Earthquakes and Structures, *Vol. 12, No. 5 (2017) 543-557*
DOI: https://doi.org/10.12989/eas.2017.12.5.543

A new approach for 3-D pushover based analysis of asymmetric buildings: development and initial evaluation

Dimitrios K. Baros[1,2a] and Stavros A. Anagnostopoulos[*1]

[1]Department of Civil Engineering, University of Patras, Patras, 26500, Greece
[2]Department of Civil Engineering, Technological Educational Institute of Western Greece, Patras, 26334, Greece

(Received September 13, 2016, Revised May 15, 2017, Accepted May 16, 2017)

Abstract. Results of an extensive study aiming to properly extend the well known pushover analysis into 3-D problems of asymmetric buildings are presented in this paper. The proposed procedure uses simple, 3 DOF, one-story models with shear-beam type elements in order to quantify the effects of inelastic torsional response of such buildings. Correction coefficients for the response quantities at the "stiff" and "flexible" sides are calculated using results from non-linear time history analyses of the simple models. Their values are then applied to the results of a simple, plane pushover analysis of the detailed building models. Results from the application of the new method for a set of three, conventionally designed, five-story buildings with high values of uniaxial eccentricities are compared with those obtained from multiple non-linear dynamic time history analyses, as well as from similar pushover methods addressing the same problem. This initial evaluation indicates that the proposed procedure is a clear improvement over the simple (conventional) pushover method and, in most cases, more accurate and reliable than the other methods considered. The accuracy, however, of all these methods is reduced substantially when they are applied to torsionally flexible buildings. Thus, for such challenging problems, use of inelastic dynamic analyses for a set of two component earthquake motions appears to be the preferable solution.

Keywords: asymmetric buildings; torsional behavior; pushover analysis; shear beam models

1. Introduction

Based on the results of ongoing research in the field of inelastic response of structures, as well as on field observations of building performance during strong earthquakes, modern Codes such as Eurocode 8 (EC8, CEN, 2004) and ASCE/SEI 41-06 (ASCE 2007), have introduced new ideas for earthquake resistant design of new buildings and for assessing the capacity of existing ones. Performance based design, a methodology applied in the past only to important industrial structures e.g., nuclear power plants and offshore platforms, is the new direction of modern codes for the design of new buildings or the capacity assessment of existing ones. This has been aided by the advances in computers, which make affordable the computationally demanding inelastic methods required to predict building response up to collapse under very strong earthquakes. Thus, the elastic methods of the past, approximating inelastic building response through the use of the so-called response reduction factors or behavior factors (in U.S. and European terminology, respectively), are included now in modern codes along with inelastic methods as alternatives. The latter are gradually becoming more popular, for the obvious reason that they account explicitly for the expected inelastic behavior of building elements under the action of design-level earthquakes.

Obviously the most accurate method to predict building response in strong earthquakes is the nonlinear (inelastic) dynamic time history analysis (NL-THA), for which the complete hysteretic force-deformation relationships of the building elements are specified and the equations of motions are solved numerically in the time domain for the prescribed motions. The lack, however, of widely accepted models of inelastic cyclic behavior of various types of building members up to failure, has limited usage of this advanced method primarily for research purposes or for very special structures, although EC8 allows it as an alternative to simpler approximate methods.

A static inelastic method that came to be known as "pushover" analysis has become very popular in the past decade or two, as a simpler static alternative to NL-THA. This inelastic approximation is similar to the elastic approximation of the dynamic time history or response spectrum analysis by the static method of equivalent lateral loads.

Pushover analysis was initially introduced as a method to identify possible weaknesses in fixed, conventionally designed, offshore structures (Kallaby and Millman 1975, Gates *et al*. 1977). The method found wider application in the field of seismic assessment of existing structures, well twenty years after its introduction. It was established as a main analytical tool in the popular FEMA 356 pre-standard (FEMA 2000) and gradually found its way into several design or assessment Standards (EC8, CEN 2004, ASCE/SEI 41-06, ASCE 2007, Greek Seismic Retrofitting Code 2012). However, this increase in the popularity of

*Corresponding author, Professor Emeritus
E-mail: saa@upatras.gr
[a]Ph.D., Adjunct Lecturer

http://www.techno-press.com/journals/eas&subpage=7 ISSN: 2092-7614 (Print), 2092-7622 (Online)

pushover analysis has resulted in the method being overused, even in problems that lie well outside its established area of application. Moreover, refinements of the pushover analysis by modifying the load pattern as the deformation progresses (e.g., Elnashai 2001, Aydinoglou 2003) deprive the method of its main advantage of simplicity and start making more attractive the dynamic NL-THA method.

Having been developed for 2-D applications, pushover analysis assumes that a dominant translation mode defines the behavior of a structure under seismic excitation. This assumption is generally valid for symmetric, low or medium rise buildings, but it is inaccurate for tall or asymmetric ones. In the latter case, it is well known that torsion affects their inelastic seismic response in an adverse and potentially unpredictable way. Moreover, the behavior patterns of buildings susceptible to torsion are usually complex and require the consideration of both components of earthquake motion in order to be accurately described. Hence, in such cases, "conventional" pushover analysis (plane analysis with triangular load distribution) is prone to failure as many researchers have pointed out (Lawson *et al.* 1994, Krawinkler and Seneviratna 1998, Elnashai 2002) and should be applied with caution or supplemented with extra analyses in order to assure conservative results.

Expanding the pushover method beyond the limits of its 2-D origins is an open issue for the engineering community. Over the past decade, researchers have explored different concepts and presented various solutions to the problem. Our initial evaluation of some of the most popular alternatives raised questions over their reliability in cases where torsion strongly affects the behavior of a building (Baros and Anagnostopoulos 2008a, b). This motivated our work to develop a new pushover based method adapted to the specific characteristics of the problem of asymmetric buildings susceptible to torsion. This paper presents results of our effort and introduces a new procedure that, in our opinion, provides an acceptable answer to the previously set requirement. First, we discuss the basis of our method and its main assumptions. Based on these considerations, the computational steps that form the backbone of the proposed procedure are presented in detail. Finally, the new pushover-based method, applied to three, one-way asymmetric buildings, is evaluated by comparing results with those from multiple NL-THA, as well as from other similar methods.

2. Background and development of the proposed method

2.1 Available procedures for the pushover-based analysis of asymmetric buildings

To extend the simple pushover method into 3-D problems of asymmetric buildings, several alternatives of varying complexity have been proposed aimed at improving results obtained by the "conventional" procedure. Some solutions use adaptive load patterns based on the yielding of the structural members, as proposed by Shakeri *et al.* (2012), who extended the adaptive pushover method introduced in Gupta and Kunnath (2000) and discussed in Elnashai (2001), Aydinoglou (2003), Antoniou and Pinho (2004). Another option was presented in Penelis (2007), following the principles of the Incremental Dynamic method introduced by Vamvatsikos and Cornell (2002). However, it is our opinion that these methods are overly complex, thus lose their main advantage over the more accurate non-linear time history analysis. Finally, multi-mode pushover analyses using properly derived force vectors (Sucuoglu and Gunay 2011, Kaatsiz and Sucuoglu 2014) or equivalent systems (Lin and Tsai 2007) yield results of improved accuracy usually at a cost of increased complexity.

A simpler solution was presented by Chopra and Goel (2004), who extended their Modal Pushover Analysis (MPA) method, introduced in Chopra and Goel (2002), into problems of asymmetric buildings. The MPA is an extension into the inelastic range of the well known elastic method of Response Spectrum Analysis (RSA). Although conceptually simple, the method lacks the theoretical background since the extension of the modal solution to inelastic problem is rather arbitrary. Moreover, the inelastic "modal" responses are obtained from multiple pushover analyses of the detailed model of the building using force and torque loads. As a result, in most cases the computational effort is substantially increased. The method, shown to give good results in cases of tall symmetric buildings (e.g., Chintanapakdee and Chopra 2003, Goel and Chopra 2004, Goel 2005), was recently extended in Reyes and Chopra (2011), for analyses of asymmetric buildings subjected to two components of ground motion. It should be noted, however, that its latest version adds extra displacement-based analyses in the proposed algorithm, which makes it more complex than the "original" MPA method. As a result, we believe that our earlier comment concerning the complexity of the adaptive pushover techniques, applies also to the extension the MPA procedure to 3-D eccentric buildings.

Another interesting approach of the pushover based analysis of asymmetric buildings is the extension of the N2 method (Fajfar 2000) into the problem under consideration. The extended N2 method that accounts for torsion in the response of asymmetric buildings has been presented in Fajfar *et al.* (2005) and more recently in Kreslin and Fajfar (2012). The procedure is based on modifying the response quantities from a "normal" N2 analysis with correction factors calculated from an elastic RSA of the building's model. This approach is based on practical considerations concerning torsional response and is supported by the findings of Perus and Fajfar (2005), and Marusic and Fajfar (2005), who investigated the response of multiple asymmetric buildings using NL-THA. Although the results presented in the aforementioned papers are indicative of the increased accuracy achieved when using the extended N2 algorithm, the inherent assumption that the torsional effects can be estimated using elastic analyses is rather arbitrary. Considering the fact that the method was tested and verified using mostly torsionally stiff buildings designed mainly for research purposes, its applicability to torsionally flexible buildings remains to be proven. However, in our opinion the extended N2 method maintains the simplicity that makes the pushover method so popular and, based on assumptions well understood and reasonable, it is worth considering. Moreover, applying correction factors on response quantities in order to account for certain aspects of a building's behavior is a usual practice in structural engineering. A similar idea has been used in the development of the method proposed herein.

2.2 Basic assumptions of the proposed procedure

An initial effort to extend the pushover method to 3-D applications of asymmetric buildings was based on the idea of applying proper combinations of increasing lateral loads simultaneously in both horizontal directions in order to account for torsion under two component motions. The idea was investigated in Baros (2014), using buildings with uniaxial eccentricities subjected to a combination of ramp-type, ground accelerations, which in turn resulted in monotonically increasing inertial forces with patterns related to the height-wise mass distribution of each building. Several spatial combinations were considered and the results were compared with those obtained from multiple NL-THA. The results indicated that this approach - equivalent to spatial combinations of pushover load patterns - may improve results in some cases, especially with torsionally stiff buildings, but may also lead to results of reduced accuracy. It was further found that the use of pushover load combinations with torsionally flexible buildings failed to properly account for the effects of torsion and gave questionable results. The failure of this approach is attributed to the dominant dynamic effects of torsion in cases of torsionally flexible asymmetric buildings, due to which the loading pattern from static torsion (increasing displacements at the so called "flexible" edge and decreasing at the "stiff" edge, Fig. 1) is not applicable.

The results of the previous investigation led to the conclusion that in order to properly assess the effect of seismic induced torsion it is important to directly consider its dynamic nature. The effects of inelastic torsion on pushover analysis results have been discussed in De Stefano

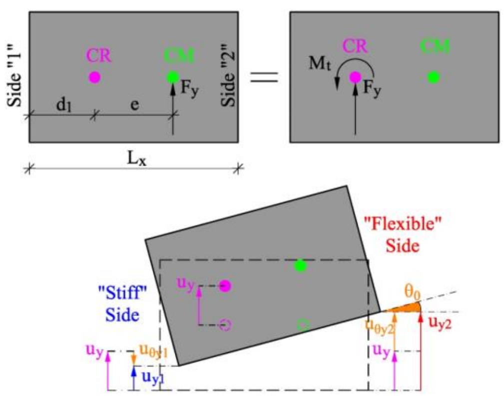


Fig. 1 "Static" representation of the torsional behavior of an asymmetric building and definition of the stiff and flexible sides

and Pintucchi (2010). Since the "simple" pushover method is just a static method, rather than trying to extend it in ways that may not be consistent with its basic principles, the procedure proposed in the present paper tries to supplement it with an easy to use dynamic addition that directly considers the non-linear, torsional behavior of the building.

The key feature of the proposed procedure lies in the use of a simple, one story, 3-DOF - two translations and one rotation - nonlinear shear-beam model (SSBM, Fig. 2) as a supplemental tool that quantifies the effects of torsion on the building examined. Shear-beam models have been used extensively in the investigation of seismic induced torsion in buildings, with results of varying accuracy. Anagnostopoulos *et al.* (2010), have shown that matching the three periods of the simple model with the three fundamental periods T_x, T_y and T_θ of the actual building, is the key for achieving response results that are qualitatively similar in both models. This may be achieved by setting the 3 elastic stiffness of the simple 3-DOF system equal to those of the actual building and then gradually reducing the translational masses and inertia of the simple model until a reasonable match of the said periods is achieved. Finally, the strength of the shear-beam type elements that represent the frames of the building should also be modified using the following reduction factors

$$\lambda_{M,x} = \frac{M_{x,SSBM}}{M_x}, \quad \lambda_{M,y} = \frac{M_{y,SSBM}}{M_y}, \quad \lambda_{Jm} = \frac{J_{m,SSBM}}{J_m} \quad (1)$$

where $M_{x,SSBM}$, $M_{y,SSBM}$ are the translational masses of the simple model along the x and y axes respectively, $J_{m,SSBM}$ its polar mass moment of inertia and M_x, M_y, J_m the respective quantities of the full, 3-D model of the building. Reducing the strength of the elements as shown in Fig. 3 is important to ensure that the (seismic force / strength) ratio of the simple model is similar to that of the full model of the building, as this feature also plays a key role on the reliability of the results obtained from the analyses of the SSBM.

Considering the above, the SSBM was selected as a means to "handle" the 3-D nature of the nonlinear torsional effects that lie beyond the field of application of the "simple" pushover algorithm. The idea was to carry out the simplest possible pushover analysis of the complex building model, e.g., using a triangular load pattern applied at the approximate Centers of Rigidity (CR) of the building floors to perform a separate analysis for each direction, thus staying with the 2-D character of the method. The results of these "simple" pushover analyses can then be corrected using appropriate factors calculated as the normalized floor displacements from a series of NL-THAs of the SSBM subjected to a set of ground motions.

The proposed procedure, called in short BA-3D, draws from the N2 method and its intuitiveness of using correction factors for the response quantities. However, it moves one step further and allows the calculation of such factors from non-linear dynamic analyses of an appropriate simple system. Obviously, when compared with the pushover procedures implemented in the previously referenced Codes

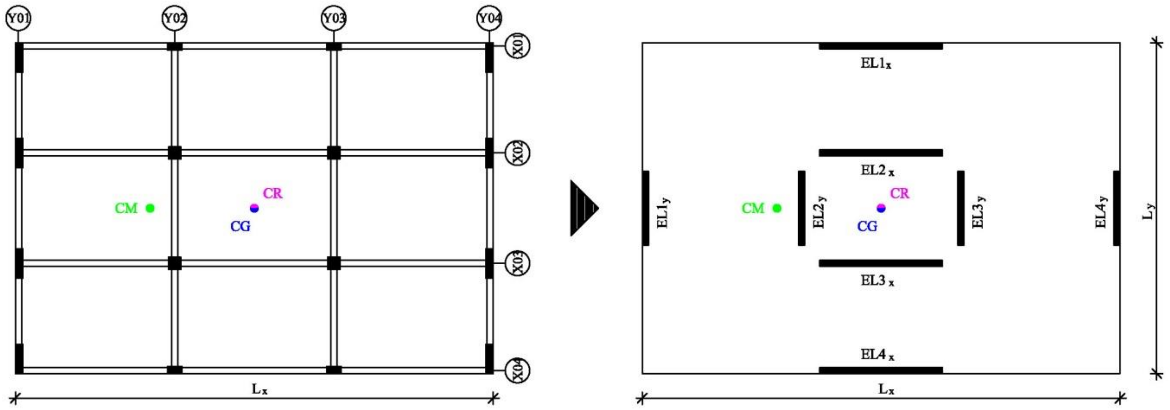


Fig. 2 Approximation of the building with a single-story, eight element shear-beam model having similar dynamic characteristics

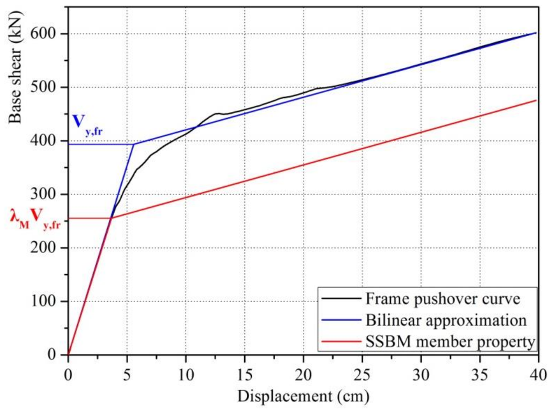


Fig. 3 Calculation of the shear-beam member properties from the idealized curve of the respective frame of the building

(EC8, CEN 2004, ASCE/SEI 41-06, ASCE 2007, Greek Seismic Retrofitting Code 2012) as well as with the N2 method, the proposed BA-3D procedure is more demanding computationally. However, the bulk of the required analyses are performed using the SSBM system, which is easy to derive and quick to analyze using practically any commercial software or even simpler available codes (e.g., AIDA, Anagnostopoulos and Roesset 1972). Thus, it is our opinion that the relatively small increase in the required computational steps is justified and balanced by the expected increase in the accuracy and reliability of the results, which can be achieved by directly considering the inelastic torsion of the building using the proposed simple model.

2.3 Step-by-step summary of the proposed BA-3D pushover method

Considering the above discussion, the BA-3D method proposed in the present paper can be summarized into the following 7 computational steps, in which 1 and 2 are needed to compute the properties of the simple SSBM system, 3 and 4 to estimate the correction factors and 5, 6, 7 to determine the final results:

1. For each individual plane frame of the building in each horizontal direction, develop the base shear - roof displacement pushover curve, by simple, plane, non-linear static analysis. Idealize each curve as a bilinear curve in order to calculate the properties (stiffness, yield point shear, Fig. 3) of each element of the equivalent SSBM system. The location of the CR axis of the building can also be approximately calculated as the stiffness (rigidity) center of the single-story simple system.

2. Modify (reduce) the mass and mass moment of inertia of the simple system in order to match its periods with the three fundamental periods T_x, T_y and T_θ, of the building as discussed in Anagnostopoulos *et al*. (2010). Use the values of $\lambda_{M,x}$, $\lambda_{M,y}$ (Eq. (1)) in order to modify the yield point shear (i.e., the strength) of the elements of the simple model in each horizontal direction (Fig. 3). Thus, the (seismic force / yield shear) ratio of the SSBM is equal to that of the detailed model of the building.

3. Perform a series of non-linear dynamic analyses of the SSBM model using an adequate number of accelerograms. The accelerograms should be properly scaled according to the applicable code and the considered earthquake intensity. Both components of each acceleration record should be applied simultaneously. The required number of records as well as the scaling process depends on the applicable Code. Due to the simplicity and intuitiveness of the SSBM, the non-linear analyses required in this step can be performed easily using any commercial or academic nonlinear analysis software. This observation is important since it implies that the proposed procedure is consistent with the role of the pushover method as an every-day practical analysis tool.

4. Compute the following correction factors $f_{T,i}$ that quantify the effect of torsion on the non-linear response of the i-th frame of the building

$$f_{T,i} = \frac{\bar{u}_{max,i}}{\bar{u}_{max,CR}} \tag{2}$$

where, $\bar{u}_{\max,i}$ is the average of the maximum displacements at the i-th element and $\bar{u}_{\max,CR}$ the average of the maximum displacements of the SSBM at the CR, as calculated by the non-linear time history analyses. Hence, for the stiff and flexible sides of the building the respective correction factors $f_{T,SS}$ and $f_{T,FS}$ can be defined as follows

$$f_{T,SS} = \frac{\bar{u}_{max,SS}}{\bar{u}_{max,CR}}, \quad f_{T,FS} = \frac{\bar{u}_{max,FS}}{\bar{u}_{max,CR}} \tag{3}$$

where, $\bar{u}_{\max,SS}$ and $\bar{u}_{\max,FS}$ are the average values of the maximum displacements at the stiff and flexible sides of the SSBM, respectively.

5. Perform "simple" pushover analyses of the full model of the building by applying loads separately in each horizontal direction. The loads should be applied on the previously defined CR axis of the building in order to ensure its purely translational response, since the effect of two component motion acting simultaneously as well as torsion will be accounted for in the next steps using the correction factors defined in the previous step, Eq. (3). Thus, the analysis performed in the present step of the procedure is actually 2-D in the sense that the resulting response of the building is purely translational along each horizontal axis. It should be noted that this step does not require the development of a separate model, since these "planar" analyses can be performed using a "typical" 3-D model of the building as required by any pushover method. The simplest, triangular height-wise distribution was selected in the present paper for the pushover forces, since the height-wise distribution of the response quantities was not a subject of investigation. However, any other distribution of the said loads may also be used in this step.

6. From the pushover analysis database extract the required response quantities, namely displacements, u, and inter-story drifts, Δu, at selected locations of the building, for the step that the roof displacement reaches its target value. The target displacement d_t in pushover analyses can be calculated using approximate methods available in literature (e.g., EC8, CEN 2004, ASCE/SEI 41-06, ASCE 2007). Since, in this case, multiple NL-THAs of the buildings were performed as the basis for comparison with results from the proposed method, the average of the maximum roof displacements obtained from these analyses was used as the target displacement for all the pushover procedures under examination.

7. Finally, the response results obtained from the previous step are corrected in order to account for torsion and other aspects of the 3-D behavior of the building, using the correction factors calculated in step 4 (Eqs. (2) and (3)). For the stiff and flexible sides of the j-th floor of a building, the displacement and drift values are calculated as follows

$$u_{SS}^{j} = f_{T,SS} \cdot u_{push}^{j}, \quad \Delta u_{SS}^{j} = f_{T,SS} \cdot \Delta u_{push}^{j} \tag{4}$$

$$u_{FS}^{j} = f_{T,FS} \cdot u_{push}^{j}, \quad \Delta u_{FS}^{j} = f_{T,FS} \cdot \Delta u_{push}^{j} \tag{5}$$

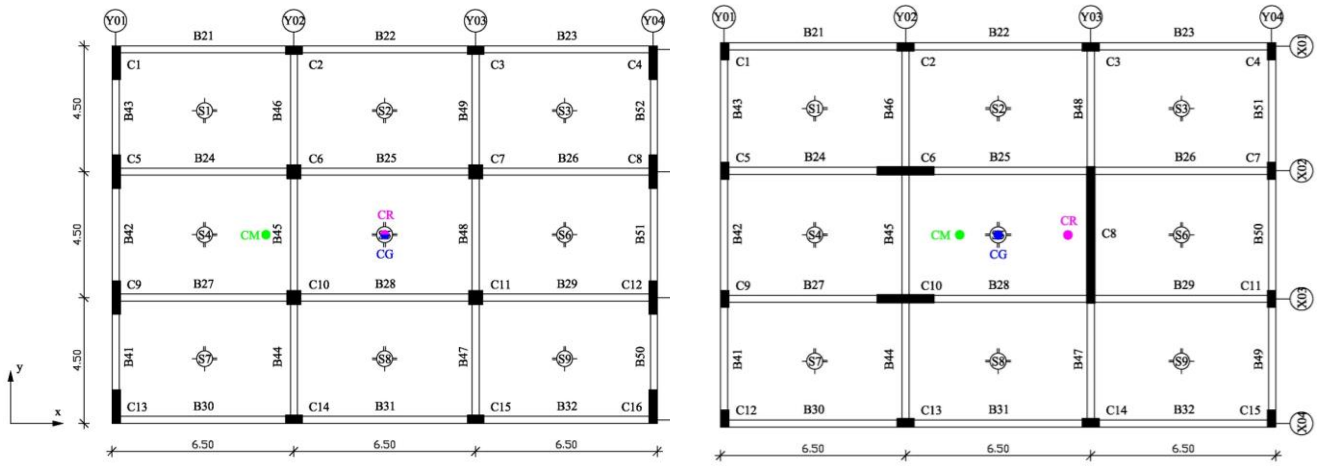
where u^j_{SS}, Δu^j_{SS} are the corrected displacement and drift values at the stiff side of the j-th floor, u^j_{FS}, Δu^j_{FS} the respective values at the flexible side and u^j_{push}, Δu^j_{push} the values of displacements and drifts obtained from step 6 from the pushover database at each location. It should be noted that the application of the pushover lateral loads at the CR leads to a practically 2-D analysis of the building. This implies that the values of u^j_{push} and Δu^j_{push} will be similar for all the examined locations of the same floor and along the same axis, e.g., $u^j_{CR,push} \approx u^j_{SS,push} \approx u^j_{FS,push}$.

3. Test buildings design and modeling assumptions

3.1 Building design and modelling considerations

The main aim of the evaluation procedure was to test the examined pushover methods under "realistic conditions". Hence, the use of "research-type", simply designed structures was excluded and the main consideration was to "produce" realistic designs, representative of actual typical residential buildings commonly found in Greece. Thus two typical story layouts of 3 different 5-story buildings were selected for our designs, the first two of which were made torsionally stiff and the third torsionally flexible. It is reminded here that torsionally stiff buildings have their two fundamental translational periods T_x and T_y longer than the lowest torsional period T_θ, and hence their response is dominated by translational motion, while the opposite happens in torsionally flexible buildings ($T_\theta > T_x$, T_y) whose motion has a significant torsional component. The difference between the two torsionally stiff buildings is in that the first, indicated as RCS-RU, is a stiffness eccentric building (its mass center, CM, coincides with the geometric center, CG, and its stiffness center, CR, shifted towards the stiff edge of the building), while the second, indicated as RCS-MU, is a mass eccentric building (its stiffness center, CR, coincides with the geometric center, CG, and CM is shifted to the left). The layout of Fig. 4(a) is for the mass eccentric building RCS-MU and that of Fig. 4(b) is for the torsionally flexible building, indicated as RCF-U. The stiffness eccentric (RCS-RU) building has the same layout as Fig. 4(a) except that its CM coincides with CG and its CR is shifted to the right of CG. The story height in all buildings is 3.20 m except for the ground story that was set at 3.60 m. Since a similar design approach was followed for all three buildings, they share several common properties and characteristics.

The buildings' structural systems consist of four moment resisting frames in each horizontal direction with bay lengths equal to 6.50 m along the x-axis and 4.50 m along the y-axis, resulting in floor plan dimensions with ratio $L_y/L_x \approx 0.70$ (Fig. 4). This was a design decision aimed to increase the effects of torsion on the external frames of the buildings, in this case those in the y-axis direction (frames Y01 and Y04, Fig. 4). All buildings have one axis of symmetry, namely the x-axis. Different design decisions led to the desired uniaxial natural eccentricity. In the case of the RCS-RU building, larger sections were selected for the members of the Y04 frame resulting in "shifting" CR towards the aforementioned side of the building ("stiff" side), while the center of mass (CM) coincides with the geometric center (CG), thus leading to a uniaxial stiffness eccentricity. A different approach was followed in the case of the RCS-MU building. This time it is the asymmetric distribution of vertical loads and respective masses that shifts the CM towards the Y01 frame ("flexible" side), whereas the member sections are symmetrically distributed,



(a) (b)

Fig. 4 Layout of the (a) RCS-MU (b) RCF-U buildings

hence the CR coincides with the CG. For the above reason, following the terminology used in building torsion, RCS-MU is characterized as a mass-eccentric building whereas RCS-RU is characterized as stiffness-eccentric. It should be noted that both RCS-RU and RCS-MU buildings were designed aiming to a value of natural eccentricity $\varepsilon \sim 0.20$, which, according to similar investigations found in the literature, is considered a generally large value associated with intense torsional effects in the buildings' seismic behavior.

In the case of the RCF-U building, the shear wall (member C8, Fig. 4) located near the CG of the floor plan and the absence of equally stiff and strong elements at the perimeter defines the location of the CR and, most importantly, the seismic behavior of the structure. Apparently the CR lies on the axis of symmetry of the building close to the strong shear wall (Fig. 4), while its distance from the CM defines a natural eccentricity, ε=0.17. The CM is also shifted from the CG of the structure due to the asymmetric distribution of the vertical loads, selected to achieve a value of natural eccentricity ε close to the target ~ 0.20. Note that the CR is generally defined only for simple, single story structures with shear-beam type elements. Here an approximate CR was calculated using an equivalent stiffness for each multistory frame, which was obtained from the linear, "elastic" part of its idealized pushover curve.

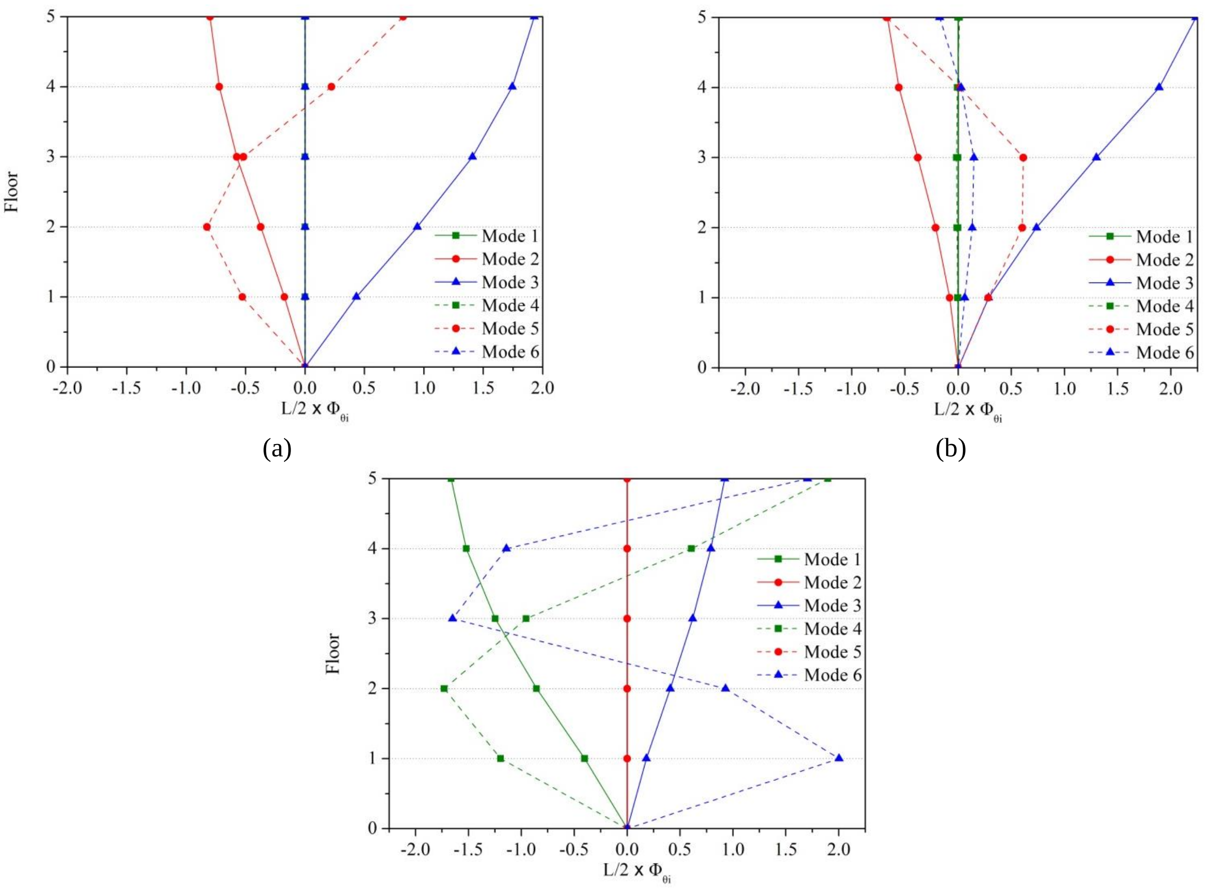
All the buildings were conventionally designed according to the Greek Codes, considering a design peak ground acceleration (PGA) of 0.16 g. The cross sections of the vertical resisting elements of the buildings are reduced over height as in normal design practice. Since the proposed method focuses on introducing inelastic torsion in the pushover algorithm, no vertical irregularities (e.g., soft stories, setbacks etc) were considered. Detailed information and design results (e.g., member reinforcement details) can be found in Baros (2014).

The buildings were analyzed using the Ruaumoko 3D code (Carr 2005). The well known plastic hinge model was used for the representation of the beam and column element inelastic behavior, with reduced, effective stiffness according to the ASCE/SEI 41-06 in order to account for cracking under intense earthquake excitations (ASCE 2007). The behavior of the thick concrete slabs was included using a kinematic diaphragm constraint for each floor of the building. Member yield moments were calculated using the EC8 (CEN 2004) equations. It should be noted that for column members, the Ruaumoko 3D beam-column element directly considers the interaction of axial loads and moments by defining proper yield moment-axial load interaction surfaces. Finally, the effect of stiffness degradation due to the effect of member cyclic deformation was included using the well known modified Takeda model (Otani 1974, Litton 1975), with the following values for its parameters: α=0.3 and β=0.

3.2 Modal analyses results

Elastic eigenvalue analyses carried out with the Ruaumoko 3D program provided the dynamic characteristics of the buildings under examination. The torsional components of the first six modes of each building are presented in Fig. 5. The fundamental bending mode of the RCS-RU and RCS-MU buildings is a pure mode in the direction of the x-axis, their axis of symmetry. The fundamental mode of the torsionally flexible RCF-U building is torsional (Fig. 6) and its second is again a pure mode along the x-axis direction.

Finally, Table 1 summarizes the first three periods of each building and their modal mass ratios along the x (M^*_x) and y-axis (M^*_y) for the first three modes, as well as those of the equivalent SSBM systems that were derived according to the proposed procedure. It should be noted that, when reducing the mass and inertia of each simple model, the main aim was to closely match the periods of the torsional and y-bending modes, which are mostly related to the torsional effects on the behavior of the building. Deviations from the x-translational modes and respective periods are, in this case, not expected to have a significant effect on the accuracy of the SSBM model results.



(c)

Fig. 5 Torsional components of the first six modes of the examined buildings (a) RCS-RU (b) RCS-MU (c) RCF-U

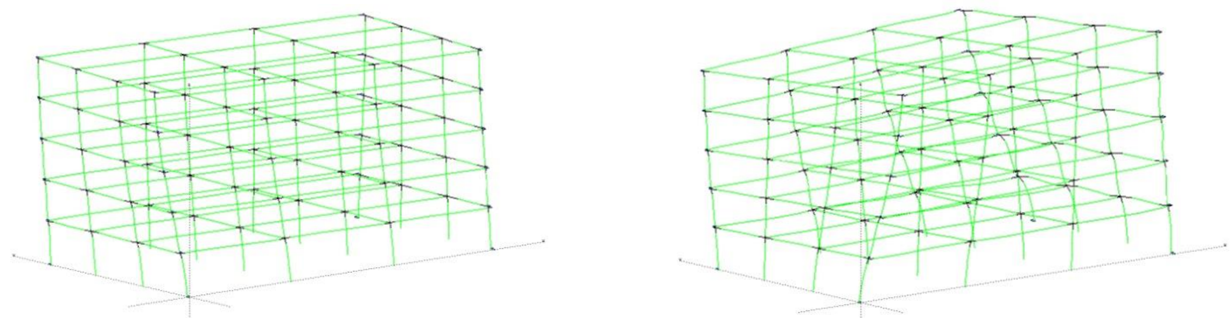
Table 1 Periods, T, and modal mass ratios, M^*, of the first three modes of each building and their respective simple, shear-beam models

Mode		Periods, T (sec)		M^*_x (%)		M^*_y (%)	
		Full	SSBM	Full	SSBM	Full	SSBM
RCS-RU							
1	T_x	1.101	1.082	86	100	~0	0
2	$\boldsymbol{T_y}$	**0.914**	**0.926**	~0	0	59	69
3	$\boldsymbol{T_\theta}$	**0.576**	**0.576**	~0	0	25	31
RCS-MU							
1	T_x	1.116	1.029	85	100	~0	0
2	$\boldsymbol{T_y}$	**0.965**	**0.969**	~0	0	58	75
3	$\boldsymbol{T_\theta}$	**0.599**	**0.600**	~0	0	17	25
RCF-U							
1	$\boldsymbol{T_\theta}$	**0.897**	**0.895**	~0	0	33	41
2	T_x	0.777	0.827	80	100	~0	0
3	T_y	0.496	0.543	~0	0	43	59

4. Non-linear analyses results and discussion

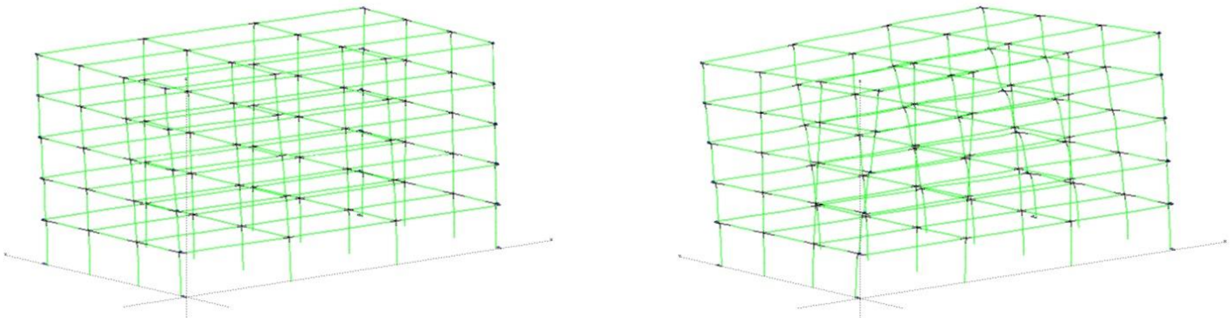
As previously discussed, the BA-3D pushover method introduced in the present paper was used for the analysis of three typical, reinforced concrete buildings, which, designed with one axis of symmetry and high values of uniaxial eccentricity, are subject to torsion. In order to assess the accuracy and reliability of the proposed procedure, NL-THAs were performed for the same buildings using an ensemble of properly selected and scaled real earthquake records. Results from these analyses are used as the basis for evaluation of the method proposed herein and its comparison with three of the best known similar methods mentioned earlier. Two groups, of five earthquake records each, were used for the NL-THAs, one consisting of pulse-type (near field) records and another of far field ones. Results obtained using the pulse-type group are presented and discussed in the following sections. The analyses using far field records led to similar observations and the results are presented in detail in Baros (2014).

The pulse-type record group which was used for the NL-THAs includes the real records summarized in Table 2. These records were scaled so that their average spectrum would match the normalized elastic design spectrum (Fig. 7) according to the EC8 (CEN 2004) requirement for recorded accelerograms. Subsequently, their intensity was increased by a factor of three in order to ensure that the members of the buildings would undergo significant inelastic deformations to compare the results of the pushover analyses well into the inelastic regime. This was important since these analyses aimed at the approximate procedures under “extreme circumstances”. It should be



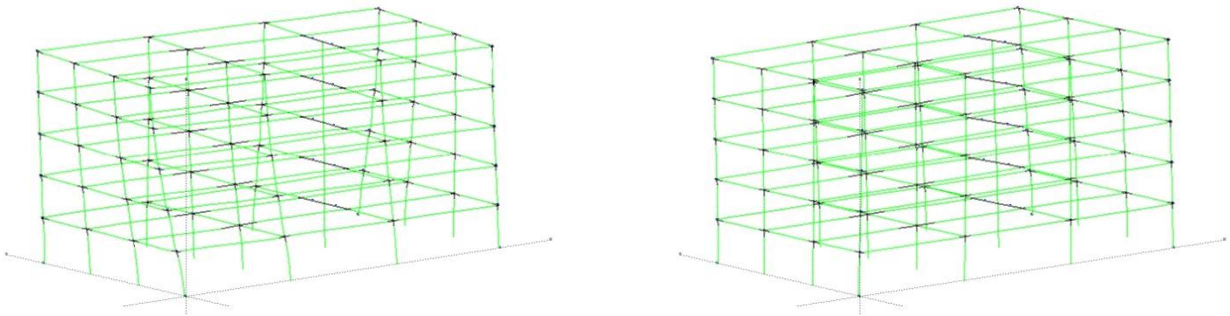
Mode 2 Mode 3

(a) RCS-RU



Mode 2 Mode 3

(b) RCS-MU



Mode 1 Mode 3

(c) RCF-U

Fig. 6 3-D views of the first modes coupling the y-translational and torsional motion of each building

noted that all the NL-THAs were performed with both components of each record acting simultaneously along the two horizontal directions. By interchanging the two motion components along the two building axes, two analyses were performed with each earthquake record and hence results from ten NL-THAs were obtained for each examined building.

Global response quantities, namely displacements and inter-story drifts at the stiff and flexible side of each building, were selected as indicative of the buildings' overall earthquake behavior and expected damage distribution. Since torsion affects the behavior of the buildings primarily along the y-axis as it has already been discussed, only y-displacements and drifts are presented here. The NL-THAs results representing the values for comparison, were calculated as the average of the respective maxima obtained from each separate analysis, i.e., these values are the means of the response quantities computed for the ten motion pairs. The (%) differences ΔR of the pushover analyses results from those obtained by the NL-THAs and used for assessing the approximate methods, were calculated as follows

$$\Delta R(\%) = 100\ \left(R_{app} - R_{THA}\right) / \ R_{THA} \tag{6}$$

where R_{app} is the response quantity as calculated from the pushover method considered and R_{THA} its mean value obtained by NL-THA for the ten motion pairs.

The proposed BA-3D pushover method was also compared to three other methods: (a) the "typical" pushover procedure along the y axis, as specified in current codes, with lateral loads applied at the CM of each floor and their values based on a "fundamental mode" height-wise distribution without any consideration for torsion, (b) the extended N2 method, also considering a "first-mode" height-wise distribution for the lateral loads with the

Table 2 Pulse-type actual earthquake records used for the NL-THAs of the buildings

Record	Component	Date	Epicentral distance (km)	PGA (g)
Aigion	L	15/6/1995	23	0.501
	T			**0.543**
Imperial Valley (Array #7)	L (140)	15/10/1979	27	0.333
	T (230)			**0.462**
Northridge (Sylmar Hospital)	L (090)	17/01/1994	15	0.592
	T (360)			**0.879**
Erzincan	L (NS)	13/03/1992	9	0.399
	T (EW)			**0.501**
Kobe Takarazuka	L (000)	16/01/1995	34	0.693
	T (090)			**0.694**

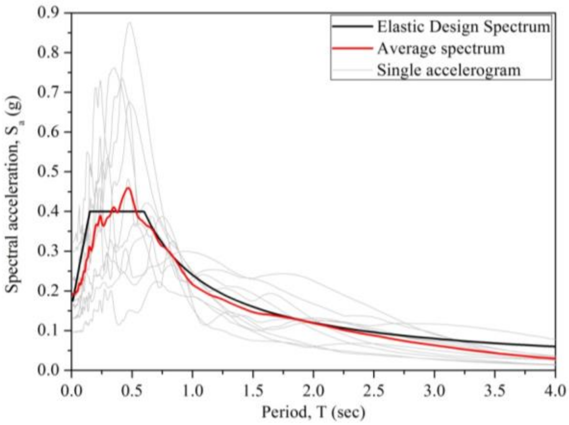


Fig. 7 Acceleration spectra of scaled pulse-type records used for the non-linear analyses

correction factors discussed in Kreslin and Fajfar (2012), and (c) the "basic" MPA for asymmetric buildings as introduced in Goel and Chopra (2004), considering the first three modes of each building. The latter was chosen instead of a more "advanced" version of the same procedure since it is, in our opinion, the most appealing version of the MPA for practical applications (more recent extensions require more complicated analyses that tend to defeat the simplicity for which the static pushover method is selected over the NL-THA).

4.1 Torsionally stiff buildings

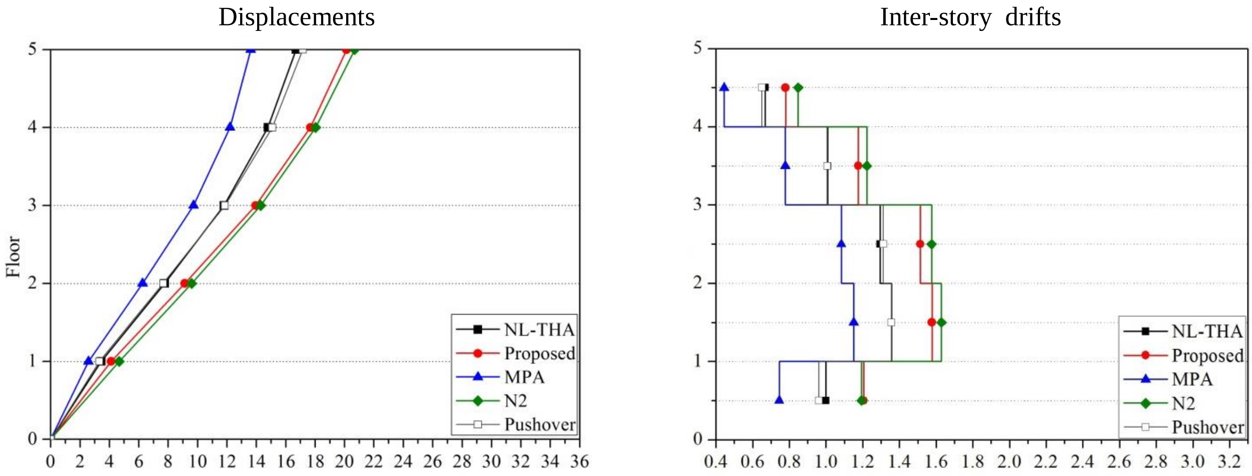
The results for the torsionally stiff buildings, RCS-RU and RCS-MU are summarized in Figs. 8 and 9. These graphs give the variation with height of total story displacements and interstory drifts for the stiff edge (top) and the flexible edge (bottom) of each building. In addition, numerical values, maximum, minimum and average for the % differences ΔR of the same response quantities, taken over the 5 stories in each case, are listed in Tables 3 and 4.

At first we must point out that the large computed displacements of both buildings are due to the very strong motions used - 3 times the design accelerations - selected to produce large inelastic deformations and thus test the method under conditions approaching collapse. Qualitatively, the deflected shapes of the two buildings are not very different but the same is not true for the distribution of their interstory drifts with height. It is interesting that the "typical" method gives deflected shapes that in three out of the four diagrams come closer to the shape by NL-THA and second closest in the fourth diagram (stiff side RCS-MU building). However, in nearly all the floors of both buildings and for both, stiff and flexible sides, the "typical" method gives interstory drifts below the NL-THA values, a fact that renders this method consistently unconservative. This is a major drawback of the method, for its use in practice.

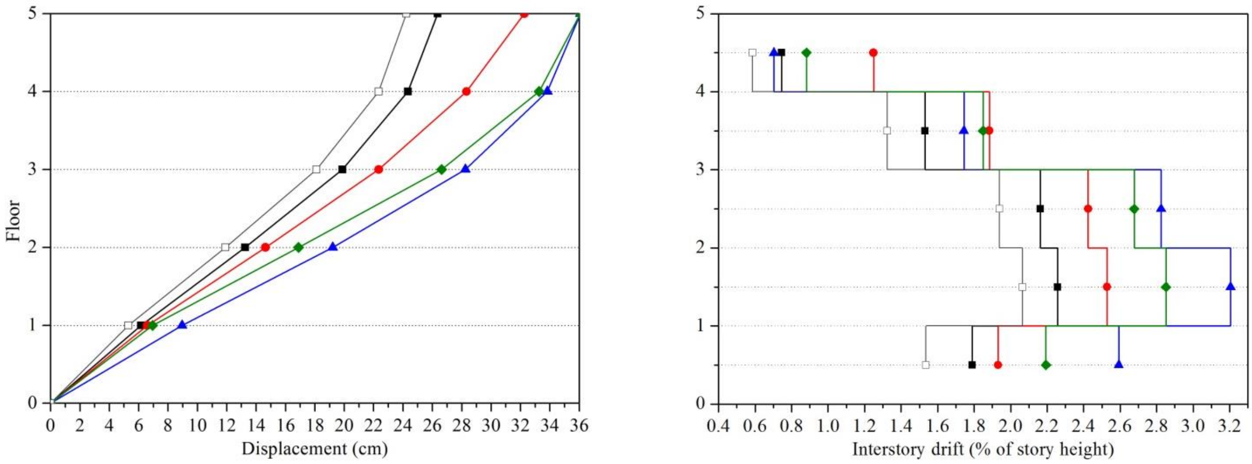
Comparing the new method, BA-3D, with the other two methods, the N2 and the MPA, we can see that overall gives results in better agreement with those from NL-THA. It is only at the flexible side of the fifth floor of the RCS-RU building that the proposed BA-3D method gives worse agreement than the N2 and MPA, while in most other cases i.e., both buildings, at both sides in all floors, its agreement with the NL-THA is better. In addition, the values by the proposed method are almost always on the conservative side as can be observed either on the graphs or from the positive signs of the ΔR values in Tables 3 and 4. This is also true with the N2 but not with the MPA method. However, the N2 method gives in most cases higher deviations than the proposed method. We note that a known systematic overestimation of response values by an approximate procedure is often desirable in practice as it provides extra safety for cases that may lie on the boundaries of the method's field of application.

The higher value of ΔR of the 5th floor interstory drift at the flexible side of the RCS-RU may be attributed to the simple triangular height-wise distribution of lateral loads used for our pushover analyses and perhaps is not related to the approach employed to account for torsion. Improving the height-wise distribution of the pushover results is a separate "challenge", not addressed in the initial form of BA-3D introduced herein. Such "corrections", however, have been included in the extended N2 method (an extra correction factor) and the MPA-3D (using an adequate number of modes) used herein.

Since both the BA-3D and the extended N2 methods use the same idea to account for torsion, i.e., they both use



(a) Stiff side



(b) Flexible side

Fig. 8 Comparison of the displacements and inter-story drifts at the stiff and flexible side of the RCS-RU building computed by NL-THA and 4 different pushover methods

Table 3 Deviations (%) of response quantities obtained from the pushover based methods for the RCS-RU building (torsionally stiff)

	Stiff Side							
	Displacements				Drifts			
	BA-3D	“Typical”	N2	MPA	**BA-3D**	“Typical”	N2	MPA
Max	**21**	1	36	-17	**20**	1	27	-15
Min	**18**	-5	21	-25	**15**	-6	19	-34
Average	**19**	2	25	-20	**16**	3	22	-23
	Flexible Side							
Max	**22**	-8	37	45	**65***	-8	26	45
Min	**7**	-13	13	37	**7**	-22	18	-6
Average	**14**	9	30	42	**23**	14	22	27

*Only for the top floor, possibly due to the height-wise distribution of pushover loads

Table 4 Deviations (%) of response quantities obtained from the pushover based methods for the RCS-MU building (torsionally stiff)

	Stiff Side							
	Displacements				Drifts			
	BA-3D	“Typical”	N2	MPA	**BA-3D**	“Typical”	N2	MPA
Max	**31**	-18	74	6	**37**	-16	64	5
Min	**23**	-36	38	3	**20**	-36	8	-11
Average	**27**	25	48	5	**27**	23	40	5
	Flexible Side							
Max	**23**	1	11	13	**22**	-1	15	12
Min	**12**	-5	-17	7	**-8**	-28	-17	-17
Average	**19**	2	10	10	**15**	10	11	9

correction factors based on the torsional response of simpler systems, any differences in results are attributed to the differences in these factors. As it has already been discussed, the factors are in both cases normalized displacements, calculated for the N2 method by elastic Response Spectrum Analysis of the full 3-D model of the building and for the BA-3D method by NL-THA of the equivalent one-story inelastic model. These normalized displacements, along with their values obtained by multiple NL-THAs of the full models of both buildings are presented

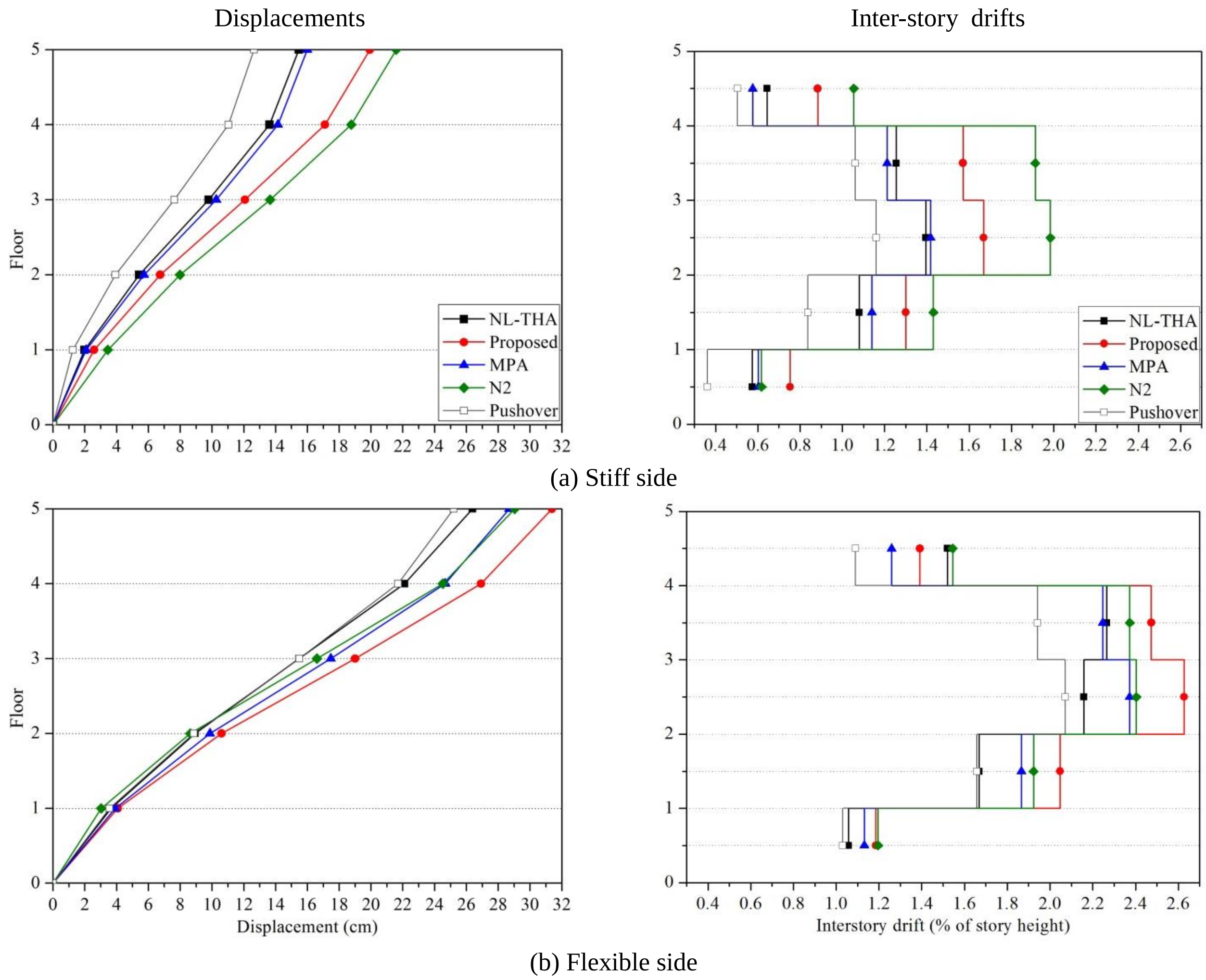


Fig. 9 Comparison of the displacements and inter-story drifts at the stiff and flexible side of the RCS-MU building computed by NL-THA and 4 different pushover methods

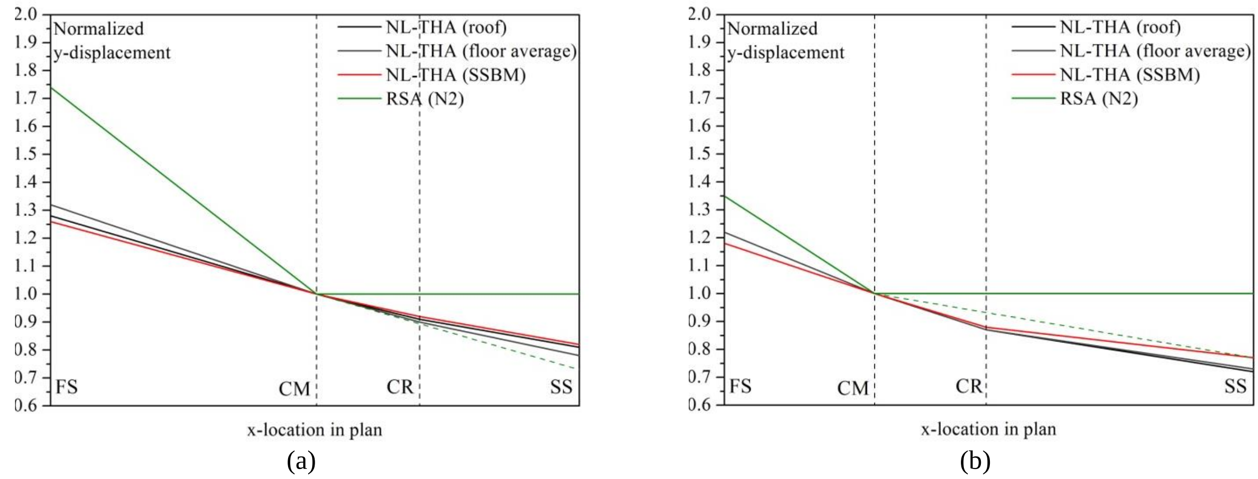


Fig. 10 Comparison of the normalized displacement values of the systems proposed in the BA-3D and N2 methods for (a) RCS-RU and (b) RCS-MU building with the actual values calculated from multiple non-linear time history analyses

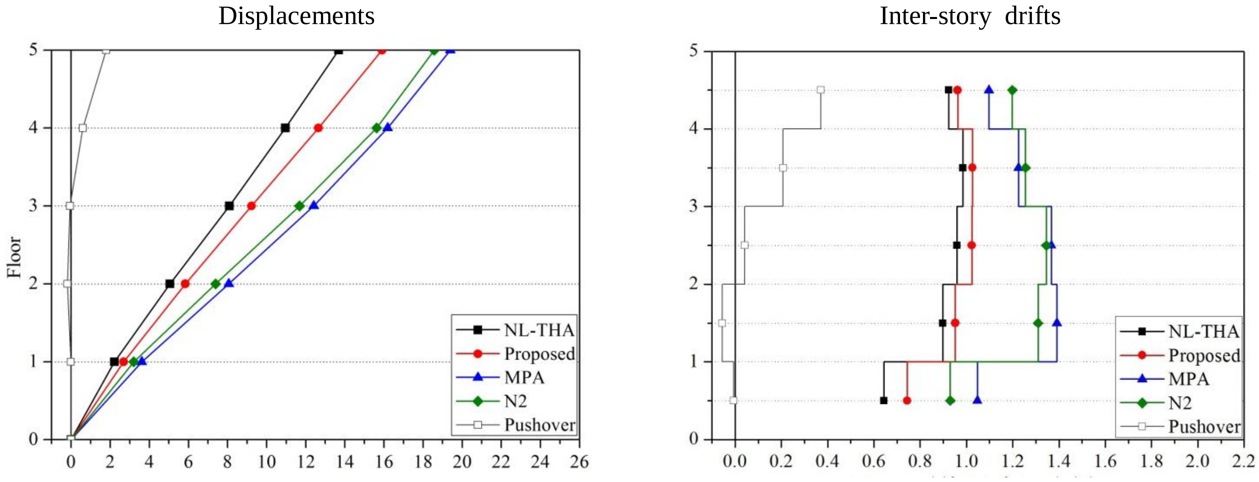
in Fig. 10. It should be noted that the correction factors of the BA-3D method are calculated by normalizing the displacements of the SSBM by those obtained at the CR (Eq. (2)).

However, normalization over the CM displacements was preferred in Fig. 10 for all the examined cases, for the sake of comparison with the N2 method. It is also noted that the N2 method does not consider de-amplification due to torsion; hence the normalized displacements on the stiff sides of the buildings are equal to 1. This conservative assumption is required since the elastic analyses employed may overestimate this effect. A similar assumption, however, was not required in the BA-3D method, since the analysis of the SSBM directly considers the effects of inelastic torsion with acceptable accuracy.

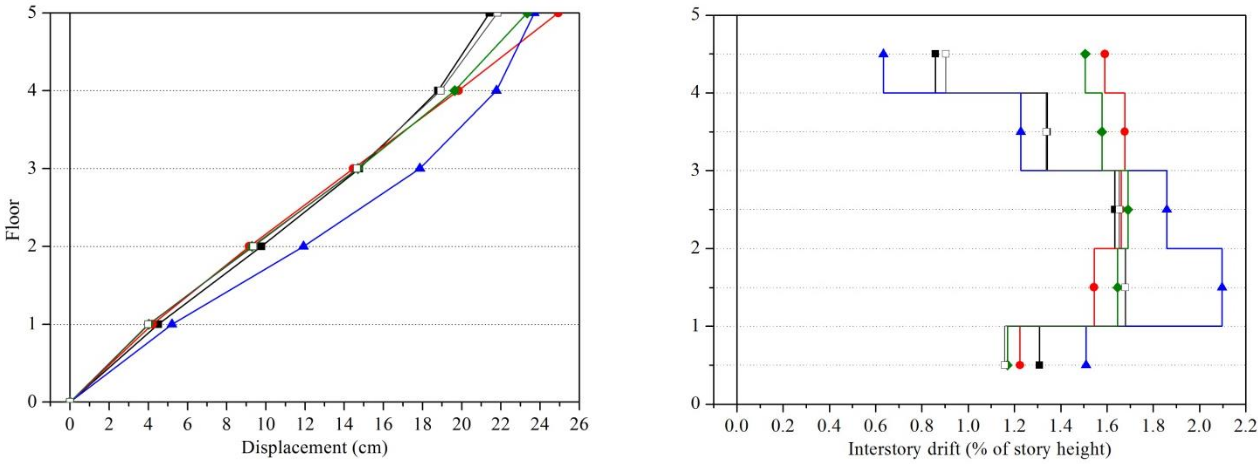
Therefore, as shown in Fig. 10, the normalized displacements used for the correction factors of the BA-3D method are very close to their actual values, whereas the N2 method's "elastic" values are significantly larger. Apparently, these are the main reasons for the often conservative response quantities observed in Figs. 9 and 10 by the N2 method and the better agreement achieved by the BA-3D procedure.

4.2 Torsionally flexible building

The inelastic response of torsionally flexible buildings may be difficult to control under strong earthquakes and for this reason modern codes impose increased requirements for their design. However, there will be cases in which such buildings may be unavoidable. Thus in our study we have included the torsionally flexible building with the layout of Fig. 4(b). The building, indicated as RCF-U, was subjected to the same set of ten earthquake motion pairs and the results - total displacements, interstory drifts and percent differences ΔR - are shown in Fig. 11 and Table 5. We observe that the "typical" method (pushover in the graphs) gives unacceptably low predictions at the stiff side and good predictions, in fact the best among all 4 methods, at the flexible side. We must note here that torsion in the "typical" procedure is accounted for in a static manner which amplifies results at the flexible side and de-amplifies them at the stiff side. However, in torsionally flexible buildings the dynamic effects of torsion may be expected to amplify results at both sides thus rendering the "typical" method inappropriate. The proposed BA-3D method gives the best results at the stiff side of all floors and good results at the flexible side, with the exception of the substantial overprediction (65%) of interstory drift at the last floor. We notice that the N2 method also overpredicts (55%) the same response quantity. On the other hand, the good agreement at the flexible side of the normalized displacements used to derive the correction factors for torsion of the BA-3D and N2 methods with the NL-THA values, Fig. 12, indicates



(a) Stiff side



(b) Flexible side

Fig. 11 Comparison of the displacements and inter-story drifts at the stiff and flexible side of the RCF-U building computed by NL-THA and 4 different pushover methods

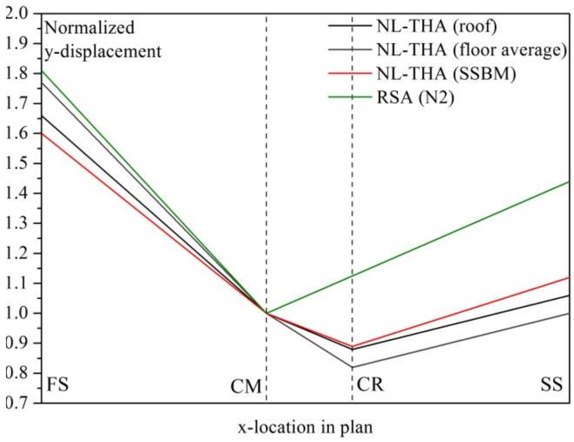


Fig. 12 Comparison of the normalized displacement values of the systems proposed in the BA-3D and N2 methods for the RCF-U building with their actual values calculated from multiple non-linear time history analyses

Table 5 Deviations (%) of response quantities obtained from the pushover based methods for the RCF-U building (torsionally flexible)

	Stiff Side							
	Displacements				Drifts			
	BA-3D	"Typical"	N2	MPA	**BA-3D**	"Typical"	N2	MPA
Max	**21**	-87	46	63	**21**	-59	46	63
Min	**14**	-103	36	42	**9**	-106	28	19
Average	**17**	97	43	53	**12**	88	38	41
	Flexible Side							
Max	**16**	2	9	22	**65**	5	55	25
Min	**-6**	-12	-11	11	**-8**	-11	-11	-26
Average	**7**	4	6	17	**25**	4	22	18

that the top floor overestimations stated above may not be associated with torsion but rather with the simple triangular distribution of the pushover loading. It is noted here that for the BA-3D procedure, no alternative load distributions with height were examined at this stage that could improve the results at the top stories. Thus, we believe that there is room to further improve the results presented in Fig. 11. As far as the MPA method is concerned, it gives also conservative results as the BA-3D and N2 do at the stiff side but at the flexible side its results are unconservative in the top two stories and quite conservative at the lower three stories. In conclusion, considering both the stiff and flexible side results, the BA-3D method overall seems to perform better than the other three methods, giving results almost always on the safe side.

Summarizing the above, it can be concluded that, for torsionally flexible buildings, the approximate, pushover methods should be used with caution and only as a supplemental tool, since they may lead to results of reduced accuracy especially at the most vulnerable to torsion flexible side of the building.

5. Conclusions

The simplicity and advantages of the static "pushover" analysis used for assessing the capacity of existing plane frames or symmetric structures, prompted researchers to (a) try improving it by modifying the original, simple triangular load distribution, and (b) extend the method to non-symmetric 3-D problems. Some of the proposed modifications, however, along these two lines, made the method too complicated to apply, and thus removed its main advantage over the more accurate inelastic dynamic analysis, normally carried out for a group of real or artificial motions. The latter is the most advanced method, as it accounts for both material and geometric nonlinearities as well as for the dynamic nature of the response.

In the present paper a new procedure, called BA-3D, was presented to extend the simple pushover analysis to one way eccentric buildings under the action of two-component earthquake motions. The method includes a simple pushover analysis with a triangular plane loading applied perpendicular to the axis of symmetry at the approximate location of the stiffness center and a subsequent correction of the computed results using factors determined to account for inelastic torsion. These factors are computed by multiple inelastic dynamic analyses of a 3-DOF, one story model of the building, determined to match its three lowest (fundamental) periods. This model maintains the layout of the resisting elements of the building whose plane stiffness and strength are determined by separate simple pushover analyses. Thus the philosophy of this method is similar to the 3-D extension of the N2 procedure, which uses also correction factors to account for torsion, except that these factors are determined by an elastic response spectrum analysis of the building.

The proposed method was tested with three, 5-story, one way eccentric buildings, two of them torsionally stiff, one with stiffness eccentricity and another with mass eccentricity, and a third, torsionally flexible building. It was also compared with the three best known extensions of pushover analysis to 3-D, namely with the "typical", the N2 and the MPA methods. The basis for comparison was results from multiple inelastic dynamic response history analyses (NL-THA).

The basic findings may be summarized as follows:

(a) The "typical" - plane - pushover analysis gives often unconservative results when applied to non symmetric 3D buildings, with large differences from results obtained by the NL-THA. Thus its use should be restricted to symmetric buildings.

(b) From the three extensions of the pushover analysis to non symmetric buildings examined, the proposed BA-3D method appears to produce conservative results more consistently than the other two (extended N2 and modified MPA) and with better overall agreement with those from NL-THA.

(c) Both the BA-3D and the extended N2 methods over predict substantially the last story interstory drift at the flexible side of the torsionally flexible building, which for the BA-3D may be attributed not to torsion but to the simple triangular distribution of the pushover lateral loading with height. This simple assumption of the lateral load distribution was considered because in its current state, our method focuses on the "challenge" of introducing inelastic torsion in a pushover based design or assessment method. Methods to improve the obtained results, especially for cases of vertically irregular and tall buildings, by varying the height-wise load distribution are currently under investigation. Thus, this type of buildings may be currently considered outside the field of application of the proposed method. On the other hand, our investigation has shown that in such cases all the examined methods should be used with extreme caution, since results of questionable accuracy may be obtained.

(d) In our opinion, all available extensions of pushover analysis for seismic assessment of torsionally flexible non symmetric buildings, including our own proposal, may lead to results of questionable accuracy and thus should be avoided. In such cases and perhaps in a few others, for which oversophisticated versions of pushover analyses have been developed defeating its main advantage of simplicity, the fully nonlinear dynamic solution for a set of properly selected ground motions should be used.

References

ASCE/SEI 41-06 (2007), Seismic Rehabilitation of Existing Buildings, American Society of Civil Engineers, Reston, Virginia, USA.

Anagnostopoulos S. A. and Roesset J. M. (1972), "Description and user's manual of the inelastic dynamic analysis program", Internal Study - Report No 16, Civil Engineering Department, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Anagnostopoulos, S.A., Alexopoulou, C. and Stathopoulos, K. (2010), "An answer to an important controversy and the need for caution when using simple models to predict inelastic earthquake response of buildings with torsion", *Earthq. Eng. Struct. D.*, **39**(5), 521-540.

Antoniou, S. and Pihno, R. (2004), "Development and verification of a displacement-based adaptive pushover procedure", *J. Earthq. Eng.*, **8**(5), 643-661.

ATC-40 (1996), Seismic evaluation and retrofit of concrete buildings, Applied Technology Council, Redwood City, California, USA.

Aydinoglu, M.N. (2003), "An incremental response spectrum analysis procedure based on inelastic spectral displacements for multi-mode seismic performance evaluation", *Bull. Earthq. Eng.*, **1**(1), 3-36.

Baros, D.K. (2014), "A new method for the extension of pushover analyses in asymmetric buildings and comparisons with existing methods", Ph.D. Dissertation, University of Patras, Patras. (in Greek)

Baros, D.K. and Anagnostopoulos, S.A. (2008a), "An assessment of static nonlinear pushover analyses in 2-D and 3-D applications", *3-D Pushover 2008*, Lisbon, Portugal.

Baros, D.K. and Anagnostopoulos, S.A. (2008b), "An overview of pushover procedures for the analysis of buildings susceptible to torsional behavior", *The 14th World Conference on Earthquake Engineering*, Beijing, China.

Carr, A.J. (2005), *Ruaumoko 3D: Inelastic Time-History Analysis of Three-Dimensional Framed Structures*, University of Canterbury, New Zealand.

Chintanapakdee, C. and Chopra, A.K. (2003), "Evaluation of modal pushover analysis using generic frames", *Earthq. Eng. Struct. D.*, **32**(3), 417-442.

Chopra, A.K. and Goel, R.K. (2002), "A modal pushover analysis procedure for estimating seismic demands for buildings", *Earthq. Eng. Struct. D.*, **31**(3), 561-582.

Chopra, A.K. and Goel, R.K. (2004), "A modal pushover analysis procedure to estimate seismic demands for unsymmetric-plan buildings", *Earthq. Eng. Struct. D.*, **33**(8), 903-927.

De Stefano, M. and Pintucchi, B. (2010), "Predicting torsion-induced lateral displacements for pushover analysis: Influence of torsional system characteristics", *Earthq. Eng. Struct. D.*, **39**(12), 1369-1394.

Eurocode 8 (2004), Design of structures for earthquake resistance, European Committee for Standardization (CEN), Brussels, Belgium.

Elnashai, A.S. (2001), "Advanced inelastic static (pushover) analysis for earthquake applications", *Struct. Eng. Mech.*, **12**(1), 51-69.

Elnashai, A.S. (2002), "Do we really need inelastic dynamic analysis?", *J. Earthq. Eng.*, **6**(Special Issue 1), 123-130.

Fajfar, P. (2000), "A nonlinear analysis method for performance-based seismic design", *Earthq. Spectra*, **16**(3), 573-592.

Fajfar, P., Marusic, D. and Perus, I. (2005), "Torsional effects in the pushover-based seismic analysis of buildings", *J. Earthq. Eng.*, **9**(6), 831-854.

FEMA-356 (2000), Prestandard and commentary for the Seismic Rehabilitation of Buildings, Federal Emergency Management Agency, Washington D.C., USA.

Gates, W.E., Marshall, P.W. and Mahin, S.A. (1977), "Analytical methods for determining the ultimate earthquake resistance for fixed offshore structures", *The 9th Annual Offshore Technology Conference*, Houston, Texas.

Goel, R.K. (2005), "Evaluation of modal and FEMA pushover procedures using strong-motion records of buildings", *Earthq. Spectra*, **21**(3), 653-684.

Goel, R.K. and Chopra, A.K. (2004), "Evaluation of Modal and FEMA pushover analyses: SAC buildings", *Earthq. Spectra*, **20**(1), 225-254.

Greek Seismic Retrofitting Code (KAN.EPE) (2012), Hellenic Organization for Seismic Planning and Protection (OASP), Athens, Greece. (in Greek)

Gupta, P. and Kunnath, S.K. (2000), "Adaptive spectra-based pushover procedure for seismic evaluation of structures", *Earthq. Spectra*, **16**(2), 367-391.

Kaatsiz, K. and Sucuoglu, H. (2014), "Generalized force vectors for multi-mode pushover analysis of torsionally coupled systems", *Earthq. Eng. Struct. D.*, **43**(13), 2025-2043.

Kallaby, J. and Millman, D. (1975), "Inelastic analysis of fixed offshore platforms for earthquake loadings", *The 7th Annual Offshore Technology Conference*, Houston, Texas.

Krawinkler, H. and Seneviratna, G.D.P.K. (1998), "Pros and cons of a pushover analysis of seismic performance evaluation", *Eng. Struct.*, **20**(4-6), 452-464.

Kreslin, M. and Fajfar, P. (2012), "The extended N2 method considering higher mode effects in both plan and elevation", *Bull. Earthq. Eng.*, **10**(2), 695-715.

Lawson, R.S., Vance, V. and Krawinkler, H. (1994), "Nonlinear static pushover analysis - why, when, and how?", *The Fifth U.S. National Conference on Earthquake Engineering*, Oakland, California.

Lin, J.L. and Tsai, K.C. (2007), "Simplified seismic analysis of asymmetric building systems", *Earthq. Eng. Struct. D.*, **36**(4), 459-479.

Litton, R.W. (1975), "A contribution to the analysis of concrete structures under cyclic loading", Ph.D. Dissertation, Department of Civil Engineering, University of California at Berkeley, California.

Marusic, D. and Fajfar, P. (2005), "On the inelastic sesimic response of asymmetric buildings under bi-axial excitation", *Earthq. Eng. Struct. D.*, **34**(8), 943-963.
Otani, S. (1974), "Inelastic analysis of R/C frame structures", *J. Struct. Div.*, ASCE, **100**, 1433-1449.
Penelis, G.G. (2007), "Contribution in the inelastic analysis of R/C buildings with irregular positioning of their stiffness elements in plan", Ph.D. Dissertation, Aristotle University of Thessaloniki, Thessaloniki. (in Greek)
Perus, I. and Fajfar, P. (2005), "On the inelastic torsional response of single-storey structures under bi-axial excitation", *Earthq. Eng. Struct. Dyn.*, **34**(8), 931-941.
Reyes, A.C. and Chopra, A.K. (2011), "Evaluation of three-dimensional modal pushover analysis for unsymmetric-plan buildings subjected to two components of ground motion", *Earthq. Eng. Struct. Dyn.*, **40**(13), 1475-1494.
Shakeri, K., Tarbali, K. and Mohhebi, M. (2012), "An adaptive modal pushover procedure for asymmetric-plan buildings", *Eng. Struct.*, **36**, 160-172.
Sucuoglu, H. and Gunay, M.S. (2011), "Generalized force vectors for multi-mode pushover analysis", *Earthq. Eng. Struct. D.*, **40**(1), 55-74.
Vamvatsikos, D. and Cornell, C.A. (2002), "Incremental dynamic analysis", *Earthq. Eng. Struct. D.*, **31**(3), 491-514.

NM