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An efficient shear deformation theory for wave propagation in functionally graded material beams with porosities

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Abstract. In this paper, an efficient shear deformation theory is developed for wave propagation analysis in a functionally graded beam. More particularly, porosities that may occur in Functionally Graded Materials (FGMs) during their manufacture are considered. The proposed shear deformation theory is efficient method because it permits us to show the effect of both bending and shear components and this is carried out by dividing the transverse displacement into the bending and shear parts. Material properties are assumed graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents; but the rule of mixture is modified to describe and approximate material properties of the functionally graded beams with porosity phases. The governing equations of the wave propagation in the functionally graded beam are derived by employing the Hamilton's principle. The analytical dispersion relation of the functionally graded beam is obtained by solving an eigenvalue problem. The effects of the volume fraction distributions, the depth of beam, the number of wave and the porosity on wave propagation in functionally graded beam are discussed in details. It can be concluded that the present theory is not only accurate but also simple in predicting the wave propagation characteristics in the functionally graded beam.

Keywords: wave propagation; functionally graded beam; porosity; higher-order shear deformation beam theories

1. Introduction

Functionally graded materials (FGMs) are a class of composites that have continuous spatially variation of material properties from one surface to another through non-uniform distribution of the reinforcement phase, and thus eliminating the interlaminar stress concentration found in laminated composites that causes many problems like delamination, initiation and propagation of cracks because of large plastic deformation at the interfaces and so on. Typically, FGMs are made of a mixture of ceramics and a combination of different metals (Ait Amar Meziane *et al*. 2014, Ahouel *et al*. 2016, Barati and Shahverdi 2016). The concept of this material was first introduced in 1984 by a group of material scientists in Japan, as ultrahigh temperature resistant material for aircraft, space vehicles and other engineering applications (Bessaim *et al*. 2013, Besseghier *et al*. 2017, Bouafia *et al*. 2017).

So the main question is an accurate description of material properties in the depth direction, to perform a satisfactory analysis of the mechanical behavior of FGM beams. Many studies on FGM structures have been studied in the literature (Bouderba *et al*. 2013, Fekrar *et al*. 2014, Bousahla *et al*. 2014, Hebali *et al*. 2014, Hamidi *et al*. 2015, Meradjah *et al*. 2015, Mahi *et al*. 2015, Larbi Chaht *et al*. 2015, Bounouara *et al*. 2016, Hebali *et al*. 2016, Chikh *et al*. 2016, Laoufi *et al*. 2016, El-Haina *et al*. 2017, Khetir *et al*. 2017, Fahsi *et al*. 2017, Menasria *et al*. 2017, Meksi *et al*. 2017). Bresse (1859), Rayleigh (1880), and Timoshenko (1921) were the pioneer investigators to include refined effects such as the rotatory inertia and shear deformation in the beam theory. The wave propagation analysis in beams made of FGM is discussed by Chakraborty and Gopalakrishnan (2003), by employing the spectral finite element method. Using the Euler–Bernoulli beam theory, Sankar (2001) presented an elasticity solution for bending of functionally graded beams by assuming that the Young's modulus of the beam varies exponentially through the thickness. Poisson's ratio was considered to be constant, while Young's modulus was supposed to change as an exponential function. By employing the Airy stress function, Zhong and Yu (2007) developed an analytical solution for cantilever beams subjected to various types of mechanical loadings. Kadoli *et al*. (2008) investigated the bending response of FG beams by utilizing higher order shear deformation and numerical method. Ould Larbi *et al*. (2013) presented an efficient shear deformation beam theory based on neutral surface position for bending and free vibration of FG beams. In the same way, Bourada *et al*. (2015) used the concept of the neutral surface position to

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develop a simple and refined trigonometric higher-order beam theory for bending and vibration behavior of FG beams. Yaghoobi and Torabi (2013a) investigated the post-buckling and nonlinear vibration of imperfect FG beams. Yaghoobi and Torabi (2013b) examined analytically the large amplitude vibration and post-buckling of FG beams resting on non-linear elastic foundations. Yaghoobi *et al*. (2014) studied also the post-buckling and nonlinear free vibration response of FG beams resting on nonlinear elastic foundation under thermo-mechanical loading using the variational iteration method (VIM). A simple refined nth-order shear deformation theory is presented by Yaghoobi and Fereidoon (2014) to discuss the mechanical and thermal buckling behaviors of FG plates supported by elastic foundation. Chakraverty and Pradhan (2014) studied the free vibration of exponential functionally graded rectangular plates in thermal environment with general boundary conditions. Belabed *et al*. (2014) proposed an efficient and simple higher order shear and normal deformation theory for FG plates. Attia *et al*. (2015) studied the free vibration behavior of FG plates with temperature-dependent properties using various four variable refined plate theories. Belkorissat *et al*. (2015) investigated the vibration properties of FG nano-plate using a new nonlocal refined four variable model. Ait Atmane *et al*. (2015) studied a computational shear displacement model for vibrational analysis of functionally graded beams with porosities. Beldjelili *et al*. (2016) analyzed the hygro-thermo-mechanical bending response of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory. Bellifa *et al*. (2016) presented static bending and dynamic analysis of FG plates using a simple shear deformation theory and the concept the neutral surface position. Bouderba *et al*. (2016) studied the thermal stability of FG sandwich plates using a simple shear deformation theory. Houari *et al*. (2016) presented a new simple three-unknown sinusoidal shear deformation theory for FG plates. Barka *et al*. (2016) analyzed the thermal post-buckling behavior of imperfect temperature-dependent sandwich FGM plates resting on Pasternak elastic foundation. Bousahla *et al*. (2016) investigated the thermal stability of plates with functionally graded coefficient of thermal expansion. Ait Atmane *et al* (2016) studied the effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations. Benbakhti *et al*. (2016) presented a new five unknown quasi-3D type HSDT for thermomechanical bending analysis of FGM sandwich plates. Draiche *et al*. (2016) used a refined theory with stretching effect for the flexure analysis of laminated composite plates. Bennoun *et al*. (2016) studied the vibration response of FG sandwich plates using a novel five variable refined plate theory. Benchohra *et al*. (2017) developed a new quasi-3D sinusoidal shear deformation theory for FG plates. Chikh *et al*. (2017) investigated the thermal buckling of cross-ply laminated plates using a simplified HSDT. Bellifa *et al*. (2017) proposed a nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams. Klouche *et al*. (2017) presented an original single variable shear deformation theory for buckling analysis of thick isotropic plates. Benahmed *et al*. (2017) developed a novel quasi-3D hyperbolic shear deformation theory for FG thick rectangular plates on elastic foundation.

The wave propagation studies are also important to understand the dynamic characteristics of FGM structure at higher frequencies due to their various real world applications. Structural health monitoring or detection of damage is one such important application. As wave propagation deals with higher frequencies, diagnostic waves can be employed to predict the presence of even minute defects, which occur at initiation of damage and propagate them till the failure of the FGM structure. In many aircraft structures, the undesired vibration and noise transmit from the source to the other parts in form of wave propagation and this requires control or reduction, which is again an important application of wave propagation studies.

The study of the wave propagation in the FG structures has received also much attention from various researchers. Chen *et al*. (2007) studied the dispersion behavior of waves in functionally graded plates with material properties varying along the thickness direction. Han and Liu (2002) investigated SH waves in FG plates, where the material property variation was assumed to be a piecewise quadratic function in the thickness direction. Han *et al.* (2001) proposed an analytical-numerical method for analyzing the wave characteristics in FG cylinders. Han *et al*. (2002) also proposed a numerical method to study the transient wave in FG plates excited by impact loads. Sun and Luo (2011a) also studied the wave propagation and dynamic response of rectangular functionally graded material plates with completed clamped supports under impulsive load. Considering the thermal effects and temperature-dependent material properties, Sun and Luo (2011b) investigated the wave propagation of an infinite functionally graded plate using the higher-order shear deformation plate theory.

However, in FGM fabrication, micro voids or porosities can occur within the materials during the process of sintering. This is due to the great difference in the temperatures of solidification of the various material constituents (Zhu *et al*. 2001). Wattanasakulpong *et al*. (2012) also gave the discussion on porosities happening in side FGM samples fabricated by a multi-step sequential infiltration technique. Therefore, it is important to take in to account the porosity effect when designing FGM structures subjected to dynamic loadings. Recently, Wattanasakulpong and Ungbhakorn (2014) studied linear and nonlinear vibration problems of elastically end restrained FG beams having porosities. Ait Yahia *et al*. (2015) studied wave propagation in order to compare different shear theories and porosities solution in FG plates. Boukhari *et al*. (2016) introduced An efficient shear deformation theory for wave propagation of functionally graded material plates.

Considering FG structural members, it is evident from the above discussed literature that there is a lack of studies on wave propagation in FG beam having porosities. Thus, the objective of this work is to investigate the influence of many parameters on the wave propagation of a FG beam having porosities. The displacement fields of the proposed theories are chosen based on a cubic variation in the in-plane displacements through the thickness. The proposed

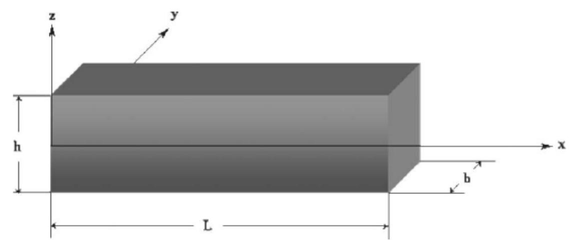


Fig. 1 Coordinates and geometry of functionally graded beam

shear deformation theory is efficient method because it permits us to show the effect of both bending and shear components and this is carried out by dividing the transverse displacement into the bending and shear parts. The governing equations of the wave propagation in the FG beam are derived by using the Hamilton's principle. The analytic dispersion relations of the FG beam are obtained by solving an eigenvalue problem. The dispersion and phase velocity curves of the wave propagation in FG beam having porosities are plotted. The influences of the volume fraction index, the depth of beam and porosity volume fraction on the dispersion and phase velocity of the wave propagation in the FG beam are clearly discussed.

2. Functionally graded plates with porosities

In this part, a FG beam fabricated from a mixture of metal and a ceramic, is considered (Fig. 1). The material characteristics of the FG beam are considered to change continuously within the thickness of the beam. In this work, an imperfect beam is supposed to contain porosities spreading across the depth due to defect during fabrication. The porosity volume fraction, α (α<<1) is assumed to vary evenly among the metal and ceramic. The modified rule of mixture proposed by Wattanasakulpong and Ungbhakorn (2014) is employed as

$$P = P_m\left(V_m - \frac{\alpha}{2}\right) + P_c\left(V_c - \frac{\alpha}{2}\right) \tag{1}$$

With $V_m+V_c=1$, and the power law of volume fraction of the ceramic is expressed by

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^N \tag{2}$$

All properties of the imperfect FGM can be obtained as

$$P = \left(P_c - P_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^N + P_m - \left(P_c + P_m\right)\frac{\alpha}{2} \tag{3}$$

It is noted that the positive real number $N(0 \le N < \infty)$ is the gradient index, and z is the distance from the mid-plane of the FG beam. When N is set to zero, the FG beam becomes fully ceramic. However, when this index takes higher values, the beam becomes fully metal.

Thus, the Young's modulus (E) and material density (ρ) can be obtained from Eq. (3) as

$$E(z) = \left(E_c - E_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^N + E_m - \left(E_c + E_m\right)\frac{\alpha}{2} \tag{4}$$

$$\rho(z) = \left(\rho_c - \rho_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^N + \rho_m - \left(\rho_c + \rho_m\right)\frac{\alpha}{2} \tag{5}$$

In this study, Poisson's ratio (v) is considered to be constant (Tounsi *et al*. 2013, Zidi *et al*. 2014, Taibi *et al*. 2015, Zemri *et al*. 2015, Mouffoki *et al*. 2017, Zidi *et al*. 2017). The material properties of a perfect FG beam can be evaluated by setting α zero.

Another scenario of porosity variation can be obtained for imperfect FGM samples which contain almost porosities spreading around the middle zone of the cross-section and the amount of porosity seems to be on the reduction to zero at the upper and lower faces of the cross-section. Based on the principle of the multi-step sequential infiltration method that can be utilized to fabricate FGM samples (Wattanasakulpong *et al*. 2012), the porosities mostly occur at the middle zone. At this zone, it is difficult to infiltrate the materials completely, while at the upper and lower zones, the process of material infiltration can be established easier and leaves less porosity. By considering this scenario, Eqs. (4)-(5) are replaced by the following forms

$$E(z) = \left(E_c - E_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^N + E_m - \left(E_c + E_m\right)\frac{\alpha}{2}\left(1 - \frac{2|z|}{h}\right) \tag{6}$$

$$\rho(z) = \left(\rho_c - \rho_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^N + \rho_m - \left(\rho_c + \rho_m\right)\frac{\alpha}{2}\left(1 - \frac{2|z|}{h}\right) \tag{7}$$

3. Fundamental equations

3.1 Basic assumptions and constitutive equations

The displacement fields of various shear deformation beam theories are chosen based on following assumptions: (1) the axial and transverse displacements are partitioned into bending and shear components; (2) the bending component of axial displacement is similar to that given by the classical beam theory (CBT); and (3) the shear component of axial displacement gives rise to the higher-order variation of shear strain and hence to shear stress through the depth of the beam in such a way that shear stress vanishes on the top and bottom surfaces. Based on these assumptions, the displacement fields of various higher-order shear deformation beam theories are given in a general form as

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{8a}$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{8b}$$

where u_0 is the mid-plane displacement of the beam in the x direction, w_b and w_s are the bending and shear components of transverse displacement, respectively; and $f(z)$ is a shape function determining the distribution of the transverse shear strain and shear stress through the depth of the beam. The shape functions $f(z)$ are chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the beam, thus a shear correction factor is not required. In this study, these shape functions are chosen based on the third-order shear deformation theory (TSDT) of Reddy (2000). This equation is expressed as

$$f(z)=\frac{4z^3}{3h^2} \tag{9}$$

The nonzero linear strains associated with the displacement field in Eq. (8) are

$$\varepsilon_x=\varepsilon_x^0+z\,k_x^b+f(z)k_x^s,\quad \gamma_{xz}=g(z)\gamma_{xz}^0 \tag{10}$$

where

$$\varepsilon_x^0=\frac{\partial u_0}{\partial x},\quad k_x^b=-\frac{\partial^2 w_b}{\partial x^2},\quad k_x^s=-\frac{\partial^2 w_s}{\partial x^2},\quad \gamma_{xz}^0=\frac{\partial w_s}{\partial x} \tag{11}$$

and

$$g(z)=1-\frac{df(z)}{dz} \tag{12}$$

By assuming that the material of FG beam obeys Hooke's law, the stresses in the beam become

$$\sigma_x=C_{11}(z)\varepsilon_x,\quad \tau_{xz}=C_{55}(z)\,\gamma_{xz} \tag{13}$$

where (σ_x, τ_{xz}) and (ε_x, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eq. (4), stiffness coefficients, C_{ij}, can be expressed as

$$C_{11}(z)=\frac{E(z)}{\left(1-\nu^2\right)}\quad\text{and}\quad C_{55}(z)=\frac{E(z)}{2(1+\nu)} \tag{14}$$

3.2 Governing equations

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as

$$0=\int_0^t(\delta U-\delta K)\,dt \tag{15}$$

where δU is the variation of strain energy; and δK is the variation of kinetic energy.
The variation of strain energy of the beam is stated as

$$\begin{aligned}\delta U&=\int_V\left[\sigma_x\,\delta\,\varepsilon_x+\tau_{zx}\,\delta\,\gamma_{zx}\right]dAdz\\&=\int_A\left[N_x\,\delta\,\varepsilon_x^0+M_x^b\,\delta\,k_x^b+M_x^s\,\delta\,k_x^s+S_{xz}^s\,\delta\,\gamma_{xz}^s\right]dA\end{aligned} \tag{16}$$

where the stress resultants N, M, and S are defined by

$$\left(N_x,M_x^b,M_x^s\right)=\int_{-h/2}^{h/2}(1,z,f)\sigma_x dz,\ \text{and}\quad S_{xz}=\int_{-h/2}^{h/2}g\,\tau_{zx}dz \tag{17}$$

The variation of kinetic energy is expressed as

$$\begin{aligned}\delta K&=\int_V\left[\dot u\delta\,\dot u+\dot w\delta\,\dot w\right]\rho(z)\,dV\\&=\int_A\Big\{I_0\left[\dot u_0\delta\dot u_0+\left(\dot w_b+\dot w_s\right)\left(\delta\dot w_b+\delta\dot w_s\right)\right]-I_1\left(\dot u_0\frac{\partial\delta\dot w_b}{\partial x}+\frac{\partial\dot w_b}{\partial x}\delta\,\dot u_0\right)\\&\quad-J_1\left(\dot u_0\frac{\partial\delta\,\dot w_s}{\partial x}+\frac{\partial\dot w_s}{\partial x}\delta\,\dot u_0\right)+I_2\left(\frac{\partial\dot w_b}{\partial x}\frac{\partial\delta\,\dot w_b}{\partial x}\right)+K_2\left(\frac{\partial\dot w_s}{\partial x}\frac{\partial\delta\,\dot w_s}{\partial x}\right)\\&\quad+J_2\left(\frac{\partial\dot w_b}{\partial x}\frac{\partial\delta\,\dot w_s}{\partial x}+\frac{\partial\dot w_s}{\partial x}\frac{\partial\delta\,\dot w_b}{\partial x}\right)\Big\}dA\end{aligned} \tag{18}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias defined as

$$\left(I_0,I_1,J_1,I_2,J_2,K_2\right)=\int_{-h/2}^{h/2}\left(1,z,f,z^2,z\,f,f^2\right)\rho(z)dz \tag{19}$$

Substituting the expressions for δU and δK from Eqs. (16) and (18) into Eq. (15) and integrating by parts, and collecting the coefficients of δu_0, δw_b and δw_s, the following equations of motion of the beam are obtained

$$\begin{aligned}&\delta\,u_0:\ \frac{\partial N_x}{\partial x}=I_0\ddot u_0-I_1\frac{\partial\ddot w_b}{\partial x}-J_1\frac{\partial\ddot w_s}{\partial x}\\&\delta\,w_b:\ \frac{\partial^2M_x^b}{\partial x^2}=I_0\left(\ddot w_b+\ddot w_s\right)+I_1\frac{\partial\ddot u_0}{\partial x}-I_2\nabla^2\ddot w_b-J_2\nabla^2\ddot w_s\\&\delta\,w_s:\ \frac{\partial^2M_x^s}{\partial x^2}+\frac{\partial S_{xz}^s}{\partial x}=I_0\left(\ddot w_b+\ddot w_s\right)+J_1\frac{\partial\ddot u_0}{\partial x}-J_2\nabla^2\ddot w_b-K_2\nabla^2\ddot w_s\end{aligned} \tag{20}$$

By substituting Eq. (10) into Eq. (13) and the subsequent results into Eq. (17), the stress resultants are obtained as

$$\begin{Bmatrix}N_x\\M_x^b\\M_x^s\end{Bmatrix}=\begin{bmatrix}A_{11}&B_{11}&B_{11}^s\\B_{11}&D_{11}&D_{11}^s\\B_{11}^s&D_{11}^s&H_{11}^s\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\k_x^b\\k_x^s\end{Bmatrix} \tag{21}$$

and

$$S_{xz}^s=A_{55}^s\gamma_{xz} \tag{22}$$

where A_{11}, B_{11}, D_{11}, B^s_{11}, D^s_{11}, H^s_{11} are the plate stiffness, defined by

$$\left\{A_{11}\,B_{11}\,D_{11}\,B_{11}^s\,D_{11}^s\,H_{11}^s\right\}=\int_{-h/2}^{h/2}Q_{11}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)dz \tag{23a}$$

and

$$A_{55}^s=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\nu)}\left[g(z)\right]^2dz \tag{23b}$$

By substituting Eq. (21) into Eq. (20), the governing equations can be expressed in terms of displacements (u_0, w_b and w_s) as

$$A_{11}\frac{\partial^2u_0}{\partial x^2}-B_{11}\frac{\partial^3w_b}{\partial x^3}-B_{11}^s\frac{\partial^3w_s}{\partial x^3}=I_0\ddot u_0-I_1\frac{\partial\ddot w_b}{\partial x}-J_1\frac{\partial\ddot w_s}{\partial x} \tag{24a}$$

$$\begin{aligned}&B_{11}\frac{\partial^3u_0}{\partial x^3}-D_{11}\frac{\partial^4w_b}{\partial x^4}-D_{11}^s\frac{\partial^4w_s}{\partial x^4}=\\&I_0\left(\ddot w_b+\ddot w_s\right)+I_1\frac{\partial\ddot u_0}{\partial x}-I_2\nabla^2\ddot w_b-J_2\nabla^2\ddot w_s,\end{aligned} \tag{24b}$$

$$B_{11}^s\frac{\partial^3u_0}{\partial x^3}-D_{11}^s\frac{\partial^4w_b}{\partial x^4}-H_{11}^s\frac{\partial^4w_s}{\partial x^4}+A_{55}^s\frac{\partial^2w_s}{\partial x^2}=$$

$$I_0\left(\ddot{w}_b+\ddot{w}_s\right)+J_1\frac{\partial \ddot{u}_0}{\partial x}-J_2\nabla^2\ddot{w}_b-K_2\nabla^2\ddot{w}_s \tag{24c}$$

4. Dispersion relations

We assume solutions for u_0, w_b and w_s representing propagating waves in the x direction with the form

$$\begin{Bmatrix} u_0(x,t) \\ w_b(x,y,t) \\ w_s(x,y,t) \end{Bmatrix}=\begin{Bmatrix} U\exp[i(kx-\omega t)] \\ W_b\exp[i(kx-\omega t)] \\ W_s\exp[i(kx-\omega t)] \end{Bmatrix} \tag{25}$$

where U; W_b and W_s are the coefficients of the wave amplitude, k is the wave number of wave propagation along x-axis direction , ω is the frequency.

Substituting Eq. (25) into Eq. (24), we obtain

$$\left([K]-\omega^2[M]\right)\{\Delta\}=\{0\} \tag{26}$$

Where

$$\{\Delta\}=\{U,W_b,W_s\}^T, \tag{27a}$$

$$[K]=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix},\quad [M]=\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} \tag{27b}$$

in which

$$\begin{gathered} a_{11}=-A_{11}k^2 \\ a_{12}=i\,k^3\,B_{11} \\ a_{21}=-i\,k^3\,B_{11} \\ a_{13}=i\,B_{11}^s\,k^3 \\ a_{31}=-i\,B_{11}^s\,k^3 \\ a_{22}=-D_{11}\,k^4 \\ a_{23}=-D_{11}^s\,k^4 \\ a_{33}=-\left(H_{11}^s k_1^4+A_{55}^s k_1^2\right) \\ m_{11}=-I_0 \\ m_{12}=i\,I_1\,k\ ,\ \ m_{21}=-i\,I_1\,k \\ m_{13}=i\,J_1\,k\ ,\ \ m_{31}=-i\,J_1\,k \\ m_{23}=-I_0-J_2\,k^2=m_{32} \\ m_{33}=-I_0-K_2\,k^2,\ \ m_{22}=-I_0-I_2\,k^2 \end{gathered} \tag{27c}$$

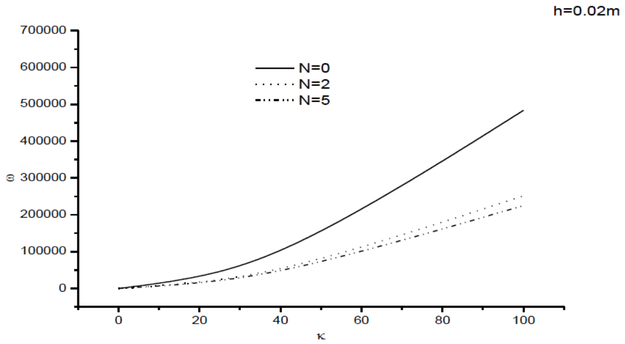
The dispersion relations of wave propagation in the functionally graded beam are given by

$$\left|\,[K]-\omega^2[M]\,\right|=0 \tag{28}$$

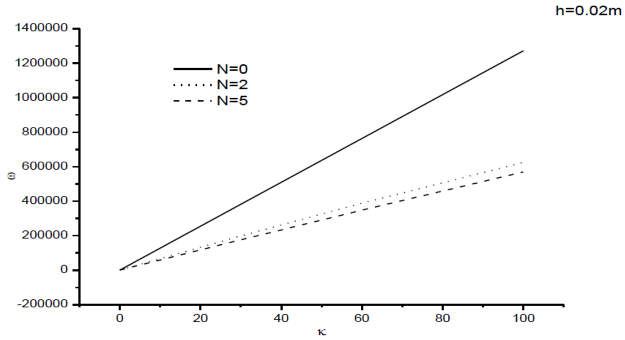
The roots of Eq. (26) can be expressed as

$$\omega_1=W_1(k),\ \ \omega_2=W_2(k)\ \text{ and }\ \omega_3=W_3(k) \tag{29}$$

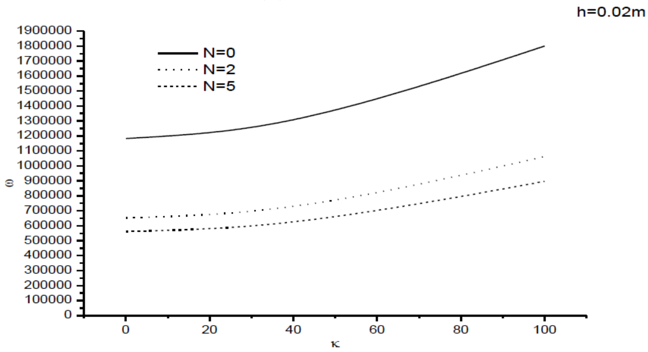
They correspond to the wave modes M_0, M_1 and M_2



(a) M_0 Mode



(b) M_1 Mode



(c) M_2 Mode

Fig. 2 The dispersion curves of the different perfect functionally graded beams

respectively. The wave modes M_0 and M_2 correspond to the flexural wave, the wave mode M_1 corresponds to the extensional wave.

The phase velocity of wave propagation in the functionally graded plate can be expressed as

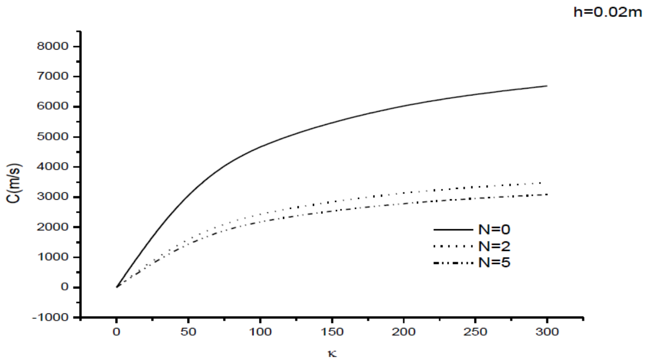
$$C_i=\frac{W_i(k)}{k},\quad (i=1,2,3) \tag{30}$$

The group velocity of wave propagation in the functionally graded beam can be expressed as

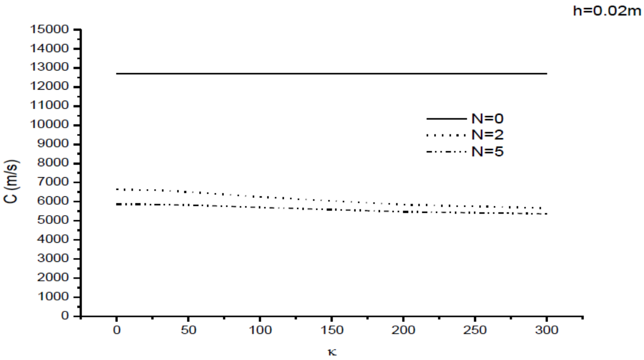
$$C_{g_i}=\frac{\partial W_i(k)}{\partial k},\quad (i=1,2,3) \tag{31}$$

5. Numerical results and discussion

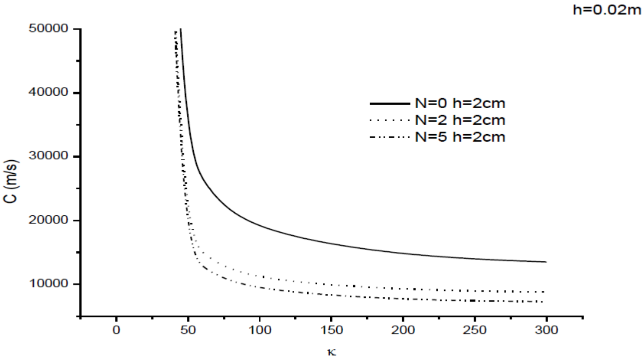
In this section, a FG beam made from Si_3N_4/SUS304, whose material properties are: E=348.43 GPa, ρ=2370



(a) M_0 Mode

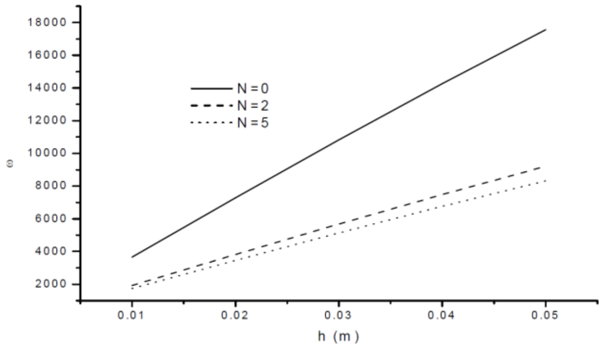


(b) M_1 Mode

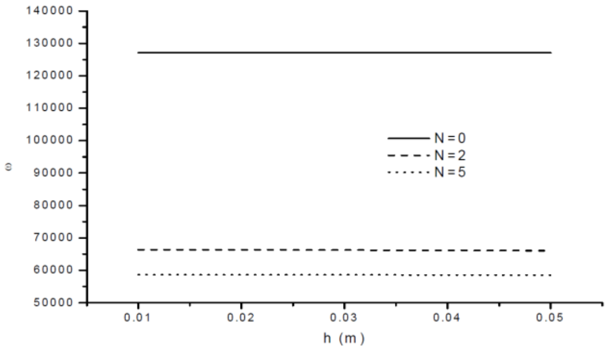


(c) M_2 Mode

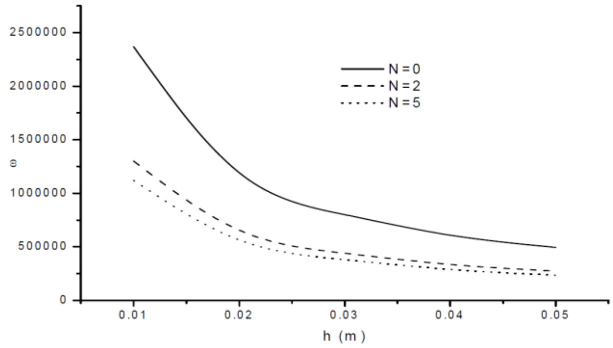
Fig. 3 The phase velocity curves of the different perfect functionally graded beams



(a) M_0 Mode



(b) M_1 Mode



(c) M_2 Mode

Fig. 4 Influence of the depth of the perfect beam on the frequency

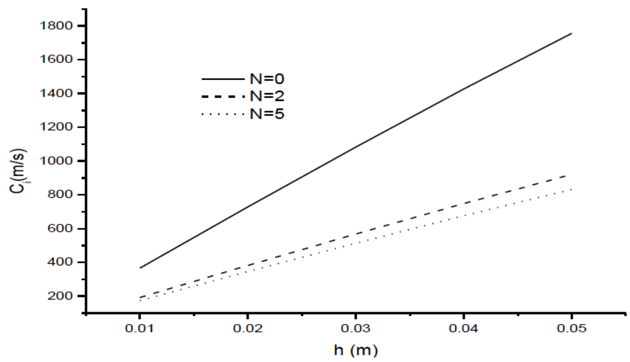
kg/m^3, ν=0.3 for Si_3N_4 and E= 201.04 GPa, ρ= 8166 kg/m^3, ν=0.3 for SUS304, are chosen. The depth of the FG beam is 0.02 m. The analysis based on the present TSDT is carried out using MAPLE.

5.1 Flawless FG beam

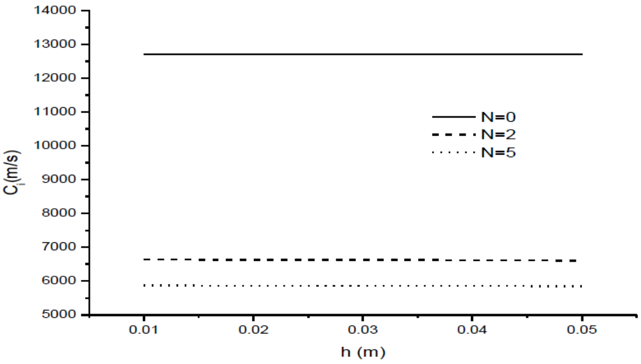
Type A: Perfect beam

First, FG beams without porosity are considered. Figs. 2 and 3 present respectively the dispersion curves and the phase velocity curves of the different perfect functionally graded beams. These curves have been obtained using the 3rd order shear deformation theory. From the dispersion curves for all the wave modes (M_0, M_1 ,and M_2), presented in Fig. 2, the higher the dispersion parameter is, the higher the frequency of the waves propagation in the perfect functionally graded beams is, whatever is the power law index. However, the increase of the power law index leads to a decrease of the frequency. As a consequence, the maximal frequency is obtained for a full ceramic beam (N=0). Fig. 3 presents the phase velocity curves of the different perfect functionally graded beams obtained using a 3rd order shear deformation beam theory for different values of the power-law index n. It can be concluded from this curve, that the phase velocity decreases when the power law index increases for the same wave number k. The phase velocity for the extensional mode M_1 of the isotropic beam (N=0) is constant contrary to the one of the non homogenous beam (N≠0). It can also be concluded that the phase velocity is maximal for the full ceramic beam (N=0).

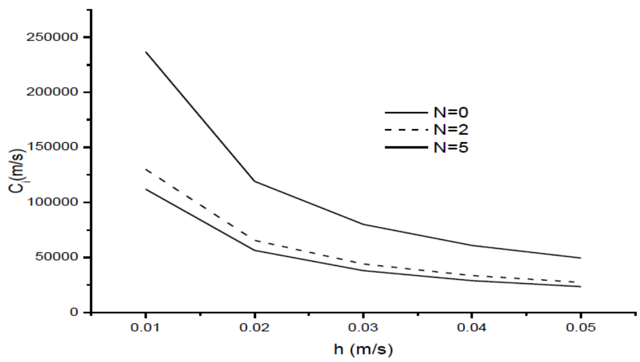
Figs. 4-5 present respectively the influence of the dispersion and the phase velocity in the perfect FG beam in function of the depth, using a third order shear deformation theory. The wave number is here taken equal to k=10. From these figures, the similarities in the dispersion and phase velocity evolutions can be put into evidence.



(a) M_0 Mode



(b) M_1 Mode



(c) M_2 Mode

Fig. 5 Influence of the depth of the perfect beam on the phase velocity

For the M_0 mode, the increase in the beam depth leads to an increase of the frequency as well as the phase velocity.

For the M_1 mode, the increase in the beam depth has no influence on the frequency and the phase velocity.

On the contrary, for the M_2 mode, the increase in the beam depth leads to a decrease of both the frequency and the phase velocity.

Type B: Imperfect beam

This part s devoted to the investigation of wave propagation in porous FG beam functionally using TSDT theory.

Two types of porosity distributions (even and uneven) across the beam thickness are considered here according to Eqs. (4) and (5)or Eqs. (6) and (7).

In Figs. 6 and 7, we present a comparison between two solutions of porosity by plotting the variation of frequency

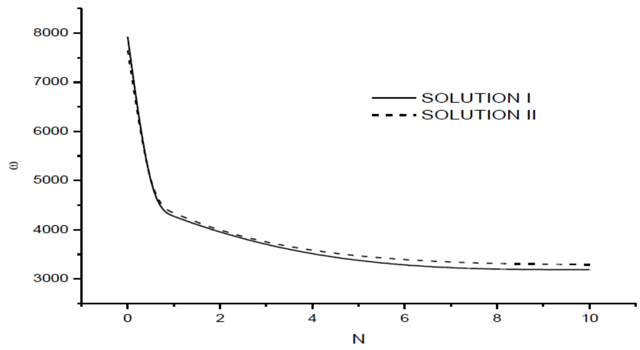


Fig. 6 Influence of low parameter material of the imperfect beam on the frequency

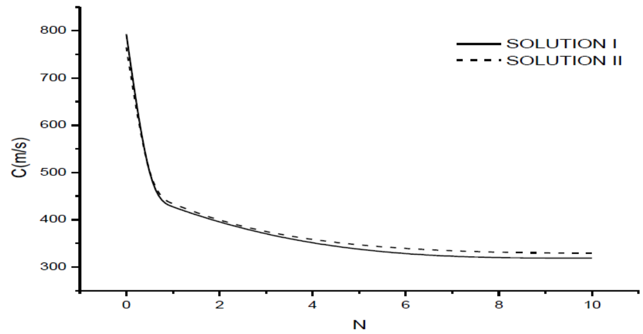
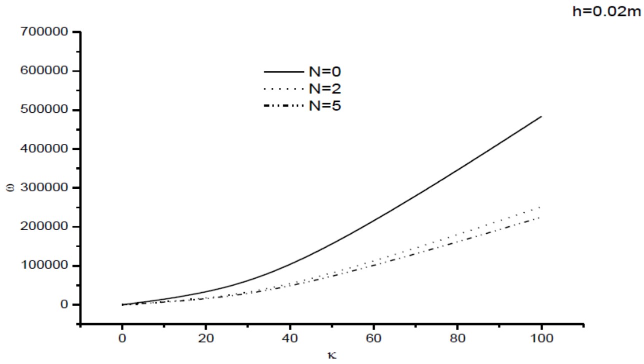


Fig. 7 Influence of low parameter material of the imperfect beam on the phase velocity

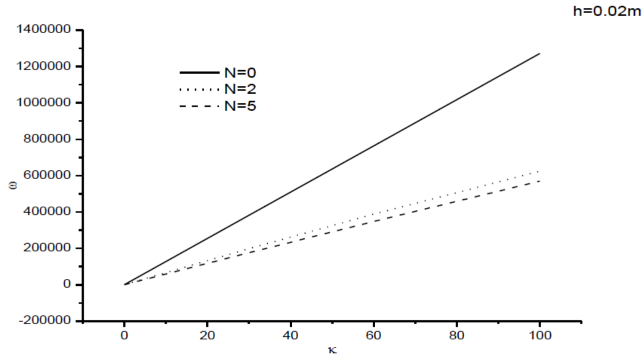
and phase velocities, versus material parameter *n*. The porosity coefficient is taken α=0.1, whereas the wave coefficient is k=10 rad/m and the thickness of the beam is h=0.01 m. It can be seen that the vibration characteristics (frequency or phase velocity) decrease with the increase of the power law index. The two solutions of porosity provide almost the same results with a slight difference in favor of the first solution described by Eqs. (4) and (5). It can be noticed from Figs. 6 and 7 that the difference between solution I and solution II is very small. This is due to distributions of porosity across thickness. Indeed the linear distribution of porosity (solution II) and constant distribution (solution I) of porosity are not considerably different to induce a high different in results.

Figs. 8 and 9 show the dispersion curves of the frequency and the phase velocity, respectively, as function of the wave number. Three values of porosity parameter are considered. The value of the power law index is taken N=2 and the deep of the beam is 0,002 m. Since the shapes of the two solutions of porosity are identical we consider only the first solution.

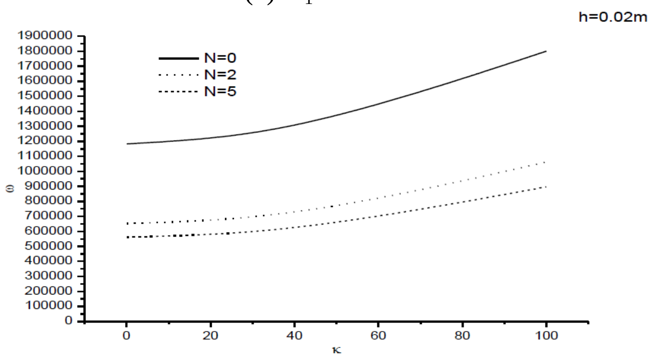
It can be seen that the porosity has a considerable effect on the frequency of the wave propagation in the FG beam for large wave numbers (k), especially for the extensional wave mode M_1. Indeed, the frequency and phase velocities are reduced when the porosity increases. It can be noticed also from Fig. 8 that at low wave numbers, the frequencies do not differ much in the case of modes M_0 and M_1. This is due to the fact that these two modes are related to the flexural and extensional wave. However, for mode M_2 at intermediate wave numbers the frequencies seem less



(a) M_0 Mode

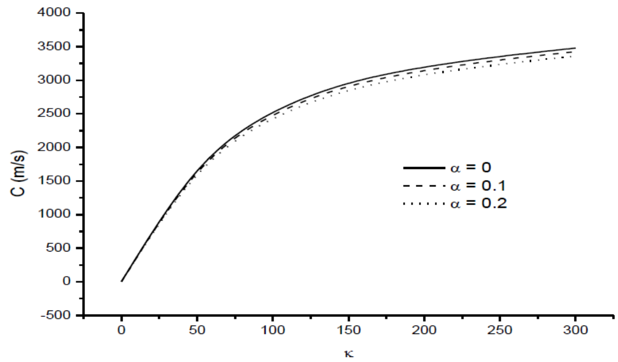


(b) M_1 Mode

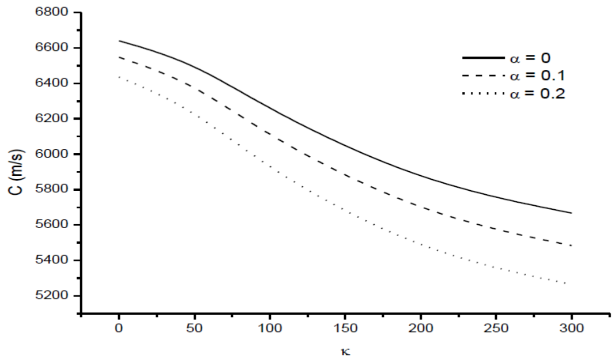


(c) M_2 Mode

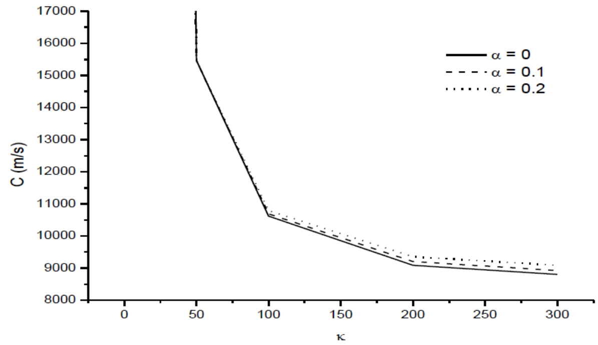
Fig. 8 Influence of porosity parameter on the frequency of imperfect FG beams for various wave numbers



(a) M_0 Mode



(b) M_1 Mode



(c) M_2 Mode

Fig. 9 Influence of porosity parameter on the phase velocity of imperfect FG beams for various wave numbers

influenced by porosity, because this mode is related to flexural wave and it contributed also to the shear mode (W_s). With the exception of the flexural wave mode M_2, the phase velocities are proportional to the porosity.

It can be concluded also that the influence of the porosity on the phase velocity is not considerable for the smaller wave number k, but this influence becomes significant with increasing the wave number.

6. Conclusions

In this research, the wave propagation of the FG porous beams with two porosity distributions is investigated using an efficient shear deformation theory. Material's properties are considered to be varied in the thickness direction based on modified rule of mixture. The governing equations of the wave propagation in the porous FG beam are derived within the framework of third-order shear deformation beam theory and by employing Hamilton's principle. The analytic dispersion relation of the porous FG beam is obtained by solving an eigenvalue problem. From the current work, it can be stated that the effect of the volume fraction distributions and porosity volume index on wave propagation in the FG beam is significant.

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