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On Propagation of Love waves in dry sandy medium sandwiched between fiber-reinforced layer and prestressed porous half-space

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Abstract. The intent of this paper is to investigate the propagation of Love waves in a dry sandy medium sandwiched between fiber-reinforced layer and prestressed porous half-space. Separate displacement components have been deduced in order to characterize the dynamics of individual materials. Using suitable boundary conditions, the frequency equation has been derived by means of separation of variables which reveals the significant role of reinforcement parameters, sandiness, thickness of layers, porosity and prestress on the wave propagation. The phase velocity of the Love wave has been discussed in accordance with its typical cases. In both cases when fiber-reinforced and dry sandy media are absent, the derived equation of Love type wave coincides with the classical Love wave equation. Numerical computations have been performed in order to graphically illustrate the dependencies of different parameters on phase velocity of Love waves. It is observed that the phase velocity decreases with the increase of parameters pertaining to reinforcement and prestress. The results have certain potential applications in earthquake seismology and civil engineering.

Keywords: Love wave; fiber-reinforced medium; dry sandy medium; prestress; phase velocity; dispersion equation

1. Introduction

The study of elastic waves finds wider applications in seismology, techtonophysics and civil engineering. Seismic waves are investigated in order to find the detailed information related to the internal structure of earth and are used in the explorations of substances such as water, oil and mineral deposits. The mathematical study is needed to enable us to gain heuristic understanding of various wave phenomena.

Earth is considered as the most complex structured elastic body and contains various materials and rocks with interesting properties such as fiber-reinforcement, anisotropy, heterogeneity, sandiness, porosity etc. The knowledge pertinent to the properties of waves in multi-layered earth's crust plays its role to understand and predict some characteristics in continental margins, mountain roots etc. Love waves are the seismic waves that causes the horizontal shifting of earth during the earthquake. A wide range of basic literatures can be found from Love (1927), Biot (1965), Gubbins (1990), Bullen (1963) and Achenbach (1973). The motion of the particle of Love wave forms a horizontal line perpendicular to the direction of propagation. It propagates transversely along surface which make us feel directly during earthquake.

The main idea of introducing a continuous self-reinforcement at every point of elastic solid was first given by Belfield *et al.* (1983). A reinforced concrete layer can be formed for all conditions of stresses that may occur in accordance with principle of mechanics.

The distinguishable property of reinforced concrete layer is that its components (steel & concrete) act together as a single body as long as they remain in the elastic state, i.e., two constituents are bonded together so that there can be no relative displacement between them. The fiber-reinforced bodies are like a unit of composite materials in which the polymer fibers are reinforced by the polymer fibers composed by same fiber. Acquiring the characteristics of high internal damping, fiber-reinforced materials deter noise transmission to neighbouring structures by absorbing the vibrational energy. Sengupta and Nath (2001) investigated the surface waves in anisotropic fiber-reinforced elastic solid media. Pradhan *et al.* (2003) studied the dispersion of Love waves in a self-reinforced layer over an elastic non-homogeneous half-space. Singh and Singh (2004) discussed the propagation of plane waves in fiber-reinforced elastic media. Chattopadhyay and Singh (2012), Chaudhary *et al.* (2005) demonstrated the behavoiur shear waves in self-reinforced media. Recently Kundu *et al.* (2014b) have shown the effect of Love waves in fiber-reinforced over nonhomogeneous half-space. Tomar and Singh (2006) analyzed the effect of corrugation between two dissimilar self-reinforced half-spaces on propagation of plane SH-waves. In recent works Kakar (2015, 2016) has elucidated the SH-wave propagation on heterogeneous and anisotropic fiber-reinforced media.

Khurana and Vasisth (2001) discussed the propagation of Love waves in an elastic sandy layer over a fluid-saturated porous solid half-space taking both the layers as prestressed medium. Tomar and Kaur (2007) exhibited the

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reflection and transmission of a plane SH-wave incident at corrugated interface between a dry-sandy half-space and an anisotropic elastic half-space. Son and Kang (2012) attempted a study on shear waves in a poroelastic sandwiched between two elastic media. In the recent study, Pal and Ghorai (2015) have shown the Love wave dispersion in prestressed sandy layer above anisotropic porous half-space under gravity.

Porous materials are often found beneath the surface of the earth in form of sandstone, limestone and other sediments permeated by groundwater or oil. A porous media is nothing but an assemblage of solid particle and pore space. A porous medium is composed by solid skeleton and pore space, air, fluid or both. The pore space remains connected. The solid skeleton consists of solid matrix and empty connected pore space. There are some reasonable ground for considering that anisotropy prevails in the continents. An obvious instance is that of the materials deposited in water. Anisotropy in the earth’s crust and upper mantle have significant effects on the propagation of Love wave. The dynamical characteristics of structured porous media is of great concern. The governing equations of motion for liquids saturated porous solids are formulated by Biot (1965). Ghorai *et al.* (2010), Ke *et al.* (2006) laid an emphasis on the surface wave propagations in fluid-saturated porous medium under certain factors. Boxberg *et al.* (2015) manifested the necessary equations to calculate the wave speeds for unsaturated porous media and tested the equations for a representative storage scenario. Kielczyn’ski *et al.* (2012) focused on the study regarding effect of viscous liquid loading on Love wave propagation. Kakar and Kakar (2015a) have well documented the torsional wave transmission on prestressed inhomogeneous medium. Chen *et al.* (2012) framed how transverse waves transmit through a plane interface between isotropic elastic and unsaturated porous elastic solid half-spaces.

Duly motivated by several evidences described which the geophysicist and civil engineers encounter during analysing the seismic waves, the authors incorporated the problem of propagation of Love wave where the crustal layer constituted with fiber-reinforced and dry sandy layer with semi-infinite prestressed porous substrate.

2. Formulation of the problem

We have considered a fiber-reinforced layer of h_1 thickness lying over a dry sandy layer of finite thickness h_2 and a prestressed porous half-space.The free surface of the fiber-reinforced layer is considered to be traction free. The z-axis is taken vertically downwards in the porous half-space. The x-axis is taken along the direction in which the wave propagates.

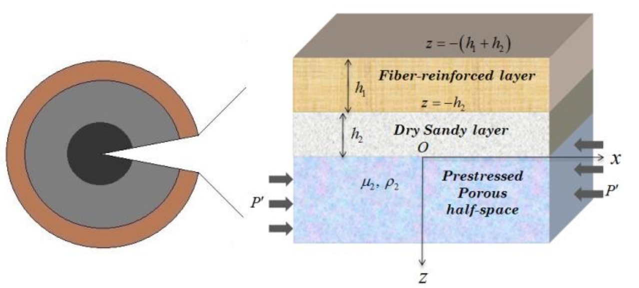

Fig. 1 Geometry of the problem

3. Dynamics of fiber-reinforced medium

The governing equation which represents a fiber-reinforced elastic body along the preferred direction $\vec{a}$ is given by Spencer (1972)

$$\begin{aligned}\tau_{ij} &= \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + \\ &2(\mu_L-\mu_T)\times(a_i a_k e_{kj} + a_j a_k e_{ki}) + \\ &\beta(a_k a_m e_{km} a_i a_j);\ i,j,k,m = 1,2,3\end{aligned} \tag{1}$$

where σ_{ij} are stress components and $e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ are infinitesimal strain components. τ_{ij} are components of stress vector, a_i are components of $\vec{a}$ and may be a function of position. Indices take the values 1,2,3 and the convention of summation is applied. $(\mu_L-\mu_T)$ is also the reinforced anisotropic elastic parameters with dimension of stress. u_i $(i=1,2,3)$ are the displacement vector components of $\vec{a}$ with respect to the all referred to rectangular Cartesian coordinate system such that $a_1^2+a_2^2+a_3^2=1$. Also α and β are the specific stress components to take into account different layers for concrete part of the composite materials.

In absence of body forces the eqn. of motion for a fiber-reinforced layer is given by

$$\frac{\partial\tau_{12}}{\partial x}+\frac{\partial\tau_{22}}{\partial y}+\frac{\partial\tau_{23}}{\partial z}=\rho\frac{\partial^2 u}{\partial t^2} \tag{2}$$

where ρ is material density of the fiber-reinforced layer. Taking $u_1\equiv 0$, $u_2\equiv u_2(x,z,t)$, $u_3\equiv 0$, we have

$$\begin{aligned}\tau_{12} &= \mu_T\left(P\frac{\partial u_2}{\partial x}+Q\frac{\partial u_2}{\partial z}\right),\quad \tau_{22}=0,\\ \tau_{23} &= \mu_T\left(P\frac{\partial u_2}{\partial x}+R\frac{\partial u_2}{\partial z}\right)\end{aligned} \tag{3}$$

where

$$\begin{aligned}P &= 1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^{\,2},\ Q=\left(\frac{\mu_L}{\mu_T}-1\right)a_1 a_3,\\ R &= 1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^{\,2}\end{aligned}$$

Using relations (3), the Eq. (2) becomes

$$P\frac{\partial^2 u_2}{\partial x^2}+2Q\frac{\partial^2 u_2}{\partial x\partial z}+R\frac{\partial^2 u_2}{\partial z^2}=\frac{\rho}{\mu_T}\frac{\partial^2 u_2}{\partial t^2} \tag{4}$$

We assume the harmonic solution of (4) as

$$u_2(x,z,t)=\phi(z)e^{ik(x-ct)} \tag{5}$$

where k is the wave number and c is the phase velocity. On substituting the relation (5) in Eq. (4) we get

$$R\frac{d^2\phi}{dz^2}+2ikQ\frac{d\phi}{dz}+k^2\left(\frac{c^2}{\beta_0{}^2}-P\right)\phi=0 \tag{6}$$

where $\omega=kc$ & $\beta_0{}^2=\sqrt{\frac{\mu_T}{\rho}}$ is the shear wave velocity of the fiber-reinforced layer. The solution of the (6) can be expressed as

$$\phi(z)=\left(Ae^{-ikm_1z}+Be^{-ikm_2z}\right) \tag{7}$$

where

$$m_1=\frac{1}{R}\left[Q+\sqrt{Q^2+R\left(\frac{c^2}{\beta_0{}^2}-P\right)}\right]\ \&$$
$$m_2=\frac{1}{R}\left[Q-\sqrt{Q^2+R\left(\frac{c^2}{\beta_0{}^2}-P\right)}\right]$$

and $\frac{c}{\beta_0}$ denotes the phase velocity ratio.

From (5) & (7), the displacement equation for the fiber-reinforced layer is given by

$$u_2=v_0(x,z,t)=\left(Ae^{-ikm_1z}+Be^{-ikm_2z}\right)e^{ik(x-ct)} \tag{8}$$

where A, B are arbitrary constants.

4. Solution for the dry sandy layer

Neglecting the body forces, the equation of motion for the Love wave propagating in a dry sandy elastic medium is given by Tomar and Kaur (2007) as

$$\frac{\mu_1}{\eta}\left[\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 v_1}{\partial z^2}\right]=\rho_1\frac{\partial^2 v_1}{\partial t^2} \tag{9}$$

where v_1 is the y-component of the displacement vector μ_1, ρ_1, η are the rigidity, density and the sandiness of the medium. Precisely when $\eta>1$ the layer corresponds to the sandy materials &$\eta=1$ corresponds to isotropic elastic solid. Considering $\beta_1=\sqrt{\frac{\mu_1}{\eta\rho_1}}$ as the shear wave velocity the Eq. (9) takes the form

$$\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 v_1}{\partial z^2}=\frac{1}{\beta_1^2}\frac{\partial^2 v_1}{\partial t^2} \tag{10}$$

Using the method of separation of variables and taking $v_1=V_1(z)e^{ik(x-ct)}$ the Eq. (10) becomes

$$\frac{d^2V_1}{dz^2}+s_1{}^2V_1=0 \tag{11}$$

where $s_1=k\sqrt{\frac{c^2}{\beta_1^2}-1}$

The solution of Eq. (10) can be written as

$$v_1=V_1(z)e^{ik(x-ct)}=\left(C\cos s_1z+D\sin s_1z\right)e^{ik(x-ct)} \tag{12}$$

where $s_1=k\sqrt{\frac{c^2}{\beta_1^2}-1}$ & $\beta_1=\sqrt{\frac{\mu_1}{\eta\rho_1}}$

5. Dynamics for prestressed porous substratum

We consider a prestressed anisotropic porous half-space. Let (u_2,v_2,w_2) are the components of the displacement vector of the solid and U_x,V_y,W_z are the same of the liquid part of the porous material in the direction of x, y, z-respectively.

Neglecting the viscosity of water the dynamic equations of motion in a porous layer under the compressive initial stress P', i.e., in the absence of body forces can be written as Biot (1965)

$$\begin{aligned}
&\frac{\partial\tau_{11}}{\partial x}+\frac{\partial\tau_{12}}{\partial y}+\frac{\partial\tau_{13}}{\partial z}-P'\left(\frac{\partial w_z{}'}{\partial y}-\frac{\partial w_y{}'}{\partial z}\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{11}u_2+\rho_{12}U_x\right)\\
&\frac{\partial\tau_{21}}{\partial x}+\frac{\partial\tau_{22}}{\partial y}+\frac{\partial\tau_{23}}{\partial z}-P'\left(\frac{\partial w_z{}'}{\partial x}\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{11}v_2+\rho_{12}V_y\right)\\
&\frac{\partial\tau_{31}}{\partial x}+\frac{\partial\tau_{32}}{\partial y}+\frac{\partial\tau_{33}}{\partial z}-P'\left(\frac{\partial w_y{}'}{\partial x}\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{11}w_2+\rho_{12}W_z\right)
\end{aligned} \tag{13}$$

and

$$\frac{\partial\tau'}{\partial x}=\frac{\partial^2}{\partial t^2}\left(\rho_{12}u_2+\rho_{22}U_x\right),\quad \frac{\partial\tau'}{\partial y}=\frac{\partial^2}{\partial t^2}\left(\rho_{12}v_2+\rho_{22}V_y\right)\ \&\quad \frac{\partial\tau'}{\partial z}=\frac{\partial^2}{\partial t^2}\left(\rho_{12}w_2+\rho_{22}W_z\right)$$

where τ_{ij} (i, j=1,2,3) are the incremental stress components of solid & τ' is the stress vector for liquid part of the porous material.

The stress vector τ' is related to the fluid pressure p by the relation $-\tau'=fp$, where f is the porosity of the media.

The angular components $w_x{}'$, $w_y{}'$ & $w_z{}'$ are defined as

$$\begin{aligned}
&w_x{}'=\frac{1}{2}\left(\frac{\partial w_2}{\partial y}-\frac{\partial v_2}{\partial z}\right),\quad w_y{}'=\frac{1}{2}\left(\frac{\partial u_2}{\partial z}-\frac{\partial w_2}{\partial x}\right),\\
&w_z{}'=\frac{1}{2}\left(\frac{\partial v_2}{\partial x}-\frac{\partial u_2}{\partial y}\right)
\end{aligned} \tag{14}$$

The mass coefficient ρ_{11}, ρ_{12}, ρ_{22} are related to the density ρ', ρ_s & ρ_w of the layer, solid & liquid respectively

$$\rho_{11}+\rho_{12}=(1-f)\rho_s,\quad \rho_{11}+\rho_{22}=f\rho_w \tag{15}$$

The mass density of the bulk material is

$$\rho'=\rho_{11}+2\rho_{12}+\rho_{22}=\rho_s+f(\rho_w-\rho_s) \tag{16}$$

These mass coefficients also obey the following inequalities

$$\rho_{11}>0,\rho_{22}>0,\rho_{12}<0,\rho_{11}\rho_{22}-\rho_{12}{}^2>0 \tag{17}$$

The stress-strain relations for the water saturated anisotropic porous layer under the normal initial stress P' are

$$\begin{aligned}\tau_{11}&=(A+P')e_{xx}+(A-2N+P')e_{yy}+\\&\quad(F+P')e_{zz}+Q\varepsilon\\\tau_{22}&=(A-2N)e_{xx}+Ae_{yy}+Fe_{zz}+Q\varepsilon\\\tau_{33}&=Fe_{xx}+Fe_{yy}+Ce_{zz}+Q\varepsilon\\\tau_{12}&=2Ne_{xy},\quad \tau_{23}=2Le_{yz},\quad \tau_{13}=2Le_{zx}\end{aligned} \tag{18}$$

where A, F, C, N & L are elastic constants for the medium, in particular N&L are the shear moduli of the anisotropic layer in x&z- directions respectively

$$e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\quad \varepsilon=\left(\frac{\partial U_x}{\partial x}+\frac{\partial U_y}{\partial y}+\frac{\partial U_z}{\partial z}\right) \tag{19}$$

The positive quantity Q is the measure of coupling between the change of volume of solid and liquid. The dynamical Eq. (13) have been constructed by coupling the Biot's dynamics for a poro-elastic medium.

For the propagation of the Love waves, we know that the direction of the displacement of the particle is parallel to the plane of propagation.

The displacement along the x- axis &z-axis vanishes as well as the rate of change along y-axis remains absent.

i.e., we have

$$\left.\begin{aligned}u_2=0,\ w_2=0,\ v_2=v_2(x,z,t)\\U_x=0,W_z=0,V_y=V(x,z,t)\end{aligned}\right\} \tag{20}$$

These conditions will yield the strain components e_{yz}&e_{xy} only and the others will remain zero. Thus

$$\tau_{23}=2Le_{yz},\tau_{12}=2Ne_{xy} \tag{21}$$

Substituting the relations (21) in Eq. (13), the equations of motion which are not automatically satisfied are

$$\left.\begin{aligned}&\frac{\partial\tau_{21}}{\partial x}+\frac{\partial\tau_{22}}{\partial y}+\frac{\partial\tau_{23}}{\partial z}-P'\left(\frac{\partial w_z'}{\partial x}\right)=\\&\qquad\qquad\frac{\partial^2}{\partial t^2}\left(\rho_{11}v_2+\rho_{12}V_y\right)\\&\frac{\partial\tau'}{\partial y}=0=\frac{\partial^2}{\partial t^2}\left(\rho_{12}v_2+\rho_{22}V_y\right)\end{aligned}\right\} \tag{22}$$

Since $v_2=v_2(x,z,t)$, $V_y=V(x,z,t)$ and using the relations from (20) & (21) the above Eq. (22) transforms into

$$\left(N-\frac{P'}{2}\right)\frac{\partial^2 v_2}{\partial x^2}+L\frac{\partial^2 v_2}{\partial z^2}=\frac{\partial^2}{\partial t^2}\left(\rho_{11}v_2+\rho_{12}V\right) \tag{23}$$

$$\frac{\partial^2}{\partial t^2}\left(\rho_{12}v_2+\rho_{22}V\right)=0 \tag{24}$$

From

$$\frac{\partial^2}{\partial t^2}\left(\rho_{12}v_2+\rho_{22}V\right)=0$$

&

$$\rho_{12}v_2+\rho_{22}V=d'\ \Rightarrow V=\frac{d'-\rho_{12}v_2}{\rho_{22}}$$

Hence the above Eq. (23) can be written as

$$\left(N-\frac{P'}{2}\right)\frac{\partial^2 v_2}{\partial x_2}+L\frac{\partial^2 v_2}{\partial z_2}=d'\frac{\partial^2 v_2}{\partial t_2} \tag{25}$$

From the above Eq. (25), we found that the velocity of the shear wave along x-direction is $\sqrt{\frac{N-\frac{P'}{2}}{d'}}$ & along z-direction is $\sqrt{\frac{L}{d'}}$

The shear wave velocity in the porous medium along the x-direction can be expressed as

$$\beta=\sqrt{\frac{N-\frac{P'}{2}}{d'}}=\beta_2\sqrt{\frac{1-\xi'}{d}} \tag{26}$$

where

$$d=\gamma_{11}-\frac{\gamma_{12}{}^2}{\gamma_{22}},\ \beta_2=\sqrt{\frac{N}{\rho'}}$$

β_2 is the velocity of the shear wave in the corresponding prestress free, non-porous, anisotropic elastic medium at x-direction.

$\xi'=\frac{P'}{2N}$ is the non-dimensional parameter due to the prestress P' &

$$\gamma_{11}=\frac{\rho_{11}}{\rho'},\gamma_{12}=\frac{\rho_{12}}{\rho'},\gamma_{22}=\frac{\rho_{22}}{\rho'} \tag{27}$$

are the non-dimensional parameters for the material of porous layer as obtained by Biot (1965).

Thus, we get the following as

(i) if $d\to 1$ the medium is non-porous solid

(ii) if $d\to 0$ the layer is fluid

(iii) if $0<d<1$ the layer is poro-elastic

For the Love wave propagation along x-direction the solution of Eq. (25) may be taken as

$$v_2\left(x,z,t\right)=V_2\left(z\right)e^{ik(x-ct)} \tag{28}$$

Putting the above value at Eq. (25) we get

$$\frac{d^2V_2}{dz^2}+q'^2k^2V_2=0 \tag{29}$$

where $q'=\sqrt{\frac{1}{L}\left(-c^2d'+N-\frac{P'}{2}\right)}$

Therefore, the solution of Eq. (29) takes of the form

$$V_2\left(z\right)=A_4e^{-q'kz}+B_4e^{q'kz} \tag{30}$$

where A_4&B_4 are arbitrary independent constants. As we are interested in the solution of Eq. (29) which is bounded and vanishes as $z\to\infty$.

So the required solution can taken as

$$v_2\left(z\right)=A_4e^{-q'kz}e^{ik(x-ct)} \tag{31}$$

The above indicates the displacement of a prestressed porous substratum where

$$q'=\sqrt{\frac{c^2d'-\left(N-\frac{P'}{2}\right)}{L}}=\sqrt{\gamma d\left(\frac{1-\xi'}{d}-\frac{c^2}{\beta_2{}^2}\right)}$$

$\gamma=\frac{N}{L},\xi'=\frac{P'}{2N}$, $\beta_2{}^2=\frac{N}{\rho}$ & k is the wave number.

6. Boundary conditions

i) At $z=-(h_1+h_2)$, i.e., the surface of the fiber-reinforced layer is free from any load implies the stress component τ_{23} must be zero, i.e.

$$\mu_2\frac{\partial v_0}{\partial z}=0 \tag{32}$$

ii) The stress components must be continuous at the interface of the media

a) At $z=-h_2$, i.e., along the interface of fiber-reinforced and dry sandy layer

$$\tau_{23}=t_{23} \tag{33}$$

b) At $z=0$, i.e., along the interrface of fiber- reinforced layer and prestressed porous media

$$t_{23}=\sigma_{23} \tag{34}$$

iii) The displacement components must be continuous at the interface of the media

a) At $z=-h_2$, i.e., along the interface of fiber-reinforced and dry sandy layer

$$v_0=v_1 \tag{35}$$

b) At $z=0$, i.e., along the interface of fiber- reinforced layer and prestressed porous media

$$v_1=v_2 \tag{36}$$

Using the boundary condition (i) on the Eq. (8) we get

$$\left(Q-m_1R\right)Ae^{ikm_1\left(h_1+h_2\right)}+\left(Q-m_2R\right)Be^{ikm_2\left(h_1+h_2\right)}=0 \tag{37}$$

Applying the boundary condition [ii(a)] on Eqs. (8) & (12) we get

$$\begin{aligned}&\mu_T\eta A(ik)\left(Q-m_1R\right)e^{ikm_1h_2}+\mu_T\eta B(ik)\times\\&\left(Q-m_2R\right)e^{ikm_2h_2}-\mu_1s_2C\sin\left(s_2h_2\right)-\\&\mu_1s_2D\cos\left(s_2h_2\right)=0\end{aligned} \tag{38}$$

From the boundary condition [ii(b)] and using it on (12) & (31) we get

$$\mu_1s_2D+L\eta A_4kq'=0 \tag{39}$$

Applying the boundary condition [iii(a)] on Eqs. (8) & (12) we get

$$Ae^{ikm_1h_2}+Be^{ikm_2h_2}-C\cos\left(s_2h_2\right)+D\sin\left(s_2h_2\right)=0 \tag{40}$$

From the boundary condition [iii(b)] and using it on (12) & (31) we get

$$C-A_4=0 \tag{41}$$

Eliminating A,B,C,D & A_4 from Eqs. (37) to (41) we get

$$\begin{vmatrix}\left(Q-m_1R\right)e^{ikm_1\left(h_1+h_2\right)} & \left(Q-m_2R\right)e^{ikm_2\left(h_1+h_2\right)} & 0 & 0 & 0\\ \mu_T\eta(ik)\left(Q-m_1R\right)e^{ikm_1h_2} & \mu_T\eta(ik)\left(Q-m_2R\right)e^{ikm_2h_2} & -\mu_1s_2\sin\left(s_2h_2\right) & -\mu_1s_2\cos\left(s_2h_2\right) & 0\\ 0 & 0 & 0 & \mu_1s_2 & L\eta kq'\\ e^{ikm_1h_2} & e^{ikm_2h_2} & -\cos\left(s_2h_2\right) & \sin\left(s_2h_2\right) & 0\\ 0 & 0 & 1 & 0 & -1\end{vmatrix}=0$$

which implies

$$\tan\left(s_2h_2\right)=\frac{\mu_1s_2k\eta\left\{Lq'-T\mu_T\tan\left(kh_1\frac{T}{R}\right)\right\}}{\left\{\mu_1{}^2s_2{}^2+LT\mu_Tk^2\eta^2q'\tan\left(kh_1\frac{T}{R}\right)\right\}}$$

the above equation reduces to

$$\tan\left[kh_2\sqrt{\frac{c^2}{\beta_1{}^2}-1}\right]=\frac{\mu_1\eta\sqrt{\frac{c^2}{\beta_1{}^2}-1}\left\{Lq'-T\mu_T\tan\left(kh_1\frac{T}{R}\right)\right\}}{\left\{\mu_1{}^2\left(\frac{c^2}{\beta_1{}^2}-1\right)+L\eta^2q'T\mu_T\tan\left(kh_1\frac{T}{R}\right)\right\}} \tag{42}$$

which is the dispersion equation of Love type waves.

Where

$$q' = \sqrt{\frac{c^2 d' - \left(N - \frac{P'}{2}\right)}{L}} = \sqrt{\gamma d \left(\frac{1-\xi'}{d} - \frac{c^2}{\beta_2^{\,2}}\right)}$$

and

$$P = 1 + \left(\frac{\mu_L}{\mu_T} - 1\right) a_1^{\,2}, \quad T = \sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}$$

7. Particular cases

7.1 Case I

If we take $a_1 = 1, a_2 = 0, a_3 = 0$ & $\mu_L \to \mu_T \to \mu_0$ which implies $P \to 1, Q \to 0, R \to 1$ then the dispersion Eq. (42) reduces to

$$\tan\left[kh_2\sqrt{\frac{c^2}{\beta_1^{\,2}} - 1}\right] = \frac{\mu_1 \eta \sqrt{\frac{c^2}{\beta_1^{\,2}} - 1}\left\{L\sqrt{\frac{Nd}{L}\left(\frac{1-\xi'}{d} - \frac{c^2}{\beta_2^{\,2}}\right)} - \mu_0 \sqrt{\frac{c^2}{\beta_0^{\,2}} - 1}\tan\left(kh_1\sqrt{\frac{c^2}{\beta_0^{\,2}} - 1}\right)\right\}}{\mu_1^{\,2}\left(\frac{c^2}{\beta_1^{\,2}} - 1\right) + L\eta^2 \mu_0 \sqrt{\frac{Nd}{L}\left(\frac{1-\xi'}{d} - \frac{c^2}{\beta_2^{\,2}}\right)} \times \sqrt{\frac{c^2}{\beta_0^{\,2}} - 1}\tan\left(kh_1\sqrt{\frac{c^2}{\beta_0^{\,2}} - 1}\right)} \tag{43}$$

which is the Love type wave equation in homogeneous layer and free from fiber-reinforcement.

7.2 Case II

If $\eta = 1$ i.e., when the intermediate layer becomes isotropic elastic solid material, the dispersion Eq. (42) of Love type wave becomes

$$\tan\left[kh_2\sqrt{\frac{c^2}{\beta_1^{\,2}} - 1}\right] = \frac{\mu_1 \eta \sqrt{\frac{c^2}{\beta_1^{\,2}} - 1}\left\{L\sqrt{\frac{Nd}{L}\left(\frac{1-\xi'}{d} - \frac{c^2}{\beta_2^{\,2}}\right)} - \sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}\,\mu_T \tan\left(kh_1 \frac{\sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}}{R}\right)\right\}}{\mu_1^{\,2}\left(\frac{c^2}{\beta_1^{\,2}} - 1\right) + L\mu_T \sqrt{\frac{Nd}{L}\left(\frac{1-\xi'}{d} - \frac{c^2}{\beta_2^{\,2}}\right)} \times \sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}\tan\left(kh_1 \frac{\sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}}{R}\right)} \tag{44}$$

7.3 Case III

When d=1&P'=0 i.e., ξ=0 then the Eq. (42) transforms into

$$\tan\left[kh_2\sqrt{\frac{c^2}{\beta_1^{\,2}} - 1}\right] = \frac{\mu_1 \eta \sqrt{\frac{c^2}{\beta_1^{\,2}} - 1}\left\{\sqrt{NL\left(1 - \frac{c^2}{\beta_2^{\,2}}\right)} - \sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)} \times \mu_T \tan\left(kh_1 \frac{\sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}}{R}\right)\right\}}{\mu_1^{\,2}\left(\frac{c^2}{\beta_1^{\,2}} - 1\right) + \eta^2 \sqrt{NL\left(1 - \frac{c^2}{\beta_2^{\,2}}\right)}\,\mu_T \times \sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}\tan\left(kh_1 \frac{\sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}}{R}\right)} \tag{45}$$

which is the dispersion equation of Love type waves in non-porous, prestress free semi-infinite medium.

7.4 Case IV

When $P' = 0$ & $N \to L \to \mu_2$ the dispersion Eq. (42) converts to

$$\tan\left[kh_2\sqrt{\frac{c^2}{\beta_1^{\,2}} - 1}\right] = \frac{\mu_1 \eta \sqrt{\frac{c^2}{\beta_1^{\,2}} - 1}\left\{\mu_2\sqrt{d\left(\frac{1}{d} - \frac{c^2}{\beta_2^{\,2}}\right)} - \sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}\,\mu_T \tan\left(kh_1 \frac{\sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}}{R}\right)\right\}}{\mu_1^{\,2}\left(\frac{c^2}{\beta_1^{\,2}} - 1\right) + \mu_2\eta^2 \sqrt{d\left(\frac{1}{d} - \frac{c^2}{\beta_2^{\,2}}\right)}\,\mu_T \times \sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}\tan\left(kh_1 \frac{\sqrt{Q^2 + R\left(\frac{c^2}{\beta_0^{\,2}} - P\right)}}{R}\right)} \tag{46}$$

which is the dispersion equation of Love type waves in homogeneous elastic layer free from prestress.

7.5 Case V

Combining all the cases,when $\mu_L \to \mu_T \to \mu_0$,

Table 1 Values of various dimensionless parameters for the dispersion Eq. (42)

Fiber-reinforced medium	$\mu_L = 7.07\times10^9\, N/m^2$	$\mu_T = 3.5\times10^9\, N/m^2$	$\rho_0 = 1600 kg/m^3$
Dry Sandy medium	----	$\mu_1 = 6.54\times10^{10}\, N/m^2$	$\rho_1 = 3409 kg/m^3$
Prestressed porous medium	$L = 0.1387\times10^{10}\, N/m^2$ $N = 0.2774\times10^{10}\, N/m^2$	$\rho_{11} = 1.926137\times10^3\, Kg/m^3$ $\rho_{12} = -0.002137\times10^3\, Kg/m^3$ $\rho_{22} = 0.215337\times10^3\, Kg/m^3$	$f = 0.26$

$P\to0, Q\to0, R\to1$ $\eta=1$ & $d=1, \xi=0,$ $N\to L\to\mu_2$ the Eq. (42) transfers into

$$\tan\left[kh_2\sqrt{\frac{c^2}{\beta_1^2}-1}\right] = \frac{\mu_1\sqrt{\frac{c^2}{\beta_1^2}-1}\left\{\mu_2\sqrt{\left(1-\frac{c^2}{\beta_2^2}\right)} - \mu_0\sqrt{\left(\frac{c^2}{\beta_0^2}-1\right)}\tan\left(kh_1\sqrt{\frac{c^2}{\beta_0^2}-1}\right)\right\}}{\mu_1^2\left(\frac{c^2}{\beta_1^2}-1\right) + \mu_2\mu_0\sqrt{\left(1-\frac{c^2}{\beta_2^2}\right)}\sqrt{\frac{c^2}{\beta_0^2}-1}\tan\left(kh_1\sqrt{\frac{c^2}{\beta_0^2}-1}\right)} \tag{47}$$

which is the dispersion equation of Love type wave when the upper layer is homogeneous and free from reinforcement, intermediate layer is isotropic elastic and the lower half-space is homogeneous, non-porous and prestress free.

7.6 Case VI

If we take h_1=0 i.e., in the absence of fiber-reinforced layer the Eq. (42) reduces to

$$\tan\left[kh_2\sqrt{\frac{c^2}{\beta_1^2}-1}\right] = \frac{\mu_2\sqrt{1-\frac{c^2}{\beta_2^2}}}{\mu_1\sqrt{\frac{c^2}{\beta_1^2}-1}} \tag{48}$$

which is the classical equation of Love wave.

7.7 Case VII

When $h_2 = 0$ i.e., in the absence of dry sandy layer the dispersion Eq. (42) reduces to

$$\tan\left[kh_1\sqrt{\frac{c^2}{\beta_0^2}-1}\right] = \frac{\mu_2\sqrt{1-\frac{c^2}{\beta_2^2}}}{\mu_0\sqrt{\frac{c^2}{\beta_0^2}-1}} \tag{49}$$

which is also the classical equation of Love wave.

8. Numerical calculations & discussion

In order to elaborate the effect of fiber-reinforcement, anisotropy, sandiness & porosity on the dispersion of Love waves numerical computations have been introduced with different values of parameters. The following illustrations have been performed by considering the values of reinforcement parameters & density of upper layer from Kundu *et al.* (2014a), rigidity and density of dry sandy layer from Gubbins (1990), whereas the parameters for the porous medium are taken from Chattaraj and Samal (2013).

The effect of parameters have been explored in Figs. 2-6. All of the figures have been plotted with vertical axis as dimensionless phase velocity c/β_1 against horizontal axis as dimensionless wave number *kH*.

Figure 2 explores the effect of reinforcement parameters in the fiber-reinforced medium in presence of compressive initial stress in the substratum. The curves are plotted considering the different values of reinforcement

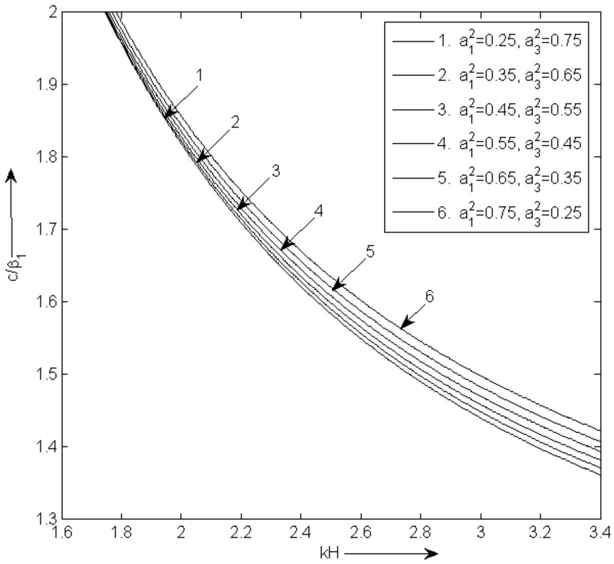

Fig. 2 Dimensionless phase velocity c/β_1 against dimensionless wave number kH for different values of a_1^2 & a_3^2

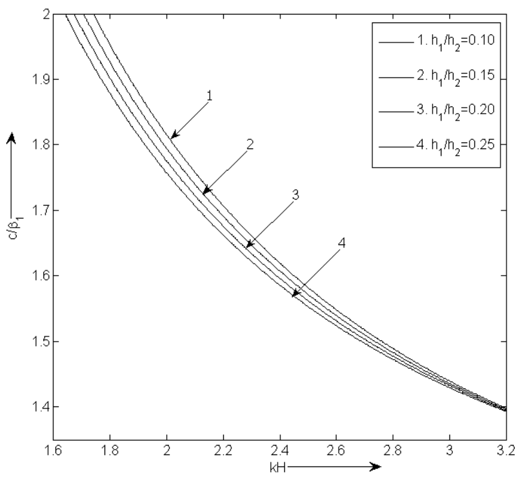

Fig. 3 Dimensionless phase velocity c/β_1 against dimensionless wave number *kH* for different values of h_1/h_2

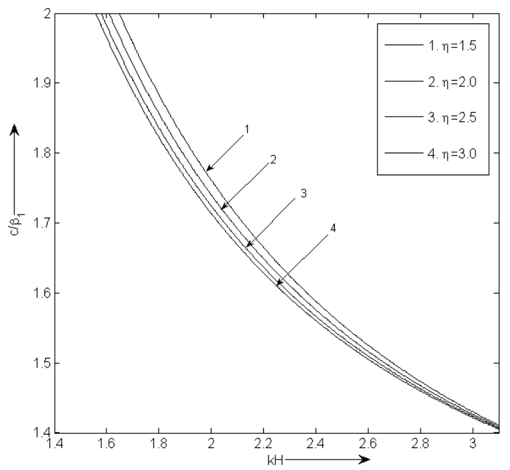

Fig. 4 Dimensionless phase velocity c/β_1 against dimensionless wave number *kH* for different values of η

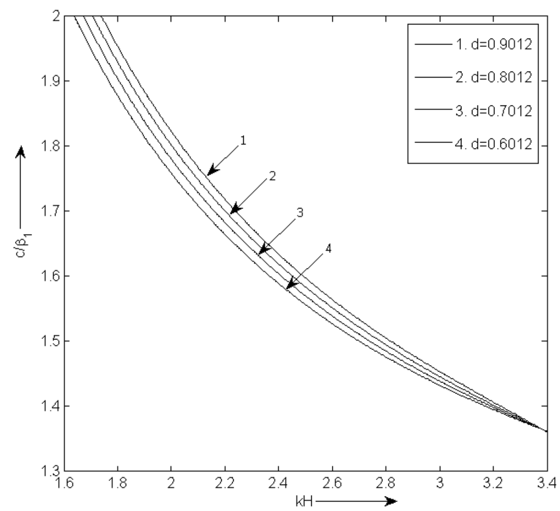

Fig. 5 Dimensionless phase velocity c/β_1 against dimensionless wave number *kH* for different values of porosity parameter *d*

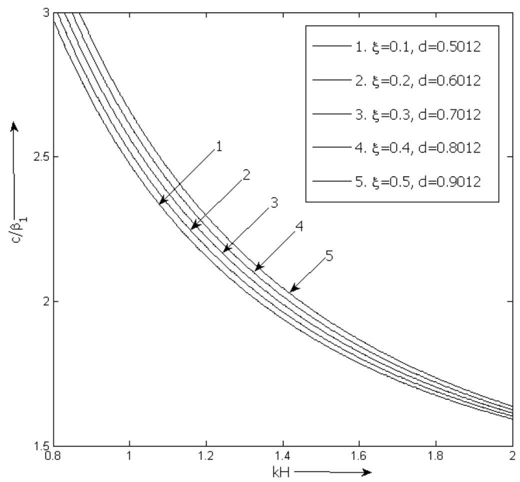

Fig. 6 Dimensionless phase velocity c/β_1 against dimensionless wave number *kH* for different values of both compressive prestress ξ & porosity parameter *d*

parameters a_1, a_3 and with fixed values of h_1/h_2, sandiness, porosity & prestress as 0.1, 1.8, 0.9012 & 0.2 respectively. The values of a_1^2 & a_3^2 for curve 1, curve 2, curve 3, curve 4, curve 5 and curve 6 are taken as 0.25, 0.35, 0.45, 0.55, 0.65 and 0.75 and 0.75, 0.65, 0.55, 0.45, 0.35 and 0.25 respectively. From that figure we can conclude that the velocity of Love wave increases with increase of a_1 and decrease of a_3.

Fig. 3 demonstrates the effect of h_1/h_2 i.e., the ratio of the thickness of fiber-reinforced & dry sandy media on the phase velocity of Love wave with the fixed values of reinforcement parameters, sandiness, prestress & porosity as (0.25, 0.75), 2, 0.2 & 0.9012 respectively. The values of h_1/h_2 for curve 1, curve 2, curve 3 and curve 4 have been taken as 0.10, 0.15, 0.20 and 0.25 respectively. The curves show that the phase velocity of Love wave decreases with the increase h_1/h_2.

Fig. 4 specifies the influence of sandiness parameter η of the dry sandy layer on the propagation of Love wave. The curves are plotted taking the different values of the sandiness parameter η and with constant values of reinforcement parameters, ratio of thickness of layers, porosity & prestress as (0.25, 0.75), 0.3, 0.9012 & 0.2 respectively. The values of η are taken as 1.5, 2, 2.5, 3 for the curves 1, 2, 3, 4 respectively. From the figure we can conclude that as the sandiness increases the velocity of the Love wave decreases accordingly.

Fig. 5 traces the influence of porosity parameter *d* on the phase velocity of Love wave. The curves are plotted using the values of the porosity for the substratum with fixed values of reinforcement parameters, sandiness, h_1/h_2& prestress as (0.25, 0.75), 7, 0.2 & 0.2 respectively. The values of d are taken as 0.9012, 0.8012, 0.7012, 0.6012 and are shown by the curves 1, 2, 3, 4 respectively. From the figure we can conclude that the phase velocity of Love wave decreases with the decrease of porosity of the substratum.

Fig. 6 signifies the joint effect of prestress & porosity on the concerned medium. The curves are plotted taking the increasing values of both d& prestress (ξ) with some constant values of reinforcement parameters, sandiness &h_1/h_2 as (0.25, 0.75), 4 & 0.7 respectively. The values for ξ are taken as 0.1, 0.2, 0.3, 0.4 & 0.5 whereas the values of porosity parameter d are taken as 0.5012, 0.6012, 0.7012, 0.8012 & 0.9012 are represented in curves 1, 2, 3, 4 & 5 respectively. It is worthy to mention that the curves clarify the fact that the increase of both prestress & porosity parameters jointly increase the velocity of Love wave.

9. Conclusions

Considering the presence of prestress in the substratum, the influence of reinforce parameters on propagation of Love wave has been discussed theoretically by introducing a three layered earth model. Dispersion equation are deduced in case of

(i) upper layer is homogeneous free from reinforcement

(ii) intermediate layer converts into isotropic elastic solid material

(iii) lower substratum is free from porosity & prestress

(iv) lower substratum is homogeneous & stress free

(v) upper layer is homogeneous and free from reinforcement, intermediate layer is isotropic elastic, lower half-space is homogeneous, non-porous and prestress free

(vi) upper layer is neglected

(vii) intermediate layer is neglected.

The mathematical model investigates the propagation of Love waves in sandy medium sandwiched between fiber-reinforced and prestressed porous media in detail. Solution for the all the media have been deduced separately in compact form. Phase velocity for the Love waves has been computed numerically and the effect of different parameters are studied graphically using MATLAB software. From the comparative study of above Figs. 2-6 we conclude that

(1) Dimensionless phase velocity c/β_1 of Love wave decreases with the increase of dimensionless wave number kH

(2) Phase velocity of Love wave increases when reinforcement parameters a_1^2 increases where a_3^2 decreases

(3) Phase velocity of Love wave decreases with gradual increase of the sandiness parameter

(4) Dimensionless phase velocity of Love wave decreases when the ratio of the thickness of layers h_1/h_2 increases

(5) Phase velocity of Love wave decreases with the subsequent decrease of porosity parameter d

(6) Dimensionless velocity of Love wave increases when both the values of porosity parameter d and prestress ξ increases.

Finally our computed dispersion equation coincides with the classical dispersion equation of Love wave when any of the two layer is neglected. The results may help in the selection of proper structural materials concerned with civil engineering. The outcomes of the study projects a theoretical framework of the proposed model and can reveal the propagation characteristics of Love waves with a clear insight in exploring the natural resources lying in form of mineral deposits beneath the earth's surface. Moreover the results from this investigation can be utilized in designing disaster-resistant heavy constructions along the seashores.

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