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Effects of dead loads on dynamic analyses of beams

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Abstract. The effect of dead loads on dynamic responses of uniform elastic beams is examined by means of a governing equation which takes into account initial bending stress due to dead loads. First, the governing equation of beams which includes the effect of dead loads is briefly presented from the author's paper (Takabatake 1990). In the formulation the effect of dead loads is considered by strain energy produced by conservative initial stresses produced by the dead loads. Second, the effect of dead loads on dynamical responses produced by live loads in simply supported beams and clamped beams is confirmed by the results of numerical computations with the Galerkin method and Wilson-θ method. It is shown that the dynamical responses, like dynamic deflections and bending moments produced by dynamic live loads, are decreased in a heavyweight beam when the effect of dead loads is included. Third, an approximate solution for dynamic deflections including the effect of dead loads is presented in closed-form. The proposed solution shows good in agreement with results of numerical computations with the Galerkin method and Wilson-θ method. Finally, a method reflecting the effect of dead loads for dynamic responses of beams on the magnitude of live loads is presented by an example.

Keywords: beams; dead load; initial stress; vibration; dynamic analysis; Galerkin equation; linear and nonlinear; live load; natural frequency; safety.

1. Introduction

Theoretically, reinforced concrete and steel structures should have the same degree of safety. However, the collapse of heavy reinforced concrete structures caused by snow loads on roofs occurs less frequently than that of steel structures. This fact shows that the collapse of structures caused by snow loads cannot be sufficiently explained by a consideration due to a heavy snowfall exceeding the maximum depth for design. The significant difference between reinforced concrete and steel structures is in magnitude of the dead loads. Beams, like structures, are always subjected to dead loads. The inherent property of dead loads has bending stresses in beams subjected to dead loads only, in which the bending stresses are conservative initial stresses. This conservative initial bending stresses are large on heavyweight beams and small on lightweight beams. Then, the author (Takabatake 1990) considered the assumption that dead loads of structures are an important factor; and demonstrated the effect of dead loads in static elastic beams. The effect of dead loads takes the additional strain energy, which is produced as multiplying conservative initial bending stresses due to dead loads by strains due to live loads, into consideration in addition to the well-known strain

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energy produced by live loads. It has been shown that this strain energy produced by the conservative initial stresses minimizes live load deflections and live load bending moments. We call such phenomena the effect of dead loads.

The author (Takabatake 1991) demonstrated the effect of dead loads on the natural frequencies of elastic beams and proposed a closed-form approximate solution of the natural frequency of simply supported beams. This new attention became the important jumping-off point for an extension of elementary beam theory and it extended to finite-element method by a beam element with the effect of dead loads (Zhou and Zhu 1996, Zhou 2002). The existence that the initial bend in a beam due to dead load increases the natural frequencies of the lateral vibration was also suggested by the other work (Kelly, Sackman, and Javid 1991). The author (Takabatake 1992) presented the effect of dead loads on the dynamic response of a uniform elastic rectangular plate and clarified the physical factors governing the effect. Mostaghel and Yu (1995) showed that preforming a thin plate into any shape has the effect of increasing its natural frequencies by means of a large deflection theory for thin plates and the principle of conservation of energy.

The author considers that the effect of dead loads exists on dynamic beams subjected to dynamic live loads, too. When an elastic beam is subjected to dynamic live loads, the beam vibrates from the static deformed state produced by dead loads; and the vibration should include an effect produced by the conservative initial bending stresses due to dead loads. However, due to little being known concerning its nature, this effect of dead loads is currently ignored in structural designs. If the effect of dead loads on dynamic beams is better understood, it will be possible to more accurately estimate the magnitude of live loads; thus, the safety factors for heavyweight and lightweight structures will be equalized; and real safe structural designs will be made possible.

Although there are numerous studies concerning static and dynamic problems of beams, as shown in the previous works (Hayashikawa *et al.* 1985, Oliveira 1982, Stephen 1981, Takabatake 1983, 1987 and Wang *et al.* 1981); a study concerning the interaction between dead loads and live loads is not found in the discussion of dynamic problems.

The purpose of this paper is to clarify the effect of dead loads on dynamic problems of elastic beams. First, the governing equation of beams, in which the effect of dead loads is included, is summarized from the author's work (Takabatake 1990). Second, the effect of dead loads on dynamic problems is clarified from results of numerical computations using Galerkin method and Wilson-θ method. Third, a closed-form solution for dynamic deflections of beams is presented from the governing equation including the effect of dead loads. The results of numerical computation using the proposed closed-form solution will show to be good in agreement with the results obtained from the linear acceleration method. Last, a method that reflects the effect of dead loads on dynamic live loads is presented by an example.

2. Governing equations of beams included the effect of dead loads

The equation of motion of damped beams with the effect of dead loads included is summarized from the extension of the author's work (Takabatake 1990) presenting the equation of motion of undamped beams including the effect of dead loads.

In Fig. 1, a beam is shown in Cartesian coordinate system: the x axis passes through the centroidal axis of the beam; and the y and z axes are the principal axes of the beam. The beam is straight without initial imperfections. Only transverse loads are considered. The static transverse

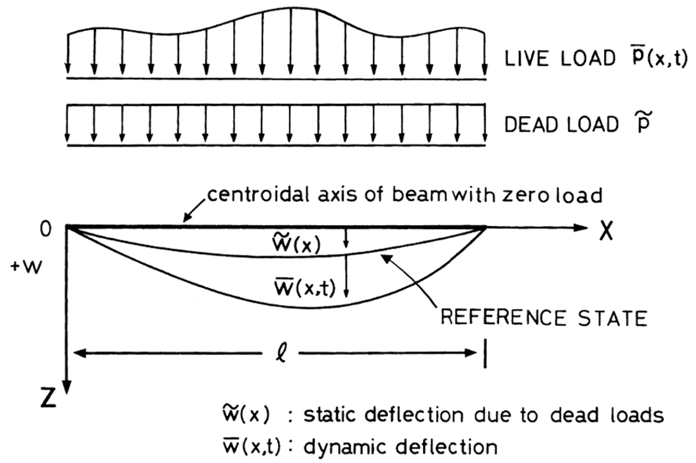

Fig. 1 Coordinate and load distribution of beam

deflections, $\tilde{w}$, are produced by dead loads, $\tilde{p}$, per unit length. This deformed state is defined as the reference state. Dynamic live loads, $\bar{p}$, always act on this reference state and produce dynamic deflections, $\bar{w}$, measured from the reference state. The deflections and transverse loads are considered positive when they point in the positive direction of the z axis.

The strain-displacement relations for the Bernoulli-Euler beams are expressed from the previous theory (Washizu 1982) by

$$\varepsilon_x = -zw'' + \frac{1}{2}(w')^2 \quad \text{(nonlinear)} \tag{1}$$

$$\varepsilon_x = -zw'' \quad \text{(linear)} \tag{2}$$

in which ε_x = strain of the beam; w = deflection; and primes indicate the differentiation with respect to x.

The strain energy, U, can be written as

$$U = \bar{U} + \tilde{U} \tag{3}$$

in which the strain energies, $\bar{U}$ and $\tilde{U}$, are defined by

$$\bar{U} = \frac{1}{2}\iiint \bar{\sigma}_x \bar{\varepsilon}_x dx\, dy\, dz \tag{4}$$

$$\tilde{U} = \iiint \tilde{\sigma}_x \bar{\varepsilon}_x dx\, dy\, dz \tag{5}$$

Here, $\bar{\sigma}_x$ = the bending stresses produced by live loads; $\tilde{\sigma}_x$ = the conservative initial bending stresses produced by dead loads; and $\bar{\varepsilon}_x$ = the normal strains produced by live loads. The following linear stress-strain relations are used

$$\bar{\sigma}_x = E\bar{\varepsilon}_x \tag{6}$$

$$\tilde{\sigma}_x = E\tilde{\varepsilon}_x \tag{7}$$

in which E = the Young's modulus; and $\tilde{\varepsilon}_x$ = the normal strains produced by dead loads. For the live load strain energy, $\overline{U}$, the liner strain-displacement relation given in Eq. (2) is used. On the other hand, for the dead load strain energy, $\tilde{U}$, the nonlinear strain-displacement relation given in Eq. (1) is used to introduce the effect of dead loads, because the static displacements $\tilde{w}$ are previously known. It should be noted that the effect of dead loads considered in this paper is not the well-known stiffening caused by mid-plane stretching in beams, because the current beam is considered only in the bending state. Hence, the strain energy, U, can be written as

$$U = \int_0^l \left[\frac{1}{2}EI(\overline{w}'')^2 + \frac{EA}{4}(\tilde{w}')^2(\overline{w}')^2 + EI\tilde{w}''\overline{w}''\right]dx \tag{8}$$

in which A = the cross-sectional area of beams; I = the principal moment of inertia; and l = the length of the beam.

The variation of the potential energy produced by external loads and damping forces of the beam can be expressed by

$$\delta V = -\int_0^l (\overline{p}+\tilde{p})\,\delta\overline{w}\,dx + \int_0^l c\dot{\overline{w}}\;\delta\overline{w}\;dx \tag{9}$$

in which c = a damping coefficient; and dots denote the differentiation with respect to time t.

Also, the kinetic energy, T, is given as

$$T = \int_0^l \frac{\rho A}{2}(\dot{\overline{w}})^2 dx \tag{10}$$

in which ρ = the mass density of the beam material.

Using Hamilton's principle with Eq. (8) to Eq. (10) and taking variations with respect to $\overline{w}$, the following result is obtained

$$\begin{aligned}\delta I = & \int_{t_2}^{t_1}\int_0^l \left\{-\rho A\ddot{\overline{w}} - c\dot{\overline{w}} - (EI\overline{w}'')'' + \frac{1}{2}\left[EA(\tilde{w}')^2\overline{w}'\right]' - \left(EI\tilde{w}''\right)'' + \left(\overline{p}+\tilde{p}\right)\right\}\delta\overline{w}\,dx\,dt \\ & + \int_{t_1}^{t_2}\left\{\left[\left(EI\overline{w}''+EI\tilde{w}''\right)' - \frac{EA}{2}\left(\tilde{w}'\right)^2\overline{w}'\right]\delta\overline{w}\right\}_0^l dt \\ & - \int_{t_1}^{t_2}\left[\left(EI\overline{w}''+EI\tilde{w}''\right)\delta\overline{w}'\right]_0^l dt = 0\end{aligned} \tag{11}$$

in which δ = a variational operator taken during the indicated time interval. In the reference state, the following equilibrium equation and boundary conditions for the simply supported beam and clamped beam, respectively, must exist

$$(EI\tilde{w}'')'' - \tilde{p} = 0 \tag{12}$$

$$\tilde{w} = 0 \quad \text{or} \quad \tilde{w}''' = 0 \quad \text{at } x = 0 \text{ and } x = l \tag{13a}$$

$$\tilde{w}' = 0 \quad \text{or} \quad \tilde{w}'' = 0 \quad \text{at } x = 0 \text{ and } x = l \tag{13b}$$

Incorporating the above into Eq. (11), the governing equation of beams with the effect of dead loads included can be written as

$$\rho A\ddot{\bar{w}} + c\dot{\bar{w}} + (EI\bar{w}'')'' - \frac{1}{2}[EA(\tilde{w}')^2\bar{w}']' = \bar{p} \tag{14}$$

for the equation of motion and

$$\bar{w} = 0 \qquad \text{or} \qquad \bar{w}''' - \frac{EA}{2}(\tilde{w}')^2\bar{w}' = 0 \qquad \text{at } x = 0 \text{ and } x = l \tag{15a}$$

$$\bar{w}' = 0 \qquad \text{or} \qquad \bar{w}'' = 0 \qquad \text{at } x = 0 \text{ and } x = l \tag{15b}$$

for the simply supported beam and clamped one, respectively. Note that Eqs. (14) and (15) are linear with respect to the unknown dynamic deflections, $\bar{w}$, because the static deflections, $\tilde{w}$, are initially known. The terms including the static deflections connect with the effect of dead loads on the dynamic deformation. In subsequent developments, for simplicity the beam is assumed to be of a uniform cross section.

3. Dynamic analyses using the Galerkin method

The equation of motion of beams which includes the effect of dead loads is solved by means of the Galerkin method. The method of separation of variables is employed assuming that

$$\bar{w}(x,t) = \sum_{n=1} \bar{w}_n(t) f_n(x) \tag{16}$$

in which $\bar{w}_n(t)$ = the unknown displacement coefficients with respect to time t; and $f_n(x)$ = shape functions satisfying the specified boundary conditions of beams. The following functions represent $f_n(x)$ for simply supported beams and clamped beams

$$f_n(x) = \sin\frac{n\pi x}{l} \qquad \text{for simply supported beams} \tag{17a}$$

$$f_n(x) = \sin\frac{\pi x}{l}\sin\frac{n\pi x}{l} \qquad \text{for clamped beams} \tag{17b}$$

Employing Eq. (16) into Eq. (14), the Galerkin equation can be written as

$$\delta\bar{w}_n : \sum_{\bar{n}=1}^{\infty} \bar{w}_{\bar{n}}[A_{n\bar{n}}] = -\sum_{\bar{n}=1}^{\infty}\left[\ddot{\bar{w}}_{\bar{n}}\frac{\rho A}{EI} + \dot{\bar{w}}_{\bar{n}}\frac{c}{EI}\right]\int_0^l f_n f_{\bar{n}} dx + \int_0^l \frac{\bar{p}}{EI} f_n dx \tag{18}$$

in which $A_{n\bar{n}}$ are defined as Eq. (18) may be solved by means of the Wilson-θ method.

$$A_{n\bar{n}} = \int_0^l f_n f_{\bar{n}}''''dx - \frac{1}{r^2}\int_0^l \tilde{w}' \tilde{w}'' f_n f_{\bar{n}}' dx - \frac{1}{2r^2}\int_0^l (\tilde{w}')^2 f_n f_{\bar{n}}'' dx \tag{19}$$

Once the dynamic displacements, $\bar{w}$, are determined from Eq. (16), the bending moments $\bar{M}$ due to live loads are determined from $\bar{M} = -EI\bar{w}''$.

4. Numerical results

The effect of dead loads on dynamic beams is discussed from results of numerical computations using the above formulations. Numerical computations are conducted for the following data: Young's modulus $E = 206$ GN/m^2; span length $l = 8$ m; $I = 2.04 \times 10^{-4}$ m^4 and the radius of gyration $r = 0.131$ m for simply supported beams while $I = 0.612 \times 10^{-4}$ m^4 and $r = 0.104$ m for clamped beams. An external live load without mass is assumed to be a harmonic load

$$\bar{p}(x, t) = \bar{p}_x \sin \omega_p t \tag{20}$$

and the maximum live load $\bar{p} = 11.8$ kN/m, in which $\bar{p}_x(x) =$ a function of x; and $\omega_p =$ a frequency of external load. The frequency ω_p is assumed to be half the first natural frequency of each beam. For uniformly distributed dynamic live loads whose intensity is $\bar{p}_0$, the relation $\bar{p}_x = \bar{p}_0$ exists.

Figs. 2 and 3 show dynamic deflections $\bar{w}$ at the midpoint of simply supported beams with the ratios of live load to dead load $\bar{p}/\tilde{p} = 0.5$ and $\bar{p}/\tilde{p} = 0.1$, respectively, under the constant live load $\bar{p} = 11.8$ kN/m, in which the damping constant $h = 0.05$ and the mass density of the beams is only

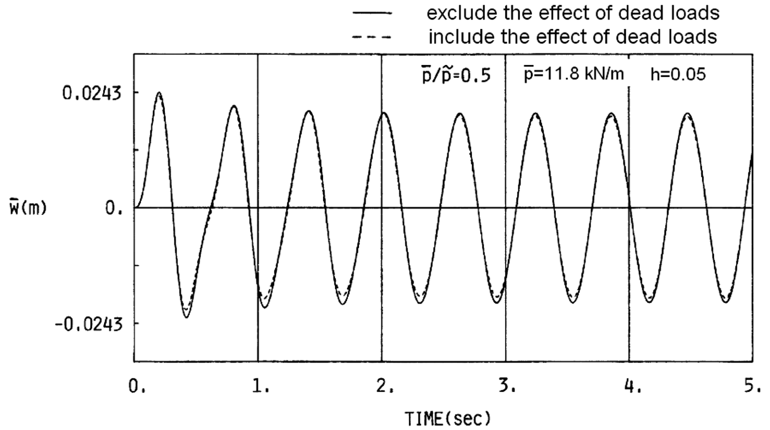

Fig. 2 Dynamic deflections $\bar{w}$ of the simply supported beam ($\bar{p}/\tilde{p} = 0.5$)

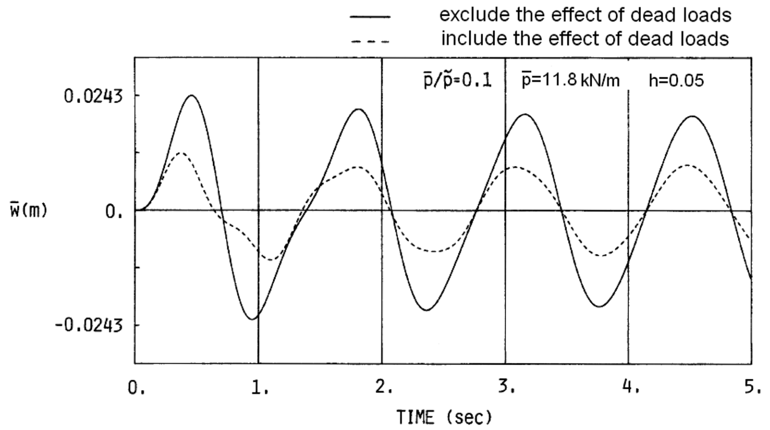

Fig. 3 Dynamic deflections $\bar{w}$ of the simply supported beam ($\bar{p}/\tilde{p} = 0.1$)

the dead load mass. The results show that the dynamic deflections including the effect of dead loads are minimized by considering the effect of dead loads. The effect of dead loads is larger in heavyweight beams than in lightweight beams. Fig. 4 shows dynamic deflections $\bar{w}$ at the midpoint of clamped beams with $\bar{p}/\tilde{p}$ and $h = 0.05$, in which the mass density of the beams is also only the dead load mass. The results show that the dynamic deflections are decreased by considering the effect of dead loads but the effect is not significant than in simply supported beams.

Although the above results have been represented for beams with only the dead load mass for the mass density, similar results may be obtained for beams with the mass density consisting of dead load mass and live load mass. Next, varying continuously the magnitude of dead loads in simply supported beams subjected to a harmonic live load, the effects of dead loads on dynamic deflections

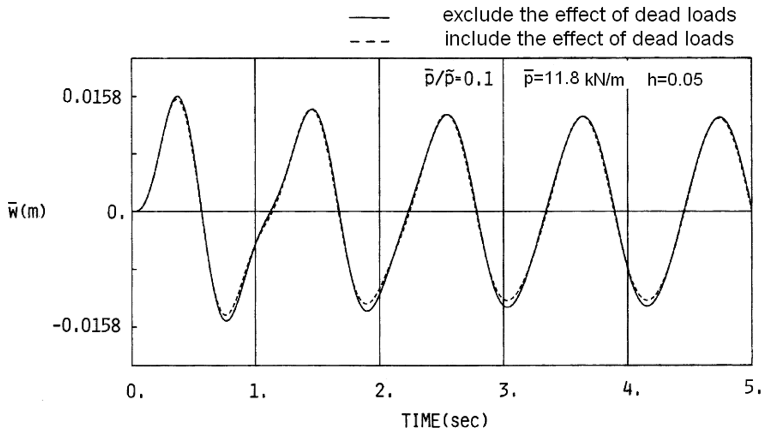

Fig. 4 Dynamic deflections $\bar{w}$ of the clamped beam ($\bar{p}/\tilde{p} = 0.1$)

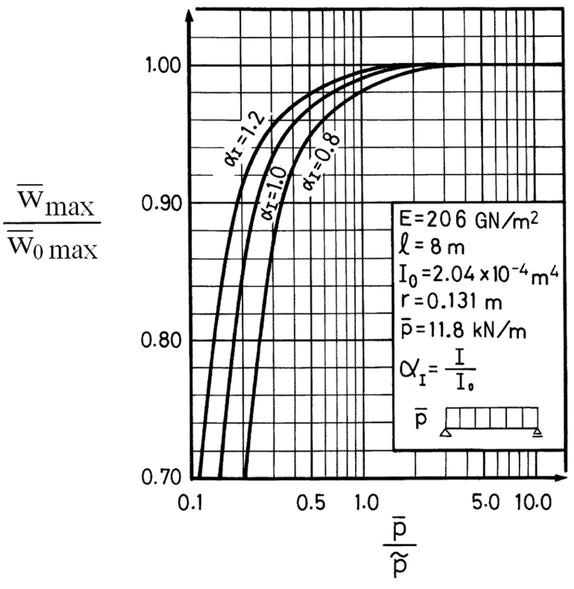

Fig. 5 Relationships between $\frac{\bar{w}_{max}}{\bar{w}_{0max}}$ and $\frac{\bar{p}}{\tilde{p}}$

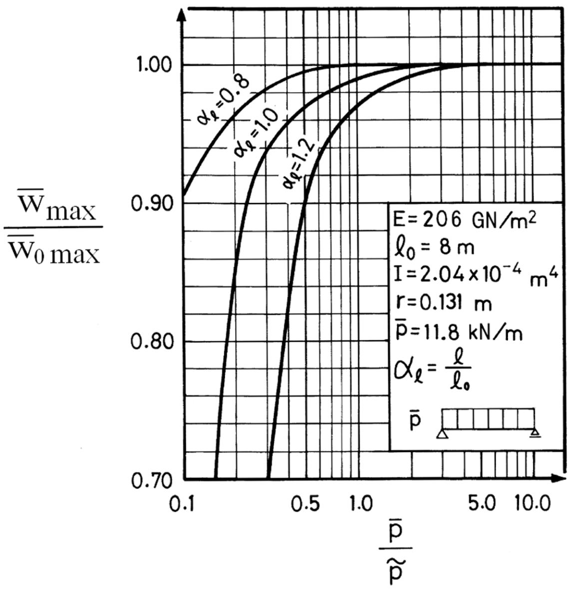

Fig. 6 Relationships between $\frac{\bar{w}_{max}}{\bar{w}_{0max}}$ and $\frac{\bar{p}}{\tilde{p}}$

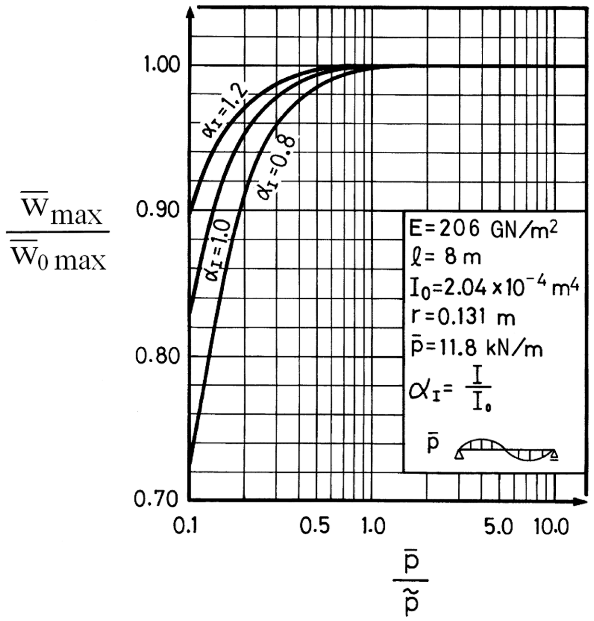

Fig. 7 Relationships between $\frac{\overline{w}_{max}}{\overline{w}_{0max}}$ and $\frac{\overline{p}}{\tilde{p}}$

in beams with a damping constant $h = 0.05$ are shown in Fig. 5 to Fig. 7, in which $\overline{w}_{max}$ = the maximum value of dynamic deflections including the effect of dead loads; and $\overline{w}_{0max}$ = the maximum value of dynamic deflections excluding the effect of dead loads. Fig. 5 shows the effect of dead loads in beams with three different moments of inertia, in which the mass density of beams excludes the live load mass. The results show that the effect of dead loads is significant on the beam with the smallest moment of inertia. For example, this figure shows that for the beam with $\alpha_I = 1.0$ and the load-ratio $\overline{p}/\tilde{p} = 0.5$ the maximum values of the dynamic deflections and bending moments reduce to be 0.967 times of that without considering the dead load effect. Similarly, Fig. 6 shows the effect of dead loads produced by the variation of the span length. The effect of dead loads is more remarkable on long span beam. Fig. 7 shows the effect of dead loads for beams subjected to asymmetric, sinusoidal live loads. It follows that the effect of dead load is more significant in the weak beam.

5. Approximate solutions

The effect of dead loads on dynamic beams subjected to dynamic live loads has been shown from the preceding numerical results. For practical applications in structural analyses, the closed-form solution is presented by employing suitable assumptions into Eq. (14). The preceding results have shown that the effect of dead loads is remarkable in simply supported beams but negligible in clamped beams. Hence, the closed-form solution for simply supported beams is considered. Eq. (14) can be rewritten as

$$\frac{\rho A}{EI}\ddot{\overline{w}} + \frac{c}{EI}\dot{\overline{w}} + \overline{w}'''' + R(x, t) = \frac{\overline{p}}{EI} \tag{21}$$

in which $R(x, t)$ is defined as

$$R(x,t) = -\frac{1}{2r^2}[2\tilde{w}'\tilde{w}''\overline{w}' + (\tilde{w}')^2\overline{w}''] \tag{22}$$

Before presenting the dynamic solution, natural frequencies ω_n of beams with the effect of dead loads included will be determined from the following equation

$$\frac{\rho A}{EI}\ddot{\overline{w}}(x,t) + \overline{w}(x,t)'''' + R(x,t) = 0 \tag{23}$$

The method of separation of variables uses

$$\overline{w}(x,t) = \overline{W}(x)\phi(t) \tag{24}$$

in which $\overline{W}(x)$ = displacement coefficient; and $\phi(t)$ = a function of time t. The term $R(x, t)$ can be also applied the method of separation of variables, because the static defections $\tilde{w}(x)$ in $\overline{R}(x,t)$ are independent of time t.

$$R(x,t) = \overline{R}(x)\phi(t) \tag{25}$$

Hence, in which $\overline{R}(x)$ = a function of x, defined as

$$\overline{R}(x) = -\frac{1}{2r^2}[2\tilde{w}'\tilde{w}''\overline{W}' + (\tilde{w}')^2\overline{W}''] \tag{26}$$

Substituting Eqs. (24) and (25) into Eq. (23), the equations for $\overline{W}(x)$ may be obtained as

$$\frac{EI}{\rho A}[\overline{W}(x)'''' + \overline{R}(x)] = \omega^2\overline{W}(x) \tag{27}$$

in which ω^2 = a constant.

The n - th natural function of simply supported beams is assumed to be

$$\overline{W}(x) = \sin\frac{n\pi x}{l} \tag{28}$$

On the other hand, $\overline{R}(x)$ is expanded in Fourier series expansions

$$\overline{R}(x) = \sum_{n=1} R_n \sin\frac{n\pi x}{l} \tag{29}$$

in which R_n = Fourier coefficients. The substitution of Eqs. (28) and (29) into Eq. (27) yields

$$\frac{EI}{\rho A}\left[\left(\frac{n\pi}{l}\right)^4\sin\frac{n\pi x}{l} + \sum_{n=1} R_n\sin\frac{n\pi x}{l}\right] = \omega^2\sin\frac{n\pi x}{l} \tag{30}$$

Hence, the n - th natural frequency, ω_n, including the effect of dead loads becomes

$$\omega_n = \omega_{0n}\sqrt{1 + \left(\frac{l}{n\pi}\right)^4 R_n} \tag{31}$$

in which ω_{0n} = the n - th natural frequency excluding the effect of dead loads, as given by

$$\omega_{0n} = \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \tag{32}$$

From Eq. (29), the Fourier coefficients R_n are

$$R_n = \frac{2}{l}\int_0^l \overline{R}(x) sin\frac{n\pi x}{l} dx \tag{33}$$

$\overline{R}(x)$ is an unknown function of the static deflections, $\tilde{w}(x)$, and of displacement coefficient, $\overline{W}(x)$, of unknown dynamic deflections, $\overline{w}(x)$, which are determined from Eq. (23) including the term $R(x)$. So, for simplicity, replacing $\overline{w}(x)$ in $\overline{R}(x)$ with the known displacement coefficient, $\overline{W}_0(x)$, excluding the effect of dead loads in the same beam, $\overline{R}(x)$ becomes a function of the known $\tilde{w}$ and $\overline{W}_0(x)$. Then, $\tilde{\omega}$ and $\overline{W}_0(x)$ are expressed in the Fourier series expansions

$$\tilde{w} = \sum_{n=1} \tilde{w}_n \sin\frac{n\pi x}{l} \tag{34}$$

$$\overline{W}_0 = \sum_{n=1} \sin\frac{n\pi x}{l} \tag{35}$$

in which the Fourier coefficients $\tilde{w}_n$ take, from Eq. (12), the values

$$\tilde{w}_n = \frac{\tilde{p}_n}{EI}\left(\frac{l}{n\pi}\right)^4 \tag{36}$$

in which the Fourier coefficients $\tilde{p}_n$ for uniform dead loads given by $\tilde{p} = \tilde{p}^*$ take the following value

$$\tilde{p}_n = \begin{cases} \dfrac{4\tilde{p}*}{n\pi} & \text{for odd} \\[2ex] 0 & \text{for the other} \end{cases} \tag{37}$$

Substituting Eqs. (34) to (36) into Eq. (26), the value of $\overline{R}(x)$ is obtained. And applying this value to Eq. (33), R_n are determined. Hence from Eq. (31), the n - th natural frequency including the effect of dead loads is given as

$$\omega_n = \omega_{0n}\sqrt{1 + k_\omega G_n} \tag{38}$$

in which the non-dimensional coefficients k_ω and G_n are defined as

$$k_\omega = \beta\left[\frac{\tilde{p}_0 l^4}{rEI}\right]^2 \tag{39}$$

$$G_n = \sum_{\tilde{n}=1}{}^*\sum_{\tilde{\tilde{n}}=1}{}^*\sum_{\overline{n}=1} \frac{16\overline{n}}{\pi^{10} n^4 (\tilde{n})^4 (\tilde{\tilde{n}})^4} \times \left[2\tilde{\tilde{n}}\, F_{sc}\left(n,\tilde{\tilde{n}};\tilde{n},\overline{n}\right) + \overline{n}\, F_{sc}\left(n,\overline{n};\tilde{n},\tilde{\tilde{n}}\right)\right] \tag{40}$$

in which the superscript, *, in sum indicates to take the sum of only odd numbers. The notation $F_{sc}(n,\tilde{\tilde{n}};\tilde{n},n)$ is defined as

$$F_{sc}\left(n,\tilde{\tilde{n}};\tilde{n},\bar{n}\right)=\int_0^1 \sin n\pi\xi\ \sin\tilde{\tilde{n}}\pi\xi\ \cos\tilde{n}\pi\xi\ \cos\bar{n}\pi\xi\ d\xi \tag{41}$$

in which $\xi = x/l$. The values of G_n are found to be

$$\left.\begin{aligned} G_1 &= 1.52503\times10^{-4}\\ G_2 &= 0.22901\times10^{-4}\\ G_3 &= 0.10016\times10^{-4}\\ G_4 &= 0.0034186\times10^{-4}\\ G_5 &= 0.0022899\times10^{-4}\end{aligned}\right\} \tag{42}$$

The results of numerical computations using Eq. (38) are examined to show excellent agreement with the results using the Galerkin method.

Next, an approximate dynamic solution including the effect of dead loads will be considered. The method of separation of variables for $R(x, t)$ in Eq. (21) is expressed as

$$R(x,t) = \sum_{n=1}\bar{R}_n(x)\phi_n(t) \tag{43}$$

Then, the general solution of Eq. (21) may be written as

$$\bar{w}(x,t) = \sum_{n=1}\bar{W}_n(x)\phi_n(t) \tag{44}$$

in which $\bar{W}_n(x)$ = natural functions satisfying the following differential equation

$$\frac{EI}{\rho A}[\bar{W}_n(x)'''' + \bar{R}_n(x)] = \omega_n^{\,2}\bar{W}_n(x) \tag{45}$$

Substituting Eqs. (43) and (44) into Eq. (21) and employing the relation of Eq. (45) into the result, we have

$$\sum_{n=1}^{\infty}\left[\bar{W}_n(x)\ddot{\phi}_n(t) + \frac{c}{\rho A}\bar{W}_n(x)\dot{\phi}_n(t) + \omega_n^{\,2}\bar{W}_n(x)\phi_n(t)\right] = \frac{\bar{p}(x,t)}{\rho A} \tag{46}$$

Multiplying Eq. (46) by $\bar{W}_m(x)$ given in Eq. (28) and integrating it from 0 to l, we have

$$\ddot{\phi}_n(t) + 2h\omega_n\dot{\phi}_n(t) + \omega_n^{\,2}\phi_n(t) = \frac{2}{\rho A l}Q_n(t) \tag{47}$$

in which the damping constant, h, and the notation $Q_n(t)$ are defined as

$$\frac{c}{\rho A} = 2h\omega_n \tag{48}$$

$$Q_n(t) = \int_0^l \bar{p}(x,t)\overline{W}_n dx \tag{49}$$

The general solution of Eq. (47) is

$$\phi_n(t) = e^{(-h\omega_n t)}[A_n \sin\omega_{Dn}t + B_n \cos\omega_{Dn}t] + \frac{2}{\rho A l \omega_{Dn}} \int_0^t Q_n(\tau) e^{[-h\omega_n(t-\tau)]} \sin\omega_{Dn}(t-\tau) d\tau \tag{50}$$

in which A_n and B_n = constants; and ω_{Dn} are defined as

$$\omega_{Dn} = \omega_n \sqrt{1-h^2} \tag{51}$$

For harmonic live loads as given in Eq. (20), Eq. (49) becomes

$$Q_n(t) = \sin\omega_p t Q_{nx} \tag{52}$$

in which Q_{nx} are defined as

$$Q_{nx} = \int_0^l \bar{p}_x(x)\overline{W}_n(x) dx \tag{53}$$

Now, for simplicity, assuming the following initial conditions

$$\left.\begin{aligned} \bar{w}(x,0) &= 0 \\ \dot{\bar{w}}(x,0) &= 0 \end{aligned}\right\} \tag{54}$$

the deflections $\bar{w}(x,t)$ can be written as

$$\begin{aligned} \bar{w}(x,t) = \frac{1}{\rho A l}\sum_{n=1} \frac{Q_{nx}}{\omega_{Dn}} \Bigg\{ & \frac{h\omega_n \cos\omega_p t + (\omega_p + \omega_{Dn})\sin\omega_p t}{(h\omega_n)^2 + (\omega_p + \omega_{Dn})^2} - \frac{h\omega_n \cos\omega_p t + (\omega_p - \omega_{Dn})\sin\omega_p t}{(h\omega_n)^2 + (\omega_p - \omega_{Dn})^2} \\ & + e^{(-h\omega_n t)}\left[\frac{(\omega_p + \omega_{Dn})\sin\omega_{Dn}t - h\omega_n\cos\omega_{Dn}t}{(h\omega_n)^2 + (\omega_p + \omega_{Dn})^2} + \frac{(\omega_p - \omega_{Dn})\sin\omega_{Dn}t + h\omega_n\cos\omega_{Dn}t}{(h\omega_n)^2 + (\omega_p - \omega_{Dn})^2}\right]\Bigg\}\overline{W}_n \end{aligned} \tag{55}$$

The deflections for undamped beams are easily obtained by putting $h = 0$ in Eq. (55). The well-known deflections excluding the effect of dead loads are obtained by replacing ω_n with ω_{0n}.

Comparing the results obtained using the proposed solution with the preceding results obtained using the Galerkin method, the proposed solution can show well the dynamic behavior of beams with the effect of dead loads included. Fig. 8 shows dynamic deflections of beams with $h = 0.05$ and $\bar{p}/\tilde{p} = 0.1$.

6. Example

The effect of dead loads on dynamic behavior of elastic beams is reflected on the modification of dynamic live loads by the following procedure:

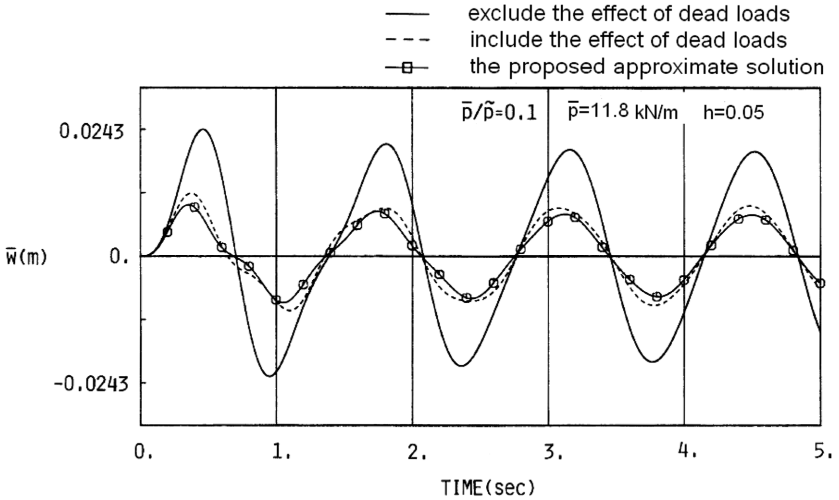

Fig. 8 Relationships between the approximate solution and numerical results

STEP 1. Consider to design a steel beam subjected to dead loads $\tilde{p}$ and dynamic live loads $\bar{p}$. Since live loads acting on current beam are prescribed from design codes, the designer cannot select the value. However, dead loads of the beam can be determined by the designer. When the structural cost is lowered, a beam is designed with the smallest cross section, and its dead loads are made minimum. The maximum value $\bar{w}_{max}(\tilde{p},\bar{p})$ of dynamic deflections $\bar{w}(x,t)$ for the beam, subjected to dead loads $\tilde{p}$ and dynamic live loads $\bar{p}$, is determined from Eq. (55). The maximum value includes the effect of dead loads produced by the selected dead loads $\tilde{p}$. If for the current beam figures like Figs. 5-7 are previously prepared, the maximum vale is obtained easily from the figures without numerical computation.

STEP 2. Due to the effect of dead loads, the action of dynamic live loads is reduced on heavyweight beams more than on lightweight beams. It is assumed that a reference ratio of dynamic live loads to dead loads for current beam is already presented, with regard to the live load problem to design safe beams. This ratio will be determined from wide investigation of structures collapsed by live loads, or experiments. However, in the present state it is unclear. Therefore, for practical use, it is recommended that the reference magnitude of dead loads which gives the reference ratio of live loads to dead loads takes the magnitude of dead loads of reinforced concrete beams subjected to the same dynamic live load. The reference magnitude of dead loads of safe beams is denoted by $\tilde{p}_0$. Thus, the maximum value $\bar{w}_{max}(\tilde{p}_0,\bar{p})$ of dynamic deflections for the beam, subjected to the reference dead loads $\tilde{p}_0$ and dynamic live loads $\bar{p}$, is also determined from Eq. (55). Its value also includes the effect of dead loads produced by the reference dead loads $\tilde{p}_0$.

STEP 3. When the dead loads $\tilde{p}$ which are selected by the designer are smaller than the reference dead loads $\tilde{p}_0$, the maximum dynamic deflection $\bar{w}_{max}(\tilde{p},\bar{p})$ of the beam with $\tilde{p}$ is larger than $\bar{w}_{max}(\tilde{p}_0,\bar{p})$ of the safe beam with $\tilde{p}_0$, because the dynamic deflections produced by dynamic live loads are subjected to the effect of dead loads. This implies that live loads acting on the lightweight beam with light dead loads $\tilde{p}$ must rise by the ratio, $\bar{w}_{max}(\tilde{p},\bar{p})/\bar{w}_{max}(\tilde{p}_0,\bar{p})$ to require the same degree of safety for heavyweight and lightweight beams. Thus, the magnitude of live loads may be estimated with considering the effect of dead loads. The beam with the dead loads $\tilde{p}$ is designed by this magnitude of live loads. On the other hand, when $\tilde{p}$ is larger than $\tilde{p}_0$, the magnitude of live loads is unchanged.

Thus, it will be possible to change the present value of live loads to an accurate value that has a positive effect on beams. Consequently, this method will have the same effect when used on live loads for steel beams and reinforced concrete beams. The present paper does not recommend increasing the dead loads of structures; to prevent the collapse of beams due to dynamic live loads, it is absolutely imperative that the safety factor for lightweight beams is raised to coincide with the safety factor for heavyweight beams.

7. Conclusions

The effect of dead loads on elastic beams subjected to dynamic live loads has been presented by means of the strain energy produced by initial bending stresses produced by dead loads. It has been clarified from both the results of numerical computation with the Galerkin method and the closed-form solution proposed here, which includes the effect of dead loads, that the effect of dead loads reduces the action of dynamic live loads on elastic beams and is larger on heavyweight beams than on lightweight ones. Last, it has been shown how to apply the effect of dead loads to the modification of live loads.

For flexural problem of beams with simply supported or clamped boundary conditions, the current boundary conditions derived from Hamilton's principle, as shown in Eqs. (13) and (15), are the well-known usual equations and the longitudinal constrain due to the existence of dead load is free. The phenomenon examined here has been remarkable on dynamic response of simply supported beams with heavy dead load. This suggests that since lightweight beams have greater influence on the dynamic response due to the same live load than heavyweight beams and the influence of live load is noticeable for lightweight beam designed with low safety factors. This study will also be applicable to dynamic responses of bridges.

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