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Seismic vibration control of bridges with excessive isolator displacement

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Abstract. The effectiveness of base isolation (BI) systems for mitigation of seismic vibration of bridges have been extensively studied in the past. It is well established in those studies that the performance of BI system is largely dependent on the characteristics of isolator yield strength. For optimum design of such systems, normally a standard nonlinear optimization problem is formulated to minimize the maximum response of the structure, referred as Stochastic Structural Optimization (SSO). The SSO of BI system is usually performed with reference to a problem of unconstrained optimization without imposing any restriction on the maximum isolator displacement. In this regard it is important to note that the isolator displacement should not be arbitrarily large to fulfil the serviceability requirements and to avoid the possibility of pounding to the adjacent units. The present study is intended to incorporate the effect of excessive isolator displacement in optimizing BI system to control seismic vibration effect of bridges. In doing so, the necessary stochastic response of the isolated bridge needs to be optimized is obtained in the framework of statistical linearization of the related nonlinear random vibration problem. A simply supported bridge is taken up to elucidate the effect of constraint condition on optimum design and overall performance of the isolated bridge compared to that of obtained by the conventional unconstrained optimization approach.

Keywords: seismic vibration; base isolation; bridges; stochastic structural optimisation; constrained optimisation

1. Introduction

Bridges are the most important part in transportation system and also the most vulnerable component to seismic excitation. The damages of bridges after a large magnitude earthquake not only disrupt the transportation facility, but also limit the post-earthquake management by severely restricting the movement of emergency vehicles and disaster relief and restoration operation. A large number of bridges in the past have suffered damages during some major earthquakes. The

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failure of bridges in the 1995 Hyogo-Ken Nanbu earthquake,1994 Northridge earthquake, 1989 Loma Prieta earthquake, 1971 San Fernando earthquake are noteworthy in this regard (Moehle and Eberhard 2000). The predominant causes of failures were envisaged as the failure of the columns following the pounding at expansion hinges and bearing failure. The bridge column failure is the most common cause of bridge collapse due to inadequate detailing which limits the inelastic deformation capacity and shear strength of columns. In current design practice, bridge superstructures are mainly designed to behave elastically during earthquake as the failure of superstructures affect the functionality of bridges for a long period. Failure of foundations cannot be easily assessed and retrofitted. Hence, bridge substructure is the only component that can be allowed to dissipate input seismic energy through inelastic action. However, the dissipation of input seismic energy through inelastic deformations distributed over the structural elements cannot be fully relied in case of bridge structures. This is primarily because, unlike building structures, bridges lack redundancy because of their inherently simple structural configurations. Lack of redundancy prevents bridges from availing adequate ductility demand enforced by strong motion from large earthquakes. Thus, vibration control technology is gaining momentum for improving functionality of bridge structures during and succeeding an earthquake.

The effectiveness of isolation devices for aseismic design of bridges and their performances have been extensively studied in the past (Turkington *et al.* 1988, Li 1989, Kunde 2006, Dicleli and Buddaram 2006, Abdel 2009, Madhekar and Jangid 2009, Ozbulut and Hurlebaus 2011, Bhuiyan and Alam 2013, Dezfuli and Alam 2014). A comprehensive review on the subject can be obtained in Kunde and Jangid (2003).These studies provide good insight into the behaviour of such systems. The effectiveness of sliding bearing (Constantinou *et al*. 1993, Wang *et al.*1998), rubber bearings and hysteretic dampers (Pagnini and Solari 1999, Jangid 2008a), shape memory based rubber bearing (Mishra *et al.* 2016) also have been reported. It is well established that the performance of isolator in bridge largely depends on the characteristics of the isolator parameters such as the yield strength for lead rubber bearings (LRBs) (Jangid 2008a), optimal damping for resilient friction bearing and optimal friction for friction pendulum system (Jangid 2008b). The response evaluations in the process of optimization in these studies are largely based on the deterministic transient response of BI system under several real and or simulated earthquake ground motions. Usually, a standard nonlinear optimization problem is formulated to minimize the stochastic response of structure, referred as SSO (Nigam 1972). The optimization study is based on minimizing superstructure response, such as top floor acceleration. The reduction of force transmitted to the superstructure (i.e., reduction in acceleration) is achieved by filtering effect of isolator due to its elongated time period, achieved through its flexibility. Large flexibility, almost inevitable, causes large shear deformation to the isolator. But, the displacement of an isolator must not be arbitrarily large. The isolator displacement should be limited to fulfil the serviceability requirements as well as reducing the possibility of pounding to the adjacent units (Jankowski *et. al.* 1998, 2000, Zhu 2004). However, the conventional SSO of BI system, as mentioned above, has been performed with reference to a problem of unconstrained optimization. Such SSO approach does not impose any restrictions on the maximum amplitude of isolator displacement. The consideration of relative motion of damper in tuned mass damper design (Marano *et al.* 2007), in liquid column damper design (Chakraborty *et al.* 2012) and also in BI design (Das *et al.* 2015) for seismic vibration control of building frame are notable. But, the effects of excessive isolator displacement of BI system for bridges in seismic vibration mitigation are not studied; though the possibility of pounding to the adjacent unit appeared to be more critical for bridge structures.

With the above in view, the present study is intended to study the effect of possible excessive displacement of isolator in optimum performance of BI system for seismic vibration mitigation of bridges. In doing so, the necessary response of the isolated bridge required for solving the optimization problem is obtained in the framework of statistical linearization of the related nonlinear random vibration problem. The constrained optimum design parameters, thus obtained, are compared to the conventional unconstrained optimum parameter. The disparity of performance of the isolated bridge adopting constrained and unconstrained optimum parameters is demonstrated by considering a LRB isolator, isolating a deck from the pier of a simply supported bridge system.

2. Stochastic response of bridge deck isolated by LRB

A typical bridge deck isolated by LRB placed between the top of the piers and the decks is shown in Fig. 1(a). The type of bridge considered herein consists of discontinuous decks simply supported on the piers in each span and on the abutments at the ends. One typical such deck-pier system is considered in Fig. 1(b) for analysis. The equivalent mechanical model of the deck-pier system is depicted in Fig. 1(c).The associated bi-linear force-deformation behaviour is depicted in Fig. 1(d). The pier is discretized into number of nodes with lateral degrees of freedom. The masses of each segment are assumed to be distributed between the two adjacent nodes in the form of point masses, while the deck is treated as rigid. The equation of motion of the idealized such system can be written as

$$\left[M_p\right]\left\{\ddot{x}_p\right\}+\left[C_p\right]\left\{\dot{x}_p\right\}+\left[K_p\right]\left\{x_p\right\}-\{\psi\}F_p=-\left[M_p\right]\{r\}\ddot{x}_g \tag{1}$$

in which $[M_p]$,$[C_p]$ and $[K_p]$ are respectively, the mass, damping and stiffness matrix of size $n\times n$ of the pier which is free at the top. In the present problem, the number of nodes considered in the pier is five (n=5). The relative acceleration, velocity and displacement of the pier with respect to ground are denoted as $\left\{\ddot{x}_p\right\}$,$\left\{\dot{x}_p\right\}$ and $\left\{x_p\right\}$. The displacement vector of different discretized nodes of the pier is $\{x_p\}=\{x_1,x_2,\ldots\ldots,x_n\}^{\mathrm{T}}$. The force exerted by the LRB at the pier level is given by F_p. A vector $\{\psi\}$ of size $nx1$ and the form $\{1 \;\; 0 \;\; \ldots \;\; 0\}^T$ i.e., all terms zero except the first one is introduced to write the F_p in vector form which is compatible with the dimension of the vectors used in Eq. (1). The vector $\{r\}=\{1 \;\; 1 \;\; \ldots \;\; 1\}^{\mathrm{T}}$ is the influence coefficient vector and $\ddot{x}_g$ is the ground acceleration of the earthquake. The equation of motion for the rigid deck can be written as,

$$m_d\ddot{x}_d+F_p=-m_d\ddot{x}_g \tag{2}$$

where, m_d is the mass of the bridge deck, $\ddot{x}_d$ is the acceleration of the deck relative to the ground. F_p is the restoring force of the isolator described by differential Bouc-Wen model (Bouc 1967, Wen 1976, Ismail *et al.* 2009). The force can be expressed in terms of the relative displacement and velocity of the deck and deck-pier system as

$$F_p=c_b\dot{u}_p+\alpha k_b u_p+(1-\alpha)F_y Z_p \tag{3}$$

where, c_b represent the viscous damping of the LRB, k_b is the initial stiffness of the LRB, α is an index representing the ratio of the post to pre yield stiffness of the LRB. $\dot{u}_p$ and u_p represents the relative velocity and displacement in the isolator, respectively. $u_p=x_d-x_1$, x_1 is the displacement of the top node of the pier and x_d is the displacement of the deck relative to the ground.

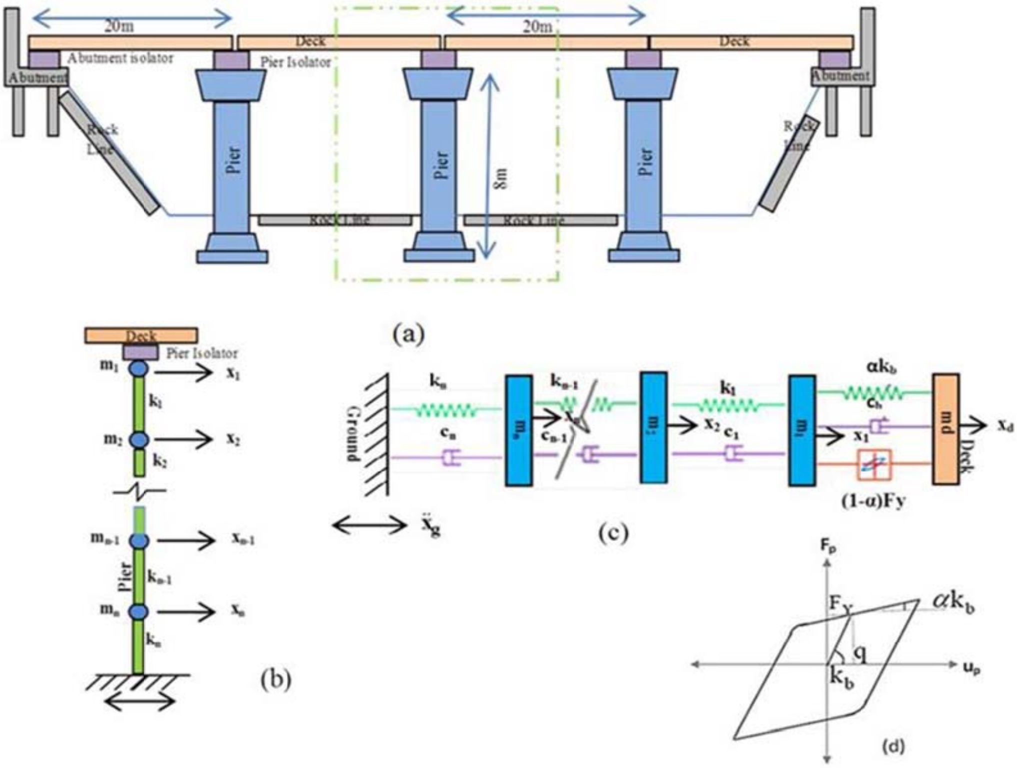

Fig. 1 (a) Isolated bridge deck-bearing-pier system (b) Simple structural model for the isolated bridge (c) Equivalent mechanical model and (d) The bilinear model

F_Y is the yield strength of the isolator. Z_p is a variable quantifying the hysteretic response of the isolator, expressed through Bouc-Wen model as

$$q\dot{Z}_p = -\gamma\left|\dot{u}_p\right|Z_p\left|Z_p\right|^{\eta-1} - \beta\dot{u}_p\left|Z_p\right|^{\eta} + \delta\dot{u}_p \tag{4}$$

where, q is the yield displacement of the isolator. The four parameters β,γ,η and δ appear in Eq. (4) and parameter α in Eq. (3) control the shape of the hysteretic loop. The parameter η characterises the transition from elastic to plastic phase. Note that as $\eta\rightarrow\infty$ (infinity) the behaviour becomes bilinear. With increasing value of η the elastic to plastic transition becomes increasingly sharp. However, due to smooth nature of the of the Bouc-Wen model this transition is smooth. The ideal sharp bilinear nature can only be attained at $\eta\rightarrow\infty$ (infinity). However, with the presently adopted value of η=1, the smooth transition can adequately be taken close enough to the ideal bilinear behaviour. The parameter β depicts the nature of the model e.g., β>0 implies hardening and β<0 yields softening. The parameters γ and δ control the shape and size of the hysteresis loop. The parameters adopted in the present study are: α=0.05, β=γ=0.5, δ=1 and η=1, which corresponds to the bi-linear force deformation characteristics as shown in Fig. 1(d). The stiffness and damping of the isolator is selected to provide the restoring force. The post-yield stiffness (αk_b) of the isolator is selected in order to provide specific isolation time period, T_b=$2\pi\sqrt{m_d / \alpha k_b}$ i.e., ω_b=$2\pi/T_b$ is the circular frequency of the isolator. Most of the published literatures prefer this definition, instead of effective stiffness based on secant modulus. It may be noted here that the isolators are designed to have quite low yield strength so that it can plastically deforms at reasonably moderate level of seismic excitations, yet strong enough to avoid yielding under nominal lateral loading conditions i.e., an earthquake of very low intensity. This will also ensure elongated period of isolation which is not possible to realize with un-yielded high initial stiffness. In fact, in some devices (such as super-elastic Shape Memory Alloy based isolators) an amount of pre-stretch is also applied to get rid of high value of initial pre-yield strength. Therefore, during strong shaking, as the isolator mobilizes its yield strength to yield, it will remain almost entirely in the post yield regime during the course of strong ground motion and the effective stiffness can reasonably be taken as the post yield strength. This is the reason why the post yield strength is commonly adopted to define the period of isolation, not the effective stiffness. The viscous damping of the isolator is given by, c_b=$2\xi_b m_d \omega_b$ in which ξ_b is the viscous damping ratio of the isolator. The yield strength is conveniently normalized with respect to the deck weight (W=$m_d g$) i.e., yield strength F_0=F_Y/W, g is the gravitational acceleration.

The stochastic analysis of the system under random earthquake requires number of recorded ground motions at a site. However, in absence of sufficient statistical data, available stochastic models for earthquake loading are utilized. A widely adopted model for stationary ground motion ($\ddot{x}_g$) is obtained by filtering a white noise process acting at the rock bed through a filter representing the soil. This is the well acclaimed Kanai-Tajimi model (Kanai 1957, Tajimi 1960) characterizing the input frequencies of earthquakes for a wide range of practical situations. While subjected to white noise rock bed motion, this filter gives output of colour noise having Kanai-Tajimi Power Spectral Density (PSD). The filter equations are expressed as

$$\ddot{x}_g = \ddot{x}_f + \ddot{w} \tag{5a}$$

$$\ddot{x}_f + 2\xi_g \omega_g \dot{x}_f + \omega_g^2 x_f = -\ddot{w} \tag{5b}$$

Substituting Eq. (5b) in Eq. (5a) yields

$$\ddot{x}_g = -2\xi_g \omega_g \dot{x}_f - \omega_g^2 x_f \tag{6}$$

in which $\ddot{w}$ is the white noise intensity at the rock bed having PSD of S_0, ω_g and ξ_g are the characteristic frequency and damping of the soil strata over the rock bed and underlying the pier. $\ddot{x}_f$, $\dot{x}_f$ and x_f are the acceleration, velocity and displacement response of the filter.

The nonlinear force-deformation characteristic of the LRB as represented by Eq. (4) is too complicated to be readily incorporated in the state-space formulation for evaluating the response of BI system accounting for fluctuation involve due to system parameter uncertainty. The statistic response evaluation is thus conducted by utilizing statistical linearization technique (Roberts and Spanos 1990). The equivalent linear form of the nonlinear Eq. (4) is obtained as

$$q\dot{Z}_p + C_e \dot{u}_p + K_e Z_p = 0 \tag{7}$$

where, C_e and K_e are the equivalent damping and stiffness obtained by the least square error minimization among the linear and nonlinear terms of Eqs. (7) and (4). For η=1, the equivalent damping and stiffness of the isolator can be obtained in closed form as

$$C_e = \sqrt{\frac{2}{\pi}}\left\{\gamma \frac{E\left[\dot{u}_p Z_p\right]}{\sqrt{E\left[\dot{u}_p^2\right]}} + \beta\sqrt{E\left(Z_p^{\,2}\right)}\right\} - \delta,\; K_e = \sqrt{\frac{2}{\pi}}\left\{\gamma\sqrt{E\left[\dot{u}_p^2\right]} + \beta \frac{E\left[\dot{u}_p Z_p\right]}{\sqrt{E\left[Z_p^{\,2}\right]}}\right\} \tag{8}$$

where, $E[\ ]$ is the expectation operator. It is noted that even though the differential Bouc-Wen model of the isolator is stochastically linearized for easy incorporation in the state space formulation, the relevant equivalent damping (C_e) and stiffness (K_e) are still functions of the system response. This implies that the nonlinearity of isolator is still present.

The deck, pier and filter equations are now rearranged to express those in state space form suitable for stochastic response evaluation. Substituting Eqs. (3) and (6) in Eq. (2) and normalizing with respect to m_d, the equation of pier i.e., Eq. (2) can be written as

$$\ddot{x}_d = -\frac{c_b}{m_d}\dot{u}_p - \frac{\alpha k_b}{m_d}u_p - (1-\alpha)\frac{F_y}{m_d}z_p + 2\xi_g\omega_g\dot{x}_f + \omega_g^2 x_f \tag{9}$$

Multiplying both sides of Eq. (1) with $[\mathrm{M}]^{-1}$ and substituting the expression of $\ddot{\mathrm{x}}_{\mathrm{g}}$ from Eq. (6), Eq. (1) can be rewritten as

$$\begin{aligned}\left\{\ddot{x}_p\right\} = &-\left[M_p\right]^{-1}\left[C_p\right]\left\{\dot{x}_p\right\} - \left[M_p\right]^{-1}\left[K_p\right]\left\{x_p\right\} - \left[M_p\right]^{-1}\{\psi\}F_p \\ &+ \{r\}\left(2\xi_g\omega_g\dot{x}_f + \omega_g^2 x_f\right)\end{aligned} \tag{10}$$

The linearized equation for the hysteretic isolator, obtained through stochastic linearization depicted by Eq. (6) can be rewritten as,

$$\dot{Z}_p = -\frac{C_e}{q}\dot{u}_p - \frac{K_e}{q}Z_p \tag{11}$$

The Eqs. (9) to (11) and (6) are expressed in state space form, where the state vector is defined as $\{Y\} = \left\{x_d \quad \{x_p\} \quad Z_p \quad x_f \quad \dot{x}_d \quad \{\dot{x}_p\} \quad \dot{x}_f\right\}^T$. The state space form of the equation of motion becomes

$$\frac{d}{dt}\{Y\} = [A]\{Y\} + \{w\} \tag{12}$$

Where, $[A]$ is the augmented system matrix and $\{w\}$ is a vector containing the terms of the rock bed white noise excitations, expressed as $\{w\} = [0 \quad \{0\} \quad 0 \quad 0 \quad 0 \quad \{0\} \quad -\ddot{w}]^T$. $\{Y\}$ has the length of (2N+5), N is the number of structural degrees of freedom. The detail of the augmented system matrix $[A]$ is provided in the Appendix. The response of the system can be evaluated by solving Eq. (12) by numerical Runge-Kutta integration method. In stochastic analysis, rather than the response, the statistics such as covariance of responses are evaluated. Assuming the stochastic response processes to be Markovian, the evolution equation for the response covariance matrix of the state vector can be readily obtained as (Lutes and Sarkani 1997)

$$\frac{d}{dt}[C_{YY}] = [A][C_{YY}]^T + [C_{YY}][A]^T + [S_{ww}] \tag{13}$$

in which the elements of $[C_{YY}]$ consist of the covariance of responses as $C_{Y_iY_j} = E\left[Y_iY_j\right]$ and $\left[S_{ww}\right]$ is a matrix containing the terms of rock bed white noise excitation. Following the structure of the vector $\{w\}$, matrix $\left[S_{ww}\right]$ has all terms zero except the last diagonal as $2\pi S_0$. The details of $\left[S_{ww}\right]$ matrix are given in Appendix. The above matrix equation can be readily solved to obtain the covariance of responses. The response statistics of the derivative process of the response (such as acceleration $\{\ddot{x}_p\}, \ddot{x}_d$) is then obtained as

$$\left[C_{\dot{Y}\dot{Y}}\right] = [A][C_{YY}]^T[A]^T + \left[S_{ww}\right] \tag{14}$$

The Root-Mean-Square (RMS) of individual response is obtained as $\sigma_{Y_i} = \sqrt{C_{Y_iY_j}}$. The absolute acceleration $\left(\ddot{u}_d\right)$ of the deck is the sum of the relative acceleration of the deck and the ground obtained as

$$\ddot{u}_d = \ddot{x}_d + \ddot{x}_g \tag{15}$$

The RMS of the absolute deck acceleration are obtained by summing up the RMS acceleration of the deck and the ground as

$$\boldsymbol{\sigma}_{\ddot{\boldsymbol{u}}_{\boldsymbol{d}}} = \sqrt{\sigma_{\ddot{x}_d}^2 + \sigma_{\ddot{x}_g}^2} \tag{16}$$

The RMS of the relative displacement of bearing on the top of the pier is obtained as

$$u_p = \left(x_d - x_1\right) \tag{17}$$

$$\sigma_{u_p} = \sqrt{\sigma_{x_d}^2 + \sigma_{x_1}^2 - 2cov\left(x_d, x_1\right)} \tag{18}$$

The stochastic response analysis presented herein is based on the assumption that the excitations and the responses are Gaussian and stationary stochastic process. Extension to non-stationary earthquake model can be made by evaluating the time varying response statistics. This requires time dependent characteristic frequency and damping of ground to be incorporated in the analysis procedure, presented earlier.

3. Optimization of BI system

The stochastic dynamic response evaluation of the bi-linear isolator presented in the previous section clearly revealed that the characteristic parameters of an isolator are the isolation time period (T_b), coefficient of viscous damping (ξ_b) and normalized yield strength (F_0) of the isolator. The previous studies (Baratta and Corbi 2004, Jangid 2010) indicate that the RMS acceleration of structure decreases with the increase of isolation period and damping. On the other hand, the bearing displacement increases with the increase in isolation period. This is obvious as the increase of time period indicate more flexible isolator i.e., more capability to plastically deform leading to more energy absorbing capability. Thus, the earthquake force transmitted to the structure is reduced at the expense of increasing relative displacement of the bearings. However, with increase in the yield strength of LRB, the RMS deck acceleration reduces first and attains a minimum value, and then it increases with the increase of yield strength. This indicates that there exists optimum yield strength of LRB system for which the acceleration of the superstructure attains the minimum value. Thus, normalized yield strength, F_0 is taken as the design variable in the present study. In this regard it is to be noted that the relative displacement of the LRB has a practical limitation. Therefore, in designing the isolation system a compromise is necessary between transmitted earthquake forces and relative bearing displacements.

3.1 Stochastic optimization of BI system

The optimum design of BI system is usually obtained by minimizing the vibration effect of structure under stochastic earthquake load. The optimization problem for a structure subject to random load like earthquake could be formulated as the search of a suitable set of variables collected in the so called design vector over an admissible domain. For structures subjected to stochastic excitation, the measure of performance can be given in terms of mean square responses (displacement, acceleration, stress etc.). The failure probability of the structure or the total life-cycle cost of the structure can also be used as the performance index. The conventional SSO problem, so defined, is usually transformed into a standard nonlinear programming problem. One of the much used approaches is to minimize the RMS response of structure. The response or reliability being nonlinear functions of the design variables requires the solution of a nonlinear optimization problem termed as SSO. More details may be found elsewhere (Crandall 1960, Jensen 2006, Taflanidis 2008).The deck RMS acceleration is considered as the objective function in the present study. The SSO problem is stated as following

$$\text{Find } \mathrm{F}_0 \text{ to minimize } \sigma_{\ddot{u}_d} \tag{19}$$

The optimization problem, stated above, can be solved by readily available optimization algorithm.

3.2 Constrained stochastic optimization of BI system

The optimization of BI system under stochastic earthquake load as presented above is basically an unconstrained SSO which does not impose any restriction to determine the optimum value of the normalized yield strength, F_0. As already discussed, such an approach cannot incorporate the limited values of the isolator displacement. Keeping in view that isolator displacement is an important quantity in practical design of BI system; the problem is reformulated as constrained non-linear optimization problem. The peak value of the isolator displacement u_p (denoted as P_{u_p}) can be given by (Lutes and Sarkani 2004, Sun 2006)

$$P_{u_p} = \bar{k}\sigma_{u_p} \tag{20}$$

In which, $\bar{\mathrm{k}}$ is the peak factor which can be obtained as

$$\bar{k} = \sqrt{2\,ln(\nu T\,)} + \frac{0.577}{\sqrt{2\,ln(\nu T\,)}} \tag{21}$$

In which, T is the duration of the ground motion and ν is a factor defined as

$$\nu = \frac{\sigma_{\ddot{u}_p}}{\sigma_{u_p}} \tag{22}$$

where, $\sigma_{\ddot{u}_p}$ is the RMS acceleration of the isolator. The optimization problem is redefined by incorporating the peak isolator displacement constraint as following

$$\text{Find } \mathrm{F}_0 \text{ to minimize } \sigma_{\ddot{u}_d} \text{ such that } P_{u_p} \le d \tag{23}$$

where, *d* is the maximum allowable isolator displacement. The values of *d* will be governed by design considerations. The optimization problems cited above is solved by using the gradient based standard nonlinear optimization routine available in the MatLab Optimization Toolbox. However, for more complex configuration, genetic algorithm based search techniques are robust choice as such approach is independent from the initial choice of solution and also does not require information on the gradient of the objective function.

4. Numerical study

The simply supported bridge as shown in Fig. 1(a) is taken up to study the effect of constraint of isolator displacement on the optimum performance of seismic vibration mitigation of bridge isolated by LRB type isolator. The mass of the deck is 144×10^3 kg and the mass of each pier is 21.6×10^3 kg. The typical span length of each simply supported deck is 20 m, and each pier is 8 m tall. The moment of inertia and Young's modulus of the piers are given as 0.1 m^4 and 20.67×10^9 N/m^2, respectively. The isolated bridge is modelled as a six degree of freedom system with the LRB system. Unless specifically mentioned, the damping in the deck and piers is taken as 2% of the critical value in all the modes of vibration and the time period of the pier is assumed as 0.3 sec. The time period and the viscous damping of the LRB are taken as 2.5 sec and 5%, respectively. The yield displacement (*q*) of the isolator is considered to be 0.025 m. The mean values of the parameters characterizing the stochastic earthquake load are taken as: $\omega_f = 5\pi$ rad/sec, $\xi_f = 0.6$, $S_0 = 0.05 m^2/s^3$ and T=20 sec. With these mean data, the RMS acceleration of deck of the bridge without isolator is 6.99 m/s^2.

The optimum yield strength of the LRB is obtained by solving both the unconstrained optimization problem described by Eq. (19) and constrained optimization problem defined by Eq. (23) with varying allowable isolator displacement for different time period of the pier. The results are depicted in Fig. 2. The respective accelerations reduction ratios of the deck are shown in Fig. 3. The RMS acceleration reduction ratio is defined as the ratio of the RMS acceleration of the deck of the isolated bridge to that of the bridge without isolator.

It may be noted that the efficiency of the BI system is less i.e., RMS acceleration reduction is less when the constraint condition is considered in the optimization procedure compare to the unconstrained case for all three time periods. The isolator time period, damping and intensity of earthquake are taken as, 2.5 s, 5% and 0.05 m^2/s^3, respectively for developing these plots.

The results clearly indicate the disparities between the constrained and the unconstrained values of the optimal parameters. The constrained optimal yield strength is consistently found to be higher than the respective unconstrained values. This is due to the fact that to avoid the constraint violation (i.e., isolator displacement exceeds the permissible value) higher yield strength is necessary in order to keep the isolator displacement within the permissible range. However, for

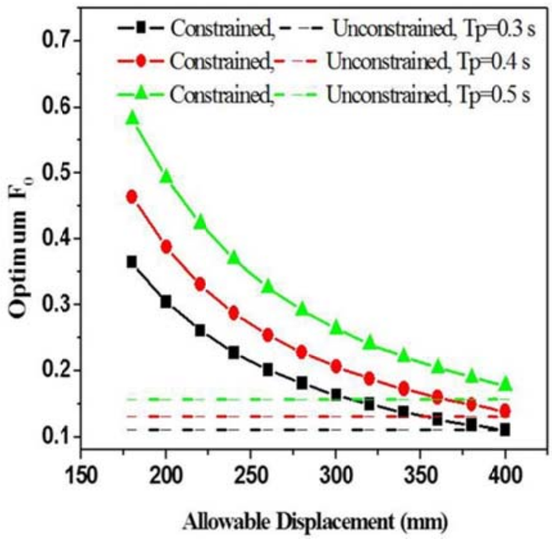

Fig. 2 The comparison of optimum isolator normalised yield strength with and without isolator displacement constraint for different time period of pier

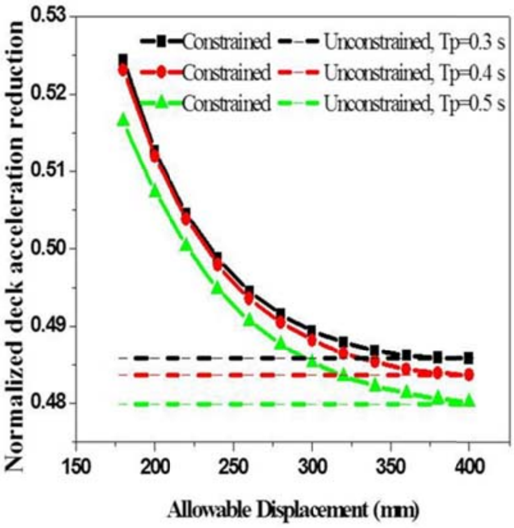

Fig. 3 The comparison of normalized deck RMS acceleration with and without isolator displacement constraint for different time period of pier

higher allowable isolator displacement (400 mm or more) the constrained and unconstrained cases overlap implying constraint inactive regime and as such there will be no importance of constrained optimum design. But for bridge, 400 mm displacement of LRB may be a critical issue and indicate the importance of considering the effect of constrained on isolator displacement.

The variation of optimum isolator normalised yield strength with and without isolator displacement constraint for different intensity of earthquake are compared in Fig. 4. The optimal yield strength of the isolator increases with increasing seismic intensity. This is obvious as increasing earthquake intensity transfer more energy to the structures resulting increasing superstructure acceleration. Thus, the higher yield strength value is sought to offset the increased acceleration. The corresponding acceleration reduction ratios of deck are shown in Fig. 5. Here also, it is following the same trend i.e., higher acceleration of deck when the allowable displacement is less but as the allowable displacement is increasing the effect of constrained optimisation is decreasing. Moreover, in case of higher seismic intensity the gap between constrained and unconstrained optimisation value exits with large allowable displacement but for small intensity of earthquake the gap between these two narrows down.

The variations of optimum yield strength and associated RMS acceleration reduction are further studied with respect to varying time period in Figs. 6 and 7 and for varying damping of the LRB in Figs. 8 and 9 for different allowable displacement of the isolator. The time period of the pier and seismic intensity are taken as 0.3 sec and 0.05 m^2/s^3, respectively. The effect of isolator displacement constraint is clearly evident from these plots with regard to optimum yield strength of the BI system and its performance for a wide range of parameter variations. The results reported in Fig. 8 show that for unconstrained case or larger allowable isolator displacement, the normalized yield strength decreases with increasing level of damping of the isolator. This is obvious as damping helps in dissipating energy and thereby less F_0 can achieve same level of

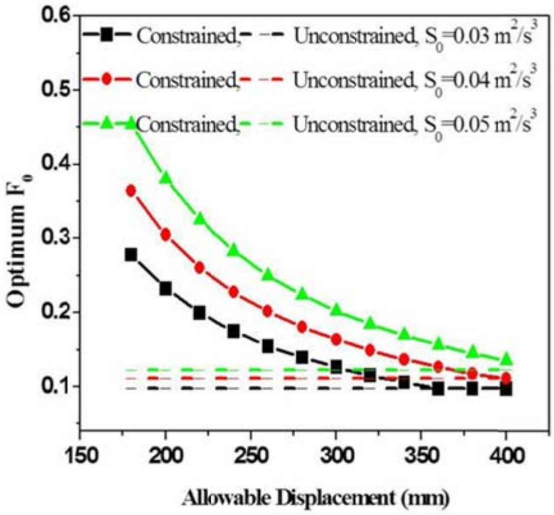

Fig. 4 The comparison of optimum isolator normalised yield strength with and without isolator displacement constraint for different intensity of earthquake

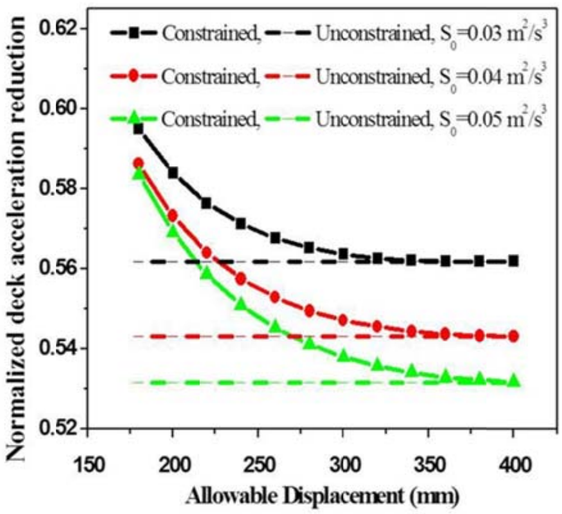

Fig. 5 The comparison of normalized deck RMS acceleration with and without isolator displacement constraint for different intensity of earthquake

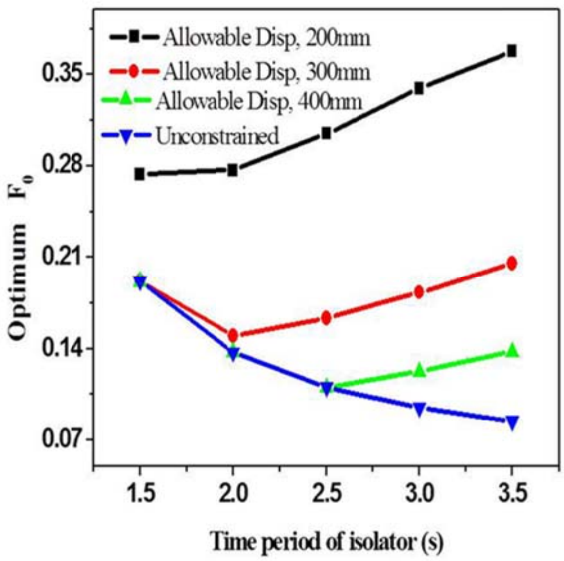

Fig. 6 The variation of isolator normalised yield strength with varying level of time period of isolator with and without isolator displacement constraint

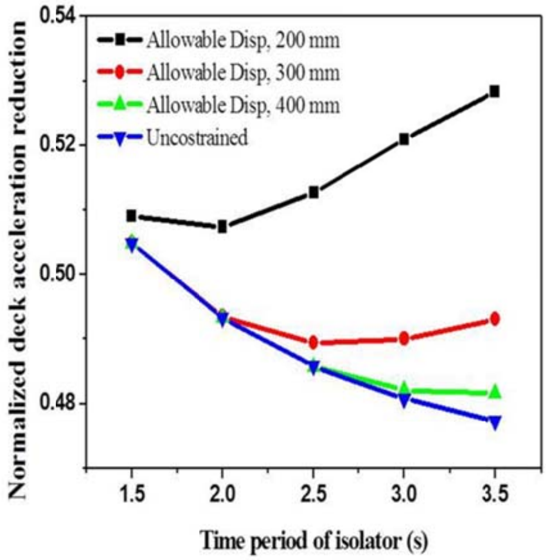

Fig. 7 The variation of normalized deck RMS acceleration with varying level of time period of the isolator with and without isolator displacement constraint

reduction. However, the decreasing trend of F_0 gradually diminishes when the allowable displacement becomes smaller as the constraint becomes more stringent with less allowable isolator displacement. Thereby, to minimize the response of the structure i.e., to achieve same

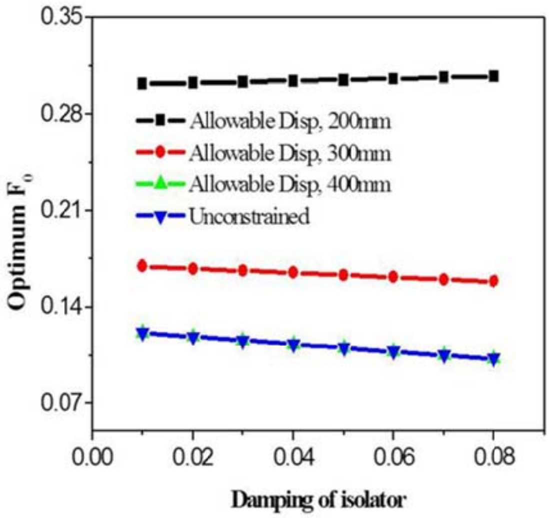

Fig. 8 The variation of isolator normalised yield strength with varying level of damping of isolator with and without isolator displacement constraint

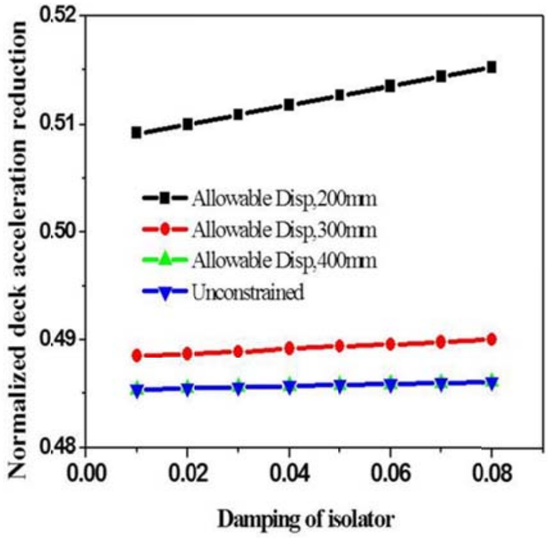

Fig. 9 The variation of normalized deck RMS acceleration with varying level of damping of isolator with and without isolator displacement constraint

level of reduction more F_0 is sought with respect to earlier (unconstrained or more allowable displacement cases). Thus depending on the activation of constraint, the optimal yield strength might increase or decrease. It may be noted from Fig. 9 that the reduction is practically remains same for different damping level for a given allowable displacement. But the optimum F_0value gets adjusted (increases or decreases) with different damping level of the isolator to yield the optimum response reduction. Therefore, the optimum parameter and the performance behaviour of the isolated bridge considering constrained SSO modify the usual response behaviour of the system without considering the effect of excessive isolator displacement.

5. Conclusions

The optimal performance of deck pier system of a bridge isolated by LRB type isolator is studied considering the limitation on excessive isolator displacement. For this, the optimum yield strength to minimize the maximum response of the bridge is obtained by imposing a constraint of excessive isolator displacement on the optimization procedure. The performance of the isolated bridge possible to achieve by considering such constraint is compared with that of achieved by usually adopted unconstrained optimization approach. The proposed constraint optimization results are in parity with the well known facts in the application of BI system considering unconstrained optimization procedure. However, when the maximum isolator displacement is taken into consideration in the optimization procedure, there is a definite change in the optimum value of the yield strength of the LRB. The constrained optimal yield strength is consistently found to be higher than the respective unconstrained values and respective reduction in the isolator displacement is substantially less than that of the unconstrained system. However, for higher allowable isolator displacement, the results of the constrained and unconstrained cases practically overlap indicates about the constraint inactive regime and as such there will be no importance of constrained optimum design. But for a bridge with large displacement of LRB may be a critical issue and need to check the optimum solution with constrained case. The observations made here are for stationary earthquake load model and needs further study for realistic non-stationary earthquake load model.

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Appendix

The augmented system matrix [A] is given as,

$$[A]=\begin{bmatrix} 0 & \{0\}^T & 0 & 0 & 1 & \{0\}^T & 0 \\ \{0\} & [0] & \{0\} & \{0\} & \{0\} & [I] & \{0\} \\ 0 & \{0\}^T & \dfrac{-K_e}{q} & 0 & \dfrac{-C_e}{q} & \dfrac{C_e\{\psi\}^T}{q} & 0 \\ 0 & \{0\}^T & 0 & 0 & 0 & \{0\}^T & 1 \\ \dfrac{-\alpha k_b}{m_d} & \dfrac{\alpha k_b\{\psi\}^T}{m_d} & \dfrac{-(1-\alpha)F_y}{m_d} & \omega_g^2 & \dfrac{-c_b}{m_d} & \dfrac{c_b\{\psi\}^T}{m_d} & 2\xi_g\omega_g \\ \dfrac{\alpha k_b\{\psi\}}{\left[M_p\right]} & \dfrac{-\left[K_p\right]-\alpha k_b\{\psi\}\{\psi\}^T}{\left[M_p\right]} & \dfrac{\{\psi\}(1-\alpha)F_y}{\left[M_p\right]} & \{r\}\omega_g^2 & \dfrac{c_b\{\psi\}}{\left[M_p\right]} & \dfrac{-\left[C_p\right]-c_b\{\psi\}\{\psi\}^T}{\left[M_p\right]} & \{r\}2\xi_g\omega_g \\ 0 & \{0\}^T & 0 & -\omega_g^2 & 0 & \{0\}^T & -2\xi_g\omega_g \end{bmatrix}$$

where {0} denotes the null vector of size $nx1$; [0] and [I] denotes the null and identity matrix, respectively, of size nxn; and the [Mp] in the denominator indicates the pre-multiplication to the numerator quantity by $[\mathrm{Mp}]^{-1}$.

The matrix $\left[S_{ww}\right]$ for the rock bed seismic motion, characterized by the white noise of intensity of S_0 is expressed as

$$\left[S_{ww}\right]=\begin{bmatrix} 0 & 0 & .. & 0 & 0 \\ 0 & 0 & .. & 0 & 0 \\ : & : & .. & : & : \\ 0 & 0 & .. & 0 & 0 \\ 0 & 0 & .. & 0 & 2\pi s_0 \end{bmatrix}$$