007, El-Tawil *et al.* 2007, Sasani *et al.* 2007, Sasani and Sagiroglu 2008, Sasani and Kropelnicki 2008, Talaat and Mosalam 2009, Khandelwala *et al.* 2009, Masoero *et al.* 2010, Hayes Jr *et al.* 2012, Helmy *et al.* 2012, Hafez *et al.* 2013, Karbassi and Nollet 2013, Orton and Kirby 2013, Stinger and Orton 2013). However, there are a few resear ches that have studied the 3D progressive collapse mechanism of the buildings in the presence of the earthquake loads and torsion effects. Rotational friction dampers were evaluated for resistance t o progressive collapse in the presence of the earthquake loads (Kim *et al.* 2011). Progressive collapse mechanism of the RC frames and RC shear walls were numerically simulated using fiberbeam-element and multi-layer-shell-element models under earthquake loads (Lu *et al.* 2008a, Lu *et al.* 2013). The simulation of nonlinear behavior of reinforced concrete structural elements were

done via considering the cyclic behavior under coupled bending moment- axial force and shear force, contact among the structural elements during the collapse and breakdown of the structural elements at ultimate states (Lu *et al.* 2008c, Lu *et al.* 2011).

To compare the progressive collapse potential of the frames and buildings, 3D and 2D macro models were simulated with various assumptions by Alashker *et al.* (2011). Four typical 10-story steel buildings, seismically designed in 2D and 3D macro models, were investigated under columns removal. According to the results, floor systems have considerable effects on the collapse distribution in 3D macro models. Because of the floor system contribution effects, 3D models were valuable and conservative in progressive collapse assessment in comparison with the 2D models.

Progressive collapse potential of the regular and irregular buildings has been investigated in 30story symmetric and asymmetric structural models once with RC cores and once again with braced cores as a result of the columns removal (Kim and Hong 2011). Results showed that the potential of progressive collapse varies according to the variation of the removed column locations. If the location of the removed column is in the tilted side of the structures, the potential of progressive collapse is increased in the asymmetric buildings. The plastic hinges formed in the removed column bays and adjacent bays demonstrated that the other elements of the structural system involve in resisting against the progressive collapse. Therefore, because of the contribution effects of other elements, the potential of progressive collapse in irregular buildings were not very large in comparison with the regular buildings.

Karimiyan *et al.* (2013a) investigated the progressive collapse mechanism of 3-story RC symmetric and asymmetric buildings with different levels of mass eccentricity in the presence of earthquake loads. According to the results, the building behavior with respect to the number of beam and column collapsed hinges was observed similar to the trend of story drifts of the mass centers. Results also showed that the increase of story drifts of the stiff edges is relatively similar to those of the flexible edges and both are greater than those of the mass centers. Furthermore, with increasing the mass eccentricity from 0% to 25%, the potential of progressive collapse increases around 41% because of the resonance effects.

Seismic collapse propagation of 6-story RC symmetric and asymmetric buildings with different levels of mass eccentricity was investigated in Karimiyan *et al.* (2013b). According to the results, collapse propagation process depends on the level of the mass eccentricity and there are collapse distribution patterns which can be used to predict progressive collapse scenarios in beam and columns elements of the similar symmetric and asymmetric buildings. Results also showed that damage concentrates with a high probability in places with larger mass concentration.

Although there are many buildings collapsed by the earthquakes, the distribution of the collapse has not been studied clearly in seismic design or evaluation of the structures. In this study the propagation of collapse in presence of the earthquake loads is studied by continuing NLTHA even if a number of beam and column elements exceed their collapse limit state. It has been shown that the irregularity in plan of the buildings causes more concentration of damage in one side of those buildings. So, it is expected that the extent of asymmetry in a building increases the progressive collapse potential of the building. In the present study, to investigate the effect of asymmetric 6-story reinforced concrete ordinary moment resisting frame buildings are considered. First, a symmetric building is designed based on ACI (2005), and then by introducing mass eccentricities of 5%, 15%, and 25% in the symmetric structural model, asymmetric version of the model buildings are created. Then, they are analyzed using a set of 2-component earthquake records.

2. Modeling

The sample models considered in this research are 6-story symmetric and asymmetric RC ordinary moment resisting frame buildings (see Fig. 1). These models have 3 bays with span of 5m center to center in two directions X and Z. The height of the stories is 3.5m. The design dead load and live load are 5.3 KN/m^2 and 1.96 KN/m^2 , respectively.

The irregular structural models are derived from the regular model with changing the mass distr ibution in such a way that an equal one way mass eccentricity being produced in the X direction of the all floors. Four building models are studied in this research with various mass eccentricities of 0%, 5%, 15% and 25%.

A few special buildings have mass asymmetry values more than 25% and this fact motivated us to study the effect of 25% mass asymmetry in our study as the most amount of irregularity in the building.

6-story buildings which are used in many countries as residential, office and commercial mid rise buildings are considered in this study. Unfortunately, most of these buildings may be old and show a weak resisting ability. This may calls for presenting helpful criteria to evaluate the progressive collapse potential of the mid rise RC buildings.

The probability of collapse in ordinary moment resisting frame building is greater than those w hich are designed with special or moderate moment resisting frames. Consequently, ordinary mom ent resisting frame buildings are studied in this study to evaluate the potential of progressive collap se in comparison with the other building types. It is worth to declare that in this research, the infills effect is not taken into consideration and rigid diaphragm assumption has been made.

The parameter Fc in this research is constant and equal to 28 MPa according to the common design codes. In other words, the effect of various concrete strength values has not been studied in this research.

Longitudinal bars are the same in all structural elements of 4 regular and irregular buildings. In other words, beam and column sections are fixed and with increasing the mass eccentricity levels, there is no change in beam and column sections of all symmetric and asymmetric buildings. Therefore, structural elements in irregular buildings are same as the regular one. So, in this research, mass eccentricity levels are the main variables in the building models and with increasing the mass eccentricity levels, there is no change in the longitudinal bars of the structural elements of regular and irregular buildings.

Similar to the longitudinal bars, the transverse confinement bars are the same in all structural elements of 4 symmetric and asymmetric buildings. So, if longitudinal bars or transverse confinement bars changed, collapse mechanisms would change, accordingly.

The floor is considered as a rigid in plane of the slab and the effect of the floor slabs is considered in determining stiffness of the beams (Lignos and Krawinkler 2012, Lignos and Krawinkler 2013, FEMA P695 2009, Haselton *et al.* 2007, Haselton and Deierlein 2007, Haselton *et al.* 2008).

Previous researches have shown that the careful and proper selection of the element model is vital to conveniently simulate the collapse of the buildings (FEMA P695 2009). In 2005, Ibarra, Medina and Krawinkler developed the element model which is used to simulate the global collapse of the reinforced concrete frame buildings. Fig. 2 shows the modified Ibarra-Krawinkler virgin curve and the relevant definitions. The main aspects of the model such as the capping point, where monotonic strength loss begins and the post-capping negative stiffness, enable us to model the strain-softening behavior associated with the concrete crushing, rebar buckling and fracture or



Fig. 1 View of the structural models

bond failure (FEMA P695 2009).

Using fiber element models which can capture the cracking behavior and spread of plasticity throughout the element have been investigated to simulate the cyclic response of reinforced concrete beam and column elements. But according to the currently available researches, the fiber models are not capable to simulate the strain-softening behavior relevant to the rebar buckling and consequently, cannot reliably simulate the flexural collapse in RC frames (FEMA P695 2009, Haselton *et al.* 2007, Ibarra and Krawinkler 2005).

Also, currently available steel material models are not able to replicate the behavior of rebar as it buckles and fractures. Due to this limitation, current fiber models were judged inadequate for simulating collapse in RC and steel buildings (Ibarra *et al.* 2005, Haselton and Deierlein 2007, Haselton *et al.* 2007, and FEMA P695 2009).

In this research, concentrated plasticity has been used to describe the inelastic behavior with th e zero length rotational springs which are in both ends of the elastic beam and column elements, in such a way that the rotational behaviour of the plastic regions is based on the hysteretic response o f modified Ibarra, Medina and Krawinkler deterioration model. The rotational behaviour of the con centrated plastic hinges are specified using the experimental relations which were developed by Li gnos and Krawinkler (2012 and 2013) according to the results of the comprehensive experimental t ests in RC beam and column elements. These experimental relations can be used from Haselton an d Deierlein (2007) and FEMA P695 (2009), alternatively.

Therefore, the dimension effect of the plastic hinge in RC beams and columns has been contem plated in the computation of the rotational behaviour of the concentrated plastic hinges (Biskinis a nd Fardis 2009, Panagiotakos and Fardis 2009, Karimiyan and Moghadam 2013a and 2013b).



Fig. 2 Virgin curve of modified Ibarra-Krawinkler model and the relevant definitions (Ibarra and Krawinkler 2004, Ibarra *et al.* 2005, Lignos 2008, Lignos *et al.* 2008, Krawinkler *et al.* 2009, Haselton *et al.* 2009)



Fig. 3 Monotonic and cyclic behavior of component model used in this study (Ibarra and Krawinkler 2005, Haselton *et al.* 2007, Haselton *et al.* 2008, FEMA P695 2009, Zareian *et al.* 2009, Zareian *et al.* 2010)

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According to the modified Ibarra-Krawinkler element model shown in Fig. 2, when the value of parameter κ is equal to zero, the load bearing capacity of the beam or column elements becomes zero, too. Fig. 3 shows the modified Ibarra-Krawinkler element model with $\kappa=0$. According to Fig. 3, as zero strength corresponds to the $\Theta_{\rm u}$, when the value of Θ for a hinge reaches to value of $\Theta_{\rm u}$, the related beam or column element is not capable to bear the existing loads, meaning that the relevant beam or column element is automatically eliminated from the structural system. Nonlinear time history analysis continues (without the eliminated element) in the residual structural system till the value of Θ in the second beam or column element reaches to its value of Ou. Then similar to the first element, it will be eliminated automatically from the structural system. This process will continue till the structural system becomes unstable. Consequently, we can simulate the progressive collapse mechanism in the beam and column elements of the buildings one after another. It is worth to mention that Θu is calculated according to the modified Ibarra-Krawinkler element model for each hinge, and is inputted in OPENSEES structural models. In this research a hinge is considered as the collapsed one if its rotation exceeds the extreme value of Θu . It should be noted that there are many beam and column plastic hinges that passed over Θy and even Θ cap which can cause general instability of the structure. But they have not been considered as the collapsed one. Therefore collapse has been defined in our study in two forms

- 1. Collapse of the beam and column hinges: were defined in the beam and column elements according to the modified Ibarra, Medina and Krawinkler deterioration model, and
- 2. Collapse of the buildings (general instability).

3. Pushover analyses

To present more details about the building models in this research, symmetric and asymmetric building models have been pushed in perpendicular direction of the mass eccentricity (Z direction).



Fig. 4 Pushover curves for symmetric and asymmetric buildings with the mass asymmetry of 5%, 15% and 25%

Mass eccentricity in plans	T1	T2	Т3	T4
%0	1.443	1.345	1.238	0.5
%5	1.472	1.4	1.277	0.504
%15	1.607	1.485	1.105	0.534
%25	1.593	1.457	0.933	0.557

Table 1 The main periods of the building models (sec)

Pushover curves for symmetric and asymmetric building models with different mass asymmetry of 0%, 5%, 15% and 25% and the main periods of those models are demonstrated in Fig 4 and Table 1, respectively.

4. Time history analyses

2-component earthquake records are used to perform NLTHA based on FEMA P695, Table A-4C, as shown in Table 2, using OPENSEES (Version 2.2.2) software. All earthquake records are applied on the buildings in two horizontal directions Z and X in such a way that the Z component is stronger than X component in all records.

Two hinges are considered at both ends of all beams and columns elements in OPENSEES structural models.

To increase the probability of collapse, one after another, in the beam and column elements of the buildings, the intensive effects of the earthquake loads should be applied on the structural elements. Therefore, the PGA levels are increased by incremental dynamic analyses (IDA) in such a way that besides the formation of collapsed hinges, the whole building becomes unstable. In this way, the probability of collapse will increase in the whole buildings (FEMA P695 2009). So, the incremental dynamic analysis is repeated for each record. Therefore, many different seismic hazard levels are adopted for each of 22 ground motion records.

As an example, Fig. 5 illustrates the progressive collapse mechanism of an asymmetric building with mass eccentricity of 25% in the presence of the ground motion record #1148. According to this figure, the sequence of the collapsed hinges which are formed from the first hinge to the major portion of the building is determined via tracing the assigned number to each collapsed hinge. In this way we can identify the collapse distribution pattern under various levels of mass eccentricities. The same procedure is repeated for the other ground motion records and subsequently, collapse propagation and the number of collapsed hinges are obtained in the beam and column elements of the symmetric and asymmetric buildings. To have a visual sense on the collapse tendency in the building, the first 20 collapsed hinges are shown with different colors varying from dark to light. A darker color assigned to a hinge on this spectrum demonstrates that it has been collapsed in the earlier stage of NLTHA in comparison with the lighter ones. Therefore, the probability of collapse in the hinges with a smaller index and darker color are more than the other hinges.

The response spectra of both horizontal components of the 22 earthquake records considered in this study are shown in Figs. 6a and 5b.



Fig. 5 Pushover curves for symmetric and asymmetric buildings with the mass asymmetry of 5%, 15% and 25%



Fig. 6 Pseudo acceleration spectrum of the 22 earthquake records, (a) X Components, (b) Z Components



Table 2 Summary of the used PEER NGA Database information and Parameters of Recorded Ground Motions for the Far-Field Record Set (FEMA P695 2009)

	PEER-NGA Record Information			Recorded Motions		
ID	Records	Lowest	The Names of Horizontal Record		PGA max	PGV max
No.	Seq. No.	Freq (Hz.)	Component 1	Component 2	(g)	(cm/s.)
1	953	0.25	NORTHR/MUL009	NORTHR/MUL279	0.52	63
2	960	0.13	NORTHR/LOS000	NORTHR/LOS270	0.48	45
3	1602	0.06	DUZCE/BOL000	DUZCE/BOL090	0.82	62
4	1787	0.04	HECTOR/HEC000	HECTOR/HEC090	0.34	42
5	169	0.06	IMPVALL/H-DLT262	IMPVALL/H-DLT352	0.35	33
6	174	0.25	IMPVALL/H-E11140	IMPVALL/H-E11230	0.38	42
7	1111	0.13	KOBE/NIS000	KOBE/NIS090	0.51	37
8	1116	0.13	KOBE/SHI000	KOBE/SHI090	0.24	38
9	1158	0.24	KOCAELI/DZC180	KOCAELI/DZC270	0.36	59
10	1148	0.09	KOCAELI/ARC000	KOCAELI/ARC090	0.22	40
11	900	0.07	LANDERS/YER270	LANDERS/YER360	0.24	52
12	848	0.13	LANDERS/CLW-LN	LANDERS/CLW-TR	0.42	42
13	752	0.13	LOMAP/CAP000	LOMAP/CAP090	0.53	35
14	767	0.13	LOMAP/G03000	LOMAP/G03090	0.56	45
15	1633	0.13	MANJIL/ABBARL	MANJIL/ABBAR-T	0.51	54
16	721	0.13	SUPERST/B-ICC000	SUPERST/B-ICC090	0.36	46
17	725	0.25	SUPERST/B-POE270	SUPERST/B-POE360	0.45	36
18	829	0.07	CAPEMEND/RIO270	CAPEMEND/RIO360	0.55	44
19	1244	0.05	CHICHI/CHY101-E	CHICHI/CHY101-N	0.44	115
20	1485	0.05	CHICHI/TCU045-E	CHICHI/TCU045-N	0.51	39
21	68	0.25	SFERN/PEL090	SFERN/PEL180	0.21	19
22	125	0.13	FRIULI/A-TMZ000	FRIULI/A-TMZ270	0.35	31

5. Study of the number of beam and column collapsed hinges

Fig. 7 shows the total number of beam and column collapsed hinges for each of 22 ground motion records in the building models with 0%, 5%, 15% and 25% of mass eccentricities.

From Fig. 7, as the mass eccentricity varies, the total number of collapsed hinges gets larger under the earthquake records #1148, 1158 and 900 rather than the other earthquake records. A comparison between the main periods of the symmetric and asymmetric buildings in Table 1 and the acceleration response spectra of the records in Fig. 6 shows that these records are the ones which have high spectral acceleration values in the main periods of the buildings. Therefore, there is no special trend in the number of collapsed hinges in the structures subject to these records with variation of the mass eccentricity.

As said before, the purpose of this study is to investigate the potential of progressive collapse when mass eccentricity increases in buildings. So, to find a relationship between the amount of increment in the mass asymmetry and the number of collapsed hinges, the total number of collapsed hinges under all of 22 earthquake records is compared with each other under various levels of mass eccentricities. Summing the number of beam and column collapsed hinges over the total 22 earthquake records for each mass asymmetry is shown in Fig. 8.

According to Fig. 8, the number of beam and column collapsed hinges reduces when the mass eccentricity increases in plans. The reduction in the number of beam and column collapsed hinges with increasing the value of mass eccentricity, can also be shown in an alternative way. The total number of beam and column collapsed hinges in the structures with various mass asymmetry are compared with each other for each earthquake record.

As said before, different mass eccentricities of 0%, 5%, 15% and 25% have been considered in this study. Therefore, we have three reduction/increment relations among the number of collapsed hinges under these mass eccentricities with portion of 1/3 (= 33.33%) for each relation.

As an example, based on Fig. 7, under earthquake record #1633, the number of collapsed hinges is 34, 13, 2 and 4 in buildings with various mass eccentricities of 0%, 5%, 15% and 25%, respectively. These statistics show that there are two reduction relations in the total number of the collapsed hinges; one is when passing from 0% to 5% and the other one is from 5% to 15%. There is also one increment relation when passing from 15% to 25%.

Mass eccentricity: 0%	5%	15%	25%	
Number of collapsed hing	es: 34	13	2	4
	Q	D C		D

Reduction relation reduction relation increment relation

As a result, when mass eccentricity increases from 0% to 25%, the decrement percentage in the number of collapsed hinges becomes 66.67% (Two 33.33% for reduction relations from 0% to 5% and 5% to 15%). As this case shows, varying the number of collapsed hinges have more downward trend with increasing the mass eccentricity.

This process is repeated for the other ground motion records. Fig. 9 shows the percentages of decrease in the number of collapsed hinges for 22 ground motion records when the mass eccentricity increases in the buildings.

Fig. 9 shows that for most of records (almost 73%), the number of collapsed hinges or equivalently, the progressive collapse potential of buildings, decreases as the mass eccentricity increases in plans.



Fig. 7 The total number of the beam and column collapsed hinges in the presence of 22 earthquake records in the structural models with various mass eccentricities of 0%, 5%, 15% and 25%



Fig. 8 The total number of beam and column collapsed hinges for 22 ground motion records in various levels of mass eccentricity



Fig. 9 The percentages of decrease in the total number of collapsed hinges for 22 earthquake records with the mass eccentricity increasing in plans

6. Study of the story drifts

Story drifts and displacements are also among common acceptance criteria in the building codes and guidelines. To find a relationship between the story drifts and the number of collapsed hinges, the drifts of mass centers and different edges of the asymmetric buildings are investigated to estimate the behavior of the buildings with various levels of mass eccentricities. The farthest edge and the closest edge to the mass center are defined here as the stiff edge and the flexible edge, respectively. Therefore, we can highly simplify the evaluation of progressive collapse in the moment resisting frame buildings.

The results of the nonlinear time history analyses illustrate that when the mass eccentricity increases, the maximum story drifts in the mass center, stiff and flexible edges decrease. Fig. 10 shows the relationship between the mass asymmetry and the average maximum story drifts of the mass centers, stiff and flexible edges over 22 earthquake records.

According to the Fig. 10, when the mass eccentricity increases from 0% to 25%, the average maximum story drifts of the mass centers, stiff and flexible edges over all 22 earthquake records decreases.

Increasing the value of mass eccentricity makes the structural system unstable earlier, with a lower number of the beam and column collapsed hinges. In other words, increasing the mass eccentricity levels causes earlier instability and collapse of the asymmetric building, with a lower number of the collapsed hinges, in comparison with the symmetric one. Reducing the values of story drifts and durations of NLTHA, approve this conclusion when the mass eccentricity increases. Therefore, we can conclude that increasing the mass eccentricity from 0% to 25%, results in earlier collapse and instability of the entire buildings and decreasing trend of the number of collapsed hinges which is necessary to fail the whole structures. Meaning that, in asymmetric building, with increasing the mass asymmetry, instability and collapse of the entire building occurs earlier but is not very progressive. In other words, the potential of progressive collapse decreases when the mass eccentricity increases in asymmetric buildings.

This is the reason of why reduction in the number of collapsed beam and column hinges happens when the mass eccentricity increases in plans.

The reduction percentages in drifts of the mass centers, stiff and flexible edges are separately shown in Fig. 11, when the mass asymmetry increases in plans of the structural models.

The reduction percentages shown in Fig. 11 have been calculated as follows: We first compare the maximum story drifts of the flexible edges in the first stories of the buildings under various levels of mass eccentricities. Then, the same procedure is repeated for the other stories, too. Averaging over all stories data yields the reduction percentage in drifts of the flexible edges for each record, separately. Such a process is repeated for the stiff edges and the mass centers and subsequently, the reduction percentages in the drifts of the stiff edges and the mass centers are calculated for all records, separately.

From Fig. 11, for majority of the earthquake records (almost 73%), with increasing the level of mass eccentricity, the percentages of decrease in the story drifts of the flexible edges are lower than those in the mass centers and the stiff edges. This figure also shows that the percentages of decrease in the story drifts of the stiff edges are approximately closer and similar to the story drifts of the mass centers.

Combining the data of Figs. 9 and 11, we obtain Fig. 12. As this figure shows, with increasing the mass asymmetry, for most of records, the percentages of decrease in the number of collapsed



Fig. 10 The average maximum story drifts of the mass centers, stiff and flexible edges in various mass eccentricities over 22 earthquake records



Fig. 11 The percentages of decrease in the maximum story drift of the mass centers, stiff and flexible edges with the mass eccentricity increasing in plans



Fig. 12 The percentages of decrease in the number of collapsed hinges and the maximum story drifts of the mass centers, stiff and flexible edges with the mass eccentricity increasing in plans



Fig. 13 The percentages of decrease in the number of collapsed hinges and the average maximum story drifts in the mass centers and the stiff edges with the mass eccentricity increasing in plans

hinges are greater than the percentages of decrease in story drifts of the flexible edges, and are also closer and similar to the percentages of decrease in the story drifts of the mass centers and the stiff edges. In other words, the trend of decrease in the beam and column collapsed hinges is similar to that is observed in the story drifts of the mass center and the stiff edges. Such a similarity demonstrates that the average of the maximum story drifts in the mass centers and stiff edges approximately estimates the percentages of decrease in the number of beam and column collapsed hinges. Fig. 13 shows the percentages of decrease in the number of collapsed hinges and the average of maximum story drifts in the mass centers and the stiff edges.

As Fig. 13 shows, for the most of earthquake records, when the mass eccentricity increases, the percentages of decrease in the number of collapsed hinges are approximately similar to the percentages of decrease in the average maximum story drifts of the mass centers and the stiff edges.

Consequently, to estimate the potential of progressive collapse in the similar buildings, in place of using a time consuming and difficult method which works based on the number of collapsed hinges, we can concentrate on the story drifts of the mass centers, stiff and flexible edges. With these conditions that the percentages of decrease in the number of collapsed hinges be greater than the percentages of decrease in drifts of the flexible edges and be closer to the percentages of decrease in the average story drifts of the mass centers and the stiff edges.

7. Study of the absorbed energy

After evaluation of the progressive collapse mechanism and story drift behavior of the symmetric and asymmetric buildings, progressive collapse index is calculated based on the study of total number of collapse hinges and the absorbed energy in beam and column collapsed hinges. Absorbed energy is obtained by investigation of the moment-rotation history of collapsed hinges, by summing over all multiplications of $\Delta\theta$ in the corresponding moments. Therefore, the hysteretic behavior of collapsed hinges is investigated in symmetric and asymmetric buildings. Fig. 14 shows the absorbed energy in the collapsed hinges of the symmetric and asymmetric buildings with



Fig. 14 The absorbed energy in collapsed hinges of the symmetric and asymmetric buildings with mass asymmetry of 5%, 15% and 25% in presence of the 22 earthquake records



Fig. 15 Absorbed energy in collapsed hinges of the symmetric and asymmetric buildings with different mass asymmetry of 5%, 15% and 25% in presence of the earthquake records, separately

different mass asymmetry of 5%, 15% and 25% in presence of the 22 earthquake records.

As can be seen, Fig. 14 is similar to Fig. 7 which demonstrates the total number of collapsed hinges due to 22 earthquake records with different mass eccentricity. Therefore, it can be concluded that the absorbed energy behavior of the building models is similar to the total number of the collapsed hinges behavior when the mass eccentricity increases. Also, results show that the tendency of absorbed energy is similar to the trend of the total number of collapsed hinges formed in the presence of each earthquake records. Fig. 15 shows the absorbed energy in symmetric and asymmetric buildings with different mass asymmetry of 5%, 15% and 25% in the presence of each earthquake records.

Consequently, the decreasing or increasing trend of the absorbed energy is similar to the trend of total number of collapsed hinges.

Mass eccentricity	5%	15%	25%
PCI_collapsed_hinges	0.77	0.64	0.47
PCI_absorbed_energy	0.61	0.65	0.38
PCI	0.69	0.64	0.43

Table 3 The progressive collapse indices for asymmetric buildings with mass eccentricity of 5%, 15% and 25% in the presence of all 22 earthquake records

8. Progressive collapse index

As the aim of this research is the investigation of margin of safety against the progressive collapse mechanism in asymmetric buildings in comparison with the symmetric one, according to the results, collapse indices are presented for the asymmetric buildings based on the symmetric one. These indices are defined as follows:

PCI= Progressive collapse index

PCI_collapsed_hinges=Progressive collapse index based on the total number of collapsed hinges

PCI_absorbed_energy=Progressive collapse index based on the absorbed energy

TNCHA=total number of collapsed hinges in asymmetric buildings

TNCHS=total number of collapsed hinges in symmetric building

AEA=absorbed energy in asymmetric buildings

AES=absorbed energy in symmetric building

The following indices can be defined accordingly:

$PCI_collapsed_hinges = TNCHA/TNCHS$ (1)

$$PCI_absorbed_energy = AEA/AES$$
 (2)

$$PCI = 0.5 \times (PCI_collapsed_hinges+PCI_absorbed_energy)$$
 (3)

Table 3 shows the progressive collapse indices (PCI) for the asymmetric building models with different mass eccentricity of 5%, 15% and 25% in the presence of all 22 earthquake records.

From the results of Table 3 it can be seen that with increasing the mass eccentricity level, PCI value decreases. Meaning that, with increasing the mass eccentricity, the progressive collapse decreases.

It is also possible to estimate the progressive collapse index for the similar 6 story asymmetric buildings with different mass eccentricity such as 10% and 20% with interpolation of the above progressive collapse index values.

Results of this study are practical and can be used to design similar symmetric and asymmetric buildings resisting against the progressive collapse mechanism. For example, since the story drifts and displacements are among the acceptance criteria in many codes and guidelines, the results of story drifts in the mass centers, stiff and flexible edges are applicable and useful to estimate the progressive collapse potential of the similar buildings. According to the results, the average maximum story drifts of the mass centers and the stiff edges can predict the progressive collapse potential of the similar buildings. According to the results, the average maximum story drifts of the mass centers and the stiff edges can predict the progressive collapse index.

In field of collapse, our study was inspired by the studies of FEMA P695 (2009), Haselton (2007), Haselton and *et al.* (2007 and 2008), Ibarra and Krawinkler (2004), Ibarra and *et al.* (2005), Lignos and Krawinkler (2008 and 2013). However, all of these studies only investigate the collapse in 2D models. Our study extents these studies to the case of 3D models. In other word, the modified Ibarra-Krawinkler hysteretic model which has been used in the above references is used in the current study with preserving all assumptions and conditions in 3 dimensional buildings. So, their studies constitute a foundation for control and comparison of our results. During the review process of the paper, we search a lot through literature of asymmetric buildings to find similar studies. But, to the best of our knowledge, there was no study accomplished in progressive collapse mechanism of the 3 dimensional asymmetric buildings in the presence of the earthquake loads.

9. Conclusions

- Evaluation of the progressive collapse in 6-story symmetric and asymmetric reinforced concrete ordinary moment resisting frame buildings shows that the building behavior with respect to the decrease in the number of beam and column collapsed hinges is approximately closer and similar to the trend of decrease in the average maximum story drifts of the mass centers and the stiff edges.
- With increasing the value of mass eccentricity, decrement in the story drifts of the flexible edges is lower than the decrement in story drifts of the stiff edges and the mass centers. Moreover, the decrement in the story drifts of the stiff edges is closer and similar those of the mass centers.
- Increasing the mass eccentricity levels causes earlier instability and collapse of the asymmetric buildings. In other words, the number of collapsed hinges which is necessary to fail the asymmetric buildings decreases when the mass eccentricity increases in plans. Therefore, with mass eccentricity increment from zero to 25 percent, instability and collapse of the entire building occurs earlier with lower potential of the progressive collapse.
- As an alternative to a much difficult-to-calculate local response parameter of the number of collapsed hinges, the story drift, as a global response parameter, measures the potential of progressive collapse more easily.

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