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Effects of dead loads on dynamic analyses of beams subject to moving loads

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Abstract. The effect of dead loads on dynamic responses of a uniform elastic beam subjected to moving loads is examined by means of a governing equation which takes into account initial bending stresses due to dead loads. First, the governing equation of beams which includes the effect of dead loads is briefly presented from the author's paper (1990, 1991, 2010). The effect of dead loads is considered by a strain energy produced by conservative initial stresses caused by the dead loads. Second, the effect of dead loads on dynamical responses produced by moving loads in simply supported beams is confirmed by the results of numerical computations using the Galerkin method and Wilson-θ method. It is shown that the dynamical responses by moving loads are decreased remarkably on a heavyweight beam when the effect of dead loads is included. Third, an approximate solution of dynamic deflections including the effect of dead loads for a uniform beam subjected to moving loads is presented in a closed-form for the case without the additional mass due to moving loads. The proposed solution shows a good agreement with results of numerical computations with the Galerkin method and Wilson-θ method. Finally it is clarified that the effect of dead loads on elastic uniform beams subjected to moving loads acts on the restraint of the transverse vibration for the both cases without and with the additional mass due to moving loads.

Keywords: beams; dead load; initial stress; vibration; dynamic analysis; Galerkin equation; linear and nonlinear; live load; moving load; safety

1. Introduction

The collapse of structural buildings due to snow loads on roofs is repeated every year. This failure occurs concentrically in steel structures than in reinforced concrete structures. This fact points out that the collapse of structures caused by snow loads cannot be sufficiently explained by a consideration due to a heavy snowfall exceeding the maximum depth for design. The significant difference between reinforced concrete and steel structures is in magnitude of the dead loads. Beams, like structures, are always subjected to dead loads. This dead load is invariant and exists in the initial state. The inherent property of dead loads has bending stresses in beams subjected to dead loads only, in which the bending stresses are conservative initial stresses. This conservative initial bending stresses are large on heavyweight beams and small on lightweight ones. Then, the

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present author (Takabatake 1990) proposed the suggestion that dead loads of structures play an important role in structural damages; and demonstrated the effect of dead loads in static elastic beams. The effect of dead loads takes into consideration the additional strain energy, which is produced by the combination of conservative initial bending stresses due to dead loads with strains due to live loads, in addition to the well-known strain energy produced by live loads. It has been shown that this strain energy produced by the conservative initial stresses minimizes live-load-deflections and live-load-bending moments. The present author calls such phenomena the effect of dead loads.

The present author (Takabatake 1991) demonstrated the effect of dad loads on the natural frequencies of elastic beams and proposed a closed-form approximate solution of the natural frequency of simply supported beams. This new attention became the important jumping-off point for an extension of elementary beam theory and it was extended to the finite-element method by a beam element with the effect of dead loads (Zhou *et al*. 1996, Zhou 2002). The phenomenon that the initial bending in a beam due to dead loads increases the natural frequencies of the lateral vibration was also suggested by the other researcher (Kelly *et al*. 1991). The present author (Takabatake 2012, 1992) presented the effect of dead loads on the static dynamic responses of a uniform elastic rectangular plate and clarified the physical factors governing the effect. Mostaghel *et al*. (1995) showed that performing a thin plate into any shape has the effect of increasing its natural frequencies by means of a large deflection theory for thin plates and the principle of conservation of energy.

The present author (Takabatake 2010) presented that the effect of dead loads exists on dynamic beams subjected to dynamic live loads, too. When an elastic beam is subjected to dynamic live loads, the beam vibrates from the static deformed state produced by dead loads; and the vibration should include an effect produced by the conservative initial bending stresses due to dead loads. However, this effect of dead loads is currently ignored in structural designs. If the effect of dead loads on dynamic beams is better understood, it will be possible to more accurately estimate the magnitude of live loads; thus, the safety factors for heavyweight and lightweight structures will be equalized; and real safe structural designs will be made possible.

Although there are numerous studies concerning static and dynamic problems of beams, as shown in the previous works (Hayashikawa *et al*. 1985, Oliveira 1982, Stephen 1981, and Wang *et al*. 1981) no study concerning the interaction between dead loads and live loads is found in the discussion of dynamic problem subjected to moving loads.

The purpose of this paper is to clarify the effect of dead loads on dynamic problems of elastic beams subjected to moving loads. First, the governing equation of beams, in which the effect of dead loads is included, is summarized from the author's previous work (Takabatake 1990). Second, the effect of dead loads on dynamic problems of elastic beams subjected to moving loads is clarified from results of numerical computations using the Galerkin method and Wilson-θ method. Third, a closed-form approximate solution for dynamic deflections of elastic beams is presented from the governing equation including the effect of dead loads. The results of numerical computation using the closed-form solution proposed here will prove to be in good agreement with the results obtained by a step by step integration methods using the Wilson-θ method. Finally it will be clarified that the effect of dead load on elastic uniform beams subjected to moving loads restricts the transverse vibration for the both cases without and with the additional mass due to moving loads.

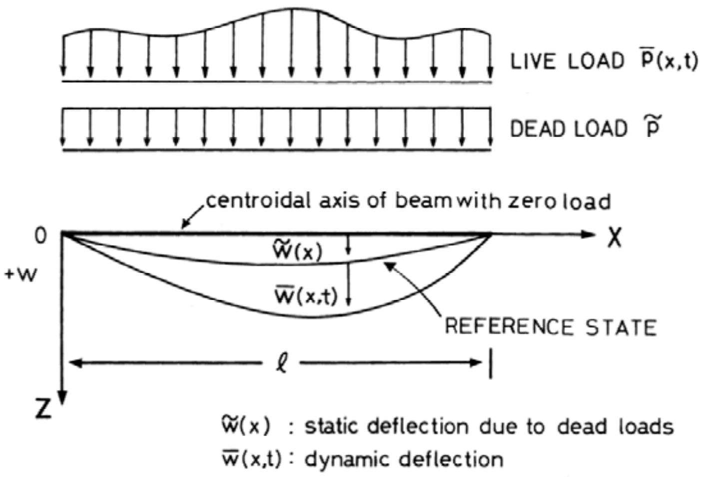


Fig. 1 Coordinate and load distribution of beam

2. Governing equations of beams included the effect of dead loads

The equation of motion of damped beams with the effect of dead loads is summarized from the extension of the author's work (1990). In Fig. 1, a beam is shown in Cartesian coordinate system: the x axis passes through the centroidal axis of the beam; and the y and z axes are the principal axes of the beam. The displacement is applicable to the Bernonlli-Euler beam theory. The relationships between stress and strain are linear. The beam is straight without initial imperfections before the action of all loads. Only transverse loads are considered. The static transverse deflections $\tilde{w}$ are produced by dead loads $\tilde{p}$ per unit length. This deformed state is defined as the reference state. Dynamic live loads $\overline{p}$ always act on this reference state and produce dynamic deflections $\overline{w}$ measured from the reference state. The deflections and transverse loads are considered positive when they point in the positive direction of the z axis.

In the reference state, the following equilibrium equation and boundary conditions for the simply supported beam and clamped beam, respectively, must exist:

$$\left(EI\tilde{w}''\right)'' - \tilde{p} = 0 \tag{1}$$

$$\tilde{w} = 0 \quad \text{or} \quad \tilde{w}''' = 0 \ \text{at} \quad x = 0 \quad \text{and} \quad x = \ell \tag{2a}$$

$$\tilde{w}' = 0 \quad \text{or} \quad \tilde{w}'' = 0 \ \text{at} \quad x = 0 \quad \text{and} \quad x = \ell \tag{2b}$$

in which $E =$ the Young's modulus and $I =$ the principal moment of inertia. On the other hand, the governing equation of beams with the effect of dead loads can be written as

$$m\ddot{\overline{w}} + c\dot{\overline{w}} + \left(E\,I\,\overline{w}''\right)'' - \frac{1}{2}\left[EA\,(\,\tilde{w}'\,)^2\,\overline{w}'\,\right]' = \overline{p} \tag{3}$$

for the equation of motion and

$$\overline{w} = 0 \quad \text{or} \quad \overline{w}''' - \frac{EA}{2}(\tilde{w}')^2\,\overline{w}' = 0 \text{ at } \quad x = 0 \quad \text{and} \quad x = \ell \tag{4a}$$

$$\overline{w}' = 0 \quad \text{or} \quad \overline{w}'' = 0 \text{ at } \quad x = 0 \quad \text{and} \quad x = \ell \tag{4b}$$

for the simply supported beam and clamped one, respectively. The mass m of the current beam per unit length in Eq. (3) is composed of the mass $\tilde{m}$ due to dead loads $\tilde{p}$ and the mass $\overline{m}$ due to moving loads $\overline{p}$. Since the moving loads $\overline{p}$ are function of x and t, the mass $\overline{m}$ depends on x and t. Note that since Eqs. (3) and (4) have nonlinear coupled form of $\tilde{w}$ and $\overline{w}$, the dynamic deflections $\overline{w}$ due to moving loads are influenced on the magnitude of the static deflections $\tilde{w}$ due to dead loads. Therefore the principle of superposition cannot apply to the current problem. Eqs. (3) and (4) become in linear with respect to the unknown dynamic deflections $\overline{w}$ because the static deflections $\tilde{w}$ are initially known. In subsequent developments, for simplicity the beam is assumed to be of a uniform cross section.

3. Dynamic analyses using the Galerkin method

The equation of motion of beams which includes the effect of dead loads is solved by means of the Galerkin method (Takabatake 2010). The method of separation of variables is employed assuming that

$$\overline{w}(x,t) = \sum_{n=1} \overline{w}_n(t)\, f_n(x) \tag{5}$$

in which $\overline{w}_n(t) =$ the unknown displacement coefficients with respect to time t; and $f_n(x) =$ shape functions satisfying the specified boundary conditions of beams. The following functions represent $f_n(x)$ for simply supported beams and clamped beams

$$f_n(x) = \sin\frac{n\pi x}{\ell} \quad \text{for simply supported beams} \tag{6a}$$

$$f_n(x) = \sin\frac{\pi x}{\ell}\sin\frac{n\pi x}{\ell} \quad \text{for clamped beams} \tag{6b}$$

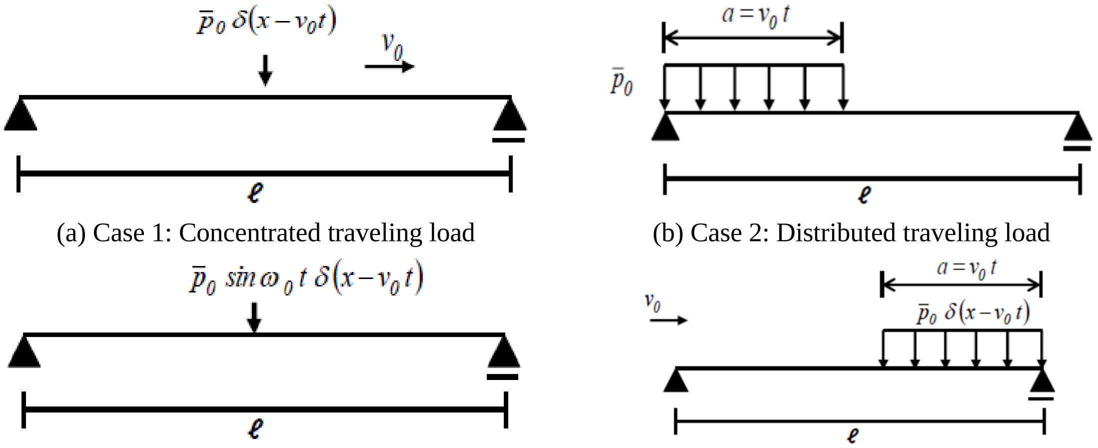
Employing Eq. (5) into Eq. (3), the Galerkin equation can be written as

$$\delta\overline{w}_n: \quad \sum_{\overline{n}=1}^{\infty}\left[\ddot{\overline{w}}_{\overline{n}}\frac{m}{EI} + \dot{\overline{w}}_{\overline{n}}\frac{c}{EI}\right]\int_0^{\ell} f_n f_{\overline{n}}\, dx + \sum_{\overline{n}=1}^{\infty}\overline{w}_{\overline{n}}\left[A_{n\overline{n}}\right] = \int_0^{\ell}\frac{\overline{p}}{EI} f_n\, dx \tag{7}$$

in which $A_{n\overline{n}}$ are defined as

$$A_{n\overline{n}} = \int_0^{\ell} f_n f_{\overline{n}}''''dx - \frac{1}{r^2}\int_0^{\ell}\tilde{w}'\tilde{w}'' f_n f_{\overline{n}}'\, dx - \frac{1}{2r^2}\int_0^{\ell}(\tilde{w}')^2 f_n f_{\overline{n}}''\, dx \tag{8}$$

in which $r =$ radius of gyration of area, $r = \sqrt{I/A}$. Eq. (7) may be solved by means of the Wilson-θ method. Once the dynamic displacements $\overline{w}$ are determined from Eq. (5), the bending moments $\overline{M}$ due to live loads are determined from $\overline{M} = -EI\,\overline{w}''$. It must be noticed that these dynamic values are influenced by the effect of the dead loads.



(a) Case 1: Concentrated traveling load (b) Case 2: Distributed traveling load

(c) Case 3: Concentrated pulsating traveling load (d) Case 4: Removal distributed traveling load

Fig. 2 Four kinds of moving loads, (a) Case 1: concentrated traveling load, (b) Case 2: distributed traveling load, (c) Case 3: concentrated pulsating traveling load and (d) Case 4: removal distributed traveling load

4. Various moving loads

This paper considers moving loads for the dynamic live loads $\bar{p}(x,t)$. The moving loads $\bar{p}(x,t)$ are explained in a Fourier series as

$$\bar{p}(x,t)=\sum_{n=1}\bar{p}_n(t)\,\sin\frac{n\pi\,x}{\ell} \tag{9}$$

Voltera and Zachmanoglou (1965) presented the various moving loads. Employing the Dirac delta function into the treatment of Voltera and Zachmanoglou, the Fourier coefficients $\bar{p}_n$ for various moving loads may be obtained as follows:

4.1 Case 1 Concentrated traveling load

Consider the case of a concentrated traveling load $\bar{p}$ advancing along a beam with constant velocity v_0 as shown in Fig. 2(a). At the instant t the moving load $\bar{p}$ is at a distance $a=v_0\,t$ from the left support. Hence the moving load $\bar{p}(x,t)$ is given in

$$\bar{p}(x,t)=\bar{p}_0\;\delta(x-v_0\,t) \tag{10}$$

in which $\delta(x-v_0\,t)$ is the Dirac delta function and $\bar{p}_0$ indicates the magnitude of the concentrated live load which is unmoving. The Fourier coefficients $\bar{p}_n$ of the moving load become

$$\bar{p}_n=\frac{1}{2}\int_0^{\ell}\bar{p}_0\,\sin\frac{n\pi(v_0 t)}{\ell}dx \tag{11}$$

4.2 Case 2 Distributed traveling load

Consider the case of a uniformly distributed live load $\bar{p}_0$ per unit length advancing along the beam with constant velocity v_0. When the head of the moving load has reached a distance, $a = v_0 t$, from the left support as shown in Fig. 2(b), the moving load distribution at this instant is

$$\begin{aligned} \bar{p}(x,t) &= \bar{p}_0 \qquad 0 \le x \le a \\ \bar{p}(x,t) &= 0 \qquad a < x \end{aligned} \tag{12}$$

in which $a = v_0 t$. Then the Fourier coefficients $\bar{p}_n$ become

$$\bar{p}_n = \frac{2\bar{p}_0}{n\pi}\left[1 - \cos\frac{n\pi(v_0 t)}{\ell}\right] \tag{13}$$

4.3 Case 3 Concentrated pulsating traveling load

Consider the case of a concentrated alternating force $\bar{p}_0 \sin\omega_0 t$ advancing along the beam with constant velocity v_0 as shown in Fig. 2(c). The moving load $\bar{p}(x,t)$ is given in

$$\bar{p}(x,t) = \bar{p}_0 \sin\omega_0 t \;\; \delta(x - v_0 t) \tag{14}$$

The Fourie coefficients $\bar{p}_n$ become

$$\bar{p}_n = \frac{1}{2}\bar{p}_0 \sin\omega_0 t \;\; \sin\frac{n\pi(v_0 t)}{\ell} \tag{15}$$

4.4 Case 4 Removal distributed traveling load

Consider the case that a uniformly distributed live load $\bar{p}_0$ per unit length advancing along the beam with constant velocity v_0 leaves out as shown in Fig. 2(d). This case is inverse with case 2. This case is found out the sliding of snow on the roof. The moving load $\bar{p}(x,t)$ is

$$\begin{aligned} \bar{p}(x,t) &= 0 \qquad 0 \le x < a \\ \bar{p}(x,t) &= \bar{p}_0 \qquad a \le x \end{aligned} \tag{16}$$

in which $a = v_0 t$. The Fourier coefficients $\bar{p}_n$ become

$$\bar{p}_n = -\frac{2\bar{p}_0}{n\pi}\left[-(-1)^n \cos\frac{n\pi(v_0 t)}{\ell}\right] \tag{17}$$

Thus, cases 1 to 4 indicate the representative moving loads which are really interesting for structural engineering. The dynamic response including the effect of dead loads due to moving loads is calculated by substituting Eq. (9) into the external load term in Eq. (7).

5. Additional mass due to moving loads

The mass in the beam theory including the effect of dead loads is composed of the mass $\tilde{m}$ of only dead loads and the mass $\overline{m}(x,t)$ of only live loads.

$$m = \tilde{m} + \overline{m}(x,t) \tag{18}$$

The mass $\tilde{m}$ is invariant and independents the time. On the other hand, the mass $\overline{m}(x,t)$ is influenced on the location of moving loads. In usual beam theory these masses depend on the dead loads and live loads as given in $\tilde{m} = \tilde{p}/g$ and $\overline{m} = \overline{p}/g$, respectively, in which g is gravity acceleration.

The additional mass $\overline{m}(x,t)$ may be expressed in the same treatment as the moving loads $\overline{p}(x,t)$. The moving additional mass $\overline{m}(x,t)$ is explained in a Fourier series as

$$\overline{m}(x,t) = \sum_{n=1} \overline{m}_n \sin\frac{n\pi x}{\ell} \tag{19}$$

in which the Fourier coefficients $\overline{m}_n$ are given from the moving loads of four cases 1 to 4 stated in the section 4.

Case 1
$$\overline{m}_n = \frac{2}{\ell}\overline{m}_0 \sin\frac{n\pi(v_0 t)}{\ell} \tag{20}$$

Case 2
$$\overline{m}_n = \frac{2\overline{m}_0}{n\pi}\left[1-\cos\frac{n\pi(v_0 t)}{\ell}\right] \tag{21}$$

Case 3
$$\overline{m}_n = \frac{2}{\ell}\overline{m}_0 \sin \omega_0 t \sin\frac{n\pi(v_0 t)}{\ell} \tag{22}$$

Case 4
$$\overline{m}_n = \frac{2\overline{m}_0}{n\pi}\left[(-1)^n + \cos\frac{n\pi(v_0 t)}{\ell}\right] \tag{23}$$

Here $\overline{m}_0 = \overline{p}_0/g$.

Hence in the calculation of step by step integration methods of Eq. (7) used Wilson-θ method, the variation with respect to the time t must be considered both the mass $m(x,t)$ and the moving loads $\overline{p}(x,t)$. However, the variation of moving additional mass $\overline{m}(x,t)$ during infinitesimal time may be assumed to be negligible.

6. Approximate solutions

The effect of dead loads on dynamic beams subjected to moving loads is given from the numerical results in Eq. (7). For the practical use let us consider the approximate solution in the closed-form solution. Eq. (7) is coupled form with respect to n and $\overline{n}$. Now assuming the uncoupled form of Eq. (7) upon the approximate orthogonal relation of the used shape functions, the Galerkin equation becomes

$$\ddot{\overline{w}}_n \frac{m}{EI}\int_0^\ell f_n f_n dx + \dot{\overline{w}}_n \frac{c}{EI}\int_0^\ell f_n f_n dx + \overline{w}_n A_{nn} = \int_0^\ell \frac{\overline{p}}{EI} f_n dx \tag{24}$$

The following notation is defined as

$$\int_0^\ell f_n f_n dx = \beta_n \tag{25}$$

in which $\beta_n =$ a constant. The substitution of Eq. (25) into Eq. (24) becomes

$$\ddot{\overline{w}}_n m + \dot{\overline{w}}_n c + \overline{w}_n A_{nn} \frac{EI}{\beta_n} = \frac{1}{\beta_n}\int_0^\ell \overline{p}\, f_n dx \tag{26}$$

Eq. (26) is exactly the differential equation with variable coefficient because the additional mass $\overline{m}(x,t)$ due to moving loads depends on the time. When the additional mass $\overline{m}(x,t)$ is far smaller than unmoving mass $\tilde{m}$ due to the dead load, the mass m may be considered as a constant which is independent of time. So, Eq. (26) may reduce to the differential equation with constant coefficients.

$$\ddot{\overline{w}}_n + 2a_n\dot{\overline{w}}_n + b_n\overline{w} = q_n(t) \tag{27}$$

in which

$$a_n = \frac{c}{2m} \tag{28}$$

$$b_n = \frac{EI}{m\ \beta_n} A_{nn} \tag{29}$$

$$q_n(t) = \frac{1}{m\,\beta_n}\int_0^t \overline{p}(x,t)\, f_n dx \tag{30}$$

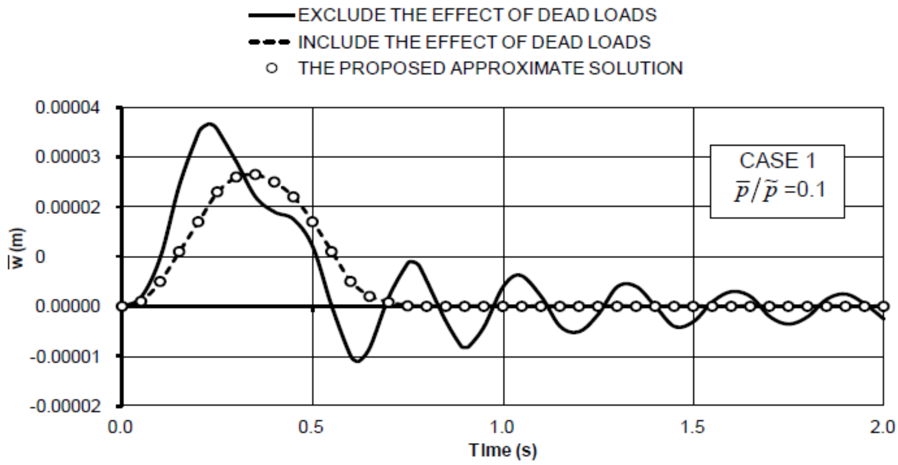
The general solution of Eq. (27) becomes

$$\overline{w}_n = e^{-a_n t}\left[C_1 \sin\alpha_{0n}t + C_2\cos\alpha_{0n}t\right] + \frac{1}{\alpha_{0n}t}\int_0^t e^{-a_n(t-\tau)}\sin\alpha_{0n}(t-\tau)q_n(\tau)d\tau \tag{31}$$

in which $\alpha_{0n} = \sqrt{b_n - a_n^2}$.

7. Numerical results

Let us examine the effect of dead loads on dynamic behavior of a simply supported elastic beams subject to four kinds of moving loads. The beam has the following properties: $E = 2.05 \times$



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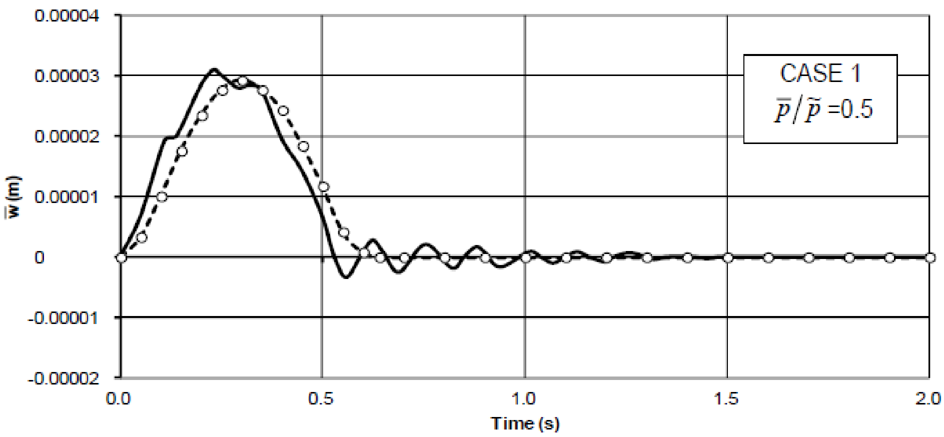

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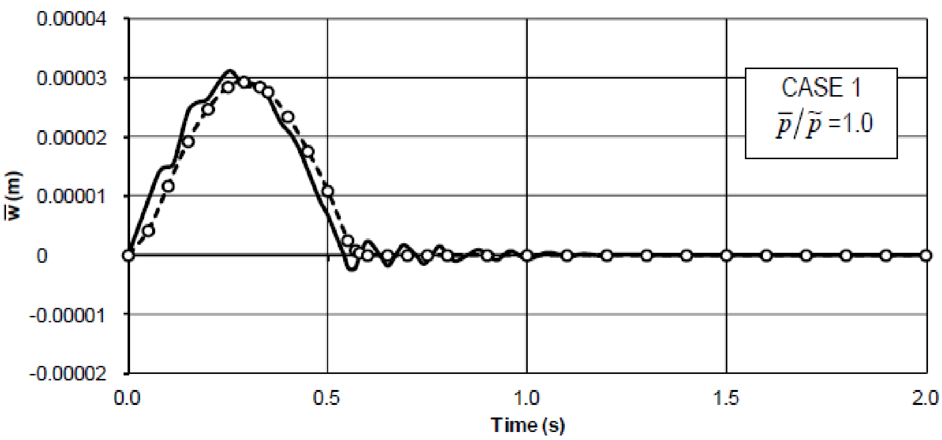
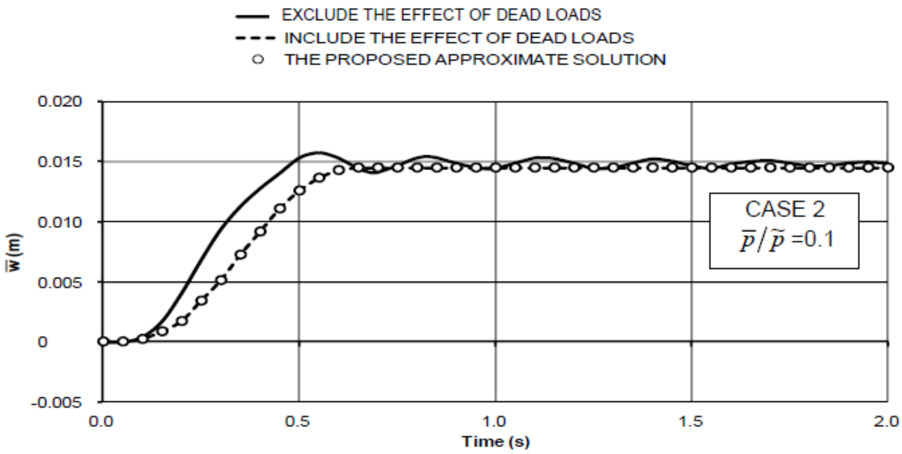


(c)

Fig. 3 Time histories of dynamic deflection $\overline{w}$ at the midspan for case 1; (a) $\overline{p}/\tilde{p}$ =0.1,(b) $\overline{p}/\tilde{p}$ =0.5, and (c) $\overline{p}/\tilde{p}$ =1.0



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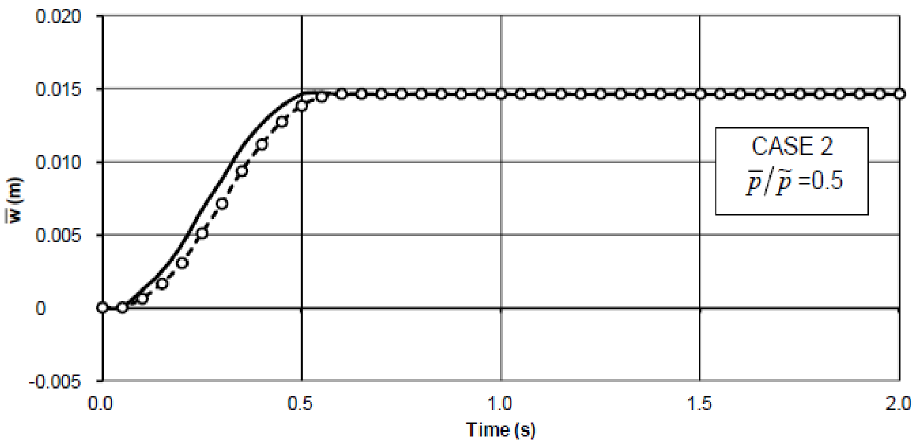

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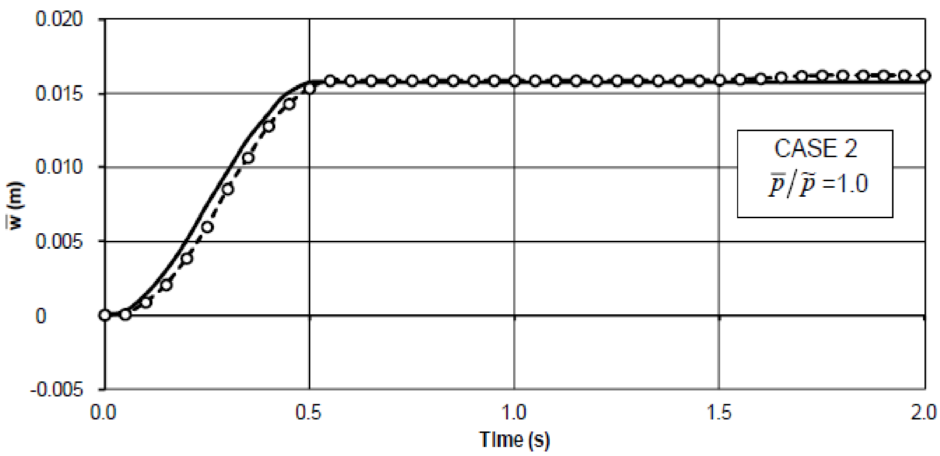
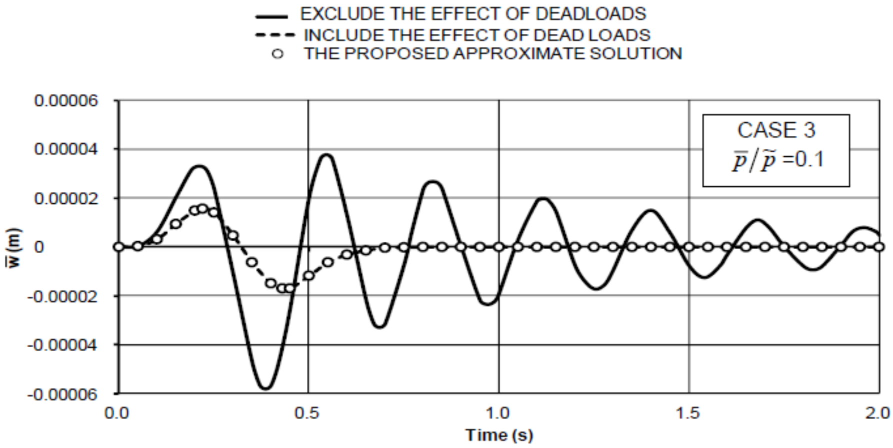


(c)

Fig. 4 Time histories of dynamic deflection $\overline{w}$ at the midspan for case 2; (a) $\overline{p}/\tilde{p}$ =0.1,(b) $\overline{p}/\tilde{p}$ =0.5 and (c) $\overline{p}/\tilde{p}$ =1.0



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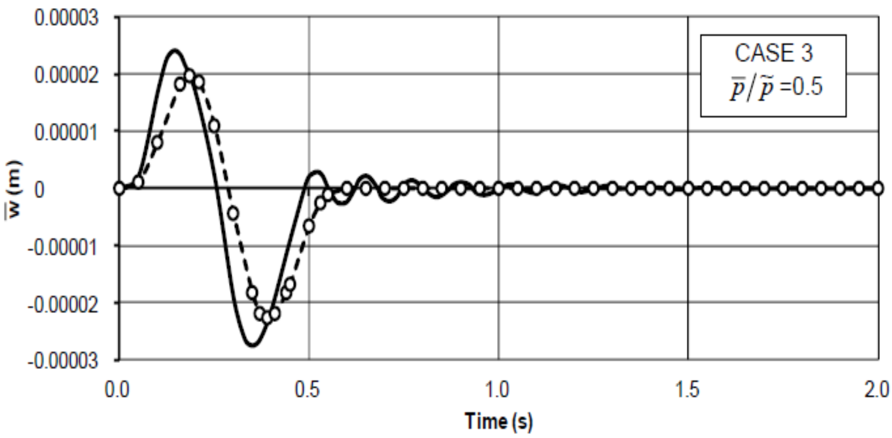

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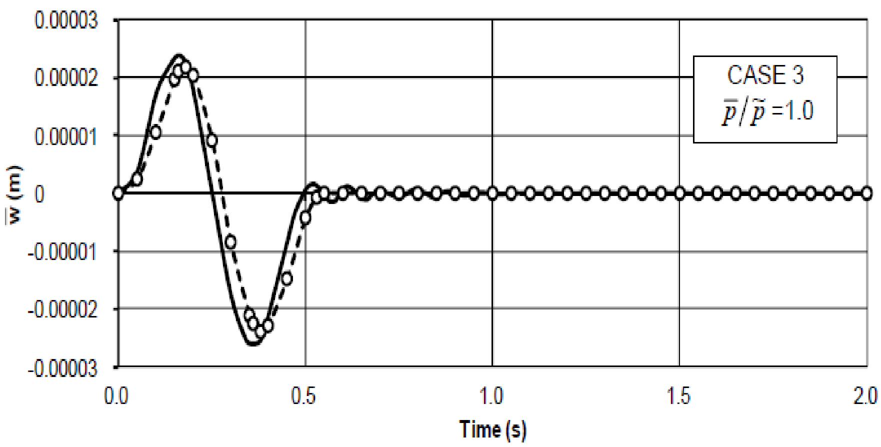
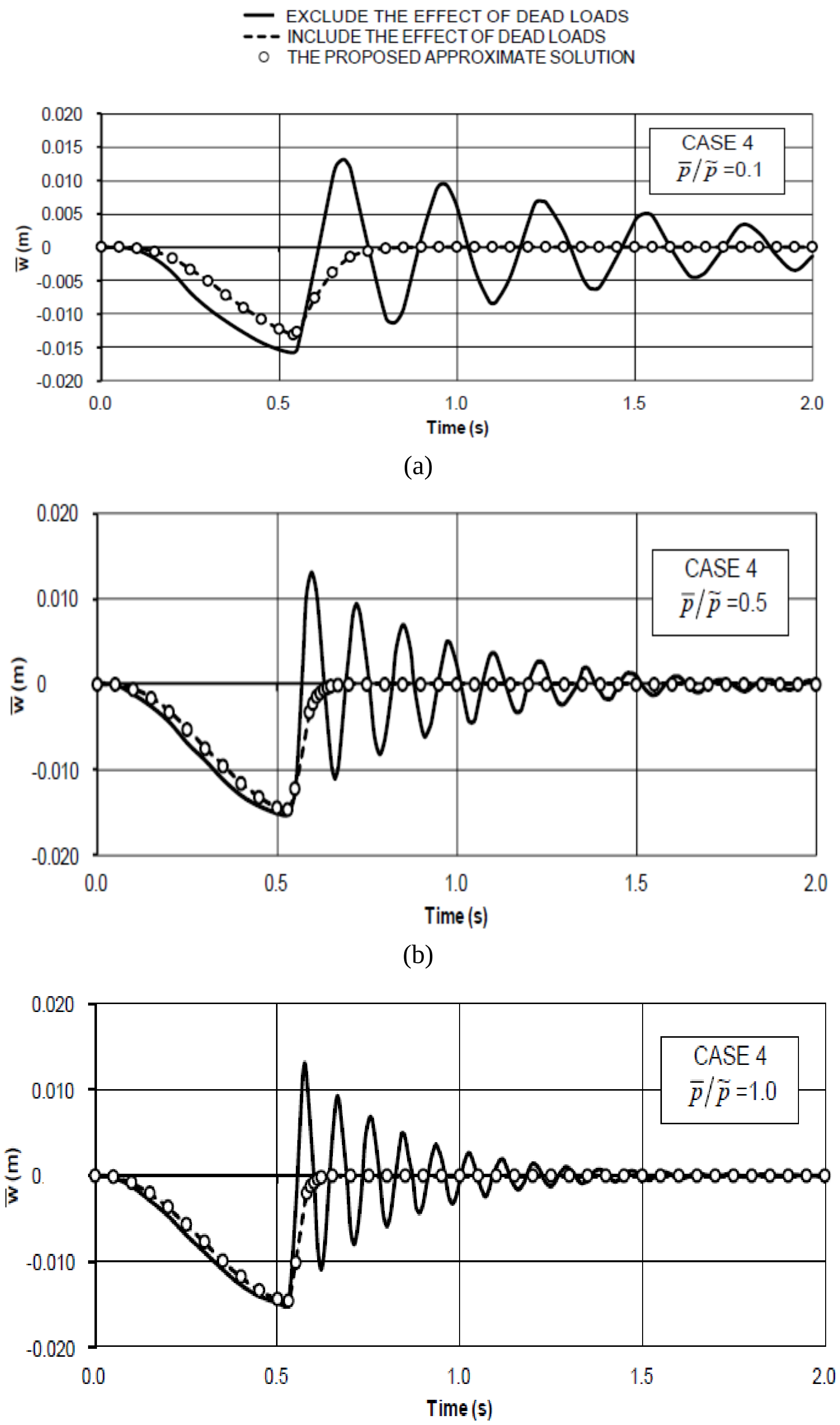


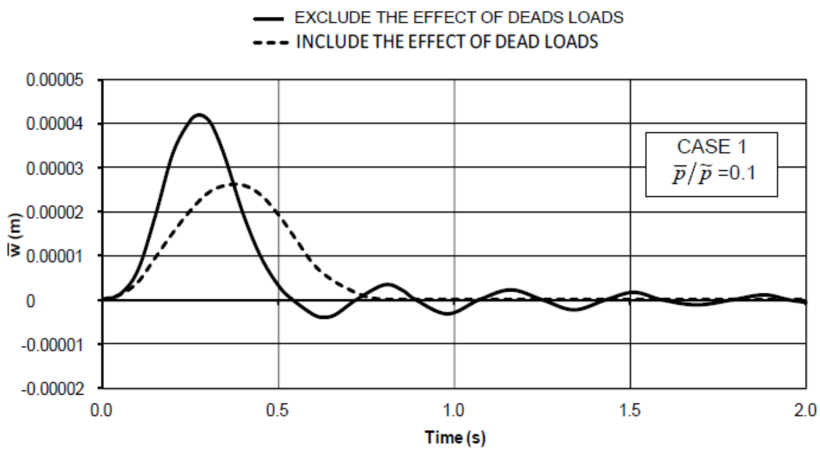
(c)

Fig. 5 Time histories of dynamic deflection $\bar{w}$ at the midspan for case 3; (a) $\bar{p}/\tilde{p}$ =0.1, (b) $\bar{p}/\tilde{p}$ =0.5, and (c) $\bar{p}/\tilde{p}$ =1.0



(c)

Fig. 6 Time histories of dynamic deflection $\overline{w}$ at the midspan for case 4; (a) $\overline{p}/\tilde{p}$ =0.1, (b) $\overline{p}/\tilde{p}$ =0.5, and (c) $\overline{p}/\tilde{p}$ =1.0



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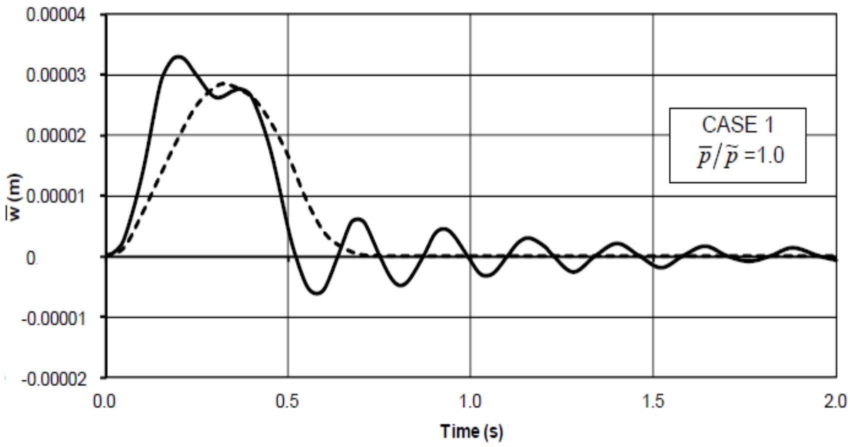


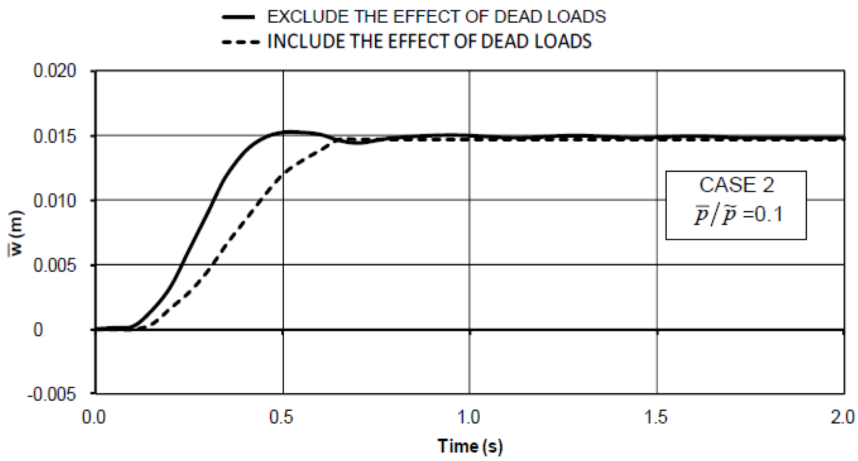
(b)

Fig. 7 Time histories of dynamic deflection $\overline{w}$ at the midspan for case 1; (a) $\overline{p}/\tilde{p}$ =0.1, (b) $\overline{p}/\tilde{p}$ =1.0

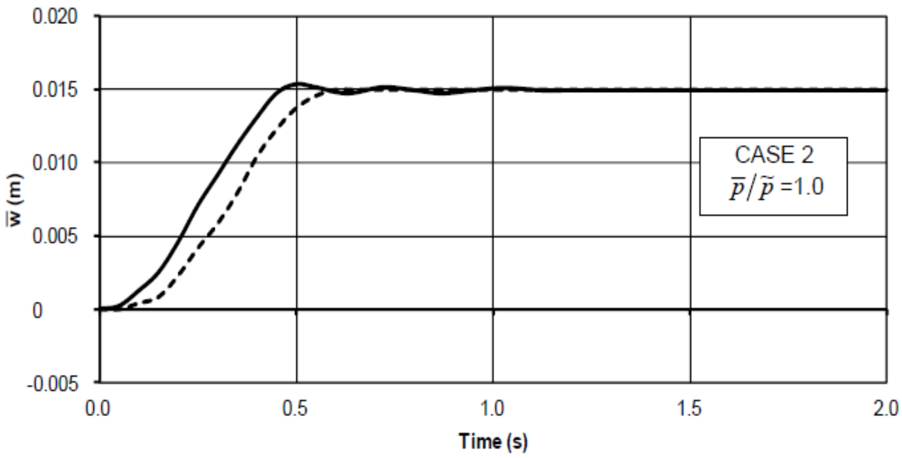
10^{11} N/m^2, $I = 2.04\text{x}10^{-4}$ m^4, $\ell = 8$ m, $A = 0.11887$ m^2, $v_0 = 15$ m/s, $\omega_0 = 4\pi$, $\tilde{p} = 1960$ N/m, and $r = 0.131$ m.

The damping constant h_1 for the first natural mode of the beam is 0.02. The damping constant h_n for the higher n-th natural mode is $h_n = h_1 \cdot \omega_n / \omega_1$, in which ω_n is the n-th natural frequency (rad/s).

The effect of dead load on the simply supported beam subject to moving loads is examined by varying the magnitude of the moving loads under the constant dead load. This is reflected by the load-ratio ($\overline{p}/\tilde{p}$). The magnitude of the moving load in the current beam of load-ratio 1.0 indicates 10 times larger than one in the load-ratio 0.1. Figs. 3(a) to (c) show the time histories of dynamic deflection $\overline{w}$ at the midspan for Case 1, in which the load-ratios take the values of 0.1, 0.5, and 1.0. The solid line displays the dynamic deflection excluded the effect of dead loads and agrees with the deflection obtained from the well-known fundamental beam theory subject to the moving load. The broken line indicates the deflection including the effect of dead loads. On the



(a)



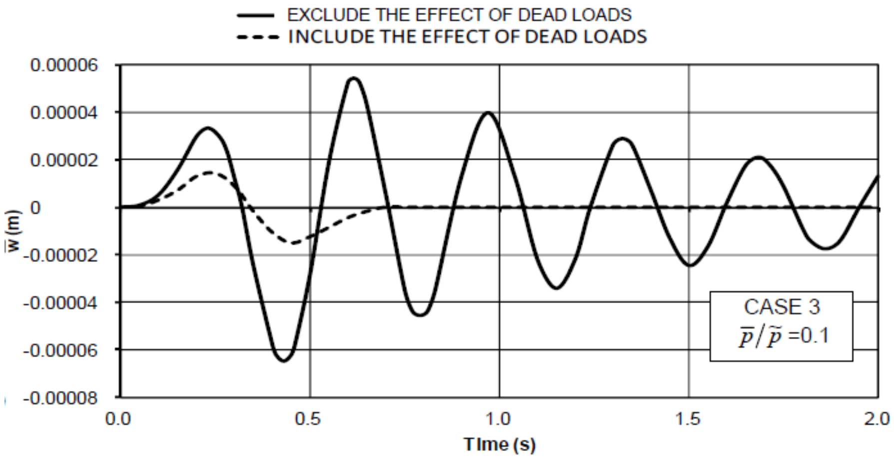
(b)

Fig. 8 Time histories of dynamic deflection $\overline{w}$ at the midspan for case 2; (a) $\overline{p}/\tilde{p}$ =0.1, (b) $\overline{p}/\tilde{p}$ =1.0

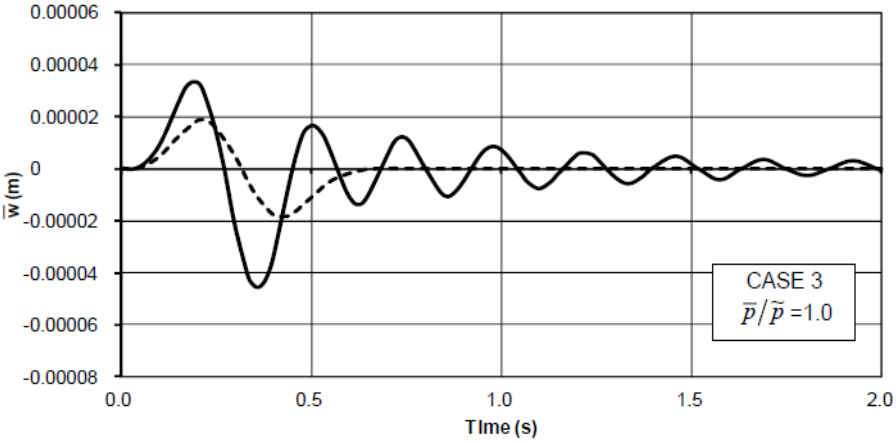
other hand, the line with circle denotes the proposed approximate solution. It is clarified from these figures that the effect of dead loads is remarkable in the case that the load-ratio is small. This implies that the dynamic behavior due to moving loads in the heavyweight beam is remarkably reduced by the effect of dead loads. Also, the approximated solution proposed here agrees with the numerical result including the effect of dead loads which is indicated by the broken line.

Figs. 4(a) to (c) show the time histories of dynamic deflection $\overline{w}$ at the midspan for Case 2. Since the uniformly distributed moving load moves from the left side to the right side with a constant velocity v_0, the dynamic deflection does not occurred the vibration. The effect of dead load is more remarkable in heavyweight beams than in lightweight ones.

Figs. 5(a) to (c) show the time histories of dynamic deflection $\overline{w}$ at the midspan for Case 3. The effect of dead loads is more remarkable on heavyweight beams than on lightweight ones. Its effect acts on the direction which constraints the vibration due to the moving loads in the heavyweight beam.



(a)



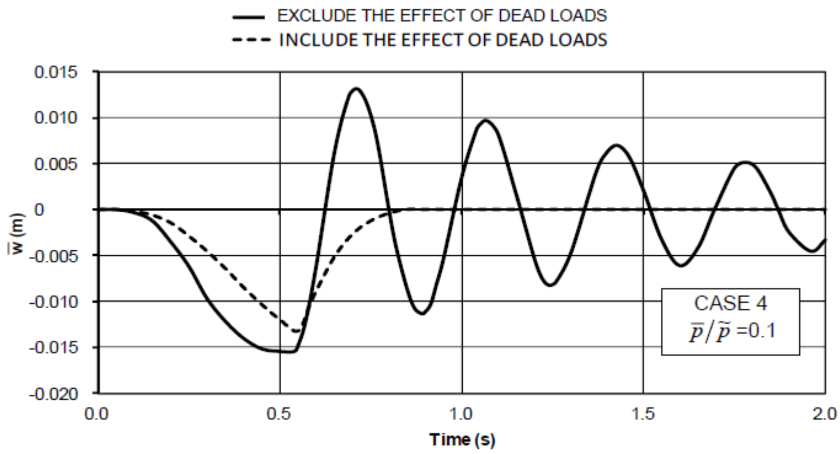
(b)

Fig. 9 Time histories of dynamic deflection $\overline{w}$ at the midspan for case 3; (a) $\overline{p}/\tilde{p}$ =0.1, (b) $\overline{p}/\tilde{p}$ =1.0

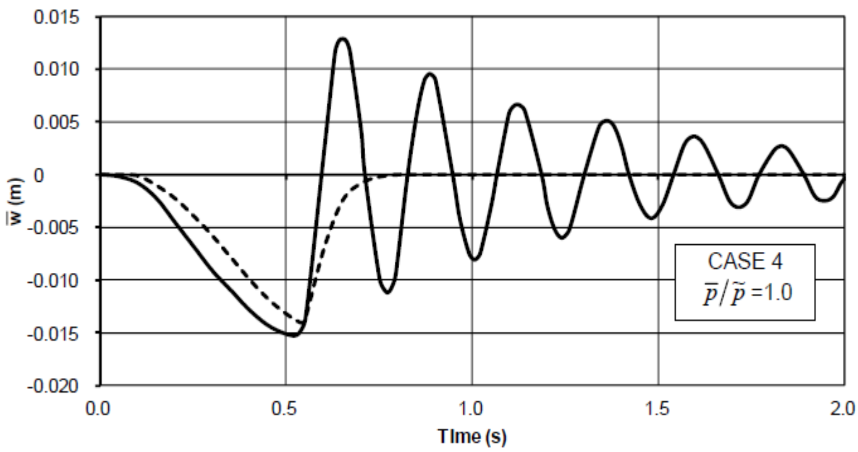
Also, Figs. 6(a) to (c) indicate the time histories of dynamic deflection $\overline{w}$ at the midspan of Case 4. The numerical results in the case are the same behavior as the above-mentioned results.

Thus, it has been clarified that the effect of dead loads has the behavior which restrains the vibration of the dynamic deflection due to moving loads. The effect is more remarkable on the heavyweight beams than on lightweight ones.

The above-mentioned results have been considered in the case that the additional mass due to the moving loads is negligible. Next we consider the effect of dead loads with the additional mass due to the moving loads. Figs. 7(a) and (b) show the time histories of the dynamic deflection $\overline{w}$ at the midspan for Case 1, in which the load-ratios take the value of 0.1 and 1.0, respectively. The solid line displays the dynamic deflection $\overline{w}$ excluding the effect of the dead loads. The broken line indicates the dynamic deflection including the effect of dead loads. These lines include the influence of the additional mass due to moving loads. It is clarified that the effect of dead loads on the dynamic deflection which includes the additional mass due to the moving loads restrains the



(a)



(b)

Fig. 10 Time histories of dynamic deflection $\overline{w}$ at the midspan for case 4; (a) $\overline{p}/\tilde{p}$ =0.1, (b) $\overline{p}/\tilde{p}$ =1.0

vibration of dynamic defection due to the moving loads and that the effect is more remarkable on heavyweight beams than on lightweight ones.

Figs. 8(a) and (b) show the time histories of dynamic deflection $\overline{w}$ at the midspan for Case 2. Figs. 9(a) and (b) and Figs. 10(a) and (b) indicate the time histories of dynamic deflection $\overline{w}$ at the midspan for Cases 3 and 4, respectively. It is clarified from the above-mentioned numerical results that the effect of dead loads which includes the additional mass due to moving loads is more remarkable on heavyweight beams than on lightweight ones.

8. Conclusions

The effect of dead loads on elastic beams subjected to moving loads has been presented by the use of moving equation including the influence of dead loads (Takabatake 1990). From the results of numerical computation using the Galerkin method and from the closed-form approximate solution including the effect of dead loads, it has been clarified that the effect of dead loads reduces the action of moving loads acting on elastic beams and is larger on heavyweight beams than on lightweight beams. It is also explained that this effect of dead loads restrains the transverse dynamic deflection $\overline{w}$ in the both cases with and without the additional mass due to moving loads. Last, it is clarified that the approximate solution proposed here for the case without the additional mass due to moving loads shows excellent agreement with the corresponding numerical results. This study will raise an important point to dynamic responses of bridges subject to moving loads. Since the action due to moving loads is reduced more effectively on heavyweight beams than on lightweight beams, the author proposes that the safety factor for lightweight beams should be raised to coincide with the safety factor for heavyweight beams in order to estimate the same action of beams due to moving loads.

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