Earthquakes and Structures, Vol. 1, No. 3 (2010) 289-306
DOI: http://dx.doi.org/10.12989/eas.2010.1.3.289

An evolutionary algorithm for optimal damper placement to minimize interstorey-drift transfer function in shear building

Kohei Fujita, Kaoru Yamamoto and Izuru Takewaki*

Department of Urban and Environmental Engineering, Kyoto University, Kyotodaigaku-Katsura, Nishikyo-ku, Kyoto 615-8540, Japan

(Received May 3, 2010 , Accepted July 8, 2010)

Abstract. A gradient-based evolutionary optimization methodology is presented for finding the optimal design of viscous dampers to minimize an objective function defined for a linear multi-storey structure. The maximum value along height of the transfer function amplitudes for the interstorey drifts is taken as the objective function. Since the ground motion includes various uncertainties, the optimal damper placement may be different depending on the ground motion used for design. Furthermore, the transfer function treated as the objective function depends on the properties of structural parameters and added dampers. This implies that a more robust damper design is desired. A reliable and robust damping design system against any unpredictable ground motions can be provided by minimizing the maximum transfer function. Such design system is proposed in this paper.

Keywords: optimal damper placement; transfer function; evolutionary optimization; gradient-based method; earthquake response.

1. Introduction

During an early stage (1980-1995) of passive structural control, the main objective was aimed at installing supplemental dampers into predetermined places from the viewpoint of performance-based design. After extensive developments of various damper systems and materials, an advanced target was set and upgraded to the development of smart and effective installation of such dampers.

In spite of the motivation inspired and directed to the smart and effective installation of dampers, the advancement in the field of optimal damper placement has been restricted. Several studies have treated this subject in the early stage. De Silva (1981) presented a gradient algorithm for the optimal design of discrete dampers in the vibration control of a class of flexible systems. Constantinou and Tadjbakhsh (1983) derived the optimum damping coefficient for a damper placed on the first story of a shear building subjected to horizontal ground motions. Gurgoze and Muller (1992) presented a numerical optimal design method for a single viscous damper in a prescribed linear multi-degree-of-freedom system. Hahn and Sathiavageeswaran (1992) performed parametric studies on the effects of damper distribution on the earthquake response of buildings and showed that, for a building with uniform story stiffnesses, dampers should be added to the lower half of the building. Tsuji and Nakamura (1996) proposed an algorithm to find both the optimal story stiffness and damper

* Corresponding author, Professor, E-mail: takewaki@archi.kyoto-u.ac.jp

distributions for a shear building subjected to the spectrum-compatible ground motions.

As another approach, Takewaki (1997) developed an optimal method for the smart and effective damper placement with the help of the concepts of inverse and optimal criteria-based design approaches. He proposed a procedure for solving a problem of optimal damper placement by deriving the optimality criteria and then by developing an incremental inverse problem approach. Subsequently, Takewaki and Yoshitomi (1998), Takewaki et al. (1999), Takewaki (2000) and Fujita et al. (2010) introduced a different algorithm based on the concept of optimal sensitivity. The optimal quantity of passive dampers is obtained automatically together with the optimal placement through this new algorithm. The essence of these approaches is summarized in the monograph by Takewaki (2009).

In the same period, significant works have been developed by many researchers (Singh and Moreschi 2001, 2002, Garcia 2001, Garcia and Soong 2002, Liu *et al.* 2003, Uetani *et al.* 2003, Wongprasert and Symans 2004, Trombetti and Silvestri 2004, 2007, Kiu *et al.* 2004, Lavan and Levy 2005, 2006a, b, Silvestri and Trombetti 2007, Tan *et al.* 2005, Liu *et al.* 2005, Levy and Lavan 2006, Marano *et al.* 2007, Aydin *et al.* 2007, Cimellaro 2007, Cimellaro and Retamales 2007, Attard 2007, Cirrtellaro and Retamales 2007, Viola and Guidi 2008, Wang and Dyke 2008). Most of these studies have developed new optimal design methods of supplemental dampers and proposed effective and useful approaches.

In this paper, an evolutionary method is proposed for finding the optimal design of viscous dampers to minimize an objective function defined for a linear multistorey structure. The maximum value along height of the transfer function amplitudes for the interstorey drifts is taken as the objective function. As a similar research, Takewaki (1997) treated the ‘sum’ along height of the maximum transfer functions (at the structural fundamental natural frequency) of interstorey drifts as the objective function. However, it seems natural to treat the maximum value along height of the transfer functions for the interstorey drifts from the viewpoint of structural safety because the interstorey drift itself at a specific storey (not the sum of the interstorey drifts) is directly related to the structural safety. Since the maximum transfer function along height is treated here, the storey number has to be identified where the amplitude of the interstorey-drift transfer function (=IDTF) is maximized. This additional manipulation may cause a difficulty, i.e. discontinuous objective functions, because the objective function changes at this point. An evolutionary algorithm to overcome such difficulty is presented here. Numerical analyses are conducted to show the validity of the proposed optimal design method.

2. Structural model with viscous dampers

Consider an N-storey planar shear building model with viscous dampers (VD) as shown in Fig. 1. Let M_i, k_{Fi}, c_{Fi} and c_{di} $(i = 1,...,N)$ denote the floor mass, the storey stiffness of the frame, the structural damping coefficient and the additional damping coefficient by the passive damper in the i-th storey, respectively.

Let $\mathbf{U}(\omega)$ and $\ddot{U}_g(\omega)$ denote the Fourier transforms of horizontal floor displacements $\mathbf{u}(t)$ and a horizontal acceleration $\ddot{u}_g(t)$ of the ground motion. The equations of motion of the building without additional VD subjected to the horizontal ground motion can be expressed in frequency domain by

$$(-\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C} + \mathbf{K})\mathbf{U}(\omega) = -\mathbf{Mr}\,\ddot{U}_g(\omega) \tag{1}$$

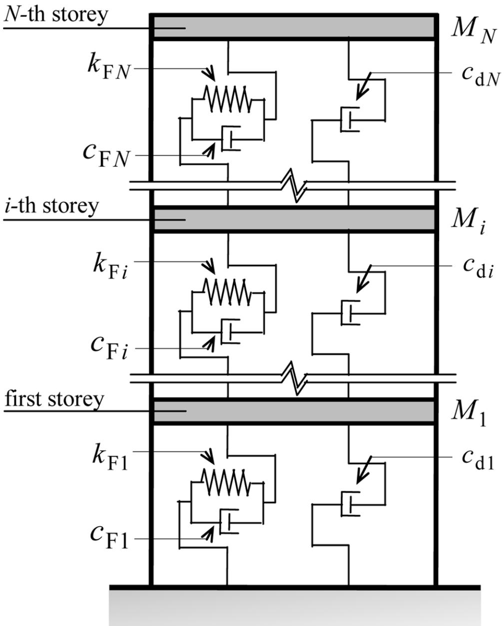


Fig. 1 N-storey building model with viscous damper

where **M**, **C** and **K** are the system mass, structural damping and stiffness matrices respectively. Furthermore $\mathbf{r} = \{1,...,1\}^T$ is the influence coefficient vector.

When the passive viscous dampers are added to the building, Eq. (1) can be modified to the following form

$$\{-\omega^2\mathbf{M} + i\omega(\mathbf{C} + \mathbf{C}_D) + \mathbf{K}\}\mathbf{U}(\omega) = -\mathbf{Mr}\ddot{U}_g(\omega) \tag{2}$$

where $\mathbf{C}_D$ denotes the damping matrix by the added VD. Eq. (2) can be described simply as

$$\mathbf{AU}(\omega) = \mathbf{B}\ddot{U}_g(\omega) \tag{3a}$$

where

$$\mathbf{A} = -\omega^2\mathbf{M} + i\omega(\mathbf{C} + \mathbf{C}_D) + \mathbf{K},\ \mathbf{B} = -\mathbf{Mr} \tag{3b, c}$$

The Fourier transforms $\mathbf{D}(\omega) = \{D_1,...,D_N\}^T$ of the interstorey drifts can then be derived as

$$\mathbf{D}(\omega) = \mathbf{TU}(\omega) \tag{4}$$

where **T** is a constant transformation matrix consisting of 1, -1 and 0.

The vector of IDTF $\mathbf{H}_D(\omega) = \{H_{D_1}(\omega), \ldots, H_{D_N}(\omega)\}^T$ can be defined by the ratio of $\mathbf{D}(\omega)$ to $\ddot{U}_g(\omega)$ as

$$\mathbf{H}_D(\omega) = \mathbf{D}(\omega)/\ddot{U}_g(\omega) \tag{5}$$

IDTF can then be derived from Eqs. (3), (4) and (5) as

$$\mathbf{H}_{\mathrm{D}}(\omega) = \mathbf{T}\mathbf{A}^{-1}\mathbf{B} \tag{6}$$

It is remarkable that, since the mean squares of the response can be evaluated by multiplying the power spectral density function of the horizontal acceleration $\ddot{u}_g(t)$ on the squared transfer function and integrating that in the frequency domain, the amplitude of the transfer function is meaningful for designing the building. In other words, when this quantity is large at a particular frequency which generally corresponds to the fundamental natural circular frequency ω_1, the response of the interstorey drift in time domain can be amplified to a larger level when the principal frequency range of the ground motion is resonant to the fundamental natural circular frequency of the building. In this paper, the objective function f to be minimized is given by the maximum value of $\mathbf{H}_D(\omega)$ with respect to both frequency and storey number.

$$f = \max_{i,\omega}\left\{\left|H_{D_i}(\omega)\right|; 1 \le i \le N\right\} \tag{7}$$

The maximum amplitude of the transfer function can be obtained approximately at the fundamental natural circular frequency ω_1. For evaluating ω_1 of the building including the added VD, there may be two different methods. The first method is to use the undamped eigenvalue analysis. The second one is to use the complex eigenvalue analysis for a nonproportionally damped system. In case of treating VD, we can assume that only the damping matrix $\mathbf{C}$ varies and the stiffness matrix $\mathbf{K}$ is constant. In previous related research (Takewaki 1997, Takewaki *et al.* 1999), the undamped fundamental natural circular frequency was used to evaluate the objective function. In this paper, the influence of using two different methods stated above is also discussed.

3. Optimal design problem

The problem of optimal damper placement for the N-storey shear building model is to find the optimal distribution of VD capacities $\mathbf{C_d} = \{c_{d1},...,c_{dN}\}$ so as to minimize the objective function f. The problem can be stated as

$$\text{Minimize} \quad : \; f = \max_{i,\omega}\left|H_{\mathrm{D}_i}(\omega)\right| \qquad (i = 1, 2, \cdots, N) \tag{8}$$

$$\text{subject to} \quad : \; \sum_{i=1}^{N} c_{\mathrm{d}i} = \overline{W} \quad (\overline{W}: \text{ prescribed value of total damper capacity}) \tag{9}$$

$$0 \le c_{\mathrm{d}i} \le \overline{c}_{\mathrm{d}i} \; (i = 1, ..., N) \tag{10}$$

The first constraint is on the total damper capacity. The second constraint is on the upper and lower bounds of the damper's capacity. In Eq. (10), $\overline{c}_{\mathrm{d}i}$ denotes the upper bound of damper capacity in the i-th storey.

The objective function f, Eq. (8), can be approximated as the value of the transfer function at the fundamental natural circular frequency ω_1. of the building. The fundamental natural circular frequency ω_1 generally varies according to the change of the additional damper capacity. Since the objective function f, Eq. (8), can be regarded as a function of ω_1 and $\mathbf{C}_\mathrm{d}$, the objective function can be described by $f(\omega_1, \mathbf{C}_\mathrm{d})$.

4. Optimality conditions

The generalized Lagrangian L for the optimal design problem can be defined as

$$L(\mathbf{C}_\mathrm{d}, \lambda, \boldsymbol{\mu}, \boldsymbol{\gamma}) = f + \lambda\left(\sum_{i=1}^{N} c_{\mathrm{d}i} - \overline{W}\right) + \sum_{i=1}^{N} \mu_i(0 - c_{\mathrm{d}i}) + \sum_{i=1}^{N} \gamma_i(c_{\mathrm{d}i} - \bar{c}_{\mathrm{d}i}) \tag{11}$$

where λ, $\boldsymbol{\mu} = \{\mu_i\}$ and $\boldsymbol{\gamma} = \{\gamma_i\}$ are the Lagrange multipliers. The principal optimality conditions for this problem without active upper and lower bound conditions on damper capacity may be derived from the stationarity conditions of $L(\boldsymbol{\mu} = \mathbf{0}, \boldsymbol{\gamma} = \mathbf{0})$ with respect to $\mathbf{C}_\mathrm{d}$ and λ.

$$f_{,i} + \lambda = 0 \quad \text{for} \quad 0 < c_{\mathrm{d}i} < \bar{c}_{\mathrm{d}i} \quad (i = 1,\ldots,N) \tag{12}$$

$$\sum_{i=1}^{N} c_{\mathrm{d}i} - \overline{W} = 0 \tag{13}$$

The symbols $(\ )_{,i}$ denotes the partial differentiation with respect to $c_{\mathrm{d}i}$.

When the other constraints on upper and lower bounds of damper's capacity are active, the optimality conditions should be modified into

$$f_{,i} + \lambda \geq 0 \quad \text{for} \quad c_{\mathrm{d}i} = 0 \tag{14}$$

$$f_{,i} + \lambda \leq 0 \quad \text{for} \quad c_{\mathrm{d}i} = \bar{c}_{\mathrm{d}i} \tag{15}$$

It is very important to note that, while a fixed objective function has been set in the previous paper (Takewaki 1997), the objective function may change depending on the magnitude of the value of the transfer function. In other words, if the maximum transfer function change from the i-th storey to the j-th storey, the meaningful first-order sensitivities should be changed from $(\max_{\omega}|H_{\mathrm{D}i}(\omega)|)_{,k}$ to $(\max_{\omega}|H_{\mathrm{D}j}(\omega)|)_{,k}$. These sensitivities are usually discontinuous.

5. Solution procedure of optimal design problem

5.1 Algorithm for optimal damper placement

A gradient-based evolutionary solution algorithm is presented in this section for the problem of optimal damper placement. Since it is quite beneficial to obtain the optimal damper placement for various capacity levels of dampers, the total damper capacity $\overline{W}$ is increased gradually. Let $\Delta\mathbf{C}_\mathrm{d} = \{\Delta c_{\mathrm{d}i}\}$ and $\Delta\overline{W}$ denote the increment of VD capacity and the increment of the sum of VD capacities, respectively. The flowchart of this solution algorithm is shown in Fig. 2. The solution procedure is summarized as follows:

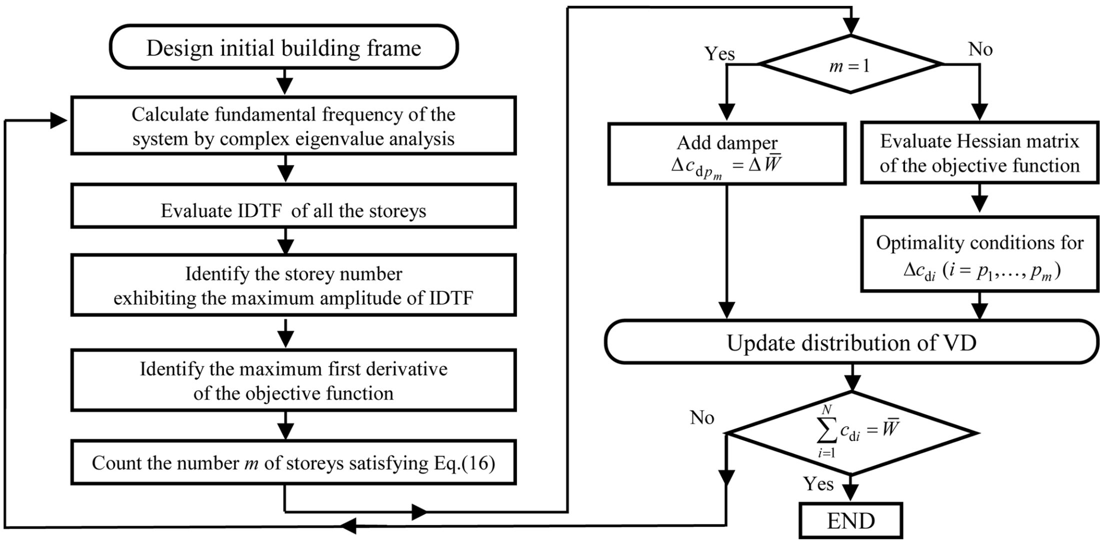


Fig. 2 Flowchart of the proposed solution algorithm

Step 0 Design the main frame without VD.
Step 1 Identify the storey where the peak amplitude of IDTF is maximized.
Step 2 Assume an increment of added VD capacity.
Step 3 Identify the location of storey p_m where the absolute value of the first-order sensitivity of the objective function f is maximized, i.e. satisfying following equation

$$f_{,p_m} = \max_i \left| f_{,i} \right| \quad (i = 1, \cdots, N) \tag{16}$$

Step 4 Count the number of optimal location of storeys identified in step 3, i.e. where the maximum absolute values of the first-order sensitivity of the objective function coincide. If m is equal to one, go to step 4A. If not, i.e. multiple p_m's exist, the optimal damper distribution can be computed in step 4B. It is important to monitor all the sensitivities related to the storey number exhibiting the maximum transfer function amplitude.
Step 4A Add VD to the optimal location of the storey identified in step 3.

$$c_{\mathrm{d}p_m} \to c_{\mathrm{d}p_m} + \Delta c_{d\mathrm{p}_m} \tag{17}$$

where $\Delta c_{\mathrm{d}p_m}$ is equal to an increment $\Delta\bar{W}$ of the added damper capacity.
Step 4B Compute the optimal damper distribution $\Delta c_{\mathrm{d}p_m}$ and update the damper placement by

$$c_{\mathrm{d}p_i} \to c_{\mathrm{d}p_i} + \Delta c_{d\mathrm{p}_i} \quad (i = 1, \cdots, m) \tag{18}$$

where $\sum_{i=1}^{m} \Delta c_{\mathrm{d}p_i} = \Delta\bar{W}$.

Step 5 Update the objective function and the sensitivities.
Step 6 Repeat the procedure from Step 4 to Step 5 until the constraint expressed by Eq. (12) is satisfied in a new storey.

The initial model is the model without VD, i.e. $c_{\mathrm{d}i} = 0$ ($i = 1,...,N$). Additional VD's are distributed via the steepest direction search algorithm (Takewaki et al. 1999, Takewaki 2009). When $\Delta\overline{W}$ is given, it is necessary to identify the optimal placement to decrease the objective function most effectively. For this purpose, the first and second-order sensitivities of the objective function with respect to the design variables $\mathbf{C_d}$ are necessary. Those sensitivities $f_{,i}$ and $f_{,ij}$ can be derived by differentiating Eq. (7) with respect to the design variable. Since the objective function can be described explicitly in Eq. (7) in the proposed gradient-based evolutionary solution algorithm, the sensitivities $f_{,i}$ and $f_{,ij}$ can be evaluated efficiently. This procedure is different from the well-known steepest descent method. Detailed expressions of the first and second-order sensitivities, i.e. the gradient vector and Hessian matrix of the objective function with respect to Δc_{d_i}, will be shown in the next section.

In step 4, if the multiple equalities of the optimality conditions, Eq. (12), are satisfied, the optimal damper placement has to be searched and the objective function has to be updated to keep the coincidence of the multiple first-order sensitivities. The objective function has to be updated by

$$f \to f + \sum_{j=p_1}^{p_m} f_{,i}\Delta c_{\mathrm{d}i} \tag{19}$$

where p_i ($i = 1,...,m$) denotes the optimal location of the storey for added damper which can be identified by the gradient vector. Since the number of unknown variables is m, m equations are required for solution.

After the multiple optimality conditions are updated, the first-order sensitivities should continue to be satisfied. To achieve successive satisfaction of the optimality conditions, the following equations can be derived.

$$\sum_{j=p_1}^{p_m} f_{,p_1 j}\Delta c_{\mathrm{d}j} = \cdots = \sum_{j=p_1}^{p_m} f_{,p_m j}\Delta c_{\mathrm{d}j} \tag{20}$$

After some manipulation in Eq. (20), m-1 equations can be derived with respect to $\Delta\mathbf{C_d}$

$$\sum_{j=p_1}^{p_m} \left(f_{,p_i j} - f_{,p_{i+1} j}\right)\Delta c_{\mathrm{d}j} = \mathrm{const.} \quad (i = 1,\cdots,m-1) \tag{21}$$

Furthermore, the increment $\Delta\mathbf{C_d}$ of added damper has to satisfy Eq. (13). In the case of the existence of the multiple equalities of the optimality conditions, Eq. (13) may be rewritten as

$$\sum_{i=p_1}^{p_m} \Delta c_{\mathrm{d}i} = \Delta\overline{W} \tag{22}$$

From Eqs. (21) and (22), we can derive the following simultaneous linear equations to obtain the increment $\Delta\mathbf{C_d} = \{\Delta c_{\mathrm{d}p_1}, \ldots, \Delta c_{\mathrm{d}p_m}\}$ of the optimal damper distribution.

$$\begin{bmatrix} f_{,p_1 p_1} - f_{,p_2 p_1} & & f_{,p_1 p_m} - f_{,p_2 p_m} \\ & \ddots & \\ f_{,p_{m-1} p_1} - f_{,p_m p_1} & & f_{,p_{m-1} p_m} - f_{,p_m p_m} \\ 1 & \cdots & 1 \end{bmatrix} \begin{Bmatrix} \Delta c_{\mathrm{d}p_1} \\ \vdots \\ \Delta c_{\mathrm{d}p_m} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ \Delta\overline{W} \end{Bmatrix} \tag{23}$$

5.2 Gradient vector and Hessian matrix of objective function with respect to damper's capacity

5.2.1 Formulation using undamped eigenvalue analysis

For evaluating the maximum amplitude of IDTF, it may be useful to utilize the value at the fundamental natural circular frequency ω_0 of the structure without VD. In this section, the gradient vector and Hessian matrix of the objective function with respect to damper's capacity $\mathbf{C_d}$ are provided by using ω_0 which is calculated by the undamped eigenvalue analysis. Since ω_0 of the frame without VD is fixed at a particular value, the first and second-order sensitivities of the objective function in Eq. (7) can be simplified to

$$f_{,i} = \left|H_{\mathrm{D}_m}(\omega_0)\right|_{,i} \tag{24}$$

$$f_{,ij} = \left|H_{\mathrm{D}_m}(\omega_0)\right|_{,ij} \tag{25}$$

where $\left|H_{\mathrm{D}_m}(\omega_0)\right|_{,i}$ and $\left|H_{\mathrm{D}_m}(\omega_0)\right|_{,ij}$ are derived as follows.

$$\left|H_{\mathrm{D}_m}(\omega_0)\right|_{,i} = \frac{1}{\left|H_{\mathrm{D}_m}(\omega_0)\right|}\left[\begin{array}{l}\mathrm{Re}\left\{H_{\mathrm{D}_m}(\omega_0)\right\}\mathrm{Re}\left\{H_{\mathrm{D}_m}(\omega_0)_{,i}\right\} \\ +\mathrm{Im}\left\{H_{\mathrm{D}_m}(\omega_0)\right\}\mathrm{Im}\left\{H_{\mathrm{D}_m}(\omega_0)_{,i}\right\}\end{array}\right] \tag{26}$$

$$\left|H_{\mathrm{D}_m}(\omega_0)\right|_{,ij} = \left[\frac{1}{2\left|H_{\mathrm{D}_m}(\omega_0)\right|}\left\{H_{\mathrm{D}_m}(\omega_0)H_{\mathrm{D}_m}^{\ *}(\omega_0)\right\}_{,i}\right]_{,j} \tag{27}$$

In this section, m is used as an arbitrary storey number. In Eqs. (26) and (27), the first and second derivatives $H_{\mathrm{D}_m}(\omega)_{,i}$ and $H_{\mathrm{D}_m}(\omega)_{,ij}$ of the transfer function can be derived as

$$H_{\mathrm{D}_m}(\omega_0)_{,i} = \mathbf{T}_m\{\mathbf{A}(\omega_0)^{-1}\}_{,i}\mathbf{B} \tag{28}$$

$$H_{\mathrm{D}_m}(\omega_0)_{,ij} = \mathbf{T}_m\{\mathbf{A}(\omega_0)^{-1}\}_{,ij}\mathbf{B} \tag{29}$$

where $\mathbf{T}_m$ is the m-th row vector in $\mathbf{T}$. In Eq. (28), the first derivative of $\mathbf{A}(\omega_0)^{-1}$ with respect to $\mathbf{C_d}$ is given by

$$\{\mathbf{A}(\omega_0)^{-1}\}_{,i} = -\mathbf{A}(\omega_0)^{-1}\mathbf{A}(\omega_0)_{,i}\mathbf{A}(\omega_0)^{-1} = -\mathrm{i}\,\omega_0\mathbf{A}(\omega_0)^{-1}\mathbf{C}_{\mathrm{D},i}\mathbf{A}(\omega_0)^{-1} \tag{30}$$

where $\mathbf{C}_{\mathrm{D},i}$ is the first derivative of the damping matrix $\mathbf{C}_\mathrm{D}$ composed by 1 and 0.

On the other hand, in Eq. (29), the second derivative of $\mathbf{A}(\omega_0)^{-1}$ with respect to $\mathbf{C_d}$ can be computed by differentiating Eq. (30) with respect to c_{d_j}.

$$\begin{aligned}&\{\mathbf{A}(\omega_0)^{-1}\}_{,ij} \\ &= \mathbf{A}(\omega_0)^{-1}\{\mathbf{A}(\omega_0)_{,i}\mathbf{A}(\omega_0)^{-1}\mathbf{A}(\omega_0)_{,j} + \mathbf{A}(\omega_0)_{,j}\mathbf{A}(\omega_0)^{-1}\mathbf{A}(\omega_0)_{,i}\}\mathbf{A}(\omega_0)^{-1}\end{aligned} \tag{31}$$

5.2.2 Formulation using complex eigenvalue analysis

For evaluating the maximum amplitude of IDTF more exactly, the natural circular frequency computed by the complex eigenvalue analysis for the whole system with additional damper is more appropriate. The objective function may then be regarded as the function of both $\mathbf{C}_d$ and ω_1 (fundamental natural circular frequency by complex eigenvalue analysis). The gradient vector and Hessian matrix of the objective function can be described as follows

$$f_{,i} = \omega_{1,i} \left| H_{\mathrm{D}_m}(\omega_1) \right|^{,\omega_1} + \left| H_{\mathrm{D}_m}(\omega_1) \right|_{,i} \tag{32}$$

$$f_{,ij} = \omega_{1,ij} \left| H_{\mathrm{D}_m}(\omega_1) \right|^{,\omega_1} + \omega_{1,i} \left| H_{\mathrm{D}_m}(\omega_1) \right|_{,j}^{,\omega_1} + \left| H_{\mathrm{D}_m}(\omega_1) \right|_{,ij} \tag{33}$$

where $(\ )^{,\omega_1}$ denotes the partial differentiation with respect to ω_1 (Takewaki 2002). The derivatives of the maximum amplitude of IDTF described by Eqs. (24)-(27) have been rewritten here.

6. Numerical examples

Numerical examples are presented for 5-storey and 10-storey shear building models to demonstrate the usefulness and validity of the proposed optimal design method. Detailed comparison is also provided with the result obtained by another general optimization methodology of "Sequential Quadratic Programming (SQP)". Further comparison with the optimized result for an H_∞ norm (Bai *et al.* 2006,Yamamoto *et al.* 2010) is made from the view point of robustness.

The structural parameters of the main frames are shown in Table 1. The floor masses and frame storey stiffnesses are identical in all the storeys. From the practical point of view, the problem of optimal damper placement for high-rise buildings should be considered. However, in this paper, for the sake of investigating the basic aspect of the proposed optimization methodology, the number of

Table 1 Structural parameters of main frame

	5-storey building	10-storey building
Floor mass [kg]	512×10^3	1024×10^3
Storey stiffness (N/mm)	1.559×10^9	1.809×10^9
Fundamental natural circular frequency without damper (rad/s)	30.96	6.281
Fundamental natural period without damper (s)	0.40	1.0

Table 2 Comparison of the maximum amplitude of transfer function

	Maximum amplitude of interstorey drift transfer function		
	(a) Undamped eigenvalue analysis	(b) Complex eigenvalue analysis	(c) Exact
$\overline{W} = 3.91 \times 10^7$ [Ns/m]	1.294×10^{-2}	1.291×10^{-3}	1.294×10^{-2}
$\overline{W} = 1.17 \times 10^8$ [Ns/m]	6.174×10^{-3}	6.348×10^{-3}	6.382×10^{-3}

storeys is given as 5 and 10. The fundamental natural period T of the building without additional dampers is given by 0.4 [s] for the 5-storey building model and 1.0 [s] for the 10-storey building model, respectively. The structural damping ratio of the main frame is assumed to be 0.02 (stiffness-proportional damping). The total damper capacity $\overline{W}$ is taken as $\overline{W} = 1.19 \times 10^8$ [Ns/m] for the 5-storey building model and $\overline{W} = 7.41 \times 10^8$ [Ns/m] for the 10-storey building model.

Table 2 shows the comparison of the maximum values of IDTF including VD ($\overline{W} = 3.91 \times 10^7$ [Ns/m] and $\overline{W} = 1.17 \times 10^8$ [Ns/m]) calculated based on (a) the undamped eigenvalue analysis, (b) the complex eigenvalue analysis and (c) the numerical search method (exact). In this example, the additional damper is assumed to be distributed to only the 1st storey. It can be observed from Table 2 that, when the quantity of the additional VD is large, the objective function computed by the complex eigenvalue analysis is more accurate than the undamped one.

Fig. 3 shows the variation of the first-order sensitivities of the objective function with respect to $\mathbf{C_d} = \{c_{d1},...,c_{d5}\}$ for the 5-storey model. The objective function in Eq. (7) has to be checked for all the storeys in the optimization procedure. If the storey number varies which makes the amplitude of IDTF maximum, i.e. the target storey for the objective function moves into other storeys, the first-order sensitivity of the objective function becomes a discontinuous function as observed in Fig. 3. This phenomenon occurs because the first-order sensitivities required in the optimality condition are different according to the target story. Therefore full monitoring is necessary of all the sensitivities related to the storey number exhibiting the maximum transfer function amplitude. Only the sensitivities related to the storey number exhibiting the maximum transfer function amplitude are meaningful. Another phenomenon of interchange of the first-order sensitivities (like a vibration phenomenon) can be avoided by decreasing the increment of the total damper capacity $\overline{W}$. However this procedure will increase the computational time.

Fig. 4 illustrates the maximum amplitudes of IDTF with respect to the varied total damping capacity. It can be observed that the maximum amplitude of IDTF changes from the first story to the second story at $\overline{W} = 3.90 \times 10^7$ [Ns/m]. Fig. 5(a) shows the absolute values of the first-order sensitivities of the objective function (the first-story maximum amplitude of IDTF) with respect to $\mathbf{C_d} = \{c_{d1},...,c_{d5}\}$ and Fig. 5(b) presents that of the objective function (the second-story maximum

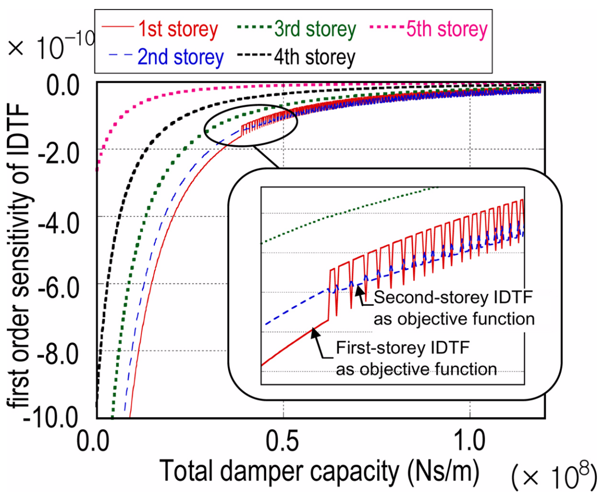


Fig. 3 Variation in the first-order sensitivities of IDTF with respect to design variable

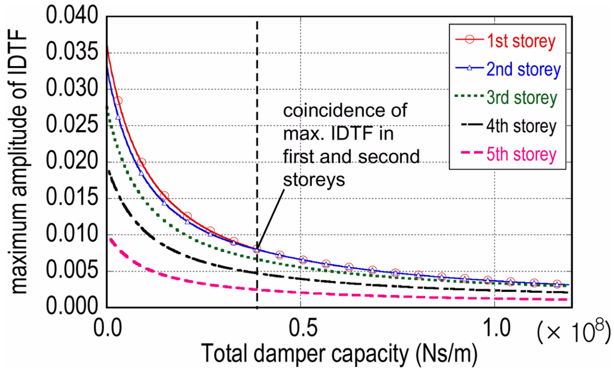
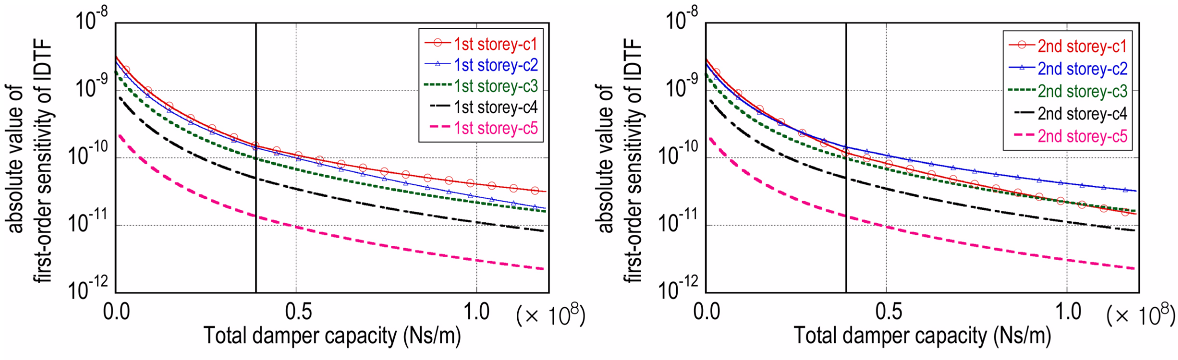


Fig. 4 Variation of the maximum amplitude of IDTF



(a) (b)

Fig. 5 Variation of the absolute value of the first-order sensitivities of IDTF with respect to design variable: (a) First-storey IDTF, (b) Second-storey IDTF

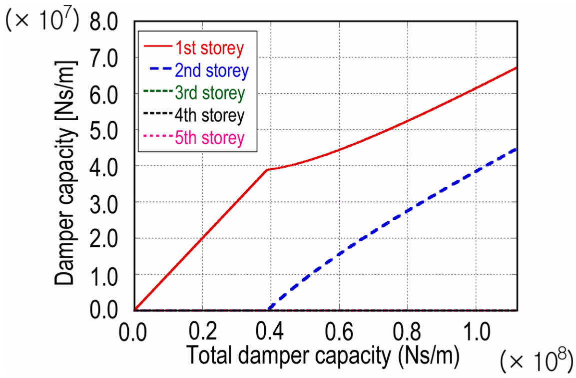


Fig. 6 Variation in the optimal damper placement for 5-storey shear building

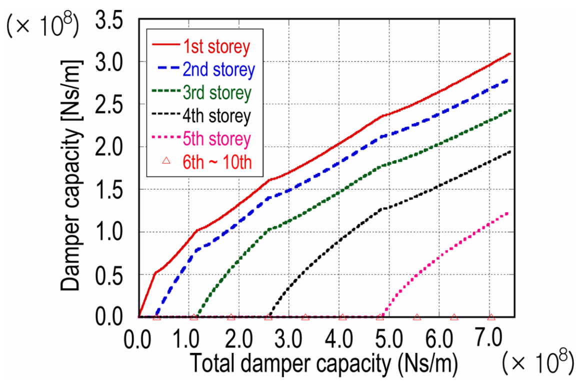


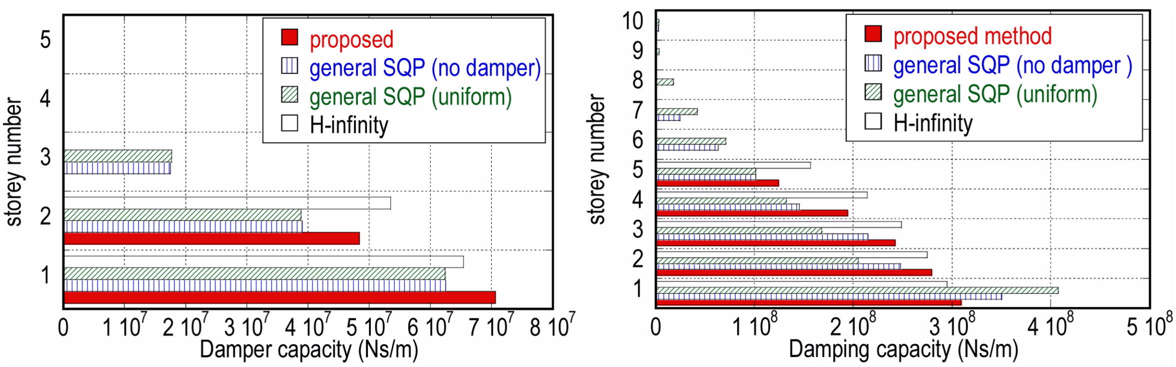
Fig. 7 Variation in the optimal damper placement for 10-storey shear building

amplitude of IDTF) with respect to $\mathbf{C}_\mathbf{d} = \{c_{d1},...,c_{d5}\}$. It is important to note that Fig. 3 drawn for the first-storey IDTF is quite insufficient and Figs. 4 and 5 for the first and second-storey IDTF are inevitable for the successful analysis of the present problem.

Figs. 6 and 7 show the variation of the optimal damper distribution for the 5-storey and 10-storey building models with respect to the varied total damping capacity, respectively. From the first-order sensitivities of the objective function in Fig. 5, it can be confirmed that VD should be distributed to the multiple storeys appropriately where the objective function as the maximum amplitude of IDTF is switched to other storeys. Fig. 7 presents the optimal damper placement for the 10-storey building.

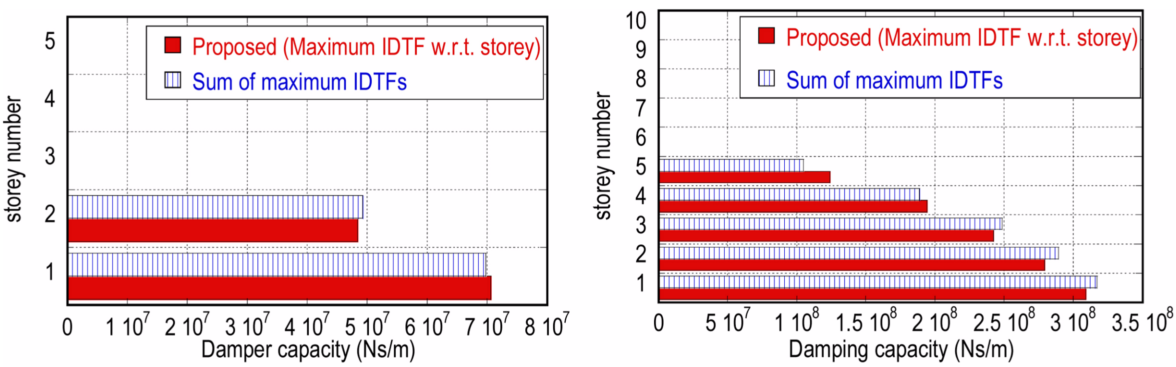
Fig. 8 shows the comparison of the optimal damper placements by the proposed methodology and a well-known general optimization methodology of "Sequential Quadratic Programming" for (a) the 5-storey building and (b) the 10-storey building. In the procedure of SQP, we need to conduct an initial guess. The initial guess of the design variable $\mathbf{C_d}$ is made as that without VD $\mathbf{C_d} = \{0,...,0\}$ and a uniform VD distribution $\mathbf{C_d} = \{\overline{W}/N,...,\overline{W}/N\}$. In Fig. 8, the optimal damper placement for an H_∞ norm (Bai *et al*. 2006, Yamamoto *et al*. 2010) is also included. The definition of the H_∞ norm is given in Appendix.

It can be observed from Fig. 8(a) that there is a little difference among the results by the two optimization methodologies and that for another objective function (H_∞ norm: Yamamoto *et al*. 2010). It is also seen that VD should be distributed also to the 3rd storey by SQP, while that phenomenon can not be seen in the damper placement by the proposed method and for another



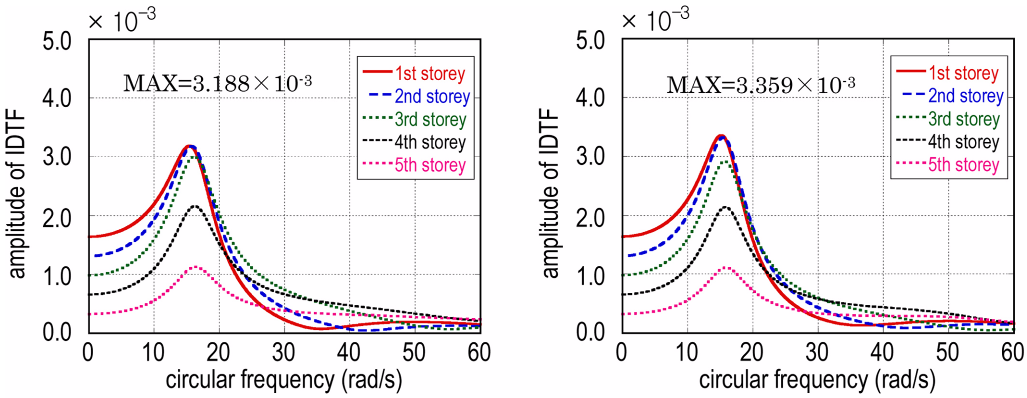
(a) (b)

Fig. 8 Comparison of the optimal damper placement by the proposed methodology with that by the general SQP (initial model: no damper or uniform placement) and that for H_∞ norm: (a) 5-storey building, (b) 10-storey building



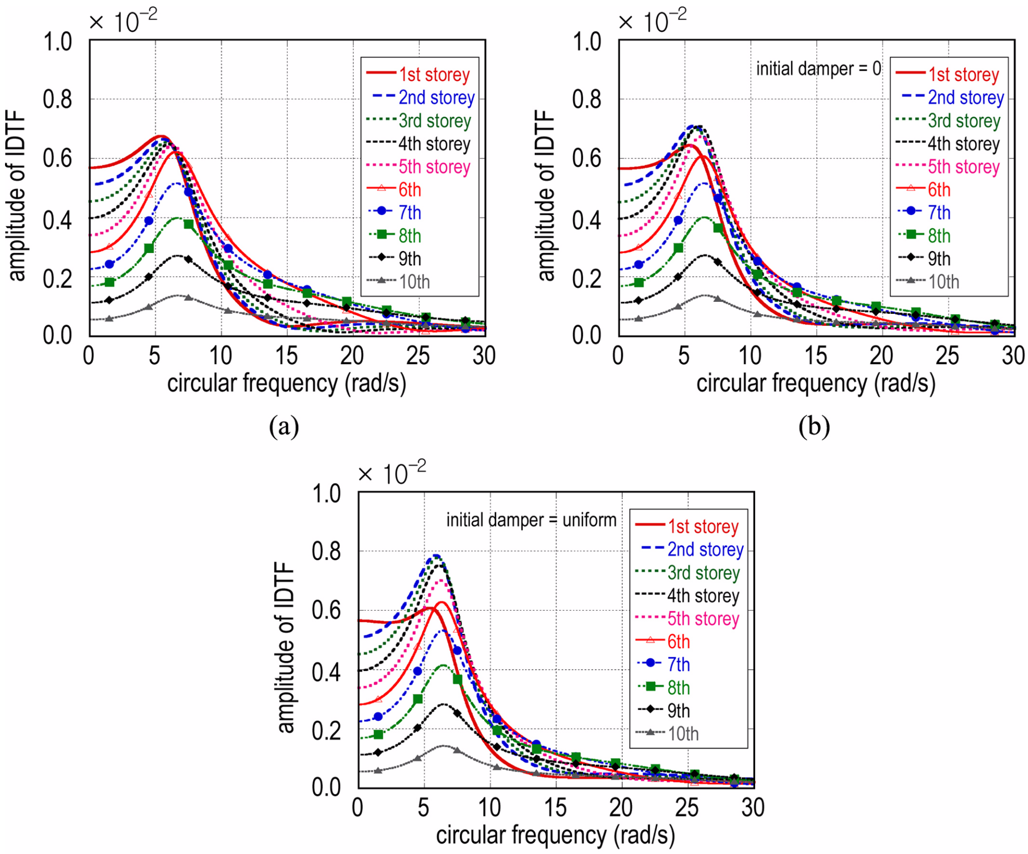
(a) (b)

Fig. 9 Comparison of the optimal damper placement by the proposed methodology with that by the previous one for sum of maximum IDTFs as objective function: (a) 5-storey building, (b) 10-storey building



(a) (b)

Fig. 10 Comparison of the amplitude of IDTF in 5-storey building model with the optimal damper placement: (a) Proposed methodology, (b) General SQP (initial model: no damper)



(a) (b)

(c)

Fig. 11 Comparison of the amplitude of IDTF in 10-storey building model with the optimal damper placement: (a) Proposed methodology, (b) General SQP (initial model: no damper), (c) General SQP (initial model: uniform placement) objective function (H_∞ norm: Yamamoto *et al*. 2010). In addition, there is no difference of the damper placements by two initial guesses. On the other hand, Fig. 8(b) indicates that there may be some remarkable points to say that (1) different solutions can be obtained by two optimization methodologies and (2) the optimal damper placement computed by SQP depends on the initial guess in a large-scale structure. In general, it appears that the optimal damper placement by the proposed method and that for another objective function (H_∞ norm: Yamamoto *et al*. 2010) are similar.

For the investigation of the effect of the objective function, Fig. 9 presents the comparison of the optimal damper placement by the proposed methodology for 5 and 10-story models with that by the previous one (Takewaki 1997) for the sum of the maximum IDTFs as an objective function. It can be observed that the effect of the objective function is rather small.

In order to demonstrate the validity of the proposed optimization methodology, we look at the amplitude of IDTF of structures with the optimal damper distributions illustrated in Fig. 8. Figs. 10(a)-(b) and 11(a)-(c) show the amplitude of IDTF for the 5-storey building and the 10-storey building with the optimal damper placement. It can be observed that the optimal damper placement obtained by the proposed methodology minimizes the maximum amplitude of IDTF most

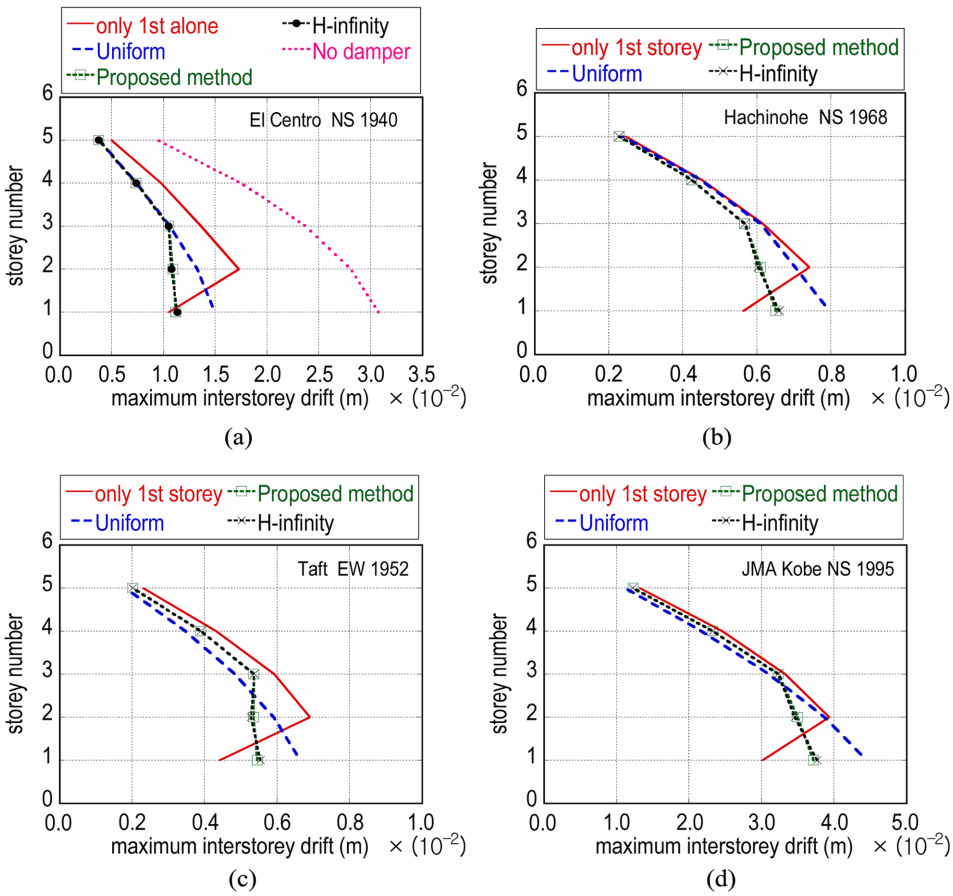


Fig. 12 Comparison of the maximum interstorey drifts, by time history response analysis, of the models with various damper distributions: (a) El Centro 1940 NS, (b) Hachinohe 1968 NS, (c) Taft 1952 EW, (d) JMA Kobe 1995 NS

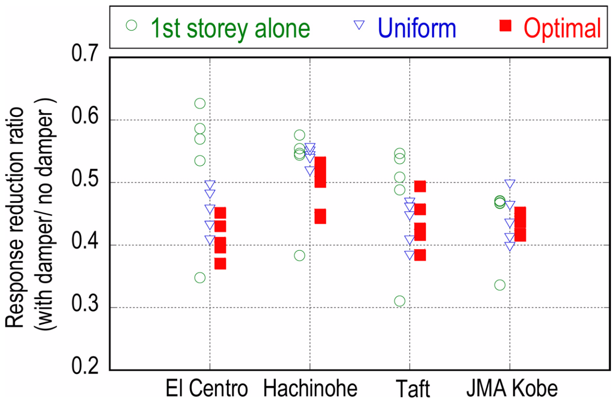


Fig. 13 Response reduction ratio of the maximum interstorey drift of the models with various placements of added damper to that of the model without damper under various recorded ground motions

effectively. Although SQP can be used easily so long as the objective function can be evaluated numerically, these figures indicate that SQP may not work well in case of poor parameter setting. In other words, the problem of convergence and initial-parameter dependence may arise in SQP. From these analyses, it can be concluded that the proposed optimization methodology can provide the optimal solution in a more accurate and reliable way than SQP.

From the practical point of view, it is quite useful to discuss what advantage can be achieved by solving the problem of the optimal damper placement for the maximum amplitude of IDTF. Figs. 12(a)-(d) show the comparison of the maximum interstorey drifts, in the 5-storey models with various damper placements, computed by the time history response analysis for recorded ground motions, (a) El Centro 1940 NS, (b) Hachinohe 1968 NS, (c) Taft 1952 EW and (d) JMA Kobe 1995 NS. Three different damper placements, i.e. the optimal placement, a 1st storey alone $\mathbf{C}_d = \{\overline{W},0,...,0\}$ and a uniform placement $\mathbf{C}_d = \{\overline{W}/5,...,\overline{W}/5\}$ are investigated. Furthermore, Fig. 13 shows the comparison of the response reduction ratio between the maximum interstorey drifts of the buildings with various added damper placements and those of the building without damper for various recorded ground motions. It can be observed from these figures that the optimally placed dampers can reduce the interstorey drifts most effectively among various damper placements.

7. Conclusions

The conclusions may be stated as follows.

(1) An evolutionary optimal placement method of viscous dampers has been presented to minimize the maximum transfer function of the interstorey drift with respect to both height and frequency for a shear building model. Since the gradient vector and the Hessian matrix of the objective function can be derived explicitly, a successive optimization methodology with respect to the varying total damper capacity can be provided.

(2) The storey number exhibiting the maximum transfer function amplitude may change storey to storey. Since the sum of the interstorey-drift transfer functions has been treated as an objective function in the former method (Takewaki 1997), this fact did not cause any problem. In the present problem, the problem of discontinuous objective functions may arise. This difficulty can be

overcome by monitoring all the sensitivities related to the storey number exhibiting the maximum transfer function amplitude.

(3) A gradient-based evolutionary optimization technique can be developed by using the Lagrange multiplier method. The satisfaction of the optimality criterion on placement of added viscous dampers has been guaranteed and this has been demonstrated through numerical examples.

(4) Numerical examples by the proposed method and detailed comparison with the results by a general optimization methodology (Sequential Quadratic Programming) have been conducted. The optimal damper placement derived by the proposed optimization methodology can produce the optimal damper placement effectively. The validity of the proposed optimization methodology has also been investigated by the time history analysis for various recorded ground motions.

(5) The comparison of the result due to the proposed method with that for another problem for the H_∞ norm demonstrated that the proposed method is almost equivalent to the method for the H_∞ norm which is believed to be robust for uncertainty.

Acknowledgements

Part of the present work is supported by the Grant-in-Aid for Scientific Research of Japan Society for the Promotion of Science (No.18360264, 21360267, 21 · 364). This support is greatly appreciated.

References

Attard, T.L. (2007), "Controlling all interstory displacements in highly nonlinear steel buildings using optimal viscous damping", *J. Struct. Eng. - ASCE*, **133**(9), 1331-1340.

Aydin, E., Boduroglub, M.H. and Guney, D. (2007), "Optimal damper distribution for seismic rehabilitation of planar building structures", *Eng. Struct.* **29**, 176-185.

Bai, Y., Grigoriadis, K.M. and Demetriou, M.A. (2006), "Damping parameter design in structural systems using an analytical H_∞ norm bound approach", *Proc. of the 45th IEEE conf. on Decision & Control,* San Diego, CA.

Cimellaro, G.P. (2007), "Simultaneous stiffness-damping optimization of structures with respect to acceleration, displacement and base shear", *Eng. Struct.*, **29**, 2853-2870.

Cimellaro, G.P., and Retamales, R. (2007). "Optimal softening and damping design for buildings", *Struct. Control Health Monitor.*, **14**(6), 831-857.

Constantinou, M.C. and Tadjbakhsh, I.G. (1983), "Optimum design of a first story damping system", *Comput. Struct.* **17**(2), 305-310.

De Silva, C.W. (1981), "An algorithm for the optimal design of passive vibration controllers for flexible systems", *J. Sound Vib.* **74**(4), 495-502.

Fujita, K., Moustafa A. and Takewaki, I. (2010), "Optimal placement of viscoelastic dampers and supporting members under variable critical excitations", *Earthq. Struct.*, **1**(1), 43-67.

Garcia, D.L. (2001), "A simple method for the design of optimal damper configurations in MDOF structures", *Earthq. Spectra*, **17**(3), 387-398.

Garcia, D.L. and Soong, T.T. (2002), "Efficiency of a simple approach to damper allocation in MDOF structures", *J. Struct. Control*, **9**(1), 19-30.

Gurgoze, M. and Muller, P.C. (1992), "Optimal positioning of dampers in multi-body systems", *J. Sound Vib.* **158**(3), 517-530.

Hahn, G.D. and Sathiavageeswaran, K.R. (1992), "Effects of added-damper distribution on the seismic response of buildings", *Comput. Struct.* **43**(5), 941-950.

Kiu, W., Tong, M., Wu, Y. and Lee, G. (2004), "Optimized damping device configuration design of a steel frame

structure based on building performance indices", *Earthq. Spectra*, **20**(1), 67-89.
Lavan, O. and Levy, R. (2005), "Optimal design of supplemental viscous dampers for irregular shear-frames in the presence of yielding", *Earthq. Eng. Struct. Dyn.*, **34**(8), 889-907.
Lavan, O. and Levy, R. (2006a), "Optimal design of supplemental viscous dampers for linear framed structures", *Earthq. Eng. Struct. Dyn.*, **35**(3), 337-356.
Lavan, O. and Levy, R. (2006b), "Optimal peripheral drift control of 3D irregular framed structures using supplemental viscous dampers", *J. Earthq. Eng.*, **10**(6), 903-923.
Levy, R. and Lavan O. (2006), "Fully stressed design of passive controllers in framed structures for seismic loadings", *Struct. Multidiscip. O.*, **32**(6), 485-498.
Liu, W., Tong, M., Wu, X. and Lee, G. (2003), "Object-oriented modeling of structural analysis and design with application to damping device configuration", *J. Comput. Civil Eng. - ASCE*, **17**(2), 113-122.
Liu, W., Tong, M. and Lee, G. (2005), "Optimization methodology for damper configuration based on building performance indices", *J. Struct. Eng. - ASCE*, **131**(11), 1746-1756.
Marano, G..C., Trentadue, F. and Greco, R. (2007), "Stochastic optimum design criterion for linear damper devices for seismic protection of building", *Struct. Multidiscip. O.*, **33**, 441-455.
Silvestri, S. and Trombetti, T. (2007), "Physical and numerical approaches for the optimal insertion of seismic viscous dampers in shear-type structures", *J. Earthq. Eng.*, **11**(5), 787-828.
Singh, M.P. and Moreschi, L.M. (2001), "Optimal seismic response control with dampers", *Earthq. Eng. Struct. Dyn.*, **30**(4), 553-572.
Singh, M.P. and Moreschi, L.M. (2002), "Optimal placement of dampers for passive response control", *Earthq. Eng. Struct. Dyn.*, **31**(4), 955-976.
Takewaki, I. (1997), "Optimal damper placement for minimum transfer functions", *Earthq. Eng. Struct. Dyn.*, **26**(11), 1113-1124.
Takewaki, I. (2000), "Optimal damper placement for planar building frames using transfer functions", *Struct. Multidiscip. O.*, **20**(4), 280-287.
Takewaki, I. (2002), "Robust building stiffness design for variable critical excitations", *J. Struct. Eng. - ASCE*, **128**(12), 1565-1574.
Takewaki, I. (2009), *Building control with passive dampers: optimal performance-based design for earthquakes*, John Wiley & Sons (Asia), Singapore.
Takewaki, I. and Yoshitomi, S. (1998), "Effects of support stiffnesses on optimal damper placement for a planar building frame", *J. Struct. Des. Tall Build.*, **7**(4), 323-336.
Takewaki, I., Yoshitomi, S., Uetani, K. and Tsuji, M. (1999), "Non-monotonic optimal damper placement via steepest direction search", *Earthq. Eng. Struct. Dyn.*, **28**(6), 655-670.
Tan, P., Dyke, SJ., Richardson, A. and Abdullah, M. (2005), "Integrated device placement and control design in civil structures using genetic algorithms", *J. Struct. Eng. - ASCE*, **131**(10), 1489-1496.
Trombetti T. and Silvestri, S. (2004), "Added viscous dampers in shear-type structures: the effectiveness of mass proportional damping", *J. Earthq. Eng.*, **8**(2), 275-313.
Trombetti, T. and Silvestri, S. (2007), "Novel schemes for inserting seismic dampers in shear-type systems based upon the mass proportional component of the Rayleigh damping matrix", *J. Sound Vib.*, **302**(3), 486-526.
Tsuji, M. and Nakamura, T. (1996), "Optimum viscous dampers for stiffness design of shear buildings", *J. Struct. Des. Tall Build.*, **5**, 217-234.
Uetani, K., Tsuji, M. and Takewaki, I. (2003), "Application of optimum design method to practical building frames with viscous dampers and hysteretic dampers", *Eng. Struct.*, **25**(5), 579-592.
Viola, E. and Guidi, F. (2008), "Influence of the supporting braces on the dynamic control of buildings with added viscous dampers", *Struct. Control Health Monitor.*, **16**(3), 267-286.
Wang, Y. and Dyke, S. (2008), "Smart system design for a 3D base-isolated benchmark building", *Struct. Control Health Monitor.*, **30**, 939-957.
Wongprasert, N. and Symans, M.D. (2004), "Application of a genetic algorithm for optimal damper distribution within the nonlinear seismic benchmark building", *J. Struct. Eng. - ASCE*, **130**(4), 401-406.
Yamamoto, K., Fujita, K. and Takewaki, I. (2010), "H_∞ optimization in damper placement for interstorey drift control of structures", *Proc. of Advances in Interaction and Multiscale Mechanics (AIMM'10)*, Jeju, Korea.

CC

Appendix: Definition of H_∞ norm

The equations of motion, in time domain, of the shear building model treated in this paper may be expressed by

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(\mathrm{t}) = -\mathbf{M}\mathbf{r}\ddot{u}_g(t) \tag{A1}$$

We now transform this system into a state space form as

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}f \\ \mathbf{y} = \mathbf{H}\mathbf{x} \end{cases} \tag{A2}$$

where

$$\mathbf{x} = \begin{bmatrix} \mathbf{u} \\ \dot{\mathbf{u}} \end{bmatrix},\ f = \ddot{u}_g,\ \mathbf{F} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix},\ \mathbf{G} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{r} \end{bmatrix}. \tag{A3}$$

The vector **y** and the matrix **H** denote the output vector and the output matrix, respectively.

The H_∞ norm of the system defined by Eq. (A2) is given by

$$\left\|\mathbf{T}\right\|_\infty = \sup_{\omega} \sigma_{\max}\left(\mathbf{T}\left(\mathrm{i}\omega\right)\right) \tag{A4}$$

Here $\mathbf{T}(s) = \mathrm{H}(s\mathbf{I} - \mathbf{F})^{-1}\ \mathbf{G}$ $(s = \mathrm{i}\omega)$ is the transfer function matrix of the system (**F**, **G**, **H**) and $\sigma_{\max}(\mathbf{A})$ denotes the maximum singular value of a matrix **A**, i.e., the square root of the maximum eigenvalue of $\mathbf{A}^*\mathbf{A}$ where $\mathbf{A}^*$ is the conjugate transpose of **A**. The maximum singular value may be regarded as the gain of an operator **A** that gives the maximum magnification against all input vectors applied to it.