

# Seismic analysis of $Al_2O_3$ nanoparticles-reinforced concrete plates based on sinusoidal shear deformation theory

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**Abstract.** In this study, nonlinear dynamic response of a concrete plate retrofit with Aluminium oxide ( $Al_2O_3$ ) under seismic load and magnetic field is investigated. The plate is a composite reinforced by Aluminium oxide with characteristics of the equivalent composite being determined using Mori-Tanka model considering agglomeration effect. The plate is simulated with higher order shear deformation plate model. Employing nonlinear strains-displacements, stress-strain, the energy equations of column was obtained and using Hamilton's principal, the governing equations were derived. Differential quadrature method (DQM) in conjunction with Newark method is applied for obtaining the dynamic response of structure. The influences of magnetic field, volume percent of nanoparticles, geometrical parameters of column, agglomeration and boundary conditions on the dynamic response were investigated. Results showed that with increasing volume percent of nanoparticles, the dynamic deflection decreases.

**Keywords:** dynamic response; nano-composite plate; DQM; magnetic field; seismic load

## 1. Introduction

Seismic analysis is a subset of structural analysis in which dynamic response of a building structure (or non-building structures such as bridges, etc.) against the earthquake is examined. This analysis is a part of the structural engineering, the earthquake engineering and seismic retrofitting of the structures which should be constructed in earthquake prone zones (Liang and Parra-Montesinos 2004).

Mechanical analysis of nanostructures has been reported by many researchers (Zemri 2015, Larbi Chaht 2015, Belkorissat 2015, Ahouel 2016, Bounouara 2016, Bouafia 2017, Besseghier 2017, Bellifa 2017, Mouffoki 2017, Khetir 2017, Boadu *et al.* 2017). Liang and Parra-Montesinos (2004) studied seismic behavior of four reinforced concrete column-steel plate under various ground motions using experimental tests. Cheng and Chen (2004), Changwang *et al.* (2010) studied seismic behavior of steel reinforced concrete column-steel truss plate. They developed a design formula for shear strength of the structure subjected to seismic activities using experimental tests. The effect of cumulative damage on the seismic behavior of steel tube-reinforced concrete (ST-RC) columns through experimental testing was investigated by Ji *et al.* (2014). Six large-scale ST-RC column specimens were subjected to high axial forces and cyclic lateral loading. The effect of plastic hinge relocation on the potential damage of a reinforced concrete frame subjected to different seismic levels was studied by Cao and Ronagh (2014) based on current seismic designs.

The optimal seismic retrofit method that uses FRP jackets for shear-critical RC frames was presented by Choi *et al.* (2014). This optimal method uses non-dominated sorting genetic algorithm-II (NSGA-II) to optimize the two conflicting objective functions of the retrofit cost as well as the seismic performance, simultaneously. They examined various parameters like, failure mode, hysteresis curves, ductility and reduction of stiffness. Liu *et al.* (2016) focused on the study of seismic behavior of steel reinforced concrete special-shaped column-plate joints. Six specimens, which are designed according to the principle of strong-member and weak-joint core, are tested under low cyclic reversed load.

In none of the above articles, the nanocomposite structure is considered. Wuite and Adali (2005) performed stress analysis of carbon nanotubes (CNTs) reinforced plates. They concluded that using CNTs as reinforcing phase can increase the stiffness and the stability of the system. Also, Matsunaga (2007) examined stability of the composite cylindrical shell using third-order shear deformation theory (TSDT). Formica *et al.* (2010) analyzed vibration behavior of CNTs reinforced composites. They employed an equivalent continuum model based on Eshelby-Mori-Tanaka model to obtain the material properties of the composite. Liew *et al.* (2014) studied postbuckling of nanocomposite cylindrical panels. They used the extended rule of mixture to estimate the effective material properties of the nanocomposite structure. They also applied a meshless approach to examine the postbuckling response of the nanocomposite cylindrical panel. In another similar work, Lei *et al.* (2014) studied dynamic stability of a CNTs reinforced functionally graded (FG) cylindrical panel. They used Eshelby-Mori-Tanaka model to estimate effective material properties of the resulting nanocomposite structure and also employed Ritz method to distinguish the instability

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regions of the structure. Static stress analysis of CNTs reinforced cylindrical shells is presented by Ghorbanpour Arani *et al.* (2015). In this work, the cylindrical shell was subjected to non-axisymmetric thermal-mechanical loads and uniform electro-magnetic fields. Eventually, the stress distribution in the structure is determined analytically by Fourier series. Buckling analysis of CNTs reinforced microplates is carried out by Kolahchi *et al.* (2013). They derived the governing equations of the structure based on Mindlin plate theory and using Hamilton's principle. They obtained buckling load of the structure by applying differential quadrature method (DQM). Dynamic response of FG circular cylindrical shells is examined by Davar *et al.* (2013). They developed the mathematical formulation of the structure according to first order shear deformation theory (FSDT) and Love's first approximation theory. Also, Kolahchi *et al.* (2016) investigated dynamic stability of FG-CNTs reinforced plates. The material properties of the plate are assumed to be a function of temperature and the structure is considered resting on orthotropic elastomeric medium. Jafarian Arani and Kolahchi (2016) presented a mathematical model for buckling analysis of a CNTs reinforced concrete column. They simulated the problem based on Euler Bernoulli and Timoshenko plate theories. Nonlinear vibration of laminated cylindrical shells is analyzed by Shen and Yang (2014). They examined the influences of temperature variation, shell geometric parameter and applied voltage on the linear and nonlinear vibration of the structure. An investigation on the nonlinear dynamic response and vibration of the imperfect laminated three-phase polymer nanocomposite panel resting on elastic foundations was presented by Duc *et al.* (2015). Van Thu and Duc (2016) presented an analytical approach to investigate the non-linear dynamic response and vibration of an imperfect three-phase laminated nanocomposite cylindrical panel resting on elastic foundations in thermal environments. Alibeigloo (2016) employed theory of piezoelectricity to study bending behavior of FG-CNTs reinforced composite cylindrical panels. They used an analytical method to study the effect of CNT volume fraction, temperature variation and applied voltage on the bending behavior of the system. Feng *et al.* (2017) studied the nonlinear bending behavior of a novel class of multi-layer polymer nanocomposite plates reinforced with graphene platelets (GPLs) that are non-uniformly distributed along the thickness direction. Duc *et al.* (2017a, b, c) studied thermal and mechanical stability of a functionally graded composite truncated conical shell, plates and double curved shallow shells reinforced by carbon nanotube fibers. Based on Reddy's third-order shear deformation plate theory, the nonlinear dynamic response and vibration of imperfect functionally graded carbon nanotube-reinforced composite plates was analyzed by Thanh *et al.* (2017). Duc *et al.* (2018) presented the first analytical approach to investigate the nonlinear dynamic response and vibration of imperfect rectangular nanocomposite multilayer organic solar cell subjected to mechanical loads using the classical plate theory.

For the first time, dynamic response of  $AL_2O_3$  nanoparticles-reinforced concrete plates subjected to seismic excitation and magnetic field is studied in the

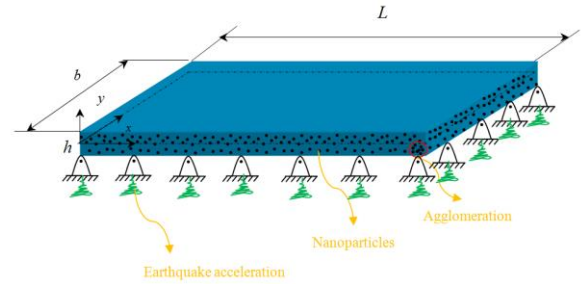


Fig. 1 Schematic figure of concrete plate reinforced by agglomerated  $AL_2O_3$  nanoparticles under magnetic field

present research. So, the results of this research are of great importance in Civil Engineering. The concrete plate is modeled by applying SSPT and the effective material properties of the concrete plate are obtained based on Mori-Tanaka model considering agglomeration of  $AL_2O_3$  nanoparticles. The dynamic displacement of structure is calculated by HDQM in conjunction with Newmark method. The effects of different parameters such as volume fraction and agglomeration of  $AL_2O_3$  nanoparticles, magnetic field, boundary conditions and geometrical parameters of concrete plate are studied on the dynamic response of the structure.

## 2. Formulation

### 2.1 Strain relations

As shown in Fig. 1, a concrete plate is reinforced by agglomerated  $AL_2O_3$  nanoparticles subjected to the earthquake load and magnetic field. The geometrical parameters of plate are length of  $L$  and thickness of  $h$ .

There are many new theories for modeling of different structures. Some of the new theories have been used by Tounsi and co-authors (Bessaim 2013, Boudarba 2013, Belabed 2014, Ait Amar Meziane 2014, Zidi 2014, Hamidi 2015, Bourada 2015, Bousahla *et al.* 2016a, b, Beldjelili 2016, Boukhari 2016, Draiche 2016, Bellifa 2015, Attia 2015, Mahi 2015, Ait Yahia 2015, Bennoun 2016, El-Haina 2017, Menasria 2017, Chikh 2017).

By applying SSPT, the displacements fields are defined as below (Simsek and Reddy 2013)

$$\begin{aligned} U_1(x, y, z, t) &= U(x, y, t) - z \frac{\partial W_b}{\partial x} - \Phi(z) \frac{\partial W_s}{\partial x}, \\ U_2(x, y, z, t) &= V(x, y, t) - z \frac{\partial W_b}{\partial y} - \Phi(z) \frac{\partial W_s}{\partial y}, \\ U_3(x, y, z, t) &= W_b(x, y, t) + W_s(x, y, t), \end{aligned} \quad (1)$$

where  $U$ ,  $V$  and  $W$  are the respective translation displacements of a point at the mid-plane of the plate in the longitudinal  $x$ , transverse  $y$  and thickness  $z$  directions. Also,  $\phi$  denotes the rotation of the cross section area and  $\Phi(z)$  is the shape function of the plate which is considered as follows

$$\Phi(z) = \left( z - \left( \frac{h}{\pi} \sin \frac{\pi z}{h} \right) \right), \quad (2)$$

However, the strain-displacement relations of the structure are given as below

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{pmatrix} + z \begin{pmatrix} k_{xx}^b \\ k_{yy}^b \\ k_{xy}^b \end{pmatrix} + \left( z - \frac{h}{\pi} \sin \frac{\pi z}{h} \right) \begin{pmatrix} k_{xx}^s \\ k_{yy}^s \\ k_{xy}^s \end{pmatrix} \quad (3a)$$

$$\begin{pmatrix} \gamma_{yz} \\ \gamma_{xz} \end{pmatrix} = \cos \left( \frac{\pi z}{h} \right) \begin{pmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{pmatrix} \quad (3b)$$

where

$$\begin{pmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{pmatrix} = \begin{pmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \end{pmatrix}, \quad \begin{pmatrix} k_{xx}^b \\ k_{yy}^b \\ k_{xy}^b \end{pmatrix} = \begin{pmatrix} -\frac{\partial^2 W_b}{\partial x^2} \\ -\frac{\partial^2 W_b}{\partial y^2} \\ -2\frac{\partial^2 W_b}{\partial x \partial y} \end{pmatrix}, \quad \begin{pmatrix} k_{xx}^s \\ k_{yy}^s \\ k_{xy}^s \end{pmatrix} = \begin{pmatrix} -\frac{\partial^2 W_s}{\partial x^2} \\ -\frac{\partial^2 W_s}{\partial y^2} \\ -2\frac{\partial^2 W_s}{\partial x \partial y} \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{pmatrix} = \frac{-4}{h^3} \begin{pmatrix} \frac{\partial W_s}{\partial y} \\ \frac{\partial W_s}{\partial x} \end{pmatrix} \quad (5)$$

## 2.2 Stress relations

The constitutive equations of the orthotropic plate are considered as below

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{zy} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{zy} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix}, \quad (6)$$

where  $C_{ij}$  are the elastic constants of the concrete plate. To obtain the effective material properties of the concrete plate and to consider the agglomeration effect, Mori-Tanaka model (Mori and Tanaka 1973) is employed which the effective Young's modulus  $E$  and Poisson's ratio  $\nu$  of the composite material are given by

$$E = \frac{9KG}{3K+G}, \quad (7)$$

$$\nu = \frac{3K-2G}{6K+2G}. \quad (8)$$

where the effective bulk modulus  $K$  and shear modulus  $G$  may be written as below (Shu and Xue 1997)

$$K = K_{out} \left[ 1 + \frac{\xi \left( \frac{K_{in}}{K_{out}} - 1 \right)}{1 + \alpha (1 - \xi) \left( \frac{K_{in}}{K_{out}} - 1 \right)} \right], \quad (9)$$

$$G = G_{out} \left[ 1 + \frac{\xi \left( \frac{G_{in}}{G_{out}} - 1 \right)}{1 + \beta (1 - \xi) \left( \frac{G_{in}}{G_{out}} - 1 \right)} \right], \quad (10)$$

The agglomeration effect can be considered based on the micro-mechanical model by introducing the two following parameters

$$\xi = \frac{V_{inclusion}}{V}, \quad (11)$$

$$\zeta = \frac{V_r^{inclusion}}{V_r}. \quad (12)$$

where  $V_r$  and  $V_r^{inclusion}$  are the total volume of nanoparticles and volume of the nanoparticles inside the inclusion, respectively. In addition,  $K_{in}$  and  $K_{out}$  are the effective bulk modulus of the inclusion and the matrix outside the inclusion, respectively. Also,  $G_{in}$  and  $G_{out}$  are the effective shear modulus of the inclusion and the matrix outside the inclusion, respectively and are given as follows

$$K_{in} = K_m + \frac{(\delta_r - 3K_m \chi_r) C_r \zeta}{3(\xi - C_r \zeta + C_r \zeta \chi_r)}, \quad (13)$$

$$K_{out} = K_m + \frac{C_r (\delta_r - 3K_m \chi_r) (1 - \zeta)}{3[1 - \xi - C_r (1 - \zeta) + C_r \chi_r (1 - \zeta)]}, \quad (14)$$

$$G_{in} = G_m + \frac{(\eta_r - 3G_m \beta_r) C_r \zeta}{2(\xi - C_r \zeta + C_r \zeta \beta_r)}, \quad (15)$$

$$G_{out} = G_m + \frac{C_r (\eta_r - 3G_m \beta_r) (1 - \zeta)}{2[1 - \xi - C_r (1 - \zeta) + C_r \beta_r (1 - \zeta)]}, \quad (16)$$

where  $C_r$  is the volume percent of nanoparticles and  $\chi_r$ ,  $\beta_r$ ,  $\delta_r$  and  $\eta_r$  can be obtained as

$$\chi_r = \frac{3(K_m + G_m) + k_r - l_r}{3(k_r + G_m)}, \quad (17)$$

$$\beta_r = \frac{1}{5} \left\{ \frac{4G_m + 2k_r + l_r}{3(k_r + G_m)} + \frac{4G_m}{(p_r + G_m)} + \frac{2[G_m(3K_m + G_m) + G_m(3K_m + 7G_m)]}{G_m(3K_m + G_m) + m_r(3K_m + 7G_m)} \right\}, \quad (18)$$

$$\delta_r = \frac{1}{3} \left[ n_r + 2l_r + \frac{(2k_r - l_r)(3K_m + 2G_m - l_r)}{k_r + G_m} \right], \quad (19)$$

$$\eta_r = \frac{1}{5} \left\{ \frac{2(n_r - l_r) + \frac{4G_m p_r}{(p_r + G_m)} + \frac{2(k_r - l_r)(2G_m + l_r)}{3(k_r + G_m)}}{+ \frac{8G_m m_r(3K_m + 4G_m)}{3K_m(m_r + G_m) + G_m(7m_r + G_m)}} \right\}, \quad (20)$$

in which  $k_r$ ,  $l_r$ ,  $n_r$ ,  $p_r$  and  $m_r$  are Hill's elastic moduli of the

reinforcing phase of the composite material. Furthermore,  $K_m$  and  $G_m$  are the bulk and shear moduli of the matrix phase which are defined as below

$$K_m = \frac{E_m}{3(1-2\nu_m)}, \quad (21)$$

$$G_m = \frac{E_m}{2(1+\nu_m)}. \quad (22)$$

where  $E_m$  and  $\nu_m$  are considered as Young's modulus and Poisson's ratio of the concrete plate, respectively. Moreover,  $\alpha$  and  $\beta$  in Eqs. (9) and (10) are given as follows

$$\alpha = \frac{(1+\nu_{out})}{3(1-\nu_{out})}, \quad (23)$$

$$\alpha = \frac{(1+\nu_{out})}{3(1-\nu_{out})}, \quad (24)$$

$$\nu_{out} = \frac{3K_{out} - 2G_{out}}{6K_{out} + 2G_{out}}. \quad (25)$$

### 2.3 Energy method

The potential strain energy stored in the structure is given as follows

$$U = \frac{1}{2} \int_A \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xx}\epsilon_{xx} + \sigma_{yy}\epsilon_{yy} + \sigma_{xy}\gamma_{xy} + \sigma_{xz}\gamma_{xz} + \sigma_{yz}\gamma_{yz}) dz dA \quad (26)$$

Substituting Eqs. (3) and (4) into Eq. (26) we have

$$\begin{aligned} U = \frac{1}{2} \int_A \left( N_{xx} \frac{\partial U}{\partial x} + N_{xy} \frac{\partial U}{\partial y} + N_{yx} \frac{\partial V}{\partial x} \right. \\ + N_{yy} \frac{\partial V}{\partial y} + Q_x \frac{\partial W_s}{\partial x} + Q_y \frac{\partial W_s}{\partial y} - M_{xxs} \frac{\partial^3 W_s}{\partial x^2} \\ - M_{yys} \frac{\partial^3 W_s}{\partial y^2} - 2M_{xys} \frac{\partial^3 W_s}{\partial y \partial x} - M_{xxb} \frac{\partial^3 W_b}{\partial x^2} \\ \left. - M_{yyb} \frac{\partial^3 W_b}{\partial y^2} - 2M_{xyb} \frac{\partial^3 W_b}{\partial y \partial x} \right) dA, \end{aligned} \quad (27)$$

where

$$\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} dz, \quad (28)$$

$$\begin{bmatrix} M_{xxb} \\ M_{yyb} \\ M_{xyb} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} z dz, \quad (29)$$

$$\begin{bmatrix} M_{xys} \\ M_{yys} \\ M_{xyb} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} f dz, \quad (30)$$

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} pdz. \quad (31)$$

By substituting Eqs. (5)-(8) into Eqs. (28)-(31), the stress resultants of the plate take the following form

$$\begin{aligned} N_{xx} = A_{11} \frac{\partial}{\partial x} U - A_{11z} \frac{\partial^2}{\partial x^2} W_b - A_{11f} \frac{\partial^2}{\partial x^2} W_s \\ + A_{12} \frac{\partial}{\partial y} V - A_{12z} \frac{\partial^2}{\partial y^2} W_b - A_{12f} \frac{\partial^2}{\partial y^2} W_s, \end{aligned} \quad (32)$$

$$\begin{aligned} N_{yy} = A_{21} \frac{\partial}{\partial x} U - A_{21z} \frac{\partial^2}{\partial x^2} W_b - A_{21f} \frac{\partial^2}{\partial x^2} W_s \\ + A_{22} \frac{\partial}{\partial y} V - A_{22z} \frac{\partial^2}{\partial y^2} W_b - A_{22f} \frac{\partial^2}{\partial y^2} W_s, \end{aligned} \quad (33)$$

$$\begin{aligned} N_{xy} = A_{44} \frac{\partial}{\partial y} U + A_{44} \frac{\partial}{\partial x} V \\ - 2A_{44z} \frac{\partial^2}{\partial x \partial y} W_b - 2A_{44f} \frac{\partial^2}{\partial x \partial y} W_s, \end{aligned} \quad (34)$$

$$Q_x = A_{55g} \frac{\partial}{\partial x} W_s + GA_{55g} \frac{\partial^2}{\partial x \partial t} W_s, \quad (35)$$

$$Q_y = A_{66g} \frac{\partial}{\partial y} W_s + GA_{66g} \frac{\partial^2}{\partial y \partial t} W_s, \quad (36)$$

$$\begin{aligned} M_{xxb} = A_{11z} \frac{\partial}{\partial x} U - B_{11} \frac{\partial^2}{\partial x^2} W_b - A_{11zf} \frac{\partial^2}{\partial x^2} W_s \\ + A_{12z} \frac{\partial}{\partial y} V - B_{12} \frac{\partial^2}{\partial y^2} W_b - A_{12zf} \frac{\partial^2}{\partial y^2} W_s, \end{aligned} \quad (37)$$

$$\begin{aligned} M_{xys} = A_{11f} \frac{\partial}{\partial x} U - A_{11zf} \frac{\partial^2}{\partial x^2} W_b - E_{11} \frac{\partial^2}{\partial x^2} W_s \\ + A_{12f} \frac{\partial}{\partial y} V - A_{12zf} \frac{\partial^2}{\partial y^2} W_b - E_{12} \frac{\partial^2}{\partial y^2} W_s, \end{aligned} \quad (38)$$

$$\begin{aligned} M_{yyb} = A_{21z} \frac{\partial}{\partial x} U - B_{21} \frac{\partial^2}{\partial x^2} W_b - A_{21zf} \frac{\partial^2}{\partial x^2} W_s \\ + A_{22z} \frac{\partial}{\partial y} V - B_{22} \frac{\partial^2}{\partial y^2} W_b - A_{22zf} \frac{\partial^2}{\partial y^2} W_s, \end{aligned} \quad (39)$$

$$\begin{aligned} M_{yys} = A_{21f} \frac{\partial}{\partial x} U - A_{21zf} \frac{\partial^2}{\partial x^2} W_b - E_{21} \frac{\partial^2}{\partial x^2} W_s \\ + A_{22f} \frac{\partial}{\partial y} V - A_{22zf} \frac{\partial^2}{\partial y^2} W_b - E_{22} \frac{\partial^2}{\partial y^2} W_s, \end{aligned} \quad (40)$$

$$\begin{aligned} M_{xyb} = 2A_{44z} \frac{\partial}{\partial y} U + 2A_{44z} \frac{\partial}{\partial x} V \\ - 2B_{44} \frac{\partial^2}{\partial x \partial y} W_b - 2A_{44zf} \frac{\partial^2}{\partial x \partial y} W_s, \end{aligned} \quad (41)$$

$$\begin{aligned} M_{xys} = 2A_{44f} \frac{\partial}{\partial y} U + 2A_{44f} \frac{\partial}{\partial x} V \\ - 2A_{44zf} \frac{\partial^2}{\partial x \partial y} W_b - 2E_{44} \frac{\partial^2}{\partial x \partial y} W_s, \end{aligned} \quad (42)$$

in which

$$(A_{11}, A_{12}, A_{22}, A_{44}) = \int_{-h}^h (C_{11}, C_{12}, C_{22}, C_{44}) dz + \int_h^{h+h_f} (Q_{11}, Q_{12}, Q_{22}, Q_{44}) dz, \quad (43)$$

$$(A_{11z}, A_{12z}, A_{22z}, A_{44z}) = \int_{-h}^h (C_{11}, C_{12}, C_{22}, C_{44}) z dz + \int_h^{h+h_f} (Q_{11}, Q_{12}, Q_{22}, Q_{44}) z dz, \quad (44)$$

$$(A_{11f}, A_{12f}, A_{22f}, A_{44f}) = \int_{-h}^h (C_{11}, C_{12}, C_{22}, C_{44}) f dz + \int_h^{h+h_f} (Q_{11}, Q_{12}, Q_{22}, Q_{44}) f dz, \quad (45)$$

$$(A_{11zf}, A_{12zf}, A_{22zf}, A_{44zf}) = \int_{-h}^h (C_{11}, C_{12}, C_{22}, C_{44}) z f dz + \int_h^{h+h_f} (Q_{11}, Q_{12}, Q_{22}, Q_{44}) z f dz \quad (46)$$

$$(A_{55g}, A_{66g}) = \int_{-h}^h (C_{55}, C_{66}) g dz + \int_h^{h+h_f} (Q_{55}, Q_{66}) g dz, \quad (47)$$

$$(B_{11}, B_{12}, B_{22}, B_{44}) = \int_{-h}^h (C_{11}, C_{12}, C_{22}, C_{44}) z^2 dz + \int_h^{h+h_f} (Q_{11}, Q_{12}, Q_{22}, Q_{44}) z^2 dz \quad (48)$$

$$(E_{11}, E_{12}, E_{22}, E_{44}) = \int_{-h}^h (C_{11}, C_{12}, C_{22}, C_{44}) f^2 dz + \int_h^{h+h_f} (Q_{11}, Q_{12}, Q_{22}, Q_{44}) f^2 dz, \quad (49)$$

The kinetic energy of the structure are defined as below

$$K = \frac{\rho}{2} \int (\dot{U}_1^2 + \dot{U}_2^2 + \dot{U}_3^2) dV \quad (52)$$

By substituting Eq. (1) into Eq. (52) we have

$$K = 0.5 \int \left[ I_0 \left( \left( \frac{\partial U}{\partial t} \right)^2 + \left( \frac{\partial V}{\partial t} \right)^2 + \left( \frac{\partial W_b}{\partial t} \right)^2 + \left( \frac{\partial W_s}{\partial t} \right)^2 \right) - 2I_1 \left( \frac{\partial U}{\partial t} \frac{\partial^3 W_b}{\partial t \partial x} + \frac{\partial V}{\partial t} \frac{\partial^3 W_b}{\partial t \partial y} \right) + I_2 \left( \left( \frac{\partial^3 W_b}{\partial t \partial x} \right)^2 + \left( \frac{\partial^3 W_b}{\partial t \partial y} \right)^2 \right) + I_3 \left( \left( \frac{\partial^3 W_s}{\partial t \partial x} \right)^2 + \left( \frac{\partial^3 W_s}{\partial t \partial y} \right)^2 \right) + 2I_4 \left( \frac{\partial^2 W_b}{\partial t \partial x} \frac{\partial^3 W_s}{\partial t \partial x} + \frac{\partial^2 W_b}{\partial t \partial y} \frac{\partial^3 W_s}{\partial t \partial y} \right) + 2I_5 \left( \frac{\partial U}{\partial t} \frac{\partial^3 W_s}{\partial t \partial x} + \frac{\partial V}{\partial t} \frac{\partial^3 W_s}{\partial t \partial y} \right) \right] dA. \quad (53)$$

where

$$(I_0, I_1, I_2, J_1, J_2, K_2) = \int_{-h}^h \rho (1, z, f, zf, z^2, f^2). \quad (54)$$

The external work due the earthquake and magnetic field can be calculated as follows (Kolahchi *et al.* 2016)

$$W = \int (ma(t) + \mu H_x^2 h \frac{\partial^2 w}{\partial x^2}) dx, \quad (56)$$

where  $m$  and  $a(t)$  are the mass and acceleration of the earth, respectively;  $\mu$  and  $H_x$  are magnetic permeability and magnetic field, respectively. To extract the governing equations of motion, Hamilton's principle is expressed as follows

$$\int_0^t (\delta U - \delta K - \delta W) dt = 0, \quad (57)$$

where  $\delta$  denotes the variational operator. Substituting Eqs. (58)-(60) into Eq. (57), the motion equations of the structure are obtained as follows

$$\frac{\partial}{\partial x} N_{xx} + \frac{\partial}{\partial y} N_{xy} - I_0 \frac{\partial^2 U}{\partial t^2} + I_1 \frac{\partial^3 W_b}{\partial x \partial t^2} + J_1 \frac{\partial^3 W_s}{\partial x \partial t^2} = 0, \quad (58)$$

$$\frac{\partial}{\partial x} N_{xy} + \frac{\partial}{\partial y} N_{yy} - I_0 \frac{\partial^2 V}{\partial t^2} + I_1 \frac{\partial^3 W_b}{\partial y \partial t^2} + J_1 \frac{\partial^3 W_s}{\partial y \partial t^2} = 0, \quad (59)$$

$$\frac{\partial^2}{\partial x^2} M_{xxB} + 2 \frac{\partial^2}{\partial x \partial y} M_{xyB} + \frac{\partial^2}{\partial y^2} M_{yyB} - K_w w - I_0 \left( \frac{\partial^3 W_b}{\partial t^2} + \frac{\partial^3 W_s}{\partial t^2} \right) - I_1 \left( \frac{\partial^3 U}{\partial x \partial t^2} + \frac{\partial^3 V}{\partial y \partial t^2} \right) \quad (60)$$

$$+ I_2 \left( \frac{\partial^4 W_b}{\partial x^2 \partial t^2} + \frac{\partial^4 W_b}{\partial y^2 \partial t^2} \right) + J_2 \left( \frac{\partial^4 W_s}{\partial x^2 \partial t^2} + \frac{\partial^4 W_s}{\partial y^2 \partial t^2} \right) = 0,$$

$$\begin{aligned} & \frac{\partial^2}{\partial x^2} M_{xxS} + 2 \frac{\partial^2}{\partial x \partial y} M_{xyS} + \frac{\partial^2}{\partial y^2} M_{yyS} + \frac{\partial}{\partial x} Q_x \\ & + \frac{\partial}{\partial y} Q_y - K_w w - I_0 \left( \frac{\partial^3 W_b}{\partial t^2} + \frac{\partial^3 W_s}{\partial t^2} \right) \\ & - J_1 \left( \frac{\partial^3 U}{\partial x \partial t^2} + \frac{\partial^3 V}{\partial y \partial t^2} \right) + J_2 \left( \frac{\partial^4 W_b}{\partial x^2 \partial t^2} + \frac{\partial^4 W_b}{\partial y^2 \partial t^2} \right) \\ & + K_2 \left( \frac{\partial^4 W_s}{\partial x^2 \partial t^2} + \frac{\partial^4 W_s}{\partial y^2 \partial t^2} \right) = 0, \end{aligned} \quad (61)$$

Also, the boundary conditions of the structure are considered as below

#### • Clamped-Clamped supported

$$W_s = W_b = U = V = \frac{\partial W_s}{\partial x} = \frac{\partial W_b}{\partial x} = 0, @ \quad x = 0, L \quad (62)$$

$$W_s = W_b = U = V = \frac{\partial W_s}{\partial x} = \frac{\partial W_b}{\partial x} = 0, @ \quad y = 0, b$$

#### • Clamped-Simply supported

$$W_s = W_b = U = V = \frac{\partial W_s}{\partial x} = \frac{\partial W_b}{\partial x}, @ \quad x = 0, L \quad (63)$$

$$W_s = W_b = U = V = M_{xxB} = M_{xxS}, @ \quad y = 0, b$$

#### • Simply-Simply supported

$$W_s = W_b = U = V = M_{yyB} = M_{yyS}, @ \quad x = 0, L \quad (64)$$

$$W_s = W_b = U = V = M_{xxB} = M_{xxS}, @ \quad y = 0, b$$

## 5. Solution procedure

In this study, DQM is applied to examine the dynamic behavior of the structure. In this numerical method, the governing differential equations of the structure turn into a set of first order algebraic equations by applying the weighting coefficients. According to DQ method, a derivative of a function at a given discrete point will be approximated as a weighted linear sum of the function values at all discrete points chosen in the solution domain. The one-dimensional derivative of the function can be expressed as follows (Kolahchi *et al.* 2016)

$$\frac{d^n f(x_i)}{dx^n} = \sum_{j=1}^N C_{ij}^{(n)} f(x_j) \quad n=1, \dots, N-1. \quad (65)$$

where  $f(x)$  is the mentioned function,  $N$  denotes number of grid points,  $x_i$  is a sample point of the function domain,  $f_i$  is the value of the function at  $i$ th sample point and  $C_{ij}$  indicates the weighting coefficients. So, choosing the grid points and weighting coefficients is an important factor in the accuracy of the results. The grid points are considered by Chebyshev polynomials as follows

$$X_i = \frac{L}{2} \left[ 1 - \cos \left( \frac{i-1}{N_x-1} \pi \right) \right] \quad i=1, \dots, N_x \quad (66a)$$

$$y_i = \frac{b}{2} \left[ 1 - \cos \left( \frac{i-1}{N_y-1} \pi \right) \right] \quad i=1, \dots, N_y \quad (66b)$$

Based on Chebyshev polynomials, the grid points are closer together near the borders and in distant parts of the borders they away from each other. The weighting coefficients may be calculated by the following simple algebraic relations

$$A_{ij}^{(1)} = \begin{cases} \frac{M(x_i)}{(x_i - x_j)M(x_j)} & \text{for } i \neq j, \quad i, j=1, 2, \dots, N_x \\ -\sum_{\substack{j=1 \\ i \neq j}}^{N_x} A_{ij}^{(1)} & \text{for } i = j, \quad i, j=1, 2, \dots, N_x \end{cases} \quad (67a)$$

$$B_{ij}^{(1)} = \begin{cases} \frac{P(y_i)}{(y_i - y_j)P(y_j)} & \text{for } i \neq j, \quad i, j=1, 2, \dots, N_y \\ -\sum_{\substack{j=1 \\ i \neq j}}^{N_y} B_{ij}^{(1)} & \text{for } i = j, \quad i, j=1, 2, \dots, N_y \end{cases} \quad (67b)$$

in which

$$M(x_i) = \prod_{\substack{j=1 \\ j \neq i}}^{N_x} (x_i - x_j) \quad (68a)$$

$$P(y_i) = \prod_{\substack{j=1 \\ j \neq i}}^{N_y} (y_i - y_j) \quad (68b)$$

By distributing the grid points in the domain by substituting Eq. (65) into the governing equations, we have

$$\left( \left[ \begin{matrix} K_L + K_{NL} \\ K \end{matrix} \right] \begin{Bmatrix} \{d_b\} \\ \{d_d\} \end{Bmatrix} + [M] \begin{Bmatrix} \{\ddot{d}_b\} \\ \{\ddot{d}_d\} \end{Bmatrix} \right) = \begin{Bmatrix} \{0\} \\ -Ma(t) \end{Bmatrix}, \quad (69)$$

in which  $[K_L]$ ,  $[K_{NL}]$  and  $[M]$  indicate linear part of the stiffness matrix, nonlinear part of the stiffness matrix and the mass matrix, respectively. Also,  $\{d_b\}$  and  $\{d_d\}$  denote boundary and domain points, respectively. To obtain the time response of the structure subjected to the earthquake loads Newmark method (Simsek 2010) is applied in the time domain. Based on this method, Eq. (69) is considered in the general form as below

$$K^*(d_{i+1}) = Q_{i+1}, \quad (70)$$

where subscript  $i+1$  denotes the time  $t=t_{i+1}$ ,  $K^*(d_{i+1})$  and  $Q_{i+1}$  are the effective stiffness matrix and the effective load vector which are given as

$$K^*(d_{i+1}) = K_L + K_{NL}(d_{i+1}) + \alpha_0 M + \alpha_1 C, \quad (71)$$

$$Q_{i+1}^* = Q_{i+1} + M \left( \alpha_0 \ddot{d}_i + \alpha_2 \dot{\ddot{d}}_i + \alpha_3 \ddot{\ddot{d}}_i \right) + C \left( \alpha_1 \dot{d}_i + \alpha_4 \dot{\ddot{d}}_i + \alpha_5 \ddot{\ddot{d}}_i \right), \quad (72)$$

where (Simsek 2010)

$$\begin{aligned} \alpha_0 &= \frac{1}{\chi \Delta t^2}, & \alpha_1 &= \frac{\gamma}{\chi \Delta t}, & \alpha_2 &= \frac{1}{\chi \Delta t}, \\ \alpha_3 &= \frac{1}{2\chi} - 1, & \alpha_4 &= \frac{\gamma}{\chi} - 1, & \alpha_5 &= \frac{\Delta t}{2} \left( \frac{\gamma}{\chi} - 2 \right), \\ \alpha_6 &= \Delta t (1 - \gamma), & \alpha_7 &= \Delta t \gamma, \end{aligned} \quad (73)$$

where  $\gamma=0.5$  and  $\chi=0.25$ . By applying the iteration method, Eq. (70) is solved at any time step and modified velocity and acceleration vectors are computed as follows

$$\ddot{d}_{i+1} = \alpha_0 (d_{i+1} - d_i) - \alpha_2 \dot{d}_i - \alpha_3 \ddot{d}_i, \quad (74)$$

$$\dot{d}_{i+1} = \dot{d}_i + \alpha_6 \ddot{d}_i + \alpha_7 \ddot{\ddot{d}}_{i+1}, \quad (75)$$

Then for the next time step, the modified velocity and acceleration vectors in Eqs. (74) and (75) are applied and all the mentioned procedures are repeated.

## 6. Numerical results

In this section, the effect of various parameters on the dynamic response of the concrete plate reinforced by AL<sub>2</sub>O<sub>3</sub> nanoparticles under seismic load and magnetic field is examined. The length and thickness of the concrete plate are  $L=3$  m and  $h=15$  cm, respectively. The elastic moduli of concrete and AL<sub>2</sub>O<sub>3</sub> nanoparticles are  $E_c=20$  GPa and  $E_r=165$  GPa, respectively. The earthquake acceleration is considered based on Kobe earthquake that the distribution of acceleration in 30 seconds is shown in Fig. 2.

### 6.1 Convergence of DQM

Fig. 3 shows the convergence of DQM in evaluating the maximum deflection of the structure versus time. As it can be seen, with increasing the number of grid points  $N$ , the maximum deflection of the structure decreases until  $N=15$ , which the results converge to a constant value. So, the results presented below are based on the number of grid

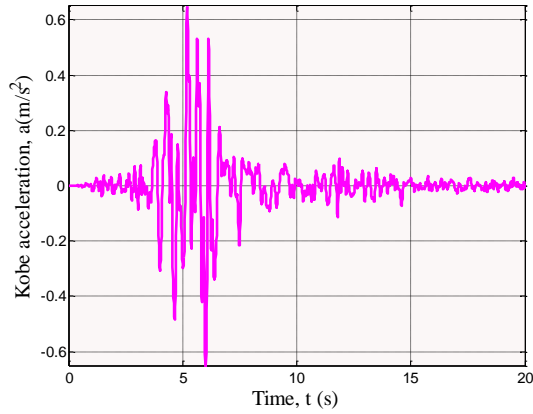


Fig. 2 Acceleration of Kobe

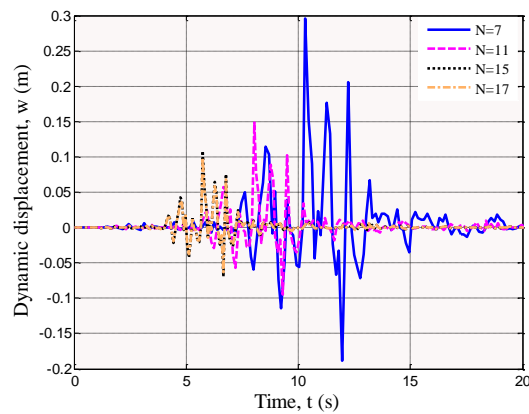


Fig. 3 Convergence and accuracy of DQM

points 15 for DQ solution method.

### 6.2 Validation of results

Since, there is not any similar work in the literature in the scope of this paper, however, validation of this work is done by comparing the numerical and analytical solutions. The results of the analytical and numerical (DQ) methods are depicted in Fig. 4. As it can be observed, the results of numerical and analytical methods are identical and therefore, the obtained results are accurate and acceptable.

### 6.3 Effect of magnetic field

Fig. 5 illustrate the effect of magnetic field on the dynamic deflection versus time. As it can be observed, the structure without magnetic field has a greater dynamic deflection with respect to the concrete plate subjected to magnetic field. The reason is that the magnetic field increases the stiffness of the structure. Fig. 5(a) shows the maximum dynamic deflection of the structure without magnetic field equal to 39 while by applying the magnetic field of 1, 5 and 10 A/m, the maximum dynamic displacement of the structure is 27.05, 18.15 and 17.97, respectively. By comparing the results, we can say that applying the magnetic field of 1, 5 and 10 A/m decreases the maximum dynamic displacement of the structure up to 30.64, 53.46 and 53.92 percent which is a remarkable result

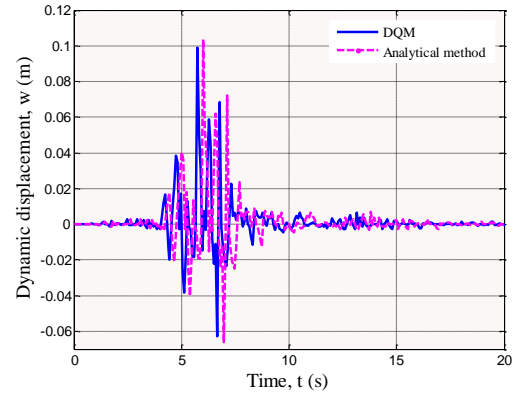


Fig. 4 Comparison of numerical and analytical results

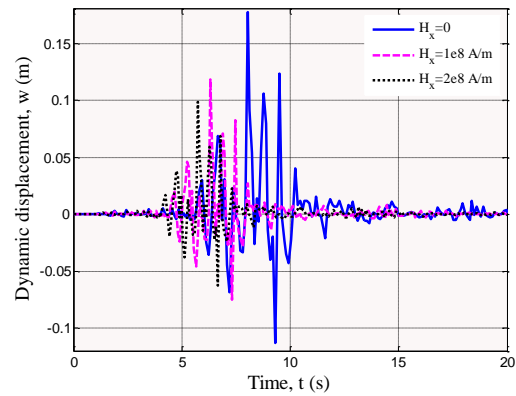
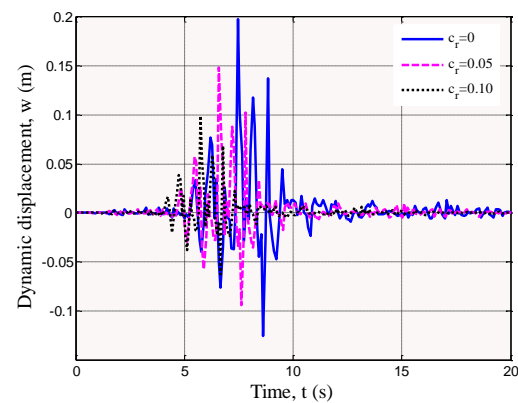


Fig. 5 The effect of magnetic field on the dynamic response of the structure

Fig. 6 The effect of  $AL_2O_3$  nanoparticles volume percent on the dynamic response of the structure

in the dynamic designing of the structures. Also it should be noted that the excessive increasing of the magnetic field increases costs while it does not have a noticeable effect on the dynamic response of the structure. Hence, the magnetic field of 5 A/m is the best choice for the present structure.

### 6.4 Effect of $AL_2O_3$ nanoparticles

The effect of  $AL_2O_3$  nanoparticles volume percent on the dynamic response of the structure is studied. Fig. 6(a)-(d). It is apparent that the maximum dynamic displacement of the structure is equals to 32.3 for the case of  $c_f=0$



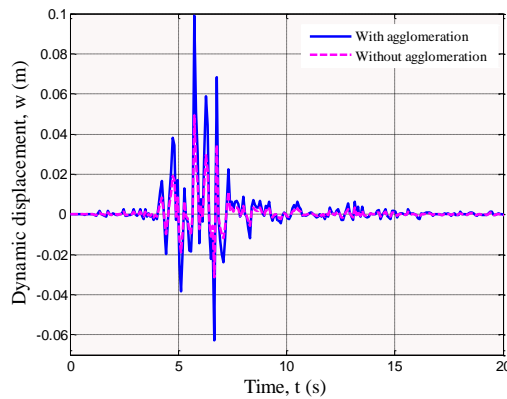


Fig. 7 The effect of  $\text{AL}_2\text{O}_3$  nanoparticles agglomeration on the dynamic response of the structure

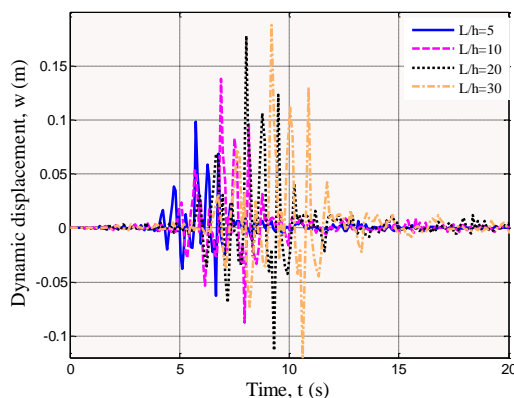


Fig. 8 The effect of concrete plate length on the dynamic response of the structure

(without  $\text{AL}_2\text{O}_3$  nanoparticles). By using  $\text{AL}_2\text{O}_3$  nanoparticles with volume fractions of 0.05, 0.1 and 0.18, the amount of maximum dynamic displacement is 29.1, 27.05 and 30.83, respectively. Therefore, using  $\text{AL}_2\text{O}_3$  nanoparticles with volume fractions of 0.05 and 0.1 increases the stiffness of the structure and reduces the maximum displacement of structure 9.91 and 16.25 percent, respectively while the volume percent of 0.18 has a converse result and 4.5 percent increase the deflection.

The agglomeration effect of  $\text{AL}_2\text{O}_3$  nanoparticles on the dynamic deflection of the structure versus time is illustrated in Fig. 7. As it can be observed, by considering the agglomeration effect, the stiffness of the structure reduces while the dynamic displacement increases. For example, in the absence of the agglomeration effect ( $\xi=1$ ), the maximum dynamic deflection of the structure is 22.22 while for  $\xi=0.5$  the maximum dynamic deflection is 27.05. The results reveal that the existence of the agglomeration changes the maximum dynamic displacement of the structure up to 21.74 percent.

#### 6.5 Effect of concrete plate length

The effect of concrete plate length on the dynamic response versus time is shown in Fig. 8(a)-(d). It can be seen that with an increase in the concrete plate length, the structure becomes softer and the dynamic deflection of the

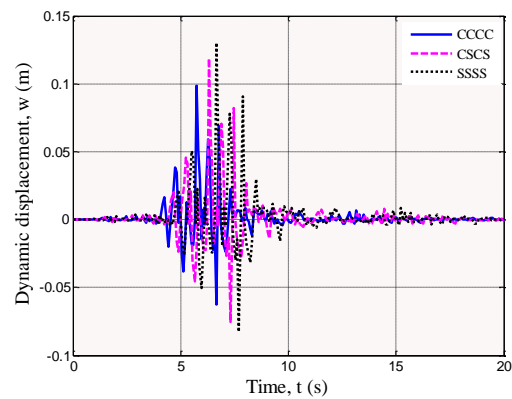


Fig. 9 The effect of different boundary conditions on the dynamic response of the structure

system increases. For example, the maximum dynamic displacements of the concrete plate increase 72.75% with increasing the length from 2 to 3 m.

#### 6.6 Effect of boundary conditions on dynamic response

Fig. 9 illustrate the effect of various boundary conditions on the dynamic response versus time. Four boundary conditions including clamped-clamped, clamped-simply, simply-simply and free-simply supported are considered. The maximum dynamic deflections of the structure for clamped-clamped, clamped-simply supported, simply-simply supported and free-simply supported boundary conditions are 13.53, 18.94, 27.05 and 32.46, respectively. As it can be observed, boundary conditions have a significant effect on the dynamic response of the system so that the structure with clamped-clamped boundary condition has the lowest displacement with respect to the other boundary conditions.

## 7. Conclusions

Seismic response of concrete plates reinforced by  $\text{AL}_2\text{O}_3$  nanoparticles was presented in this article. The structure was subjected to axial magnetic field for controlling the dynamic deflection of the structure. Based on SSPT, the structure was simulated and utilizing the energy method and Hamilton's principle, the motion equations were derived. For calculating the effective material properties of structure and considering agglomeration of  $\text{AL}_2\text{O}_3$  nanoparticles, Mori-Tanaka model was used. Applying HDQ and Newmark methods, the dynamic deflection of the structure was calculated and the effects of different parameters of  $\text{AL}_2\text{O}_3$  nanoparticles volume percent and agglomeration, magnetic field, boundary conditions and length of the concrete plate were considered on the results. Numerical results indicate that the structure without magnetic field has a greater dynamic deflection with respect to the concrete plate subjected to magnetic field so that applying the magnetic field of 5 A/m decreases the maximum dynamic displacement of the structure up to 53.46 percent. Also it



should be noted that the excessive increasing of the magnetic field increases costs while it does not have a noticeable effect on the dynamic response of the structure. Using  $Al_2O_3$  nanoparticles with volume fractions of 0.05 and 0.1 increases the stiffness of the structure and reduces the maximum displacement of structure 9.91 and 16.25 percent, respectively while the volume percent of 0.18 has a converse result and 4.5 percent increase the deflection. The results reveal that the existence of the agglomeration changes the maximum dynamic displacement of the structure up to 21.74 percent. In addition, the maximum dynamic displacements of the concrete plate increase 72.75% with increasing the length from 2 to 3 m. Furthermore, boundary conditions have a significant effect on the dynamic response of the system so that the structure with clamped-clamped boundary condition has the lowest displacement with respect to the other boundary conditions.

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