Earthquakes and Structures, Vol. 4, No. 2 (2013) 219-239
DOI: http://dx.doi.org/10.12989/eas.2013.4.2.219

An energy-based design for seismic resistant structures with viscoelastic dampers

F. Paolacci*

University Roma Tre, Department of Structures, Rome, Italy

(Received April 26, 2011, Revised April 8, 2012, Accepted May 25, 2012)

Abstract. The present paper aims at studying the seismic response of structures equipped with viscoelastic dampers (VED). The performance of such a passive control system is here analyzed using the energy balance concept, which leads to an optimal design process. The methodology is based on an energy index (EDI) whose maximization permits determination of the optimal mechanical characteristics of VED. On the basis of a single degree of freedom model, it is shown that the maximum value of EDI corresponds to a simultaneous optimization of the significant kinematic and static response quantities, independently of the input. By using the proposed procedure, the optimal design of new and existing structures equipped with VED, inserted in traditional bracing systems, are here analyzed and discussed.

Keywords: viscoelastic dampers; seismic response; energy balance; random vibrations; optimal design

1. Introduction

In the last few years a review process of the methodologies for approaching some problems of seismic analysis of structures has taken place. For example, at the beginning of the 90's a particular attention was paid on the development of concepts concerning the energy quantities involved in the structural response, from which some interesting design criteria were derived (Uang and Bertero 1988).Significant examples are found in the field of seismic passive control of structures(Ciampi *et al*. 1995, Filiatrault and Cherry 1990).

Unfortunately, for several reasons its application has suffered a stalemate, even if it was clearly demonstrated that the advantages in using it may be important. For example one of the benefits, with respect to more traditional design methodologies, is to have the possibility of optimizing the structural response by reducing both kinematic and static response quantities contemporaneously, as several papers have already shown in the past (Ciampi and De Angelis 1996, Paolacci and De Angelis 1999, Renzi *et al*. 2007). Even if this idea seems to be abandoned in favour of different approaches, for example the "Direct Displacement" approach (Ponzo *et al*. 2007, Albanesi *et al*. 2007, Kim and Choi 2003, Soda and Takahashi 2000), the author deems that it may still be considered valid for several implications, as detailed in the following.

*Corresponding author, Professor, E-mail: fabrizio.paolacci@uniroma3.it

An interesting application is represented by the seismic passive control of structures equipped with viscoelastic devices (VED).

Many design methodologies of VED already proposed in literature are based on simplified procedures (Kasai *et al.* 1993, Chang *et al.* 1996, Park *et al.* 2004). To simulate the viscoelastic behaviour, it is usually considered sufficiently accurate, at least for a preliminary design, to refer to simple models like the Kelvin or Maxwell unit (Lewandowski and Chorążyczewski 2007, Shing and Chang 2009). Moreover, it appears that design procedures which explicitly use optimization concepts based on energy considerations have been considered only occasionally in literature, (Abbass and Kelly 1993, Kasai and Munshi 1994, Paolacci and De Angelis 2001, Lee *et al.* 2005); this occurred more systematically in the case of yielding-based or friction-based dampers (Inoue and Kuwahara 1998, Ciampi *et al.* 2003, Renzi *et al.* 2007). Another characteristic aspect of VED is the strong temperature dependence of the mechanical properties of viscoelastic materials. Many papers have shown that raising temperature may produce detrimental effects on the stiffness and dissipation capability of VED (Chang *et al.* 1998a, Inaudi *et al.* 1996, Kasai *et al.* 2001). This can be taken into account in different ways. A simplified approach consists of calculating the maximum temperature increase developed during the motion to correct storage and loss modulus (Chang *et al.* 1992). A more refined approach consists in the implementation of the temperature dependency on storage and loss modulus directly in the model used for the numerical simulations. This enables to account for the influence of the instantaneous variability of temperature on the viscoelastic material properties (Tsai 1994). Actually this effect depends strongly on the level of shear strain in the VED. For example in (Inaudi *et al.* 1996, Lai *et al.* 1999) the authors have shown that for shear deformationsγ lower than 15%, the behaviour remains substantially linear and the temperature rise effect can be neglected, being the temperature increment confined in a range of 1-2°C. On the contrary, for higher values of γ the internal temperature variation can be extremely high, especially in the high frequency range, where a higher number of deformation cycles is expected. In this case a non-linear analysis is recommended, including in the model of VED the temperature rise effect.

From the above considerations, in the present paper a simplified design methodology of VED is proposed. It is formulated as an optimization problem, where the objective function is an energy index, (*EDI*), a function of both the input energy of the structure and the energy dissipation of the dampers.The EDI index differs from other energy indexes,as the absolute energy dissipation of the damper (Lee *et al.* 2005) or the simple ratio between the energy dissipation of the damper and the input energy (Abbass and Kelly 1993),because it takes into account the instantaneous variability of the energy dissipation capability of dampers with respect to the input energy; therefore it is capable to account for the instantaneous variability of structural response and the seismic input.

The temperature rise effect is not included in the adopted model, with the consequence that during the motion the material properties are considered invariable. At this end, according to the above considerations, the hypothesis of small shear deformation γ is here adopted. On the contrary, the ambient temperature variation is consideredby adopting the well known frequency-temperature equivalence principle (Ferry *et al.* 1952).

Many design procedure formulated in the past adopt iterative approaches (Chang *et al.* 1993, Xua *et al.* 2003, Lee *et al.* 2005), which are time-consuming and often provide only preliminary indications about the optimal characteristics of a VED. For this reason, in this work, a design approach, based on the seismic response of simple models, useful for a preliminary design of structures equipped with VED, is proposed.

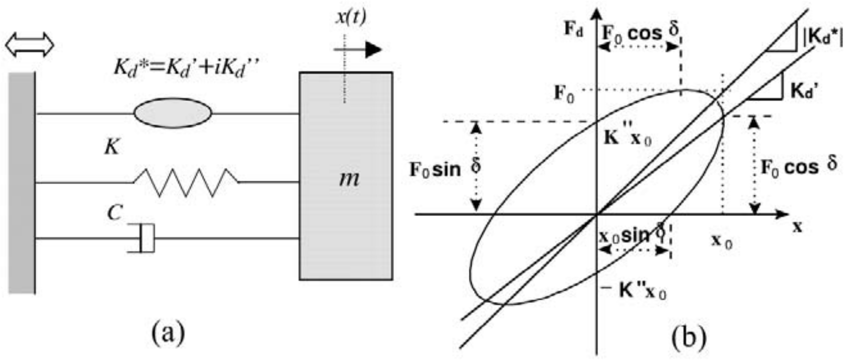


Fig. 1 (a) SDOF model of a structure equipped with VED and (b) constitutive law of VED

At this end, the dynamic response of a single degree of freedom model, representing a generic structure equipped with VED, is initially studied for harmonic and white-noise base excitations, corresponding to the limit cases of narrow-and wide-band input, respectively. Closed-forms of the response quantities, even for the energy terms, are then obtained, and synthetic indications are provided, which are for understanding the effects of more general inputs (Inaudi *et al*. 1993).

Subsequently, the case of structures equipped with viscoelastic dampers, inserted in a traditional elastic bracing system, is considered. The Maxwell model is used to simulate the dynamic behaviour of viscoelastic dampers, whoseoptimal mechanical characteristics are evaluated using the EDI criterion. The procedure leads to useful graphs both for the design of new structures and the retrofitting of existing constructions equipped with VED. It will be shown that these results can be obtained independently of the input.

2. The dynamic behaviour of systems equipped with VED

Let us consider a generic linear elastic structure equipped with damping devices, in particular VED. Since it is often possible to reduce complex dynamic systems to simpler equivalent systems, e.g. one degree of freedom systems, their behaviour can be investigated using the oscillator shown in Fig. 1a, representative of a generic structure with VED.

The frequency and the damping of the uncontrolled structure are $\omega=(K/m)^{1/2}$ and $\nu=C/2m\omega$ respectively, where K is the stiffness, m is the mass and ν is the damping ratio. The constitutive law of the dissipation device is shown in Fig. 1b, whose behaviour depends on the complex stiffness $K_d^*(i\Omega,T)=K_d'(\Omega,T)+iK_d''(\Omega,T)$. The latter is characterized by a real part and an imaginary part, named *storage modulus* and *loss modulus* respectively, which are frequency (Ω) and Temperature (T) dependent functions; their physical meaning is easily discerned from the integral formulation of the constitutive law, which, in the steady-state case, takes the following form

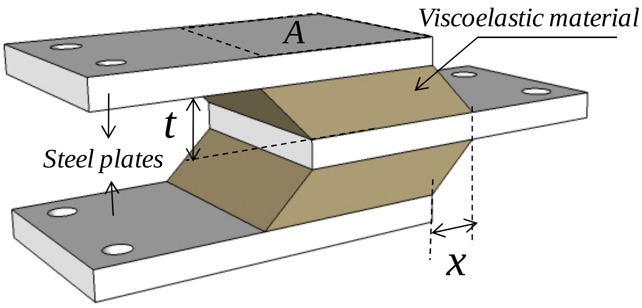


Fig. 2 A typical VED arrangement

$$F_d(x) = K_d'(\Omega,T)x(t) + K_d''(\Omega,T)\sqrt{x_0^2 - x^2(t)} \quad (1)$$

where $x(t)$ is the displacement applied to the system with amplitude x_0, and $F_d(x)$is the resultant force in the damper. It is obvious that the storage modulus is related mainly to the elastic behaviour of the device, whereas the loss modulus defines its dissipation capability, which is associated to the area of the ellipse shown in Fig. 1b. The following simple equations relate the moduli of the device to storage modulus $G'(\Omega,T)$ and loss modulus $G''(\Omega,T)$ of viscoelastic material

$$K_d'(\Omega) = \frac{G'(\Omega,T)A}{t} \quad K_d''(\Omega) = \frac{G''(\Omega.T)A}{t} \quad Tan\delta = \frac{G''(\Omega,T)}{G'(\Omega,T)} = \frac{K_d''(\Omega,T)}{K_d'(\Omega,T)} \quad (2)$$

where A is the area of the viscoelastic device and t is the total thickness of the viscoelastic layers. The term $K_d'=F_0/x_0cos\delta$ represents the stiffness calculated at the maximum displacement x_0 of the cycle, whereas $|K_d^*|$ is the modulus of the complex stiffness. It can be noticed that the maximum force F_0 does not correspond to the maximum displacement x_0 of the cycle and that this out-of-phasebehaviour is represented by the loss factor $tan\delta$.

Fig. 2 shows an example of arrangement of a viscoelastic device. It is composed by 2 layers of viscoelastic material attached to three steel plates, one internal and two external. The longitudinal differential movement x between the internal and the external plates corresponds to the relative displacement between two points of the same structure or of two different structures connected by the VED. Under a shear force F_d the damper undergoes a shear deformation $\gamma=2x/t$.

The ratio between the loss and the storage modulus of material or structure, $Tan\delta$, named *loss factor* shows, for a typical viscoelastic material, a limited variability with frequency and temperature; this characteristic can be used, as illustrated hereafter, to reduce the number of the damper parameters. Experimental tests have shown that for usual viscoelastic materials $Tan\delta$ is variable between 0.8 and 1.4 (Blondet 1994, Kasai *et al.* 2004).

As stated in the introduction, a viscoelastic material can exhibit temperature rise that may change the mechanical characteristics of the material (i.e., storage and loss modulus). This depends on the deformation level and the excitation frequency. In particular by using a simple temperature-dependent numerical model, some authors has been shown that a limited value of shear deformation (γ) limits also the temperature rise in a VED, (Inaudi *et al.* 1996, Chang *et al.* 1998b). They have shown that shear deformations $\gamma<20\%$, produce a maximum variation of internal temperature of 1-2°C,whereas a deformation level of 100% can induce a temperature rise greater than 10°C, especially for high frequencies. In addition, a so high level of deformation can induce a noticeable non-linear behaviour. Therefore, in what follows a limited deformation level is supposed and the temperature rise effect is neglected.

Consequently, the external temperature is only taken into account, considered here as design data. According to the “frequency-temperature equivalence principle” (Jones 2001, Kasai *et al.* 2001), storage modulus and loss factor can be expressed as follows

$$G'(\Omega,T) = G'(\lambda\,\Omega,T_{ref})$$

$$Tan\delta(\Omega,T) = Tan\delta(\lambda\,\Omega,T_{ref}) \tag{3}$$

where λ is the shifting factor and $\lambda\Omega$ is the equivalent frequency. Generally, the shifting factor λ is calibrated using experimental dataand decreases with temperature. For example, for T_{ref}=20°C and $Tan\delta$=1 a variation of temperature of ±10°C corresponds,for commercial VED, to a loss factor variable between 1.5 and 0.8 (Chang *et al.* 1992, Kasai *et al.* 2004).

2.1 Mechanical models for viscoelastic materials

In literature the behaviour of viscoelastic materials is modelled using different constitutive laws. One approach is based on complex rheological models obtained as a combination of simple units, such as the Maxwell or Kelvin-Voigt models. In this case the mathematical formulation leads to a time-dependent linear ordinary differential equation with constant coefficients, whose generic form is (Flugge 1967)

$$\sum_n u_n \frac{d^n\sigma(t)}{dt^n} = \sum_m u_m \frac{d^m\varepsilon(t)}{dt^m} \tag{3}$$

where *σ(t)* and *ε(t)* are the stress and the strain respectively of the material and u_n and u_m are coefficients. According to the correspondence principle, this Eq. becomes,in the frequency domain, an algebraic Eq. defining the complex stiffness $G(i\Omega,T)=G'(\Omega,T)+iG''(\Omega,T)$.This is possible by using,for example, the Fourier transform providing the solution in the frequency domain

$$\sigma(\Omega,T) = G(i\Omega,T)\varepsilon(\Omega,T) \qquad\qquad G(i\Omega) = \frac{\sum_m u_m (i\Omega,T)^m}{\sum_n u_n (i\Omega,T)^n} \tag{4}$$

Although to obtain an exhaustive realistic response a large number of coefficients (*n, m>>1*) should be used, for the purpose of the present paper, the basic Maxwell model is adopted, which is characterized by only two parameters.

In Figs. 3a and 3b, the experimental data obtained in the tests conducted at Berkeley, (Blondet 1994), for a particular viscoelastic damper produced by the 3M Company, are compared with the ones derived froma Kelvin and Maxwell model respectively, calibrated to match as well as possible the experimental results, for a frequency of 2 Hz.

In particular, while the Kelvin model exhibits a constant stiffness and a dissipation capability that increases linearly with the frequency, the Maxwell model shows a better agreement with the experimental results, in a wide range of frequencies, especially for frequencies lower than those

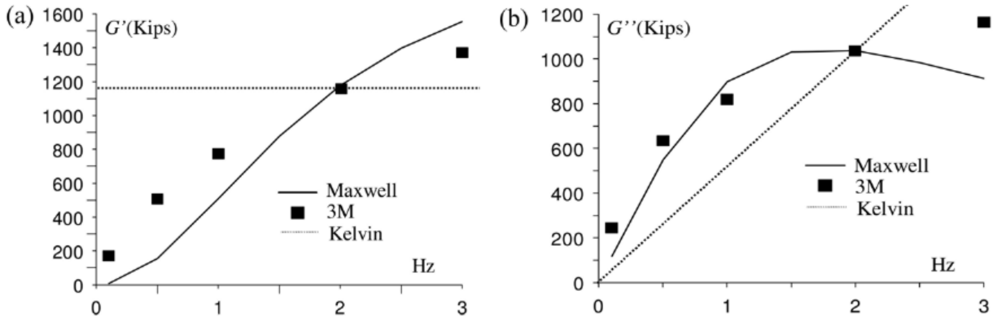


Fig. 3 (a) Storage modulus v/s Frequency and (b) Loss modulus v/s Frequency

used for the calibration (2 Hz). In fact, the stiffness increases with the frequency, with a mean deviation of 8% with respect to the experimental results; the loss modulus shows a maximum at 2 Hz after an initial increasing and presents a better agreement with the experimental results, at least in the frequency range 0-2 Hz. This suggests that reliable results may be expected also for structures, which in case of plasticization, change their frequencies during the motion.

Although there are other models based on different mathematical operators, (e.g fractional operators), capable, with few parameters, to reproduce very accurately the response of viscoelastic devices (Shen and Soong 1995), we will refer only to the previous models, leaving to future developments the use of more sophisticated ones.

2.2 Stationary response to harmonic input

Let us consider now the model of Fig. 1a, subjected to a base harmonic acceleration. The equation of motion normalized with respect to mass and maximum ground acceleration is

$$\ddot{\zeta} + \omega^2 \zeta + 2\nu\omega\dot{\zeta} + F_d(\zeta) = -e^{i\Omega t} \tag{5}$$

The solution of Eq. (5) in the time-domain assumes the form $\zeta(t)=\zeta_0 cos(\Omega t-\delta)$. The amplitude ζ_0 and the phase angle δ are respectively the modulus and the argument of the complex compliance of the system; their expressions are

$$\zeta_0(\alpha) = \frac{1}{\omega^2}\frac{1}{\sqrt{(1+\lambda_{eq}(\alpha)-\alpha^2)+(2\nu(1+\beta_{eq}(\alpha))\alpha)^2}} \tag{6}$$

$$\delta(\alpha) = Arc\,tan\left(\frac{2\nu(1+\beta_{eq}(\alpha))\alpha}{1+\lambda_{eq}(\alpha)-\alpha^2}\right) \tag{7}$$

with $\alpha = \Omega/\omega$. The non-dimensional parameters λ_{eq} and β_{eq} take into account the presence of the device and are defined as: $\lambda_{eq}=K_d'/K$ e $\beta_{eq}=\eta_d\lambda_{eq}/(2\xi\alpha)$. It is clear from Eqs. (6) and (7) that the system of Fig. 1 may be considered as a Kelvin-Voigt oscillator in which stiffness and damping are variable with the oscillation frequency of the motion. The loss factor is practically constant, at least in the range of frequencies considered, so that the parametric analysis is governed by one parameter only, related usually to the stiffness of the device.

2.3 Stationary response to white noise

A closed-form of the response is still possible by using a white noise base motion. The stationary response of a linear viscoelastic oscillator to a white noise is a zero-mean Gaussian process, characterized only by its variance. The variance σ of a generic response quantity with transfer function $H(i\alpha)$ is related to the power spectral density of white noise S_0 by the well-known relation

$$\sigma^2 = S_0 \int_{-\infty}^{+\infty} \left| H(i\alpha) \right|^2 d\alpha \tag{9}$$

Since, the transfer function modulus is the oscillation amplitude of the stationary response, given by the solution of Eq. (5), the variance of both displacement and absolute acceleration of the oscillator of Fig. 1 is given by the following relations

$$\sigma_\zeta^2 = S_0 \int_{-\infty}^{+\infty} \frac{1}{\omega^2} \frac{1}{(1+\lambda_{eq}(\alpha)-\alpha^2)+(2\nu(1+\beta_{eq}(\alpha))\alpha)^2} d\alpha \tag{10}$$

$$\sigma_{\zeta_A}^2 = S_0 \int_{-\infty}^{+\infty} \left| -K_d^*(i\alpha)C(i\alpha) \right|^2 d\alpha \tag{11}$$

where S_0 is the power spectral density of the white noise.

If the behaviour of the control device were simulated with simple models, the integrals Eqs. (10) and (11) would be calculated in closed-form or alternatively, for more sophisticated models, approximate methods could be used. For example, using Maxwell model with stiffness K_d and damping coefficient C_d, Eq. (10) turns into the following

$$\sigma_\zeta^2 = \frac{\frac{K_d}{K} C_d (K_d + CC_d) + C_d^3}{C_d (K_d + CC_d)(CK_d + C_d K + C_d K_d) - KK_d C_d^2} \tag{12}$$

2.4 The energy balance of viscoelastically damped systems

For a generic dynamic system, subjected to a base motion $W(t)$, the energy balance equation can be written as follows

$$\int_0^t \dot{\mathbf{y}}^{\mathbf{T}} \mathbf{M} \ddot{\mathbf{y}} dt + \int_0^t \dot{\mathbf{y}}^{\mathbf{T}} \mathbf{F}_R(t) dt = -\int_0^t \dot{\mathbf{y}}^{\mathbf{T}} \mathbf{M} \Gamma W(t) dt - \int_0^t \dot{\mathbf{y}}^{\mathbf{T}} \mathbf{F}_{\mathbf{C}}(t) dt \tag{13}$$

where the terms on the left hand side represent respectively the kinetic energy and the energy associated with the restoring force $\mathbf{F}_R(t)$, while on the right hand side, the energy E_{VE} associated with the control force $\mathbf{F}_C(t)$ is subtracted from the input energy. The effect of energy dissipation appears as a reduction of the input energy of the system, which depends in general, not only on the external action, but also on the dynamic characteristics of the system. In Eq. (13), Γ is a unit vector, $\mathbf{y}$ is the displacement vector and $\mathbf{M}$ the mass matrix, whereas, the first integral on the right hand side is the relative input energy E_I. The energy of the devicesis also subdivided into two contributions, one elastic $E_{VE,E}$ and the other that represents the irreversible energy $E_{VE,D}$. The latter is given by the following relationship

$$E_{VE,D} = \int_0^t F_d(\zeta)\dot{\zeta}dt - E_{VE,E} \tag{14}$$

From the application point of view it is useful to evaluate the increment of $E_{VE,D}$ in a stationary cycle. For the oscillator of Fig. 1, using Eq. (14) for $t=2\pi/\Omega$, we get the simple expression

$$E_{VE,D}^{(cycle)} = \pi K_d''(\alpha)\zeta_0^2(\alpha) = 2\pi\nu\alpha\beta_{eq}(\alpha)\zeta_0^2(\alpha) \tag{15}$$

which shows that the dissipation capability of the damper is related to the loss modulus, while for a given amplitude of the displacement, it also depends on the frequency of the base motion.

As well known, the stationary response of a viscoelastically damped oscillator to a White Noise is a zero mean Gaussian process. This implies that, within the stationary response, it is possible to identify an average cycle, similar to that of Fig. 1, with an average period equal to the natural period of the oscillator, whose maximum amplitude, x_0, may be replaced with the standard deviation of the displacement. The expected values of the several energy termsin a stationary cycle are functions of the mean square values of the kinematic quantities only, in particular displacement and velocity. As an example, for a Kelvin oscillator with frequency ω_0 and damping coefficient ν the expected values of the energies indicated in Eq. (13), for stationary conditions, have the following expressions, (Spanos 1978, Clough and Penzien 1975)

$$E[E_K] = E[\dot{y}^2]/2 = \pi S_0/(4\nu\omega_0) \tag{16}$$

$$E[E_D] = 2\pi\xi\omega_0^2 E[y^2]t = \pi S_0 t \tag{17}$$

$$E[E_E] = \omega_0^2 E[y^2]/2 = \pi S_0/(4\nu\omega_0) \tag{18}$$

where E_K, E_D, E_E are respectively, kinetic, dissipated and elastic energy of the system, while S_0 is the power spectral density of the White Noise and t is time. The mean value of the dissipated energy is therefore independent of the characteristics of the oscillator and the value, indicated in Eq. (17), is valid for all SDOF systems, (Inaudi *et al.* 1993, Karnopp 1970). Moreover, if we try to estimate the dissipated energy using Eq. (15), its average value would be double the exact value obtained with expression 17. Therefore, in the case of random excitation, it is necessary, to evaluate the dissipated energy on the basis of its general definition Eq. (15), as already shown for more complex structural situations (Inaudi *et al.* 1993).

In summary, for a generic viscoelastically damped system, subjected to White Noise excitation, it is possible to express in closed form the mean values of the energies in the balance Eq. (13) by using mean square values of displacements and velocities, the mechanical characteristics of the system, and by adopting as dominant vibration period of the system the average period of the oscillations.

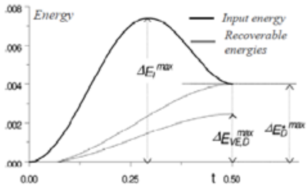


Fig. 4 Harmonic input - Stationary semi-cycle

3. Optimal design of passive control systems

As already pointed out in the introduction, the problem of the passive control of systems can be effectively formulated as optimization problem by using as objective index a functional of both dissipated and input energies. This choice is justified considering that control effectiveness does not depend only on the artificial increment of structural damping, but also on the characteristics of the external action and controlled system. In the past, an interesting design criterion based on maximization of an Energy Dissipation Index (*EDI*) was proposed (Ciampi and De Angelis 1996). This is defined, in case of stationary response to a harmonic excitation, as the ratio between the maximum increment of the energy dissipated by the control devices in a half-period, ΔE_D^{max}, (Fig. 4), and the corresponding maximum increment of the input energy in the same interval of time ΔE_i^{max}

$$EDI = \Delta E_D^{\max} \big/ \Delta E_I^{\max} \tag{19}$$

In a general case of non-harmonic response, Eq. (19) is no longer applicable. However, it is always possible to observe an oscillating behaviour of the response quantities variable with period and amplitude. This has suggested a more general definition of the index *EDI*, obtained by a weighted average of the ratios of the energy increments ΔE, by using the following expression

$$EDI = \sum\nolimits_1^n \Delta E_{D,i} \Big/ \sum\nolimits_1^n \Delta E_{I,i} \tag{20}$$

An equivalent definition may be derived for the case of stationary Gaussian response to white noise excitation, by using the average values of the individual energy terms

$$EDI = \frac{E[E_D]}{\sqrt{E[E_D^*]^2 + E[E_E + E_K]^2}} \tag{21}$$

The single terms in Eq. (21) are referred to an average equivalent half-cycle, whose maximum amplitude, (Fig. 1), is identified with the standard deviation of the displacement; moreover, as already noted, they are functions of the mean square values of displacements and velocities. The ratio between the two maximum increments in Eq. (19) is then replaced by the ratio of a proper measurement of their expected values, obtaining Eq. (21).

The maximum increment of the energy dissipated by the control devices, ($E_{VE,D}$), can be approximately determined applying definition Eq. (19), whereas $\Delta E_I^{\max}$ can be computed using the square root of the sum of the square of the recoverable energy (E_K+E_E) and of the dissipated energy (E_D^*); this last includes all sources of dissipation, such as conventional structural damping and the energy dissipation of the VED.

For example, for a Kelvin oscillator subjected to a white noise excitation, the *EDI* index assumes the simple expression

$$EDI = \frac{2\pi\nu}{\sqrt{(4\pi^2\nu^2 + 1)}} \tag{22}$$

Fig. 5 depicts the variation of Eq. (22) versus damping ratio ν; it shows a rapid increase of *EDI*, followed by an asymptotic behaviour whereby, after ν>0.4, *EDI* may be considered practically constant. Eq. (22) also appears to be in good agreement with the results obtained by using a Monte

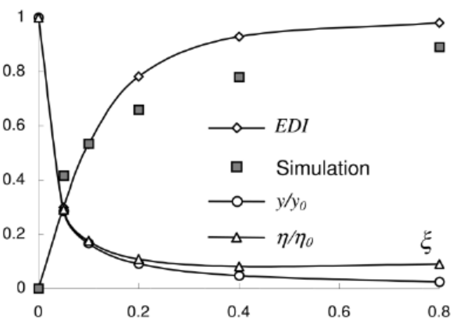


Fig. 5 White noise: Kelvin oscillator

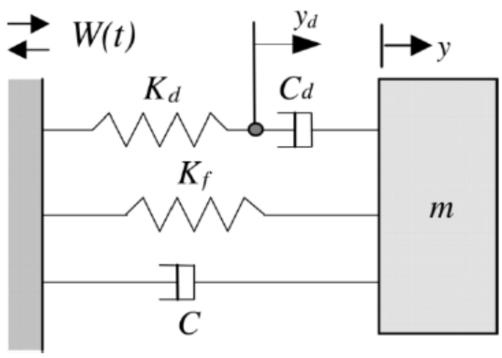


Fig. 6 Model of frame equipped with VED

Carlo simulation based on Eq. (20). The same figure illustrates the reduction of displacement (ζ/ζ_0) and total force (η/η_0); both these quantities have been normalized to the response computed for the case in which only a conventional damping, (2%), is present.

The optimal value of ν corresponds to the elbow of the *EDI* curve, (ν≅20%), where the total force shows its maximum reduction, after whichthe displacement shows no further significant decrease. A similar behaviour, which has been already shown in deterministic cases and for much more complex structural situations, confirms the multi-objective character of the *EDI* index, (Basili and De Angelis 2007).

4. Design of structures equipped with VED

One application of the proposed methodology is the evaluation of design graphs for structure equipped with VEDinstalled in special bracing systems, (Chang *et al.* 1996, Kasai and Munshi 1994, Lee *et al.* 2005). In the following, using a simple SDOF system and the *EDI* maximization criterion, design curves of structure equipped with VED inserted in traditional bracings are here proposed. In particular, both the cases of new and existing structures are here considered.

4.1 Definition of numerical model and response parameters

In order to define the optimal characteristics of viscoelastic dampers, the seismic dynamic response of the SDOF model illustrated in Fig. 6, is used. The reaction of the structure is assumed to be elasto-plastic with stiffness $K_f=\omega_f^2 m$and yielding strength F_{fy}. The control device is modelled as a Maxwell unit with stiffness K_d and damping coefficient C_d. For the dissipative bracing system, the conventional braces are assumed to be infinitely stiff, so that only the stiffness of the dissipation device needs to be taken into account. In the case of deformable braces it is sufficient to consider K_d as the entire elastic stiffness of the damper + brace ensemble.

The equation of motion, normalized with respect to the mass of the system, provides the significant parameters of the dissipation system

$$\lambda=K_d/K_f \qquad \beta=(C_d/K_d)/(2\pi/\omega_f) \tag{23}$$

The control device shows dissipative characteristics that are related to the loss factor $Tan\delta=2\pi\beta f(\beta,\lambda)$, where $f(K_d,C_d)=\omega_b/\omega_f$ is the ratio between natural frequency of the damped system and the frequency of the unbraced frame. Based on the definitions Eq. (23), the parameters λ_{eq} and β_{eq} assume the following form

$$\lambda_{eq}=\frac{4\pi^2\beta^2\alpha^2}{1+4\pi^2\beta^2\alpha^2}\lambda \quad \beta_{eq}=\frac{\pi\beta\lambda}{\nu(1+4\pi^2\beta^2\alpha^2)} \tag{24}$$

Assuming λ to be constant, it is clear that if $\beta\to\infty$ the damper degenerates into a pure elastic element ($\lambda_{eq}\to\lambda$ and $\beta_{eq}\to 0$); moreover for $\lambda\to\infty$ the viscoelastic element degenerates into a pure viscous element ($\lambda_{eq}\to\infty$ and $\beta_{eq}<\infty$).

The significant non dimensional response parameters are, besides *EDI* and the maximum displacement normalized to the corresponding value of the unbraced frame $\zeta=y/y_0$, the following other quantities

$$\mu_f=\zeta/\zeta_y \qquad \textit{Structural ductility} \tag{25}$$

$$\eta_{II}=(F_f+F_d)/(m\,\ddot{x}_{G,max}) \qquad \textit{Total force (base shear)} \tag{26}$$

$$\eta_f=F_f/(m\,\ddot{x}_{G,max})<\eta_{fy}=F_{fy}/(m\,\ddot{x}_{G,max}) \qquad \textit{Force in the frame} \tag{27}$$

where F_f and F_d are respectively the force in the frame and damper and $\ddot{x}_{G,\max}$ is the peak

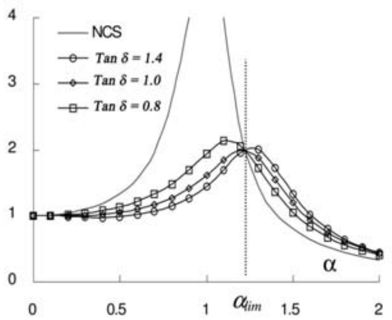


Fig. 7 Harmonic input: Frequency response function (λ=1, ξ=0.00)

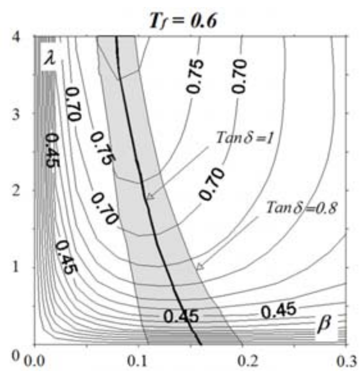


Fig. 8 Artificial accelerograms: *EDI* contour lines

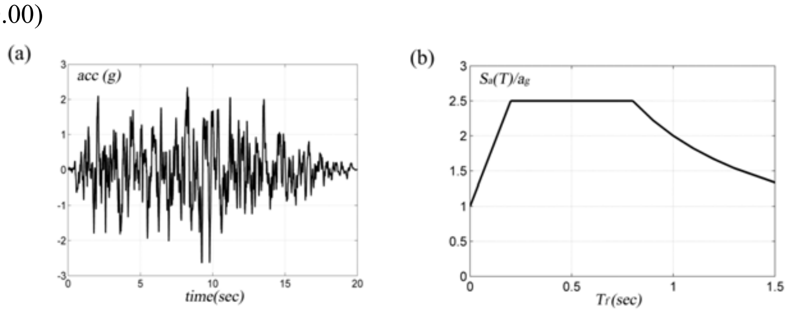


Fig. 9 (a) One of the 5 artificial accelerograms and (b) Design spectrum for the generation of the accelerograms ground acceleration.

It has already been shown that it is possible to establish a priori an optimal value for the parameter $Tan\delta$, independently of the input (Paolacci and De Angelis 1999). For the harmonic input case, Fig. 7 shows the amplification curves of the displacement as a function of the normalized frequency α, for λ=1 and three different values of $Tan\delta$. Moreover, the conventional damping ν is assumed to be zero. It is possible to observe that the control action is not always favourable. In particular a limit value $\alpha_{lim}(\lambda)$ exists, exceeding which, the response is worse than the unbraced case (NCS). The frequency α_{lim} is insensitive to the loss factor and corresponds exactly, for $Tan\delta$ =1, to the minimum value among all maxima of all the curves within the considered range of the loss factor. This suggests that a unit loss factor could be considered an optimal value. If $\nu \neq 0$ the previous conclusions donot change, with the only difference that the envelope of the maxima is no longer the decreasing branch of the unbraced frame. In addition, a limit value of α exists also for the acceleration; however for $\nu \neq 0$ this value is always greater than α_{lim}.

An interesting alternative way to prove the previous conclusion is to use the maximization criterion of the *EDI* index. Fig. 8 shows the contour lines, in the plane λ-β, of the function *EDI*, computed for a set of 5 artificially generated accelerograms compatible with EC8 spectra (C soil profile, 5% damping, see Figs. 9a and 9b).

The period of the frame is T_f= 0.6 sec. It is possible to note that the curve for $Tan\delta$=1 is the locus of the relative maxima of the function *EDI* (bold line). Actually this behaviour occurs for all the periods T_f(0.3÷1.5 sec) considered, which suggests once again unity as optimal loss factor. To complete this investigation, on the same graph the curves for $Tan\delta$=0.8 and $Tan\delta$=1.4 are also drawn.

These considerations show that λ can be considered as the main design parameter, while β is constrained to obtaining a constant value for $Tan\delta$.

According to the frequency-temperature equivalence principle and using Eq. (23), it is clear that a variation of external temperature affects both β and $Tan\delta$. In particular, being the shifting factor decreasing with temperature, atemperature increasing would decrease β and increase $Tan\delta$. In what follows a reference temperature T_{ref}=20°C and a unitary loss factor ($Tan\delta$=1) will be assumed. Considering a realistic temperature variation of ±10°C, the loss factor is considered oscillating between 0.8 (T=30°C) and 1.4 (T=10°C). Therefore the range of variation of $Tan\delta$ adopted in Fig. 8 already accounts forthe possibility that temperature mayvary within a realistic range.

4.2 Design of new structures equipped with VED

Because in designing new structure equipped with a special protection system the choice to keep the structure in the elastic range is usually adopted, in the following the dynamic behaviour of the SDOF of Fig. 6 with $F_y=\infty$ is studied.

4.2.1 Analysis of the response: Harmonic and white noise input

In the case of harmonic input the resonance phenomenon is very important and its effects can be reduced by changing damping and frequency of the system. But very often the frequency is not known and it is necessary to find a solution that reduces the response over the entire range of

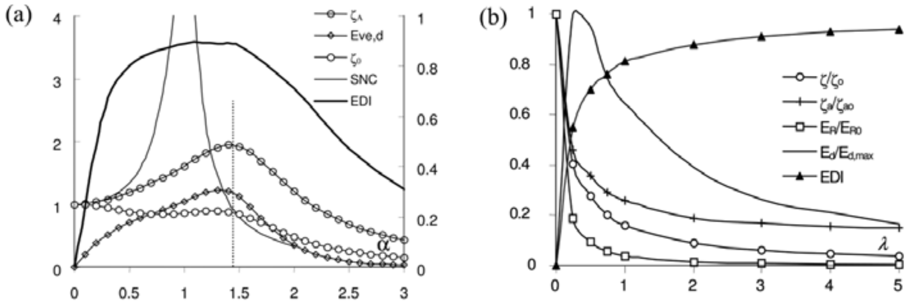


Fig. 10 (a) Harmonic base motion (λ=2, $Tan\delta$ =1) and (b) Harmonic base motion (λ=2, $Tan\delta$ =1)

frequencies. Concerning this subject, Fig. 10a shows for λ=2, ν=0.05 and $Tan\delta$=1, the different response quantities; it is clear that the presence of VED is effective in the entire range of α.

In order to evaluate the optimal value of λ, the criterion of *EDI* maximization is used. In particular, it is possible to plot the maxima of the response, corresponding to α_{lim}, versus λ, obtaining graphs like that illustrated in Fig. 10b. All quantities are normalized to the corresponding values of the unbraced system. After a rapid increase, the *EDI* index shows an asymptotic behaviour, remaining substantially unvaried for λ>2.

The same figure shows the behaviour of the dissipated ($E_{VE,D}$) and recoverable (E_R) energies. Thelatter has been obtained as a sum of kinetic and elastic energy. It is observed that a criterion, based on the dissipated energy only, fails toproduce acceptable results. In fact, it indicates as optimal relative stiffness λ=0.5, corresponding to a reduction of the response quantities of less than 25% with respect to the indication provided by the *EDI* index (λ>1).

On the contrary the reduction of the recoverable energy (E_R/E_{R0}) confirms the conclusions suggested by the proposed energy criterion.

The application of Eq. (21) permits determination of the optimal value of λ also in the random vibration case, where the average values of the energies, in the equivalent half-cycle, now assume the following expressions

$$E[E_d]=C_d E[(\zeta-\dot{\zeta}_d)^2]\pi/\omega_b \quad (28)$$

$$E[E_d^*]=CE[\dot{\zeta}^2]\pi/\omega_b+E[E_d] \quad (29)$$

$$E[E_K]=E[\zeta^2]/2 \quad (30)$$

$$E[E_E]=v^2 K_f E[\zeta^2]/2 \quad (31)$$

in which *y* and $(\zeta-\dot{\zeta}_d)$ are respectively displacement and relative velocity of the Maxwell unit.

The contour level of *EDI* in the space of the variables λ-β is illustrated in Fig. 11a. The comparison with the results of Fig. 7, even if only for a particular period, once more shows as optimal of the loss factor the unity.

Fig. 11b shows the variation of *EDI* versus λ for unit loss factor. A comparison with the results of a Monte Carlo simulation shows that expression Eq. (21) gives indications similar to those obtained numerically by using definition Eq. (20); in fact, in the field of the significant λ values, (0÷4), both curves are quantitatively comparable, even though Eq. (9) underestimate the *EDI*

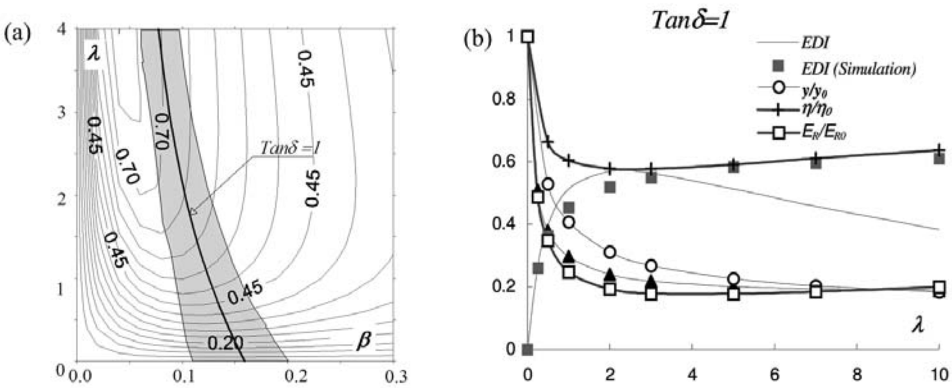


Fig. 11 (a) White noise contour line (ν=0.05) and (b) White noise (ν=0.05)

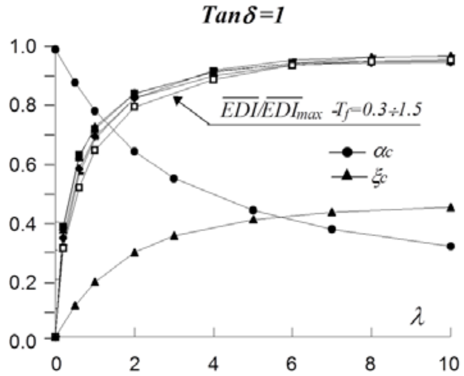


Fig. 12 Artificial accelerograms: EDI v/s stiffness ratio λ (ν=0.05)

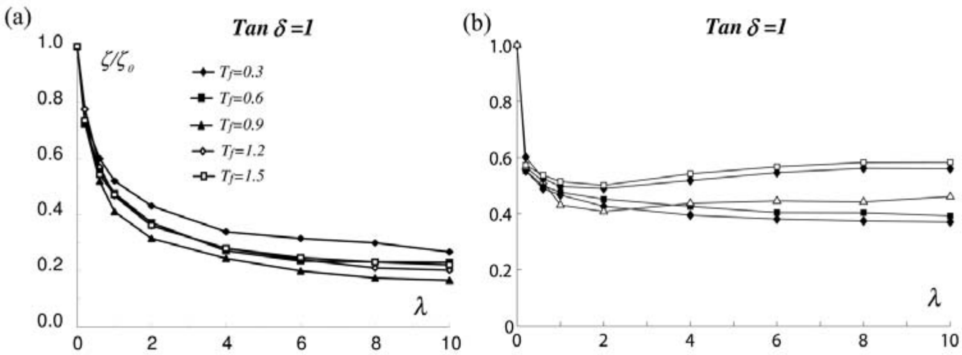


Fig. 13 Artificial accelerograms: (a) Displacement reduction and (b) Accelerograms: force reduction

values for λ> 4. Moreover the analytical curve has a maximum that coincides with the elbow of the numerical curve, where, the minimum or a stationary point of the relevant response quantities also occurs.

As a matter of fact, for λ>2, the displacement of the system no longer shows substantial reductions, while the total force presents an absolute minimum at λ=2, which might be considered an optimal value.

This same Fig. 11b shows reduction of the expected value of recoverable energy (E_R/E_{R0}), normalized to the corresponding value of the uncontrolled case. Observation confirms that, in the linear case and with constant power input (white noise), recoverable energy might be an alternative index to *EDI* for singling out optimal passive devices, since it attains minimum values, in the space of design variables, where *EDI* attains its maximum.

4.2.2 Analysis of the response: Accelerograms

In presence of a more realistic base motion, represented here by the set of the 5 synthetic accelerograms previously used, it is also possible to obtain design spectra by making a section along the curve for $Tan\delta=1$ illustrated in Fig. 7. The results are illustrated in Figs.12-14. Fig. 12 shows for different periods T_f the index *EDI* normalized with respect to the maximum, $EDI_{\max}$. It can be observed that the normalized curves are almost insensitive to the period T_f; moreover, its maximum is reached for very high values of λ and all the curves show a very pronounced elbow, so that the values of λ of interest appear to range between 0 and 4. In the Fig. 12 the corresponding equivalent damping ξ_b and the ratio between the period of the braced and unbraced structure $\alpha_b=1/\nu(\lambda,\beta)$ are also shown. It is possible to note that for $\lambda<4$ the equivalent damping and the period reduction areless than 38% and 50%, respectively. Using these results design graphs can be built by selecting the ratio $EDI/EDI_{\max}$ around the observed elbow of the $EDI-\lambda$ curves of Fig. 12. Values λ=1 or 2 may represent a good compromise between high values of *EDI* and acceptable stiffness ratios λ; the corresponding values of $EDI/EDI_{\max}$ are 0.75 or 0.85, respectively; the equivalent damping is of the order of 20%–30%, whilst the maximum reduction of displacements and base shear is about 65-75% (Fig. 13a) and 30-40% (Fig. 13b).

Fig. 14a shows the design spectrum of the base shear η_{II} computed for the above selected values of λ; for increasing values of period the effect of dissipation is maximum in the flat part of the elastic spectrum and decreases thereafter. Comparison with the elastic force spectrum shows a maximum reduction of the base shear of the order of 40% for $EDI/EDI_{\max}$=0.85. Finally, Fig. 14b shows the maximum values of frame (η_f) and dissipative brace (η_d) forces. Of course, they do not occur at the same time because of the presence of a purely viscous element in the Maxwell model that implies a difference in phase angle between the reaction of the damper and of the main structure.

The force spectra illustrated permit the design a new building, including the definition of the structural details of the dissipative bracing, depending on the corresponding maximum force η_d.

The previous observations confirm the results already obtained with simpler inputs like harmonic or white noise base motion. Thus the *EDI* maximization criterion has the important advantage of being applied independently of excitation. This makes it possible to obtain indications on the optimal design parameters, even in closed-form, by using simple external excitations and, without resorting to the many numerical analyses necessary for more complex inputs.

The effect of temperature variation in terms of displacement and total base shear is shown in Fig. 15. As stated before, the temperature is considered variable in the range 10-30°C, with T_{ref}=20°C, which correspond to $Tan\delta$ variable between 1.4~0.8, with the assumption that $Tan\delta(T_{ref})=1$.

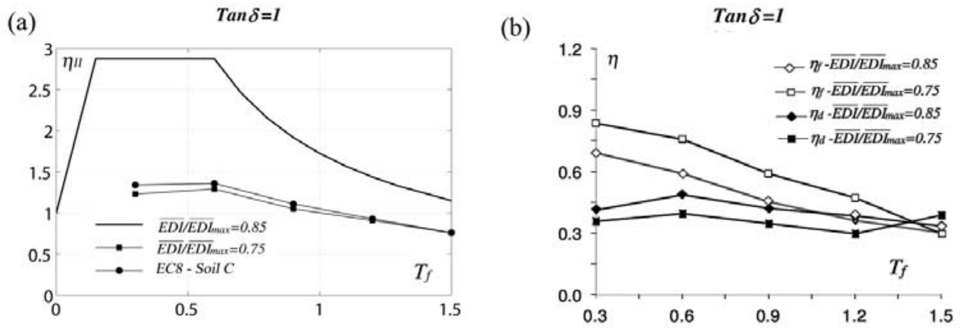


Fig. 14 Artificial accelerograms: (a) η_{II} versus T_f and (b) Accelerograms: η_f and η_d versus T_f

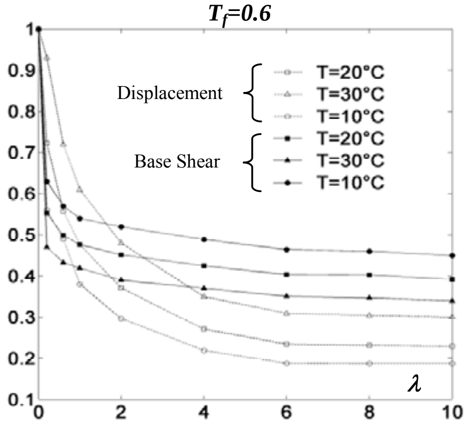


Fig. 15 Artificial accelerograms: displacements and base shear reduction versus λ for different ambient temperatures (T_f=0.6 sec)

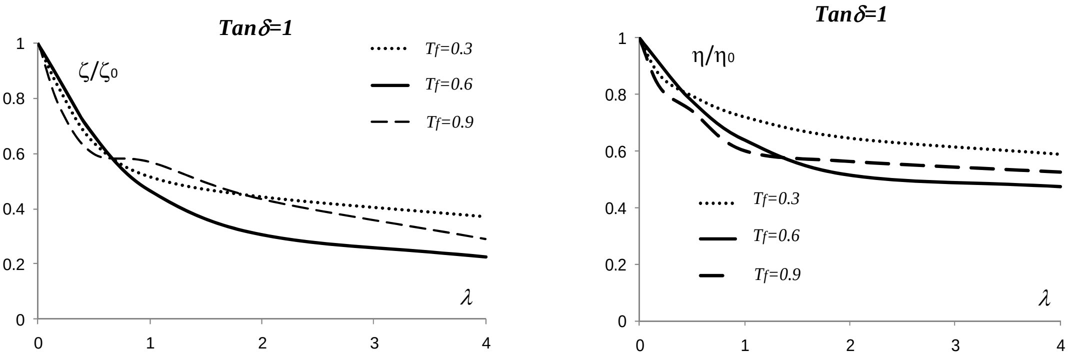


Fig. 16 El Centro record: (a) Displacement reduction and (b) Accelerograms: force reduction

A temperature decreasing (10°C) entails a decreasing of displacements (~30%) and an increasing of base shear (~15%). The opposite behaviour is shown when temperature increases, with a similar variations of displacements and base shear. These results are in good accordance with literature results (Chang *et al.* 1998a). In conclusion, the effectiveness of VED can be considered still high even in presence of important variations of ambient temperature (± 10°C).

To show the goodness of the proposed method, also in presence of natural seismic events, the response of the SDOF system of Fig. 6 to the El-Centro Earthquake (Imperial Valley, May 18 1940, 270°) is shown in terms of displacement and base shear reduction. For investigating the effectiveness of VED for structures with different dynamic properties, three different natural periods T_f have been considered: 0.3, 0.6 and 0.9 sec (see Fig. 16). In addition, a unitary loss factor has been selected, whereas the relative stiffness of VED, λ, varies within the range [0~4]. As for the artificial accelerograms the high effectiveness of viscoelastic dampers is clearly shown. For $\lambda=2$ the displacement reduction is around 60~70%, whereas the base shear can be reduced at maximum of about 50% for higher periods (0.6 and 0.9 sec), whereas for lower periods (0.3 sec) the effectiveness is reduced to 40%.

4.3 Retrofitting of existing structures using VED

The design of a seismic protection system for the retrofitting of existing structuresis usually based on the limitationof the damage in the main structure.This implies that its inelastic behaviour has to be considered. To this end, the structural reaction can be approximately considered elasto-perfectly plastic with normalized strength η_y; to quantify the damage level in the structure, in what followsthe kinematic ductility of the structure μ_f will be used, adopting values of μ_f variable

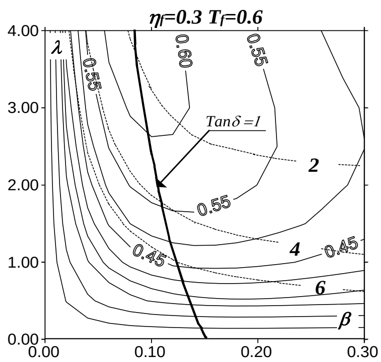


Fig. 17 *EDI* and μ_f contour lines for non-linear SDOF model ($\eta_f=0.3$, T_f=0.6 sec)

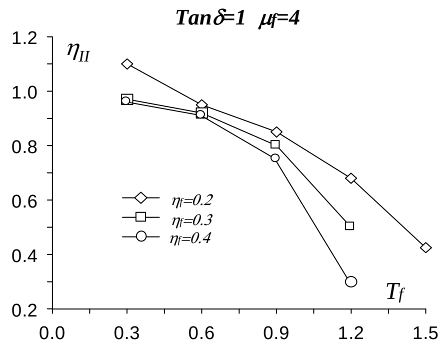
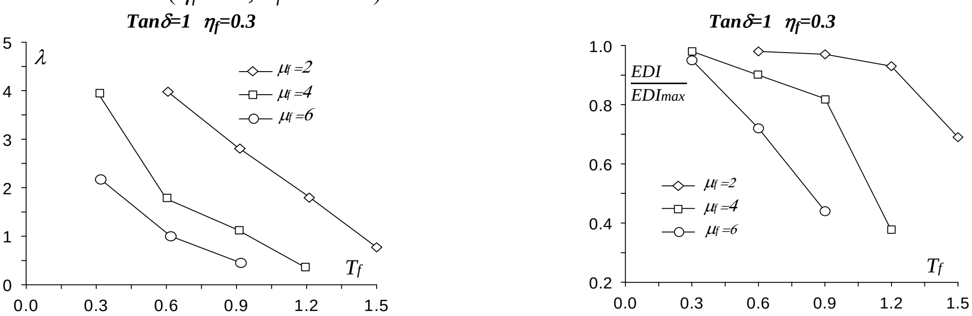


Fig. 18 η_{II}versus T_f for non-linear SDOF system



(a) (b)

Fig. 19 (a) Relative stiffness λ versus T_f and (b) Normalized *EDI* index versus T_f

between 1÷6. In any case, the choice of different parameters for the damage evaluation of the structure does not affect the conclusions.

Since the problem has many constraints, it is reasonable to select the accepted damage level μ_f and verify a posteriori the behaviour of the performance index *EDI*. Fig. 17 shows, for $\eta_f=0.3$ and a vibration period $T_f=0.6$ sec the contour lines of *EDI* and μ_f in the plane β-λ, together with the curve $Tan\delta=1$, obtained using the set of 5 synthetic accelerograms previously used. The curve $Tan\delta=1$ remains substantially unvaried with respect to the elastic case (Fig. 11), because practically independent of the frequency.

Again, the condition $Tan\delta=1$ appear to select the relative maxima of *EDI* on the surface. Therefore, the contemporarily application of the criteria $Tan\delta=1$ and μ_f =cost determines the evaluation of the design parameters. In particular the intersection point between the curve $Tan\delta=1$ and the curve $\mu_f=cost$ identify the optimal values of λ. For example, imposing a ductility $\mu_f=4$ themethod provides a value of $\lambda\sim2$.

On the basis of this criterion, design spectra of the total force η_{II} and the force in the bracings η_d can be drawn. Fig. 18 shows an example of such spectra, where the response quantity (η_{II}) is function of the elastic period of the unbraced structure T_f, and is depicted for a given level of damage μ_f and for three level of strength η_{fy}. The graphs show that a large structural strength η_{fy} do not necessarily imply a large base shear.

In Fig. 19a the design spectra of λ versus T_f and for different level of the accepted damage levels μ_f are illustrated. For increasing value of periods the values of λ decrease almost linearly up to 0, corresponding to a limit value of the period, besides which the dissipative bracing is no longer necessary. Fig. 19b shows the corresponding behaviour of the index *EDI*, that is the performance of the damper; it decreases with T_f up to 0 for a limit value of the period that depends essentially on the level of accepted damage.

Finally, different values of $Tan\delta$, and thus of ambient temperature *T* (see Section 4.1), have also been investigated, whose results are not shown here for brevity. As in the case of elastic structures, a limited sensitivity to this parameter of the response of structures retrofitted with VED has been discovered, with variations of effectiveness similar to those previously discovered for the elastic frame case.

5. Conclusions

A criterion to determine the optimal characteristics of viscoelastic devices for the seismic control of structures has been proposed and an energy-based index, (*EDI*), has been used as objective function. Optimal design of the control system is obtained by maximizing the *EDI* index. It is interesting to note that the multi-objective nature of the index induces a simultaneous reduction of both kinematic and static response quantities. Finally, for the case of a white-noise input, the design problem has been formulated by using a statistical evaluation of the significant energy response quantities and in particular of the *EDI* index.

The optimization procedure has been applied to a single degree of freedom system, representative of a structureequipped with VED; the behaviour of the latter has been modelled using a Maxwell unit. Employing both simple and more realistic inputs (artificial and natural accelerograms), it has been shown that is possible to select unity as optimal value of the loss factor.

The optimal value of the remaining parameter λ (the normalized stiffness of the bracing), has been evaluated using again the proposed optimization criterion. Comparison of the response to simple inputs, like harmonic and white-noise inputs, with the response to synthetic accelerograms, has shown that the optimal design of the viscoelastic devices is practically independent of the input. This means that it is possible to obtain preliminary indications on the optimal characteristics of the dampers, even in closed form, by using simple inputs and simple models, to check there after in a more realistic context, by analysing MDOF models, object of a further publication. These results has been used to build design graphs useful to evaluate the global parameters of a viscoelastic bracing system both for the design of new structures and the retrofitting of existing ones.

References

Abbass, H. and Kelly, J. (1993), "A methodology for design of viscoelastic dampers in earthquake resistant structures", *Report UBC-EERC-93/09*.

Albanesi, T., Bergami A.V. and Nuti, C. (2007), "Displacement-based design of buckling-restrained braces for the seismic rehabilitation of existing r.c. frames", *10th World Conference on Seismic Isolation, Energy Dissipation and Active Vibrations Control of Structures*, Istanbul, Turkey.

Basili, M. and De Angelis, M. (2007), "A reduced order model for optimal design of 2-MDOF adjacent structures connected by hysteretic dampers", *J. Sound Vib.,* **306**(1-2), 297-317.

Blondet, J.M. (1994), "Dynamic response of two viscoelastic dampers", *Report to County of Santa Clara Construction Service Division, University of California at Berkeley*, UCB/SEMM-1994/01.

Chang, K.C., Soong, T.T., Oh, S.T. and Lai, M.L. (1992), "Effect of Ambient temperature on a viscoelastically damped structure", *J. Struct. Eng.,* **118**(7), 1955-1973.

Chang, K.C., Soong, T.T., Lai, M.L. and Nielsen, E.J. (1993), "Development of a design procedure for structures with added viscoelastic dampers", *Proceedings of ATC-17-1 Seminar on Seismic Isolation, Passive Energy Dissipation, and Active Control*, San Francisco, California, **2**, *Passive Energy Dissipation, Active Control, and Hybrid Control Systems*, Applied Technology Council, Redwood City, CA, 473-484.

Chang, K.C., Lin, Y.T. and Lai, M.L. (1996), "Seismic design of structures with added viscoelastic dampers", *XI World Conference on Earthquake*, Acapulco, Mexico.

Chang, K.C., Lin, Y.T. and Lai, M.L. (1998a), "Seismic design of structures with viscoelastic dampers", *ISET J. Earthq. Technol*., **380**(35), 143-166.

Chang, K.C., Tsai, M.H., Chang, Y.H. and Lai, M.L. (1998b), "Temperature rise effect of viscoelastically damped structures under strong earthquake ground motions", *J. Mech.*, **14**(3),125-135.

Ciampi, V., De Angelis, M. and Paolacci, F. (1995), "Design of yielding or friction-based dissipative bracing for seismic protection of buildings", *Eng. Struct.*, **17**(5), 381-391.

Ciampi, V. and De Angelis, M. (1996), "Optimal design of passive control systems based on energy dissipation for earthquake protection of structures", *Proc. 3rd European Conf. on Struct. Dynamics EURODYN*, Florence, Italy.

Ciampi, V., De Angelis, M. and Renzi, E. (2003), "Passive and semi-active control of the seismic response of structures", A. Luongo, Editors. *Recent Research Developments in Structural Dynamics*, ISBN: 81-7736-186-4. Kerala (India): Research Signpost, 225-254.

Clough, R.W. and Penzien, J. (1975), *Dynamics of structures*, McGraw Hill, London.

European Committee for Standardization (CEN). (2004), *Eurocode 8 :Design of structures for earthquake resistance Part 1: General rules, seismic actions and rules for buildings*, EN 1998-1.

Ferry, J.D., Fitzgerald, E.R., Grandine, L.D. and Williams, M.L. (1952), "Temperature dependence of dynamic properties of elastomers; relaxation distributions", *J. Ind. Eng. Chem.*, **25**(4), 720-729.

Filiatrault, A. and Cherry, L. (1990), "Seismic design spectra for friction damped structures", *J. Struct. Div.-ASCE*, **166**(5), 1338-1355.

Flügge, W. (1967), *Viscoelasticity*, Blaisdell Publishing Company.

Inaudi, J.A., Zambrano, A. and Kelly, J. (1993), "On the analysis of structures with viscoelastic dampers", *UBC/EERC Report n. 93/06*.
Inaudi, J.A., Blondet, M., Kelly, J.M. and Aprile, A. (1996), "Heat generation effect on viscoelastic dampers structures", *11th Wolrd Conference on Earthquake Engineering Acapulco*, Mexico.
Inoue, K. and Kuwahara, S. (1998), "Optimum strength ratio of hysteretic damper", *Earthq. Eng. Struct. D.,* **27**(6), 577-588.
Karnopp, D. (1970), "Power balance method for nonlinear random vibration", *J. Appl. Mech.,* **34**(1), 212-214.
Kasai, K. and Munshi, J.A. (1994), "Seismic response of viscoelastic frame with yielding members", *5th US National Conference on Earthquake Engineering*, EERI, Chicago.
Kasai, K., Munshi, J.A., Lai, M. and Maison, B.F. (1993), "Viscoelastic damper hysteretic model: theory, experiment, and application", *ATC-17-1,* San Francisco, CA, 521-532.
Kasai, K., Teramoto, M., Okuma, K. and Tokoro, K. (2001), "Constitutive rule for viscoelastic materials considering temperature and frequency sensitivity", *J. Struct. Constr. Eng*., **543**, 77-86.
Kasai, K., Ooki, Y., Tokoro, K., Amemiya, K. and Kimura, K. (2004), "JSSI manual for building passive control technology part-9 time-history analysis model for viscoelastic dampers", *13th World Conference on Earthquake Engineering*, Vancouver, B.C., Canada, 1507.
Kim, J. and Choi, H. (2003), "Displacement-based design of supplemental dampers for seismic retrofit of a framed structure", *J. Struct. Eng*., **132**(6), 873-883.
Jones, D.I.G. (2001), *Handbook of viscoelastic vibration damping*, John Wiley & Sons, Chichester, UK.
Lai, M.L., Kasai, K. and Chang, K.C. (1999), "Relationship between temperature rise and nonlinearity of a viscoelastic damper", *ISET J. Earthq. Technol*., **36**(1), 61-71.
Lee, K.S., Fan, C.P., Sause, R. and Ricles, J. (2005), "Simplified design procedure for frame buildings with viscoelastic or elastomeric structural dampers", *Earthq. Eng. Struct. D*., **34**(10), 1271-1284.
Lewandowski, R. and Chorążyczewski, B. (2007), "Remarks on modelling of passive viscoelastic dampers", *9th International Conference*, Vilnius, Lithuania.
Paolacci, F. and De Angelis, M.A. (1999), "Random vibration approach to energy based design of passive control system for seismic protection of structures", *Proc. of Fourth European Conf. on Struc. Dyn. Eurodyn 99*, 299-304.
Paolacci, F. and De Angelis M. (2001), "Passive control of structures using viscoelastic devices", 7th *International Seminar on Seismic Isolation, Passive Energy Dissipation and Active Control of Vibrations of Structures*, Assisi, Italy.
Park, J.H., Kim, J. and Min, K.W. (2004), "Optimal design of added viscoelastic dampers and supporting braces", *Earthq. Eng. Struct. D*., **33**(4), 465-484.
Paolacci, F. and De Angelis, M. (1999), "Un approccio energetico al controllo passivo della risposta dinamica di sistemi a smorzamento lineare", *XIV Congresso Nazionale Associazione Italiana di Meccanica Teorica ed Applicata*, Como, Italy.
Ponzo, C.F., Dolce, M., Vigoriti, G., Arleo, G. and Di Cesare, A.A (2007), "Design procedure for energy dissipating displacement-dependent bracings system for r/c buildings", *10th World Conference on Seismic Isolation, Energy Dissipation and Active Vibrations Control of Structures*, Istanbul, Turkey.
Renzi, E., Perno, S., Pantanella, S. and Ciampi, V. (2007), "Design, test and analysis of a light-weight dissipative bracing system for seismic protection of structures", *Earthq. Eng. Struct. D*., **36**(4), 519-539.
Shen, K.L. and Soong, T.T. (1995), "Modelling of viscoelastic dampers for structural application", *J. Eng. Mech*., **121**(6), 694-701.
Singh, M.P. and Chang, T.S. (2009), "Seismic analysis of structures with viscoelastic dampers", *J. Eng. Mech*., **135**(6), 571-580.
Soda, S. and Takahashi, Y. (2000), "Performance based seismic design of building structures with viscoelastic dampers", *Proceedings of the Twelfth World Conference on Earthquake Engineering (12WCEE)*, Auckland, New Zealand.
Spanos, PT.D. (1978), "Energy analysis of structural vibrations under modulated random excitation", *J. Struct. Mech*., **6**(3), 289-302.

Tsai, C.S. (1994), "Temperature effect of viscoelastic dampers during earthquakes", *J. Struct. Eng*., **120**(2), 394-409.

Uang, C.M. and Bertero, V.V. (1988), "Use of energy as a design criterion in earthquake-resistant design", 1988 *UBC/EERC Report n. 88/18*.

Xua, Z.D., Shenb, Y.P. and Zhaoc, H.T. (2003), "A synthetic optimization analysis method on structures with viscoelastic dampers", *Soil Dyn. Earthq. Eng*., **23**(8), 683-689.

SA