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Equivalent period and damping of SDOF systems for spectral response of the Japanese highway bridges code

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Abstract. In seismic design and structural assessment using the displacement-based approach, real structures are simplified into equivalent single-degree-of-freedom systems with equivalent properties, namely period and damping. In this work, equations for the optimal pair of equivalent properties are derived using statistical procedures on equivalent linearization and defined in terms of the ductility ratio and initial period of vibration. The modified Clough hysteretic model and 30 artificial accelerograms, compatible with the acceleration spectra for firm and soft soils, defined by the Japanese Design Specifications for Highway Bridges are used in the analysis. The results obtained with the proposed equations are verified and their limitations are discussed.

Keywords: equivalent linearization, equivalent damping, equivalent period.

1. Introduction

Since performance-based design was proposed, several seismic design and assessment methodologies have been developed to fulfill its requirements. One of the most promising approaches is based on the control of displac-ements under the assumption that the damage can be managed if the deformations are controlled. In this approach, known as displacement-based design (DBD), the structure is simplified into an equivalent linear system to estimate the seismic deformation of an inelastic single degree of freedom (SDOF) system, representing the first (elastic) mode of vibration (Fig. 1). Considerable work has been carried out in the past to determine the equivalent elastic properties (period and damping) of the inelastic SDOF system. Most of the proposed equations are only defined in terms of the ductility and the initial damping of the real structure. Exceptions are the works of Kwan and Billington (2003), Blandon and Priestley (2005), and Guyader and Iwan (2006). These researchers define the equivalent damping in terms of both the ductility and the initial natural period of the oscillator. However for the equivalent damping, Guyader and Iwan (2006) limited this dependency for ductility values greater than 6.5. Blandon and Priestley (2005) do not present equations for explicit calculations of the equivalent period. Nevertheless, the equivalent period is obtained iteratively within the displacement design method. On the other side, Kwan and Billington (2003) limited the range of periods between 0.1 and 1.5 sec. In recent works, Lin and

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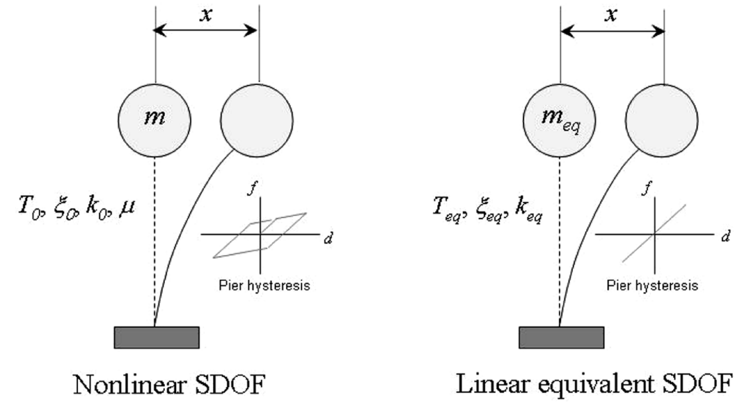

Fig. 1 Nonlinear SDOF simplified into an equivalent SDOF system

Miranda (2008) and Goda and Atkinson (2010) presented equations for the equivalent period and damping of existing structures including the initial period dependency, however, these researchers used the constant strength instead of ductility ratio as the known parameter.

The proposed equations for the equivalent properties have been derived using either of the following procedures: methods based on the harmonic response or methods based on random response. To evaluate the accuracy of the different issued equations, several comparisons of the estimations given by these approaches have been carried out. From these, one of the most remarkable results is that the equivalent period and equivalent damping should be defined in terms of the initial period. Otherwise, the dispersion and scattering of the predictions could be substantial even for relatively small mean errors and especially for large values of ductility (Miranda and Ruiz-Garcia 2002, Blandon and Priestley 2005, Degee *et al*. 2008, Zaharia and Taucer 2008). Moreover, the equivalent properties should be derived for a series of earthquakes of similar characteristics for no more than one hysteretic model (Zaharia and Taucer 2008, Goda and Atkinson 2010). In general, it has been found that, methods based on the harmonic response considerably overestimates the period shift whereas methods based on random response give more realistic results estimations (Chopra and Goel 1999). In this work, in order to overcome the underlined deficiencies in the estimations of the inelastic displacements, the equations for the equivalent damping and the equivalent period are defined as functions of the ductility and the initial period of vibration and derived by statistical procedures. Originally, the equations are intended to be used for the displacement-based design of bridges with initial period no longer than 4 sec. The estimated displacements obtained with the proposed equations are verified and discussed. Finally, a revision of the scope and limitations of the proposed equations are presented.

2. Preliminary considerations

2.1 Definition of equivalent properties

Consider an inelastic single degree of freedom (SDOF) system with structural response described by the following equation of motion

$$\ddot{x} + 2\xi_0\omega_0\dot{x} + \frac{R_F(x)}{m} = -\ddot{z}_g \tag{1}$$

In the above equation, m, ξ_0, and $R_F(x)$ are the mass, damping coefficient and the restoring force of the system, respectively, $\ddot{z}_g$ is the ground acceleration, x is the displacement of the mass relative to the ground, and by convention a single dot denotes the first derivative with respect to time, and a double dot indicates the second derivative with respect to time. The natural frequency of vibration, ω_0, is defined in terms of the initial stiffness, k_0, and the period of vibration of the system, T_0, by

$$\omega_0 = \sqrt{\frac{k_0}{m}} = \frac{2\pi}{T_0} \tag{2}$$

By substitution of the Eq. (2) into the Eq. (1), the inelastic response of the oscillator may be written in a more convenient form as

$$\ddot{x} + \left(\frac{4\pi}{T_0}\right)\xi_0\dot{x} + \frac{R_F(x)}{m} = -\ddot{z}_g \tag{3}$$

The inelastic SDOF is capable of undergoing a maximum inelastic deformation characterized by the ductility ratio, μ, which is defined as the ratio between the maximum inelastic displacement and the yielding displacement.

The exact maximum inelastic displacement can be calculated using a nonlinear analysis of Eq. (3) or can be approximated by either of the following methods: (a) those based on a displacement modification factor (Newmark and Hall 1982, Miranda 2000), and (b) those based on equivalent linearization. Only the latter will be considered in this study.

In the equivalent linearization method, the maximum inelastic displacement (Eq. 3) can be estimated from the equivalent linear model represented by

$$\ddot{x}_{eq} + \left(\frac{4\pi}{T_{eq}}\right)\xi_{eq}\dot{x}_{eq} + \left(\frac{2\pi}{T_{eq}}\right)^2 x_{eq} = -\ddot{z}_g \tag{4}$$

where T_{eq} and ξ_{eq} are the equivalent period and equivalent damping respectively, such that max $|x(T_0,\xi)| = \max|x(T_{eq},\xi_{eq})|$. In the ideal case, the equivalent displacement x_{eq} should match the inelastic displacement x. However, due to the approximated nature of equivalent linearization this will occur only when Eqs. (3) and (4) represent exactly the same system which is when $\mu = 1$. For other values of μ, the equivalent and real displacements will only match for a few cases although sufficiently accurate approximations, for engineering purposes, can be obtained.

As will be clarified in the next section, the equivalent period can be defined in terms of the original period modified by a shifted constant C as

$$T_{eq} = CT_0 \tag{5}$$

Due to the energy dissipated by the hysteretic behavior in nonlinear systems, it can be expected that T_{eq} will be longer than T_o and ξ_{eq} will be greater than ξ_0. In this study, the optimal equivalent damping and period are the combinations of T_{eq} and ξ_{eq}, which gives the best approximated value for the maximum inelastic displacement obtained from Eq. (3).

2.2 Review of previous studies

Since the first approach of equivalent linearization made by Jacobsen (1930, 1960) and the equivalent-substructure formulation made by Gulkan and Sozen (1974), extensive research has been done on expressions for the suitable equations for the equivalent properties T_{eq} and ξ_{eq}. Miranda and Ruiz Garcia (2002) presented a review of the most significant approximate methods to estimate maximum inelastic displacement demands based on equivalent linearization such as those from Rosenblueth and Herrera (1964), Gulkan and Sozen (1974), Iwan (1980) and Kowalski *et al.* (1994). In this section, from the large amount of information available in literature, only more recent references relevant to this study will be described.

Kwan and Billington (2003) using the Iwan's method and 20 ground motions records, to derive the following equations for the equivalent properties (valid for periods between 0.1-1.5 sec)

$$T_{eq} = 0.8T_0\mu^{C_1} \tag{6a}$$

$$\xi_{eq} = \frac{2C_2}{\pi}\left(\frac{T_{eq}}{T_0}\right)^2\frac{\mu-1}{\mu^2} + 0.55\left(\frac{T_{eq}}{T_0}\right)^2\xi_0 \tag{6b}$$

where the coefficients C_1 and C_2 varies according with the hysteretic system: Elastoplastic (EP), Slightly/Moderately degrading (DG), Slip (SL), Origin oriented (OO), and Bilinear elastic (BE). For instance, for EP and DG systems $C_1 = 0.5$ and $C_2 = 0.56$.

Blandon and Priestley (2005) presented the following equation for the equivalent damping derived from iterative analysis with six synthetic accelerograms

$$\xi_{eq} = \frac{a}{\pi}\left(1-\frac{1}{\mu^b}\right)\left(1+\frac{1}{(T_{eq}+C)^d}\right)\cdot\frac{1}{N} \tag{7}$$

where N is a normalizing factor and a, b, c, and d are constants which consider the hysteretic model: Takeda thin (TH), Takeda fat (TF), Bilinear inelastic (BI), EP, Ramberg Osgood (RO) and Ring Spring (RS). In this case, the effective period T_{eq} is determined by an iterative procedure during the design process. To illustrate this equation the coefficients for the TH model are considered ($a = 95$, $b = 0.5$, $c = 0.85$ and $d = 4$).

Guyader and Iwan (2006) derived equations from statistical studies, in which the mean and the standard deviation of the error in the estimation were minimized, using an ensemble of 28 far-field ground motions (varying in earthquake magnitude, soil conditions and epicentral distance). In this case the equivalent properties are defined by

$$\xi_{eq} = \xi_0 + A_G(\mu-1)^2 + B_G(\mu-1)^3 \quad \text{for} \quad \mu < 4.0 \tag{8a}$$

$$T_{eq} = T_0[1 + G_G(\mu-1)^2 - H_G(\mu-1)^3] \quad \text{for} \quad \mu < 4.0 \tag{8b}$$

$$\xi_{eq} = \xi_0 + C_G + D_G(\mu-1) \quad \text{for} \quad 4.0 \le \mu \le 6.5 \tag{8c}$$

$$T_{eq} = T_0[1 + I_G + J_G(\mu-1)] \quad \text{for} \quad 4.0 \le \mu \le 6.5 \tag{8d}$$

$$\xi_{eq} = \xi_0 + E_G\left[\frac{F_G(\mu-1)-1}{F_G(\mu-1)^2}\right]\left(\frac{T_{eq}}{T_0}\right)^2 \quad \text{for} \quad \mu > 6.5 \tag{8e}$$

$$T_{eq} = T_0\left[1 + K_G\left(\sqrt{\frac{\mu-1}{1+L_G(\mu-1)-1}} - 1\right)\right] \quad \text{for} \quad \mu > 6.5 \tag{8f}$$

In the above equations, the variables associated with the subscript G depends on the hysteretic model (BI, DG, Pinching). For the DG case, for instance: A_G = 5.6420, B_G = -1.2962, C_G = 10.182, D_G = 1.8661, E_G = 19.51, F_G = 0.38, G_G = 0.1809, H_G = -0.0366, I_G = 0.17472, J_G = 0.1640, K_G = 0.92, and L_G = 0.05.

Dwairi *et al*. (2007), by using a variation of the Jacobsen's approach derived the following equations to be used in the direct displacement-based design

$$\xi_{eq} = \xi_0 + C^h\left(\frac{\mu-1}{\mu\pi}\right)\% \tag{9}$$

where C^h is a function which varies depending on the hysteretic model considered (RS, TH, TF and EP). For instance, for the TF pattern $C^h = 65 + 50(1\text{-}T_{eq})$ if $T_{eq} < 1$ sec, and $C^h = 65$ if $T_{eq} \geq 1$ sec. The equivalent period is obtained during the design procedure by using displacement spectrum.

More recently, Zaharia and Taucer (2008) derived new coefficients for the Iwan's equations (for BI and RS systems), with a series of 30 synthetic earthquakes compatible with the Eurocode 8 spectra. The equivalent properties are defined by

$$T_{eq} = T_0[1 + A(\mu-1)^a] \tag{10a}$$

$$\xi_{eq} = \xi_0 + B(\mu-1)^b \tag{10b}$$

where A, a, B, and b are constants which consider the hysteretic model and the type of spectra. For bilinear systems and spectra type 1 the coefficients are A = 0.153, a = 1.02, B = 2.14 and b = 1.02.

For existing structures, where the strength ratio (R = elastic lateral strength/yield lateral strength) is generally known rather than the ductility ratio, Lin and Miranda (2008) derived the following equations for bilinear hysteretic systems subjected to 72 ground motions

$$T_{eq} = T_0\left[1 + \frac{m_1}{T_0^{m_2}}(R^{1.8} - 1)\right] \tag{11a}$$

$$\xi_{eq} = \xi_0 + \frac{n_1}{T_0^{n_2}}(R-1) \tag{11b}$$

The constants m_1, n_1, m_2 and n_2, are constants depending on the postyield stiffness ratios.

Goda and Atkinson (2010) proposed the following form of equations to estimate the maximum displacements demands of existing structures based on 70 earthquakes from Japan and 172 from California

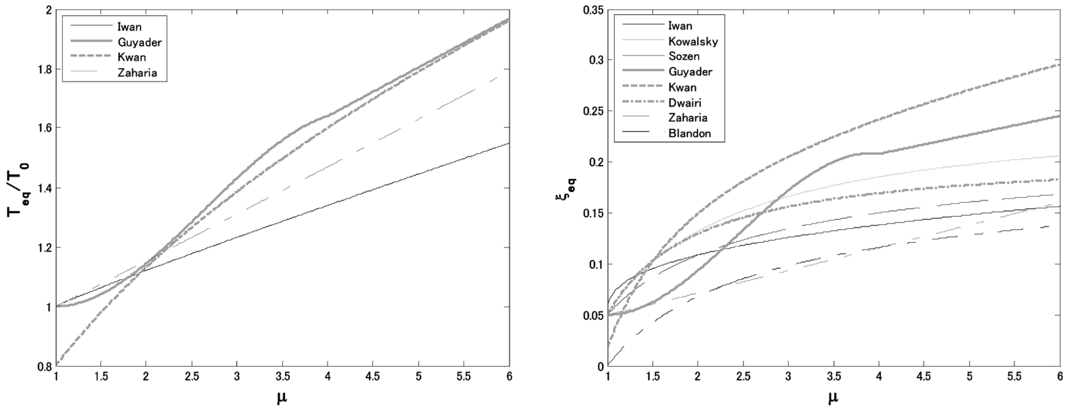

Fig. 2 Comparison of T_{eq}/T_0 and ξ_{eq} for different equivalent linearization models

$$T_{eq} = 1 + f(T_0, R; a) \tag{12a}$$

$$\xi_{eq} = 0.05 + f(T_0, R; a) \tag{12b}$$

with $f(T_0,R;a) = \exp(a_1 \min(T_0, 0.2) + a_2 T_0^{a_3} + a_4(R - 0.5)^{a_5} + a_6 T_0)$, where the vector a contains six model coefficients a_i. Eqs. (12) can be used for various hysteretic characteristics with different post-yield-to-initial stiffness ratios and degradation/pinching behavior.

Except the method from Rosenblueth and Herrera (1964), based on the steady-state harmonic response, Fig. 2 shows the variation of the ratio T_{eq}/T_0 and ξ_{eq} for the described equivalent linearization models.

3. Equivalent period and damping including initial period dependency

3.1 Form of the equations

As suggested by the most recent studies on equivalent linearization, the equivalent period and the equivalent damping should be defined in terms of the ductility ratio and initial periods of vibration. Then, hereafter the conceptual scope can be outlined as follows:

- Formulate a single expression for the equivalent period, T_{eq}, and another for the equivalent damping, ξ_{eq}, with variable coefficients that consider the type of earthquake and the class of soil.
- The equation for T_{eq} will be formulated as a function of μ and T_0 as $T_{eq} = T_{eq}(\mu, T_0)$.
- The equation for ξ_{eq} will be formulated as a function of ξ_0, μ, and T_0 as $\xi_{eq} = \xi_{eq}(\xi_0, \mu, T_0)$.
- The equations will be generated from accelerograms compatible with the acceleration spectra for strong earthquakes specified in the adopted design guidelines.

From the large number of mathematical expressions that fit the curve, a compact form that incorporates both the conceptual scope and the period dependency of the equivalent damping found in the regression analysis (also pointed out by Guyader and Iwan 2006 and Lin and Miranda 2008) is assumed as

$$T_{eq} = T_0[1 + AT_0^B(\mu^C - 1)] \tag{13a}$$

$$\xi_{eq} = \xi_0 + aT_0^b(\mu^c - 1) \tag{13b}$$

Where the A, B, C, a, b and c are constants calculated by regression analysis explained in the following section.

This general pattern of the equations was originally presented by Iwan (1980), adopted in a posterior work by Zaharia and Taucer (2008), and modified to consider the period dependency by Lin and Miranda (2008). The authors consider this form more concise and simpler than others. Moreover, the results suggest that these equations are sufficiently accurate and easy to implement for practical purposes.

3.2 Regression analysis procedure

In one of the most recurrent empirical approaches to obtain the equivalent properties (Iwan 1980, Kwan and Billington 2003, Zaharia and Taucer 2008), the values of T_{eq} and ξ_{eq} are the couple which minimizes the root-mean-square error, $\bar{\varepsilon}$, of the ratio between the averaged inelastic exact displacement and the averaged equivalent elastic displacement throughout the range of periods, i, given by

$$\bar{\varepsilon}(\varepsilon_{eq}, T_{eq}) = \sqrt{\sum_{i=1}^{N} \frac{\varepsilon_i^2}{N}} \tag{14}$$

with

$$\varepsilon(\xi_{eq}, T_{eq})_i = \frac{SD_e(T_{eq}(T_0, \mu), \xi_{eq})_i}{SD_{nl}(\mu, T_0, \xi_0)_i} - 1 \tag{15}$$

where N is the number of periods.

This approach assumes that the equivalent linear spectrum closely represents the behavior of the inelastic spectra in all the range of periods. However, this is not necessarily true. In this study, since all the earthquakes have similar characteristics, a different approach that would work better is adopted: the root-mean-square error is calculated for each period for all of the earthquakes considered (Lin and Miranda 2008, Sánchez-Flores and Igarashi 2010).

According to this and considering the objectives previously mentioned, the method consists of the following steps:

1. Set the minimum and maximum values of the equivalent damping coefficient.
2. Select the interval of periods.
3. Obtain the elastic equivalent (shifted) displacement response spectra $(SD_e)_k$, for all of the damping values throughout the range of periods, for all the artificial earthquakes in terms of the equivalent and original period, and the equivalent damping as

$$(SD_e)_k = SD_e(T_{eq}(\mu, T_0), \xi_{eq}(\xi_0, \mu, T_0))_k \tag{16}$$

4. Set the range of ductility values and the hysteretic model for the nonlinear analysis.
5. Calculate the inelastic displacement response spectra $(SD_{nl})_k$, in terms of the original period,

damping, and ductility ratio for all of the artificial earthquakes as

$$(SD_{nl})_k = SD_{nl}(T_0, \xi_0, \mu)_k \tag{17}$$

6. Calculate the difference, ε, between the inelastic and the shifted elastic spectra in the i^{th} period for each earthquake

$$\varepsilon(\xi_{eq}, T_{eq})_{ik} = \frac{SD_e(T_{eq}(T_0, \mu), \xi_{eq})_{ik}}{SD_{nl}(T_0, \xi_0, \mu)_{ik}} - 1 \tag{18}$$

7. Calculate the root-mean-square (RMS) error, ($\bar{\varepsilon}_i$), of the ratio between the inelastic and the elastic shifted spectra for the total number of earthquakes for the i period

$$\bar{\varepsilon}(\varepsilon_{eq}, T_{eq})_i = \sqrt{\sum_{k=1}^{N_k} \frac{\varepsilon_{ik}^2}{N_k}} \tag{19}$$

where N_k is the total number of earthquakes.

8. Find the optimum combination of $T_{eq}(\mu, T_0)$ and $\xi_{eq}(x_0, \mu, T_0)$, which gives the minimum RMS error. The optimal values are then substituted into the Eq. (4) to calculate the equivalent linear spectrum.
   To verify if the optimal values were properly calculated, the mean ratio between the displacements calculated by the equivalent linear system (with T_{eq} and ξ_{eq}) and those computed by the exact inelastic analysis are calculated in each period, i, with

$$\bar{E}(T_0, \mu, \xi_{eq}, T_{eq})_i = \frac{1}{N_k} \sum_{k=1}^{N_k} \frac{SD_e(T_{eq}(T_0, \mu), \xi_{eq})_{ik}}{SD_{nl}(T_0, \xi_0, \mu)_{ik}} \tag{20}$$

   If the optimal parameters are good estimations, the mean ratio in the above equation should be very close to the unity throughout the range of periods for all ductility values.
9. Perform nonlinear regression analyses to fit a curve which closely represents T_{eq} and ξ_{eq}.
10. Verification of the results obtained by mean of statistical procedures.

All of the procedures previously described are summarized in Fig. 3.

4. Equivalent properties for the Japanese design specifications

4.1 Design earthquakes

In this work, compatible earthquakes with the acceleration spectra given by the Design Specifications of the Japan Road Association (2002) are generated and used in the derivation of the equations. After the Nihonkai-chubu earthquake occurred on January 17, 1995, the Japanese Specifications were modified and the design ground motions were classified into two categories:

a) Level 1 Earthquakes. This category defines the moderate earthquakes with a high probability to occur, not considered in this study.

b) Level 2 Earthquakes. This category defines the earthquakes with less probability to occur but

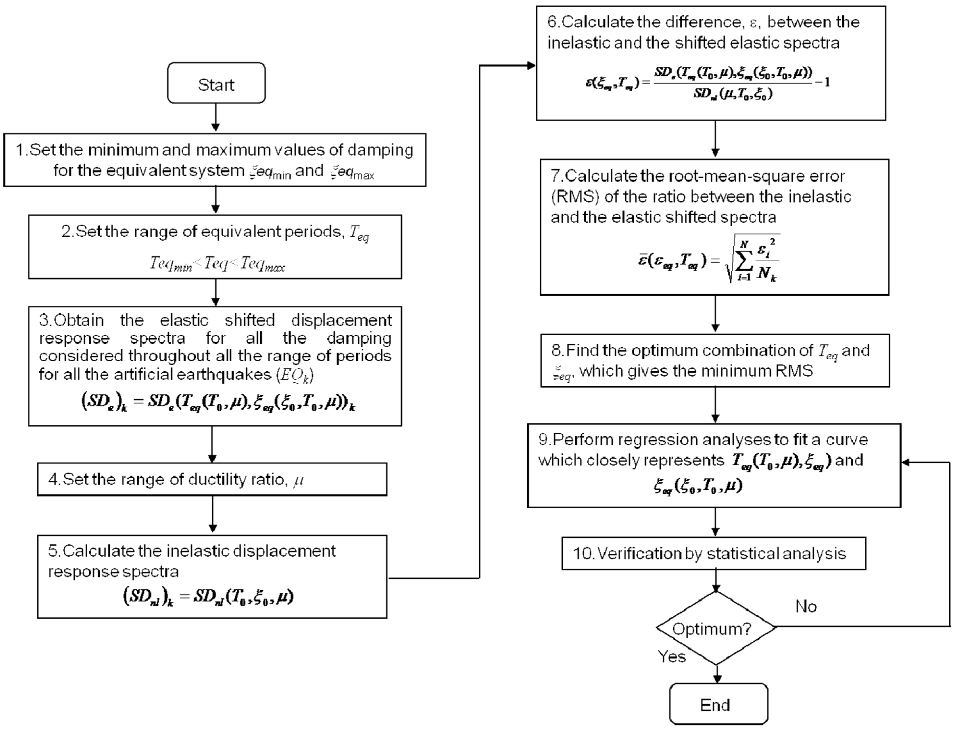

Fig. 3 Procedure followed to derive expressions for the equivalent properties

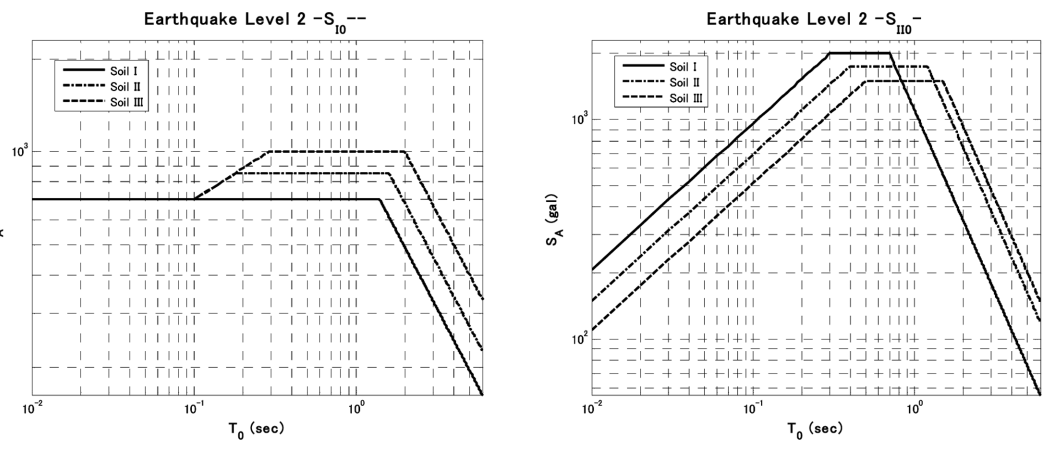

Fig. 4 Acceleration spectra

strong enough to cause critical damage. Under this category, two types of ground motion are defined: Type I, denoted by S_{I0}, the interplate-type earthquakes with magnitude around 8, and Type II, denoted by S_{II0}, near fault earthquakes of magnitude around 7.

The soils are classified into three types based on the depth of the stratus and the velocity of the wave propagation. Firm soils correspond to type I, the transition soils correspond to type II, and soft soils correspond to the type III. The results of this study are limited for soil type I and III.

The design acceleration spectra $S_A(T_0)$ for level 2 earthquakes (ξ = 5%) are shown in Fig. 4.

4.2 Response spectra compatible accelerograms

Level 1 earthquakes have been extensively recorded with high quality throughout the years. For level 2 earthquakes, however, the number of recorded motions is quite limited. In fact, Hamada (2000) documented only five earthquakes with magnitude equal or greater than 8 from 1889 to 1995, and 14 motions with magnitude equal or greater than 7 in the same period. From these, only 9 are considered type 2 and most of them occurred in the first half of the past century. Therefore, due to the lack of available records for level 2 earthquakes, a family of 30 artificial accelerograms was generated for each type of earthquake for firm and soft soils.

The earthquakes were generated to be compatible with both the acceleration $S_A(T_0)$ and the displacement spectra $S_D(T_0)$.

In the Japanese specifications, the displacement spectrum is not explicitly included. However, for elastic systems and for small or intermediate periods, it can be calculated from the acceleration spectrum by the following relationship

$$S_D(T_0) = S_A(T_0)\left(\frac{T_0}{2\pi}\right)^2 \tag{21}$$

For long periods, the displacements obtained in this manner increase indefinitely giving unrealistic predictions and some adjustments to calculate consistent acceleration-displacement spectra should be done (Boomer *et al*. 2000). Since the adjustments are beyond the scope of this work, here exclusively for the generation of artificial earthquakes, the displacement spectra is assumed to be constant beyond the corner periods calculated with the following expression (Priestley *et al*. 2007) regardless the type of soil and type of earthquake

$$T_c = 1 + 2.5(M_w - 5.7) \text{ (sec)} \tag{22}$$

where M_w is the earthquake magnitude.

The artificial motions are generated by the procedure proposed by Clough and Penzien (2003). Their duration was obtained from the ground motion samples in the seismic code, and the parameters are shown in Table 1.

Fig. 5 shows the corresponding displacement spectra in the range of periods considered and Fig. 6 shows the typical earthquakes obtained.

Table 1 Parameters for the generation of artificial earthquakes

Earthquake type	Soil type	Magnitude	T_C [sec]	Duration [sec]	Earthquakes generated
S_{I0} (S_{II0})	I	8.0 (7.0)	6.75 (4.25)	30 (30)	30 (30)
S_{I0} (S_{II0})	III	8.0 (7.0)	6.75 (4.25)	60 (30)	30 (30)
				Total	120

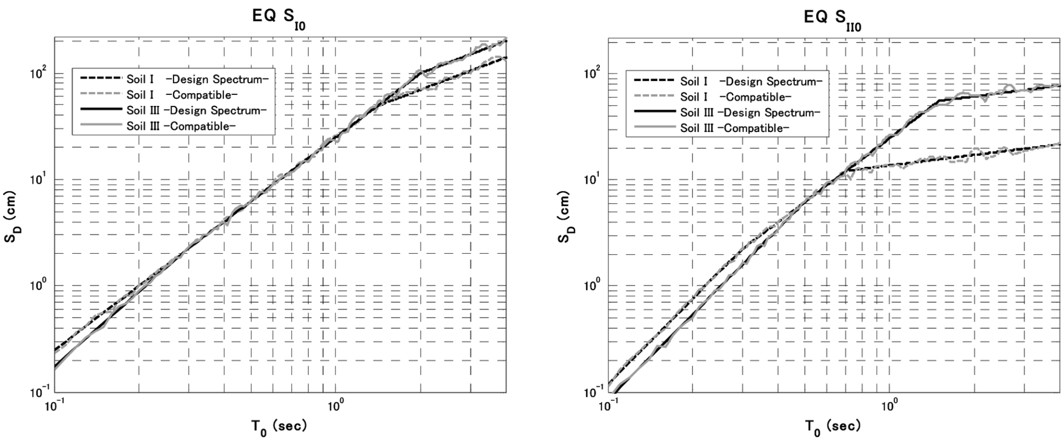

Fig. 5 Displacement spectra

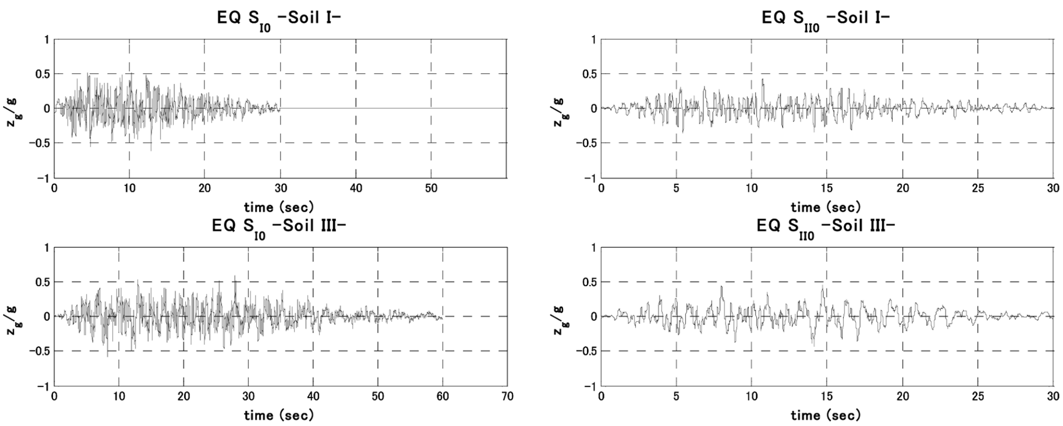

Fig. 6 Earthquakes generated

4.3 Numerical scope

The aspects considered for the time history analysis are described as follows
For nonlinear analysis:

• Seven ductility values are evaluated: 1, 1.5, 2, 3, 4, 5, and 6.
In general, ductility values beyond 6 are not common in conventional bridges. Moreover, for performance based-design where the damage should be controlled, it additional damping devices are usually included. In these cases the ductility is reduced and the damping becomes the most significant source of energy dissipation.

• The equations for equivalent properties may have different coefficients depending on the hysteretic pattern used in their derivation. Even if a single hysteretic pattern is used, the coefficients may change depending on the parameters required on its definition as reported by Lin and Miranda (2008) for a bilinear case. Therefore, a deeply study on the influence of the hysteretic pattern in equivalent linearization falls beyond the scope of this work. Thus, the time history analyses are limited to the modified Clough hysteretic behavior (Otani 1981). This model can be used to represent concrete structures that exhibit stiffness degradation under seismic loads when the behavior is primarily flexural as is expected in bridge piers. The post-yielding stiffness is set to 0.05 times the corresponding elastic stiffness.

- The initial damping coefficient ξ_0 is set to 5% to be compatible with the design guideline.
- The period range, T_0, is set from 0.1 sec to 4 sec at intervals of 0.02 sec, due to the resulting equations which are intended to be used in the design of bridges with intermediate periods. Moreover, to be consistent with the displacement spectra the initial period is set lower than that calculated with Eq. (22).

For the shifted analysis:

- Values of ξ_{eq} varying from 5% to 50% at intervals of 1% are considered.
- The range of the shifted ratio, T_{eq}/T_0, varies from 0.5 to 4 at intervals of 0.01. The maximum value is set after preliminary simulations with the largest value of ductility using the Eq. (6). The minimum value is given by the ratio T_{eq}/T_0 = 1.

5. Results

All of the steps listed in the regression analysis procedure (section 3.2) can be computed in a straightforward manner. Therefore, only the estimations of the displacements obtained for the compatible earthquakes are illustrated in this section 3.2 (Steps 7-8).

The relevant aspect is the calculation of the root-mean-square (RMS) error of the ratio between the inelastic and the elastic shifted spectra for the total number of earthquakes for each single period T_i. A matrix of RMS errors with a different period shift and equivalent damping can be computed on the basis of Eq. (19). The pair of T_{eq} and ξ_{eq} is the couple for which the error is minimum. To illustrate this, consider the elastic case and the corresponding inelastic one with μ = 1 at T_0 = 1, subjected to the earthquake S_{I0} -soil I-. Since the maximum displacement should be exactly the same for both cases, the optimal parameters are known beforehand: T_{eq}/T_0 = 1 and $\xi_{eq} = \xi_0$ = 5%. Therefore, the expected error is $\bar{\varepsilon} = 0$ as illustrated in Fig. 7.

Once the optimal parameters T_{eq}/T_0 and ξ_{eq} are calculated for all the periods and ductility ratios, they can be represented in plots similar to those shown in Fig. 8. In this figure only ductility ratios

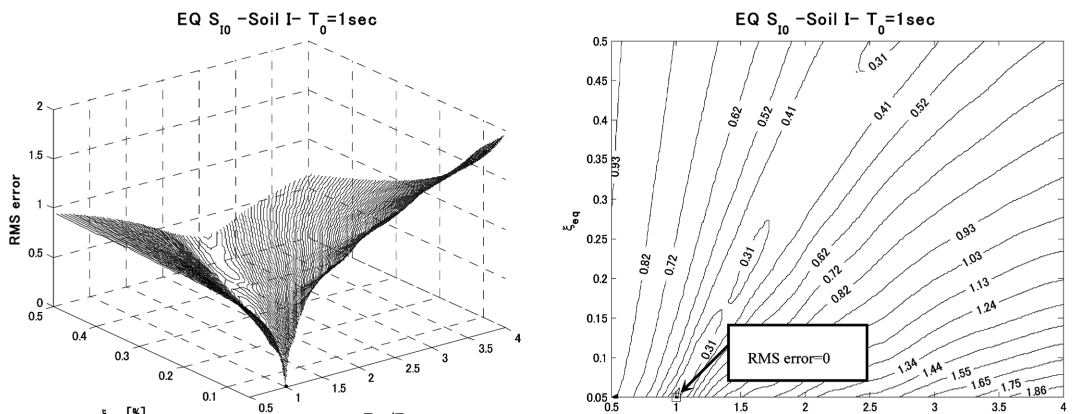

Fig. 7 Minimum SMS error in the region RMS error-ξ_{eq}-T_{eq}/T_0

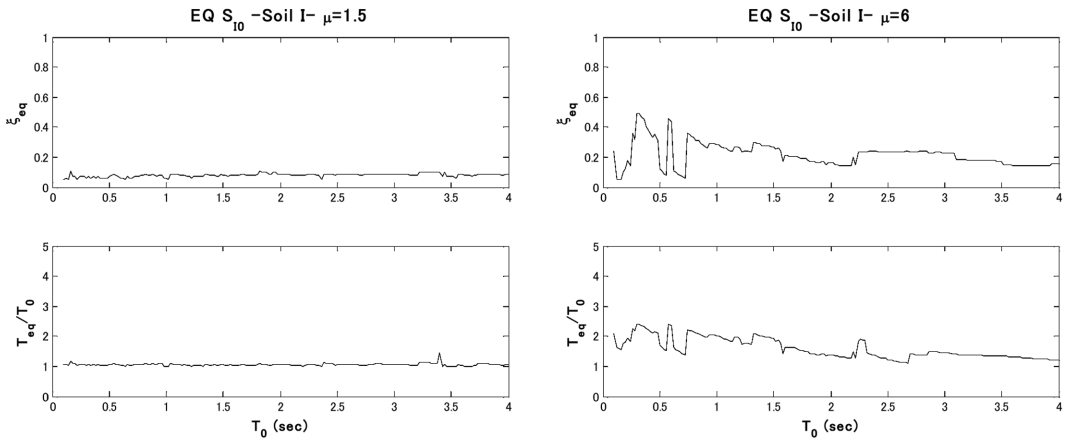

Fig. 8 Optimal period shift and equivalent damping derived from the minimum RMS error

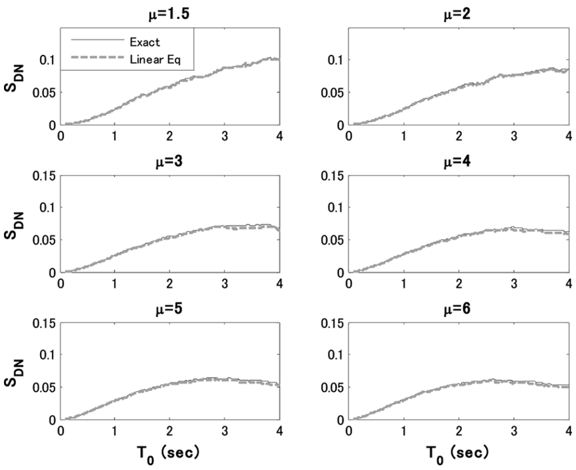

Fig. 9 Normalized displacement spectra -Earthquake S_{I0}, Soil I-

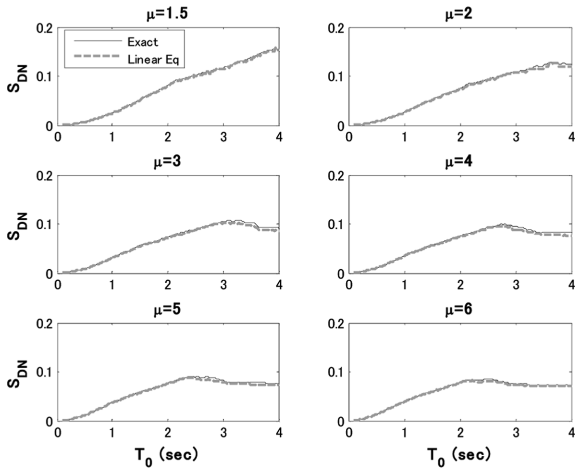

Fig. 10 Normalized displacement spectra -Earthquake S_{I0}, Soil III-

of 1.5 and 6 are shown for clarity. When the ductility ratio is 1.5, the equivalent properties are nearly constant throughout the entire range of periods. On the contrary, when the ductility ratio is increased to 6, the optimal properties depend on both initial periods and ductility ratios. For this reason, the equations proposed in the section 3.1 are defined in terms of these parameters. It can be observed (Fig. 8) that in all cases, T_{eq}/T_0 is greater than 1 and ξ_{eq} is greater than ξ_0 = 5%. This means the period of the equivalent linear system is longer, and the damping ratio of the equivalent linear system is greater than the corresponding nonlinear system.

To quantitatively measure the error in the estimation of the maximum displacement, the range proposed by Guyader and Iwan (2006) is adopted in this work. Then, hereafter the most desirable range of error values will be set between -10% and +20%. This interval considers the general preference to conservative design over unconservative design.

With the obtained equivalent properties, the normalized equivalent displacement spectra (S_{DN}), calculated with the ground acceleration normalized by the gravity (g) as $\ddot{z}_g/g$, can be obtained with

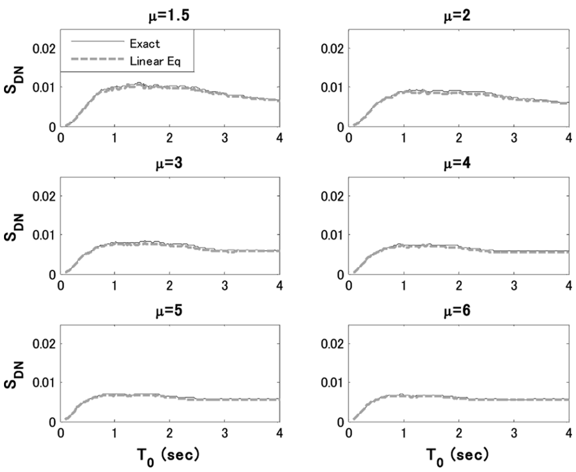

Fig. 11 Normalized displacement spectra -Earthquake S_{II0}, Soil I-

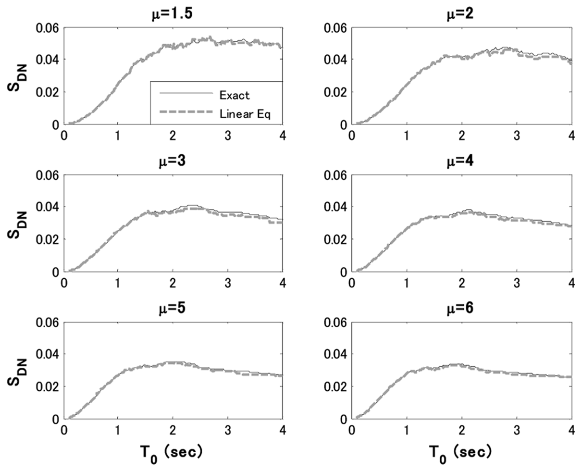

Fig. 12 Normalized displacement spectra -Earthquake S_{II0}, Soil III-

Table 2 Coefficients for the equations of the equivalent properties

Earthquake	Soil type	A	B	C	a	b	c
S_{I0}	I	0.278	-0.119	0.758	0.350	-0.038	0.210
	III	0.233	-0.102	0.884	0.642	-0.002	0.148
S_{II0}	I	0.338	-0.462	0.510	5.562	-0.235	0.018
	III	0.199	-0.150	0.897	1.233	-0.087	0.078

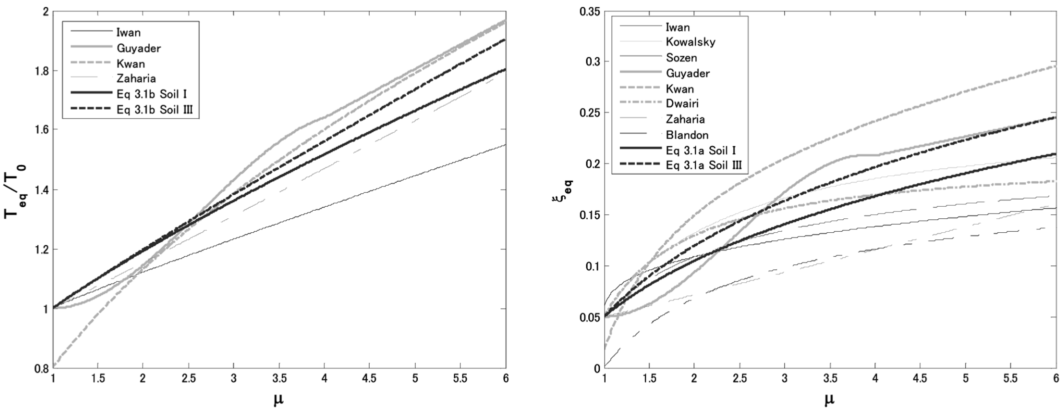

Fig. 13 Comparison of the proposed T_{eq}/T_0 and ξ_{eq} with other linearization models

Eq. (4). For comparison, the exact S_{DN} is calculated with Eq. (3) for all soils and ductility ratios (Figs. 9-12). From these figures, it can be observed that the linear equivalent spectra closely match the corresponding exact one especially for small periods. Nonetheless, there is a trend to underestimate the displacements for some period intervals. In any case, however, the error in the estimation is smaller than 5%.

After the nonlinear regression analysis to fit the curves shown in Fig. 8, the values of the constants A, B, C, a, b, and c for Eq. (13) are obtained. The corresponding values are shown in Table 2. A comparison of the proposed equations (at $T_0 = 1$) with those described in section 2.2 is show in Fig. 13.

6. Verification analysis and discussion

In this section, the accuracy and the dispersion of the results obtained with the proposed equations are investigated.

The accuracy of the estimation is evaluated by the mean error defined by Eq. (18), now denoted by E_{mean}, calculated for each earthquake and soil type. Values of E_{mean} close or equal to unity correspond to a good estimation of the maximum displacement, values greater than unity tend to overestimate the maximum displacement, and values smaller than unity tend to underestimate the maximum displacement. The approximate mean to exact displacement ratio is shown in Figs. 14 and 15 for earthquakes type I and II respectively.

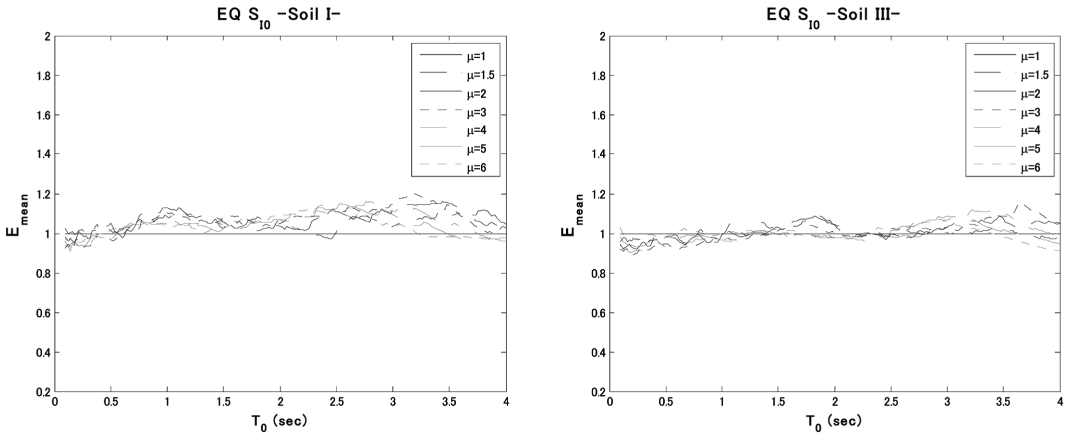

Fig. 14 Mean approximate to exact displacement ratios -Earthquake S_{I0}-

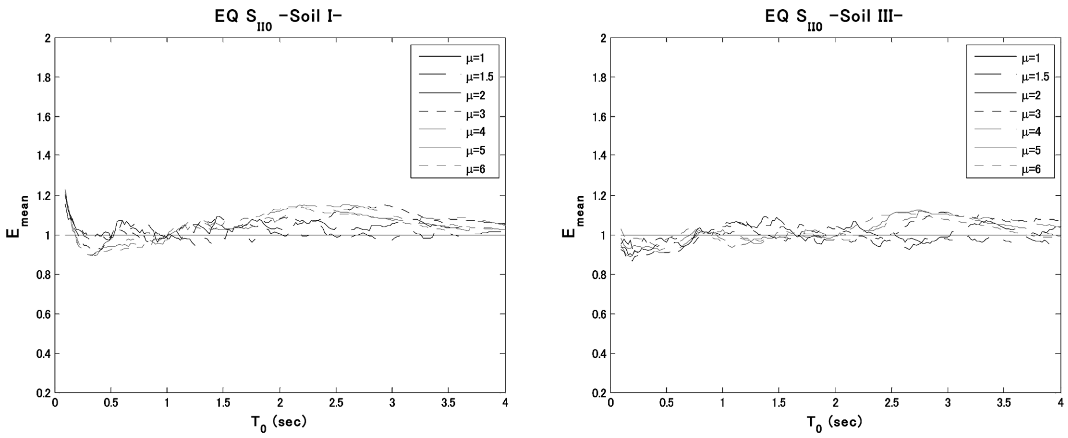

Fig. 15 Mean approximate to exact displacement ratios -Earthquake S_{II0}-

For firm soils, the equations generally tend to overestimate the maximum displacements for periods greater than 0.6 and 0.9 sec for earthquakes type I and II respectively. For the former earthquakes, the displacements are underestimated for periods smaller than 0.6. For the latter, the displacements are overestimated for all ductility ratios for periods smaller than 0.2 sec and underestimated between 0.2 and 0.5 sec. The maximum error of 20% is obtained when $\mu = 3$ at $T_0 = 3.2$ for earthquakes S_{I0}, and when $\mu = 5$ at $T_0 = 2.6$ for earthquakes S_{II0}.

For soft soils, the displacements are generally overestimated for periods larger than 1.3 for type I earthquakes, and for periods longer than 0.8 sec for type II earthquakes. For low periods, the displacement is underestimated in all cases, with the maximum error of -10% obtained when μ =3.

In all cases, the minimum and maximum errors are kept under the predefined allowable limits of -10% and 20%.

Strictly speaking, a raw comparison of these obtained results with those obtained by other methods (Section 2.2) would not be accurate. Each set of equations were derived with different number of earthquakes recorded in different types of soils. Furthermore, the hysteretic pattern adopted to model the nonlinear behavior is also different in each case. Therefore, there is no way to obtain an accurate comparison unless the input earthquakes and hysteretic models used are the same in each case. However, in order to illustrate the behavior of the estimation, a comparison between the mean errors obtained with the proposed equations and those obtained with the equations by Guyader and Iwan (2006) is presented. The expressions by Guyader and Iwan (2006) cover all of the ranges of periods and ductilities used in this study and are valid for a hysteretic pattern closely related to the modified Clough. The E_{mean} for earthquakes S_{I0} and S_{II0} are presented in Figs. 16 and 17 respectively, where the error limits (0.9 and 1.2) are also shown throughout the range of periods (0.1-4 sec).

The following trends were observed:

a) For the family of artificial earthquakes used in this study and for low ductilities (μ = 1.5 and μ = 2), the proposed equations give better estimations than those by Guyader and Iwan (2006).

b) With the exception of case S_{II0} –Soil I-, both methods give close estimations for periods until around 2 sec and μ > 2. For larger periods, however, the Guyader and Iwan's method tend to underestimate the displacements while the proposed equations tend to overestimate the displacements.

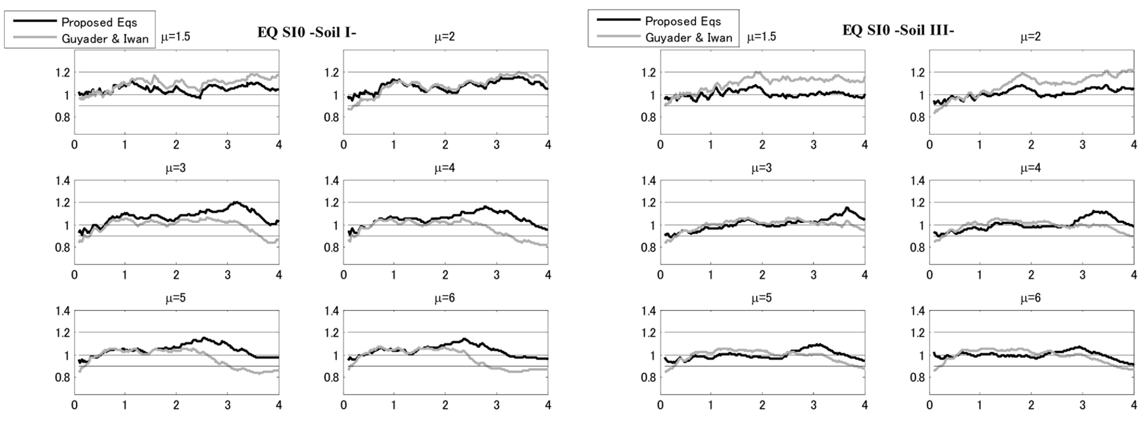

Fig. 16 Comparison of mean errors -Earthquake S_{I0}

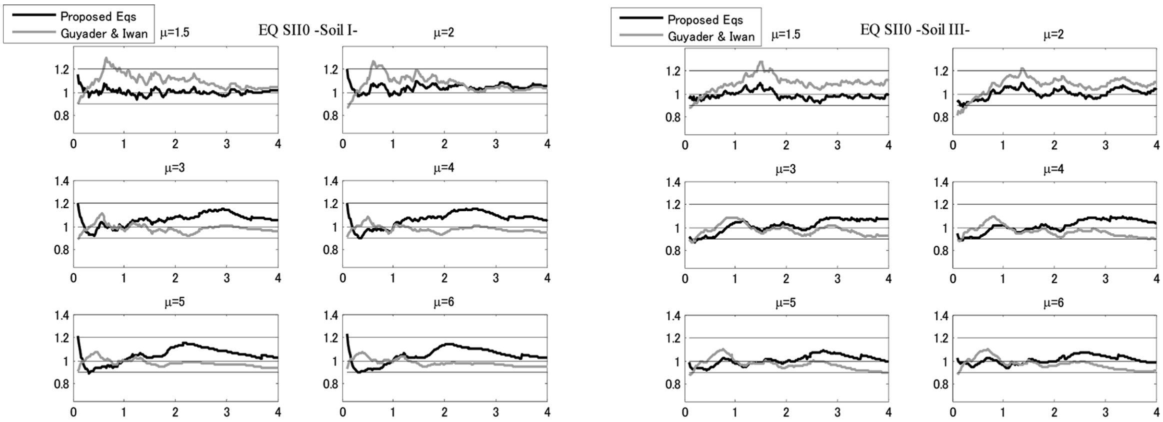

Fig. 17 Comparison of mean errors -Earthquake S_{I0}

c) For case S_{II0} –Soil I-, for $T_0 > 1.1$ sec and $\mu > 2$, although the equations from Iwan and Guyader underestimate the displacements, their absolute errors are smaller than those from the proposed method that overestimate the displacements.

d) In all cases, the proposed equations give a more conservative design than those obtained with the Guyader and Iwan's method. Furthermore, the estimations given by the proposed equations are between the predefined error limits.

These observations suggest that the equivalent properties should be derived based on three classifications: the type of earthquake, a type of hysteretic model, and specific soil conditions.

The dispersion of the estimation is evaluated with the standard errors defined as

$$\sigma_{S\varepsilon}(\xi_{eq}, T_{eq})_i = \sqrt{\frac{1}{N_k - 1}\sum_{k=1}^{N_k}\left(\frac{SD_e(T_{eq}(T_0, \mu), \xi_{eq})_{ik}}{SD_{nl}(T_0, \xi_0, \mu)_{ik}} - 1\right)^2} \tag{23}$$

The results are shown in Figs. 18 and 19. For all cases except for the earthquake S_{II0} -Soil I-, the standard errors increase with the period with the maximum located at $\mu = 3$. For the earthquake S_{II0} -Soil I-, the maximum standard errors are located between 2 and 3 seconds for ductility values greater than 2. For type III soils, the standard errors are generally uniform for small and intermediate periods throughout the range of ductility ratios.

The previously described behavior suggests the influence of the design spectra used to generate the compatible artificial earthquakes. In this work, the corner period of the displacement spectra given by Eq. (22) was defined only in terms of the magnitude, regardless the type of earthquake or soil. Since the artificial earthquakes match the displacement spectra, it seems the values of the displacement at large shifted periods are underestimated. For this reason, it is recommended that a displacement design spectra compatible with the acceleration spectra is used in the generation of artificial earthquakes.

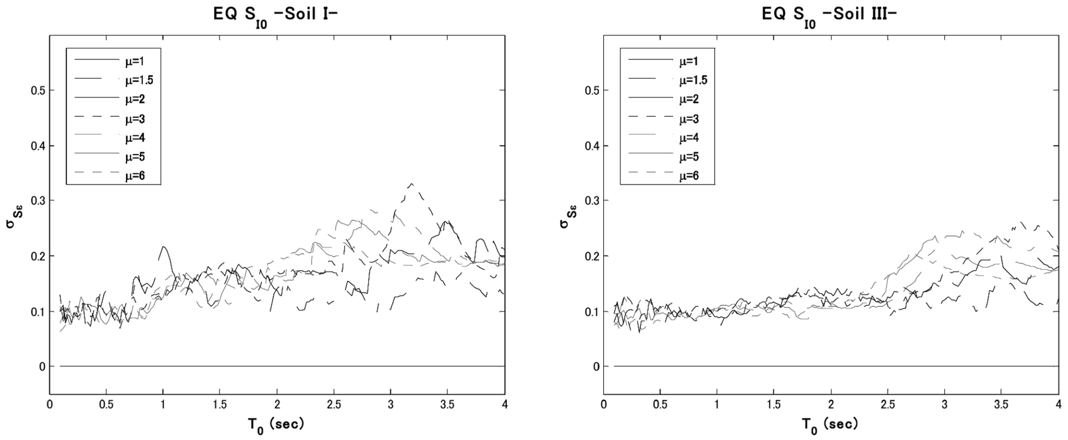

Fig. 18 Standard errors for Earthquake S_{I0}

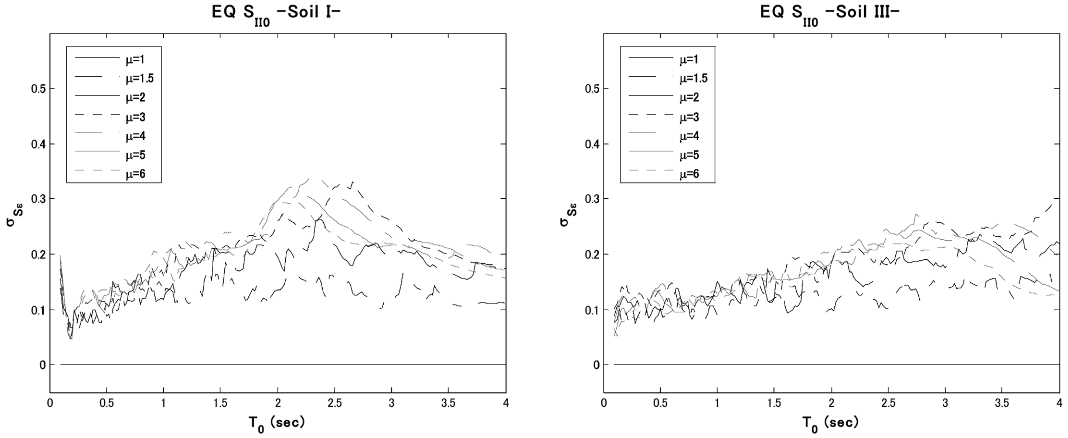

Fig. 19 Standard errors for Earthquake S_{II0}

7. Conclusions

In this study, set of equations to calculate the equivalent period and damping of SDOF systems was derived by statistical procedures on equivalent linearization. The equations were derived from nonlinear analyses of systems with modified Clough hysteretic behavior for ductility ratios values of 1, 1.5, 2, 3, 4, 5, and 6. Families of artificially generated ground motions compatible with the acceleration spectra for earthquakes level 2, as defined by the Japanese Seismic Design Specifications for Highway Bridges, were used in the time response analysis.

The results confirm the dependency on the initial period of both the equivalent period and the equivalent damping. The results indicate the proposed equations can estimate the maximum inelastic displacement between the limits of conservative errors set as -10% and 20%. In general, although there is a trend to overestimate the maximum displacements, the maximum error is always smaller than 20%. The results also suggest that the equivalent properties should be derived for specific type of earthquakes, for one hysteretic model and for specific soil conditions. Moreover, in order to capture the period shift at long periods, the use of displacement spectra compatible with the acceleration spectra is strongly recommended.

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