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Modal rigidity center: it's use for assessing elastic torsion in asymmetric buildings

George K. Georgoussis*

Department of Civil and Construction Engineering, School of Pedagogical and Technological Education (ASPETE), N. Heraklion 14121, Attica, Greece

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Abstract. The vertical axis through the modal center of rigidity (m-CR) is used for interpreting the code torsional provisions in the design of eccentric multi-story building structures. The concept of m-CR has been demonstrated by the author in an earlier paper and the particular feature of this point is that when the vertical line of the centers of mass at the floor levels is passing through m-CR, minimum base torsion is developed. For this reason the aforesaid axis is used as reference axis for implementing the code provisions required by the equivalent static analysis. The study examines uniform mixed-bent-type multistory buildings with simple eccentricity, ranging from torsionally stiff to torsionally flexible systems. Using the results of a dynamic response spectrum analysis as a basis for comparisons, it is shown that the results of the code static design are on the safe side in torsionally stiff buildings, but unable to predict the required strength of bents on the stiff side of systems with a predominantly torsional response. Suggestions are made for improving the code provisions in such cases.

Keywords: asymmetric structures; design eccentricities, modal analysis.

1. Introduction

Strong ground motions of the past have revealed that asymmetry in structures is one of the main causes of severe damage. Even a small asymmetry, which may be considered as negligible from a static point of view, can be magnified during a major ground motion resulting in serious damage. From the design point of view therefore it is necessary to know the magnitude of such torsional effects in order to be able to estimate the strength of the resisting elements that provide the lateral resistance of a given structure.

The general approach recommended for the structural design of medium size buildings is to use equivalent static loading in conjunction with design eccentricities. The equivalent static (design) force is usually determined from the acceleration design spectrum, in relation to the fundamental period of the structure. Once this force is established, code provisions require that this force be applied eccentrically to the centre of rigidity (CR) at a distance equal to the design eccentricity. The concept of CR originates from studies on single-story systems with rigid floor diaphragms, where there is always a point (CR) on the floor slab with the following property: any lateral load passing through CR causes only a translation of the slab and any torque applied on the slab causes a

* Corresponding author, Professor, E-mail: ggeorgo@tee.gr

rotation about CR. That is, CR may also be defined as the 'center of twist'. The distance between CR and the centre of mass (CM) defines the static eccentricity e_s of the system, but the code provisions require that the design eccentricity be different from e_s. This is because eccentric structures undergo not only translational oscillations but torsional oscillations as well. Such dynamic coupling can lead to increased lateral displacements in the perimeter of the structure and possibly to a frame failure and even more to building's collapse. For these reasons the code provisions for torsional eccentricities and their adequacy to provide a safe design have been the subject of many investigations the last two decades.

Modern seismic codes require that the design eccentricity, measured from CR, should be determined by the following pair of equations

$$e_{d1} = \alpha e_s + \beta b \tag{1a}$$

$$e_{d2} = \gamma e_s - \beta b \tag{1b}$$

where b is the dimension of the building perpendicular to the direction of the ground motion and α, β and γ are specified coefficients. Neglecting the last part of the right hand side of the design eccentricities above, which represents an accidental eccentricity (due to discrepancies between the mass, stiffness and strength distributions used in analysis and true distributions at the time of an earthquake), it may be seen that the term αe_s in the first of the aforementioned expressions is intended to account for amplified torsional moments arising from lack of symmetry in plan. The coefficient α takes values higher than unity and the corresponding eccentricity is critical for assessing the design strength in elements on the flexible side of the structure (Zhu and Tso 1992). The basic idea of introducing this coefficient is to match the displacements produced by the equivalent static loading (acting at an eccentricity αe_s) at the flexible side of the structure with the corresponding dynamic displacements on the same side of the system (Dempsey and Tso 1982). Similar is the meaning of coefficient γ in the second of the expressions above, which may be determined by the same procedure in respect to the stiff side of the structure. In most of the aseismic codes: $0.5 \le \gamma \le 1$, but it must be mentioned that the procedure above, in predominantly torsional systems, may lead to negative values of γ (Anastassiadis *et al.* 1998).

Once the design eccentricities have been calculated as above, the process for a structural application in single story systems is a routine procedure, since CR can be easily determined as the location of the total lateral stiffness (this gives rise to the interchanging use of the terms 'center of rigidity' and 'center of stiffness'). The same process cannot be easily applied however in multistory structures, where there is not always a vertical axis with a property similar to that of CR in single story systems. Buildings, in which the lateral resistance is provided by a single type of bents, such as moment resisting frames or flexural shear walls (Tena-Colunga and Perez-Osornio 2005), uniform over the height, are typical examples of buildings which posses a vertical axis with the aforementioned property. This axis is usually referred as the 'elastic axis', but such buildings are rare in practice and there are a number of reasons indicating that a combination of different types of bents provides a much more efficient lateral resistance during a strong ground motion (Pauley and Priestley 1992). In other words, mixed-bent-type structures are more effective in withstanding ground motions, but in these cases there is a difficulty in implementing the code provisions about the design eccentricities, as defined by Eqs. (1).

It is possible to determine a set of points located at the floor levels such that when a given distribution of lateral loading passes through them only translational movement of the floors will occur. These points, usually referred as 'rigidity centers (CRs)' (Cheung and Tso 1986), are load dependant and their space distribution is very irregular, even in uniform structures composed of different types of bents. There is only a special class of multistory buildings where these points are independent of the lateral load distribution and they all lie on a vertical axis. This is the class of buildings where the ratios between the stiffnesses of the various bents are constant over the height of the structure (proportionate structures). Interpreting the concept of eccentricity in a dynamic sense, Riddle and Vasquez (1984) also concluded that the centers of rigidity exist only in buildings with proportional framing. Similarly, it is possible to determine another set of floor points which do not undergo any translational displacement when the structure is subject to applied torques only (Tso and Cheung 1986). These are defined as 'centers of twist' (CTs) but unlike single-story systems they can not in general identified as CRs.

The lack of a generally accepted definition of the centers of rigidity of multistory buildings has led many investigators to different interpretations about their locations (Smith and Vezina 1985, Poole 1977, Humar 1984). For structural applications, Goel and Chopra (1993) proposed a method of analysis without locating CRs, while Cheung and Tso suggested that these points, as defined in their study (Cheung and Tso 1986), may be used as reference points for calculating floor eccentricities (Tso 1990). However, there is a considerable scattering in their location, even in buildings composed by structural walls, when there is a minor change in the stiffness of one wall bent (Cheung and Tso 1986). In addition to the scattering, the location of CRs is found on either side of the centre of mass of the floors, which in the case of uniform building, may be assumed to lie on a vertical line. This means that the eccentricities given by any of Eqs. (1) may take both positive and negative values along the height of the building. It is evident, that the use of CRs as reference points makes the implementation of the code provisions very difficult and is questionable whether such a procedure leads to a safe structural design. The basic question therefore, from the practicing engineer point of view, is to define a vertical axis (at least in uniform over the height structures) that when the code design eccentricities (Eqs. (1)) are implemented in relation to its location, a safe design is produced. Working on this concept Makarios and Anastassiadis (1998a,b) introduced the 'axis of optimum torsion' (Makarios 2005, 2008, Makarios *et al.* 2006). This axis has a fictitious character and can be determined on the grounds of the criterion that an in-plane lateral loading with the distribution of a 'seismic force' produces minimum rotation of the structure when is passing through this axis. It was shown by a parametric study that minimum rotation may be interpreted as zero rotation at a level equal to 80% of the total height of the structure. Therefore, using the reciprocity theorem, the location of this axis may be determined as the point of zero displacement at the aforementioned level when the structure is subjected to a set of floor torques equal in magnitude to the lateral forces at the same floors. Working on the same idea, Marino and Rossi (2004) proposed an alternative procedure to define the position of this axis by means of mathematical expressions.

The aim of this paper is dual. First, to present an alternative method for determining the axis of minimum torsional response; this, in fact, is the vertical axis (m-CR axis) through the modal center of rigidity. The concept of this center has already been demonstrated by the author in an earlier paper (Georgoussis 2009). This is the center of the element modal stiffnesses and may be determined from the first mode frequencies of the component planar bents (structural walls, moment resisting frames, coupled wall systems, wall-frame assemblies) that provide the lateral resistance of a given structure. In uniform structures, it has already been demonstrated (Georgoussis 2008, 2009) that when the vertical line of the centers of mass at the floor levels coincides with the m-CR axis, minimum base torsion is developed. Therefore, this axis may also be used as reference axis for implementing the code provisions in structural applications. The second aim is to examine whether a safe pseudo-static design is obtained when the code eccentricities are implemented in relation to this axis. The accuracy of the suggested static procedure is examined in monosysmmetric uniform multistory buildings, ranging from torsionally stiff to torsionally flexible systems. The structural system of the analyzed buildings is composed by two subsystems: structural walls and moment resisting frames, to represent common types of building structures. These building do not belong to the special class of proportionate structures, as the stiffness matrix of a wall differs from that of a moment resisting frame.

2. Determination of the m-CR axis in eccentric buildings

In eccentric building structures, the determination of m-CR requires the calculation of the first mode frequencies of the component subsystems that provide the lateral resistance in the assumed direction. Considering the monosymmetric building of Fig. 1, the practical process to determine the location of m-CR along the x-axis is as follows (Georgoussis 2009):

The first frequencies ω_{i1} (i = 1,2,...) of all the planar i-bents oriented in the y-direction are first evaluated, assuming that the mass at each level equals the mass of the corresponding story of the actual structure (in other words that the lateral resistance of the structure under consideration is provided merely by the particular i-bent, assumed to be located at the center of the floor plan. In the aforesaid figure there are a pair of structural walls (SW1 and SW2) and a pair of moment resisting frames (RF1 and RF2) along the y-direction). The location of m-CR is then determined as the ratio of the sum of the first moment of the element squared frequencies ω_{i1}^2 to the squared first mode frequency, ω_1^2, of the symmetrical counterpart structure in the assumed direction. The latter frequency can also be assessed from the ω_{i1} frequencies, by means of Southwell's formula (Newmark and Rosenblueth 1972, Jacobsen and Ayre 1958) which provides a lower, but very close,

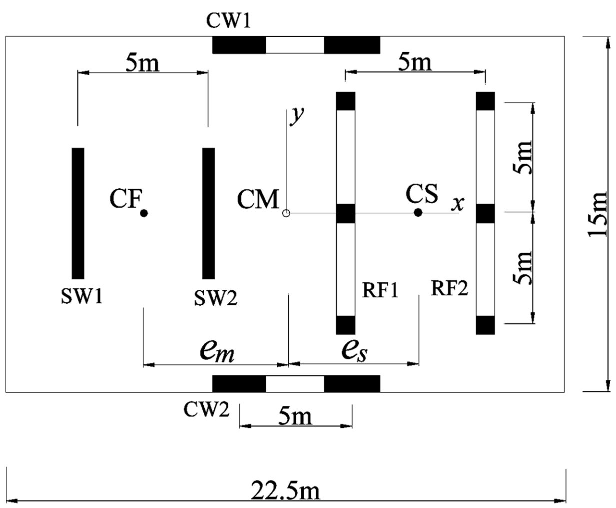

Fig. 1 Floor plan of the example structure

bound to the fundamental frequency of the complete structure, i.e.

$$\omega_1^2 = \Sigma \omega_{i1}^2 \tag{2}$$

Denoting with x_i the location of the i-bent (subsystem) in a CM reference system, the distance of m-CR from CM is given by the formula

$$e = \frac{\Sigma(x_1 \omega_{i1}^2)}{\Sigma(\omega_{i1}^2)} = \frac{\Sigma(x_1 \omega_{i1}^2)}{\omega_1^2} \tag{3}$$

An alternative method, in uniform structures, is to estimate of the element frequencies ω_{i1} by means of the continuous medium approach, as demonstrated by Heidebrecht and Stafford Smith (1973), in combination with the mathematical model proposed by Georgoussis (2006) to account for axial deformations in the vertical members of the component bents. In brief, this procedure to assess frequencies is as follows:

Assuming, again that the lateral resistance of the given structure is provided merely by the i-bent (subsystem) and denoting by I_i the sum of column inertias of this bent and by GA_{ei} its equivalent modal shear rigidity (Georgoussis 2006), the undamped equation of free motion of this subsystem is given by

$$EI_i u_i^{iv} - GA_{ei} u_i'' = \omega_i^2 m u_i \tag{4}$$

where m is the mass of the actual structure per unit height, E the modulus of elasticity, u_i the horizontal deflection of the bent and ω_i its frequency. The frequencies of the system presented by Eq. (4) can be evaluated from the formula (Heidebrecht and Stafford Smith 1973, Heidebrecht 1975)

$$\omega_i = \lambda_1 \lambda_2 \sqrt{\frac{EI_i}{mH^4}} \tag{5}$$

where

$$\lambda_1 = \beta \lambda_o$$

$$\lambda_2 = \sqrt{(\lambda_1^2 + (\alpha_{ei} H)^2)} \tag{6}$$

$$\alpha_{ei} H = \sqrt{\frac{GA_{ei}}{EI_i}} H$$

λ_o =1.875, 4.694, 7.855 for the first three modes of vibration

In the expressions above, H is the height of the structure and β a coefficient that may be assumed equal to unity when $\alpha_{ei}H$ is less than 6. For higher values of $\alpha_{ei}H$ the accurate value of β is required, especially when the fundamental frequency is to be estimated. Exact values of β, in a graphical form, are given by Georgoussis (2006) in an earlier paper.

Once the first mode frequencies ω_{i1} are determined (from Eq. (5)) for all the component subsystems in the assumed direction, the location of m-CR may again be found from Eq. (3). Note here that in slender wall bents $\lambda_1 = \lambda_2 = 1.875$, but the coefficient λ_2 receives much higher values in the case of frame bents. The first mode frequency of the symmetrical counterpart structure ω_1, of Eq. (2) may also be evaluated directly from Eqs. (5) and (6), using the total moment of inertia $I = \Sigma I_i$, instead of I_i and the overall stiffness ratio $\alpha_y H = \sqrt{\Sigma GA_{ei}/\Sigma EI_i}H$ instead of the element stiffness ratio $\alpha_{ei}H$. This is because the various types of structural bents (walls, frames, coupled walls, wall-frame assemblies) belong to the same family of shear-flexure cantilever bents and present a similar type of first mode displacements. In fact, this is the reason why Southwell's formula (Eq. (2)) provides a very good estimate of the first mode frequency of the symmetrical counterpart building structure.

It worth's mentioning here that a structural configuration of minimum base torsion may be attained when m-CR coincides with CM, that is when: $e = 0$. It has been demonstrated (Georgoussis 2008) that in such cases the resultant torsional moment developed in medium height structures is minimal and can be virtually neglected in practical problems. The condition of zero eccentricity between CM and m-CR, indicating that the CM axis passes though m-CR, can be implemented by arranging the coordinates of the center of total flexural rigidity (CF) and the center of total shear rigidity (CS), which are respectively defined as

$$e_m = \frac{\Sigma x_i EI_i}{\Sigma EI_i} \tag{7a}$$

$$e_s = \frac{\Sigma x_i GA_{ei}}{\Sigma GA_{ei}} \tag{7b}$$

to have a ratio equal to (Georgoussis 2008, 2009)

$$\frac{e_s}{e_m} = -\frac{1.875^2}{(\alpha_y H)^2} \tag{8}$$

The locations of CF and CS may be seen in Fig. 1 for the case of the example structure, which is described in detail further below. In this figure, e_m denotes the location of the center of the two structural walls (SW1 and SW2), while e_s is the center of the two moment frames (RF1 and RF2). The simple form of the Eq. (8), which provides a structural arrangement of minimum torsional response, may be used as a part of the architectural considerations in the initial stage of design, when the overall configuration of a tall building is decided. It is important for the practicing engineer, at this stage of design, to know what factors of the structure cause coupling and decide whether or not the floor plan should be rearranged without having to perform a full 3D computer dynamic analysis.

3. Systems analyzed

To investigate the accuracy of implementing the design eccentricities in respect to the proposed m-CR axis, the example structure shown in Fig. 1 was analyzed. This is a 10-story monosymmetric uniform mixed-bent-type structure with an orthogonal floor plan of 22.5×15 m, having a pair of structural walls (SW1 and SW2) and a pair of moment resisting frames (RF1 and RF2) along the y-direction and a pair of coupled wall (CW1 and CW2) bents oriented in the axis of symmetry (x-direction). Although the various resisting bents are uniform, the framing in the assumed structure is not proportional, as the stiffness matrix of a structural wall is different from that of a moment resisting frame. The structural walls are of a cross section 30×600 cm and the distance between them is 5 m. The moment resisting frames consist of three 60×60 cm columns, 5 m apart, connected by beams of a cross section 30×70 cm and the distance between them is 5 m. The coupled wall bents consist of two 30×300 cm walls at a distance of 5 m connected by 25×90 cm beams at the floor levels, located symmetrically to the axis of symmetry x. Three models of the example structure are examined. In the first model (Model 1) the pair of structural walls is located on the far left side of the floor plan, while the pair of moment resisting frames is located at the opposite side of the floor plan. The exact locations are -11.25 m and -6.25 m respectively for the two walls and 6.25 m, 11.25 m for the two frames. The coupled wall bents are located symmetrically to the axis of symmetry at a distance of 2×7.5 m. The second model (Model 2) has the same lateral stiffness along the y-direction, but as the resisting bents are closer to the centroid of the floor slab its torsional resistance is reduced. The exact locations are now: -8 m and -3 m respectively for the two structural walls, 3 m and 8 m for the two moment resisting frames, while the pair of coupled walls along the axis of symmetry is at a distance: 2×5.8 m. The third model (Model 3) is similar to the others, but its torsional stiffness is further reduced. The locations of the structural walls are: -6 m and -1 m respectively, those of the two moment frames: 1 m and 6 m, while the pair of coupled walls is at a distance: 2×5.1 m.

The total mass per floor is 270000 Kg, the radius of gyration about CM is r = 7.806 m, the story height is 3.5 m and the modulus of elasticity (E) is assumed equal to 26×10^6 KN/m^2. The center of mass at each floor lies on a vertical line passing through the centroid of the orthogonal floor plan at each level. Details of the floor plan are shown in Fig 1.

The first mode frequency of a single wall-subsystem is equal to ω_w = 3.4678/s. This is the frequency when the entire lateral stiffness is provided by a single structural wall (as in a plane structure). Similarly, the corresponding frequency of a frame-subsystem is equal to ω_f = 2.1876/s. Therefore, the first mode lateral frequency of the uncoupled counterpart structure, estimated from the approximate formula (2), is found equal to ω_1 = 5.7985/s. Accordingly, the location, e, of m-CR, in the CM reference system (determined from Eq. (3)), is equal to: -3.77 m, -2.37 m and -1.51 m for the assumed three model structures respectively. It is interesting to compare the location of this axis with that of the optimum torsion axis introduced by Makarios and Anastassiadis (1998a, b). Applying on the structure a set of torsional moments having a vertical distribution similar to that of the first mode inertia forces of the uncoupled system, the location of the latter axis is found equal to: -3.31 m, -2.02 m and -1.25 m for the assumed three model structures respectively. Evidently, the optimum axis is passing in close distance from m-CR. This is expected since, by recalling Betti's reciprocity theorem, the optimum torsion axis is also the axis though which an in-plane loading (having the first mode distribution) produces lateral displacement and no torsional response.

Having determined the location of m-CR, it is interesting to calculate the ratio, Ω, of the first mode uncoupled torsional frequency to the uncoupled lateral frequency of the system, as introduced by Tso and Dempsey (1980) in single-story systems. It has been demonstrated (Georgoussis 2009) that Ω is given as

$$\Omega = \sqrt{\frac{K_\theta^*}{K_v^*\rho^2}} = \sqrt{\frac{K_w^* - e^2 K_v^*}{K_v^*\rho^2}} = \sqrt{\frac{\Sigma(x_i^2\omega_{i1}^2 + y_j^2\omega_{j1}^2) - e^2\omega_1^2}{\omega_1^2\rho^2}} \tag{9}$$

where ω_{j1} is the first mode frequency of the j-subsystem in the x-direction, y_j its coordinate and

$\rho = \sqrt{e^2 + r^2}$ is the radius of gyration of the floor about m-CR.

In the assumed example structure there is only one type of coupled wall bents in the x-direction, and the corresponding first mode frequency of each coupled wall-subsystem was found equal to $\omega_c = 4.1076$/s. In the wide column analogy that was used to simulate this coupled wall bent, the clear span of the beam was increased by its depth (Coull and Puri 1968) in order to take into account the flexibility of the joints between the walls and the connecting beams. Subsequently, the corresponding modal ratio Ω was found equal to 1.290, 0.986, and 0.818 for the assumed three models respectively. Physically, a value of Ω higher than unity represents a structural system with the first mode being predominantly translational, while values $\Omega < 1$ correspond to systems with the first mode being predominantly torsional (Dempsey and Tso 1982). Therefore, Model 1 may be considered as a torsionally stiff system, Model 3 as a torsionally flexible system, while Model 2 reveals a 'medium' case structure, where significant coupling between the first two modes of vibration may be expected (Tso and Dempsey 1980).

The efficiency of the static procedure recommended by the building code, in relation to the m-CR axis, is examined by performing a static analysis under a lateral loading having the shape of the first mode vector of the uncoupled structure. The resultant force was taken equal to the base shear given by a response spectrum modal analysis (in relation to the design spectrum of Fig. 2) and the lateral force at each floor level was applied at a distance from the m-CR axis equal to the design eccentricity, given by the pair of Eqs. (1) neglecting the accidental part of these equations. Since the static eccentricity is equal to: $e_s = -e$, the design eccentricities are evaluated assuming that the coefficients α and γ are equal to 1.5 and 0.5 respectively, as recommended by the Greek Aseismic Code (EAK 2000) and the National Building Code of Canada (NBCC 1995). The results of this static analysis (shear and moment envelopes in the resisting bents that are furthest away from the CM) are compared with those produced by the aforementioned dynamic modal response analysis in relation to the acceleration design spectrum of Fig. 2. This spectrum varies linearly from 0.3 g to 0.75 g when the period (T) is less than 0.15 sec, it is constant when 0.15 sec$<T<$0.585 sec and it varies inversely proportionally to period when T exceeds 0.585 sec. The dynamic analysis was performed by means of the SAP2000-V11 computer program and any force quantity was obtained by the CQC rule. The data presented in Figs. 3 to 5 are based on analyses that result on the same base shear and therefore a direct comparison can be made between the accurate dynamic results and those provided by the 'equivalent pseudo-static method'.

Shear envelopes for the moment resisting frame RF2, on the far right side of the structure, are

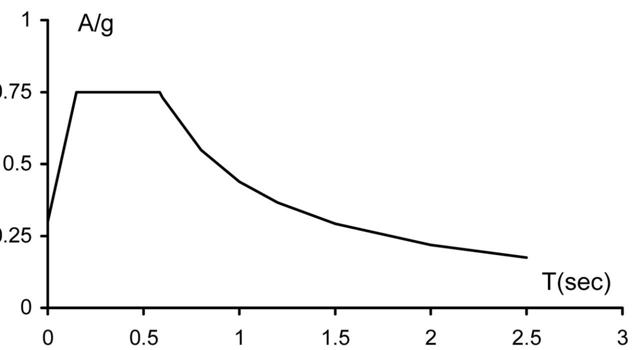

Fig. 2 Acceleration design spectrum

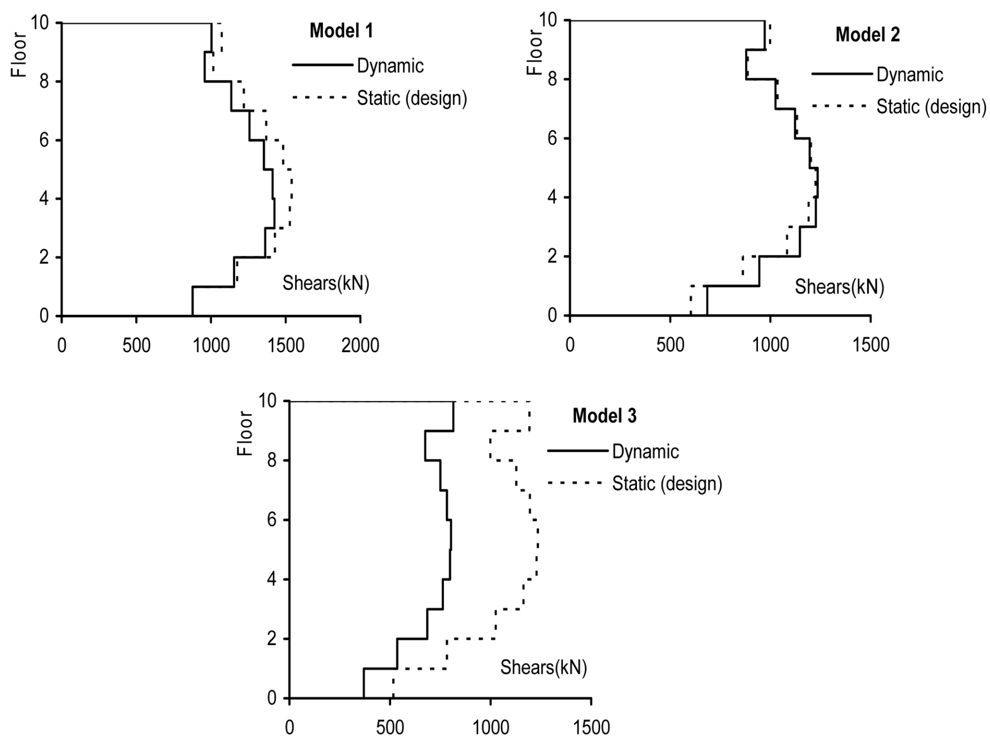

Fig. 3 Shear envelops for frame RF2

shown in Fig. 3. The static results are basically provided by the first of the design eccentricities of Eq. (1) and are considered satisfactory for structural applications. In Model 1, the static data are conservative, in all stories, representing a safe design. In Model 2 there is a negative deviation at the base of the frame of 11%, but the maximum shear force in the middle height of the bent is well estimated. In Model 3, which represents a primarily torsionally flexible system, the static results are clearly overestimating those of the dynamic analysis, by a percentage of 54%, in the middle height of the structure. It is evident that the value $\alpha = 1.5$ in the first of the design eccentricities, results to a significant overestimation of the design forces in the resisting bents on the flexible side of the building when $\Omega < 1$. Similar conclusions have also been drawn by Harasimowicz and Goel (1998) on slender flexural types of buildings.

Shear envelopes for the structural wall SW1 are shown in Fig. 4, of all model structures. In all these diagrams the static (design) results are basically given by the second of Eqs. (1). These results are well on the safe side for the case of Model 1 and this indicates that the coefficient value $\gamma = 0.5$ is adequate for torsionally stiff structures. In the case of Model 2, the static (design) results are underestimating the dynamic data by 21% at the base of this wall. This underestimating discrepancy is more notable in the case of Model 3, where it receives a value of 30%. It is evident that the code value $\gamma = 0.5$ is inadequate for structural applications on torsionally flexible systems. This becomes obvious by consideration of the procedure that provides the γ coefficient in single-story systems. The product γe_s may be seen as the 'secondary effective eccentricity' (Dempsey and Tso 1982) and is applicable for the design of resisting elements on the stiff side of the system. As it has been demonstrated, in torsionally flexible systems, the coefficient γ may obtain even negative values

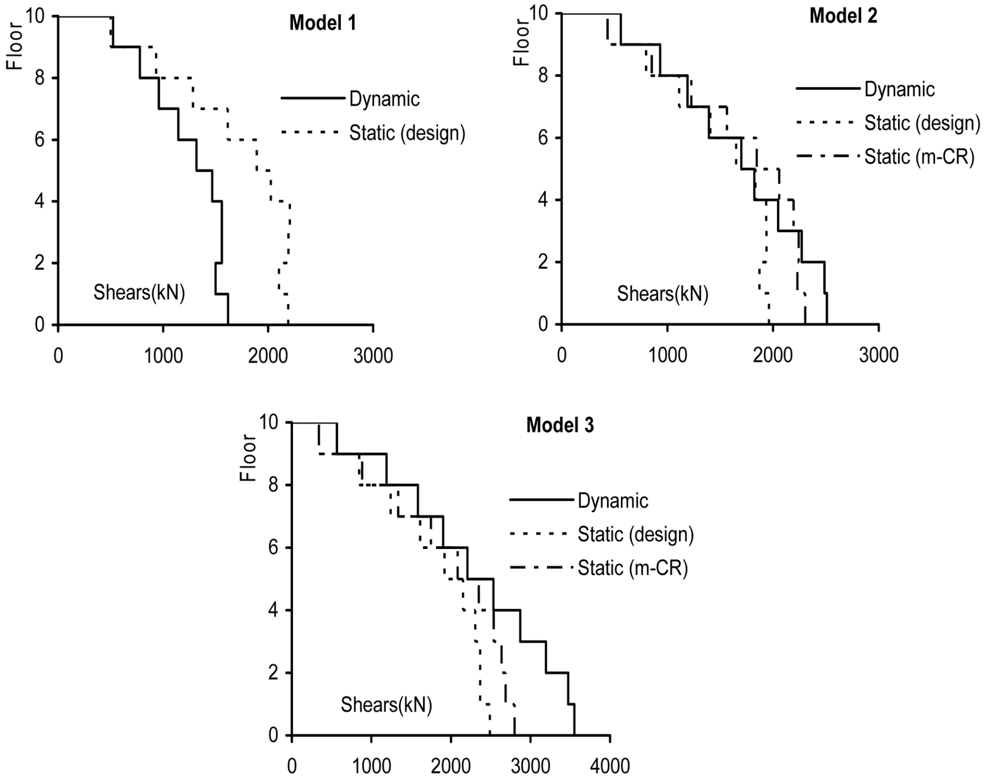

Fig. 4 Shear envelops for structural wall SW1

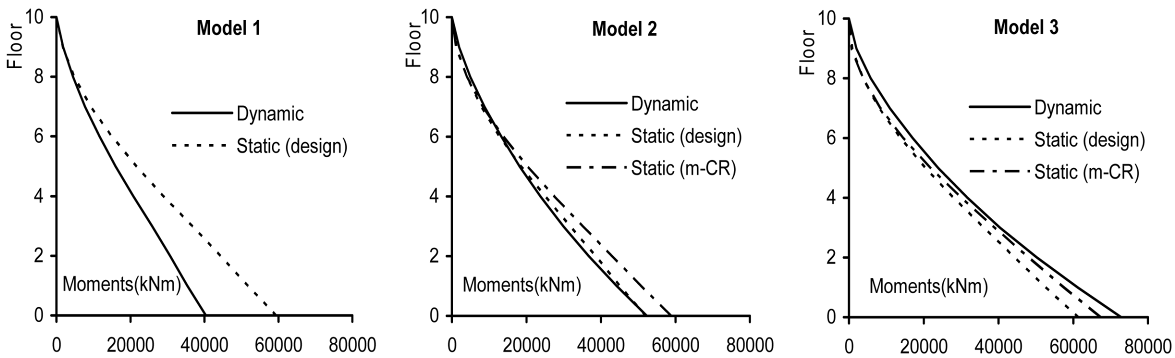

Fig. 5 Moment envelops for structural wall SW1

when the static eccentricity is rather small (Anastassiadis *et al.* 1998). Model 3 represents this sort of structures: the modal ratio of uncoupled frequencies Ω is equal to 0.818 and the normalized static eccentricity $\bar{e}_s = e_s/r$ is equal to 0.19. The static member forces are getting better when the value $\gamma = 0$ is assumed in the case of a torsionally flexible system ($\Omega < 1$). In other words when the static lateral loading is assumed to be applied through the m-CR a better profile of shear forces is obtained for the structural wall at the far left side of m-CR. This is shown by the static (m-CR) results in Models 2 and 3 of Fig. 4. The negative deviation at the base of Model 2 is now only 8%, while this deviation in Model 3 is 20%. The deviations on bending moments may be seen in Fig. 5.

The case of Model 3, which is of main concern, presents a negative error on base moments equal to -15% when γ = 0.5 (static (design) results) and an error equal to -7% when γ = 0 (static (m-CR) results). Envisaging the diagrams of Model 3, in Figs. 4 and 5, a rough conclusion that may be drawn is that the 'equivalent pseudostatic design' should be avoided in systems with Ω < 0.85.

4. Conclusions

The vertical axis through the modal center of rigidity (m-CR) is used for the design of eccentric multistory buildings. This is the center of the element modal stiffnesses and depends on the dynamic characteristics of a particular mode of vibration. It may be seen as the 'equivalent center of rigidity' for each mode of vibration, but for medium height structures, the response of which during a ground motion is basically dependent on the first mode displacements, it is adequate to restrict all the subsequent calculations on the first mode dynamic data. The location of m-CR may simply be determined from the first mode frequencies of the bent-subsystems that provide the lateral resistance of a given structure, and the vertical axis through m-CR may be used as the reference axis for implementing the code design eccentricities. This is justified by the findings that when this axis is passing through the centers of mass at the floor levels, minimum torsional coupling is developed in uniform eccentric systems. Analytical results are presented herein, obtained by implementing the static force procedures in respect to the m-CR axis in common mixed-bent-type eccentric buildings, ranging from torsionally stiff to torsionally flexible systems. The static force data are obtained by means of simple 3D static analyses, which merely require: (i) the design base shear and its in plane distribution at the floor levels of the building structure, and (ii) the design eccentricities in respect to the m-CR axis. Acknowledging the limitations of this work and the needs for further studies on different building configurations, the following conclusions may be drawn by comparing the static data, based on the coefficients α = 1.5 and γ = 0.5, with the accurate results of a dynamic response spectrum analysis:

1. The design eccentricity given from Eq. (1a), when α = 1.5, is adequate for the design of bents on the flexible side (in respect to the location of the m-CR axis) of building systems which are predominantly torsionally stiff. That is, when the ratio of the first mode uncoupled torsional frequency to the uncoupled lateral frequency, Ω, is higher than unity. This ratio is computed in the m-CR reference system and may determined from the first mode frequencies of the component bents, as in single-story systems. When: Ω < 1, the value α = 1.5 overestimates the required strength of the bents in the flexible side of the building by more than 50%.
2. For the bents on the stiff side of the building, the code eccentricity given by Eq. (1b) when γ = 0.5, proves to be adequate for systems with Ω > 1. Therefore it may be said that the code provisions (α = 1.5 γ = 0.5), are satisfactory to provide a safe design in the case of torsionally stiff structures (Ω > 1). The code accuracy is sharply reduced when the modal ratio Ω is falling below unity. In such cases and when Ω > 0.85, a relatively better estimate of the forces sustained by the bents on the stiff side of the building, may be obtained by means of a design eccentricity based on the value γ = 0. This estimate is not adequate in strict terms, but as the discrepancy is below 20%, it may be considered that it is within the accuracy limits of engineering practice.
3. The equivalent static procedure based on the design eccentricities of Eqs. (1), should be limited to systems which present a modal frequency ratio Ω > 0.85.

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