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The use of the semi-empirical method to establish a damping model for tire-soil system

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Abstract. This paper proposes a linear damping model of tire-soil system using semi-empirical method. A test rig was designed and developed to measure the vertical equivalent linear damping ratio of tire only and tire–soil system using Free-Vibration Logarithmic Decay Method. The test was performed with two kinds of tractor tires using a combination of five inflation pressure levels, two soil depths and four soil moisture contents in the paddy soil. The results revealed that the linear damping ratio of tires increased with decreasing tire inflation pressure; the linear damping ratio of tire-soil system also increased with decreasing tire inflation pressure and increased with the increasing soil depth (observed at 80 and 120 mm). It also increased with a relative increase of soil moisture contents (observed at 37.9%, 48.8%, 66.7% and 77.4%). The results also indicated that the damping ratio of tire-soil system was higher than that of tire only. A linear damping model of tire-soil system is proposed as a damping model in parallel which is established based on experimental results and vibration theory. This model will have a great significance in study of tractor vibration.

Keywords: linear damping ratio; logarithm decay method; tire-soil system; tractor vibration; semi-empirical method

1. Introduction

Establishment of a damping model for tire-soil system is very important in studying and assessing the ride comfort and safety of off-road vehicles. In order to study vibration properties of tire, soil and tire-soil system; many studies treated the tire, soil and tire soil system as a visco-elastic material, and modeled as parallel linear or nonlinear vertical models with the spring-damper in parallel (Çelebi *et al.* 2006, Nie *et al.* 2011, Das and Ramana 2011, Cuong *et al.* 2013, Damgaard *et al.* 2014, Carswell *et al.* 2015). Many studies showed that this is a quite complex and challenging task (AESCO 2005) and results vary depending on many factors such as tire characteristics, tire inflation pressure and wheel load, as well as soil conditions.

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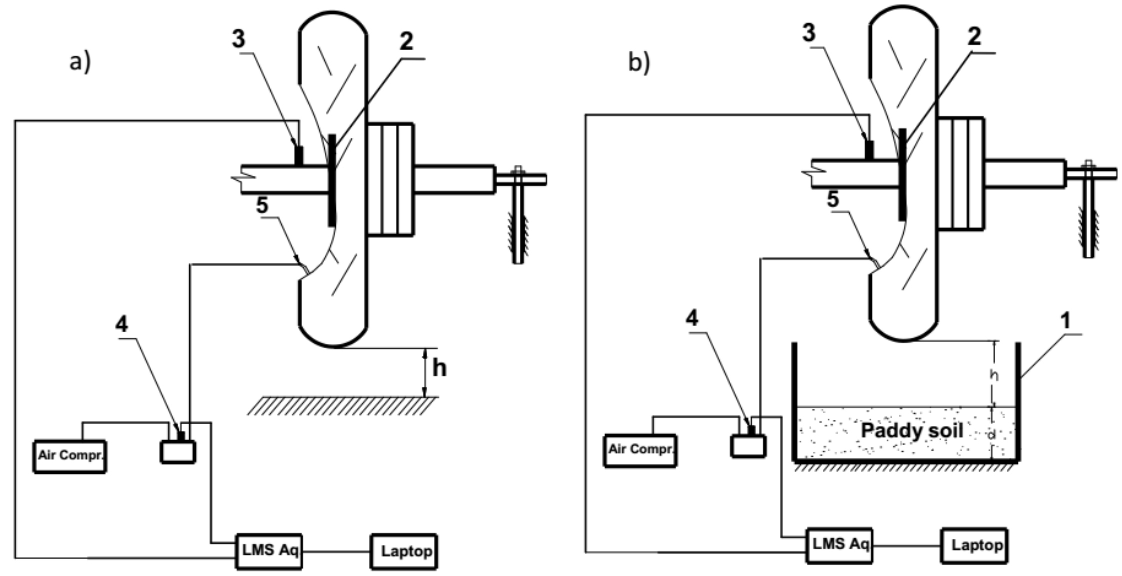


Fig. 1 Sketch of the damping ratio measurement system. (a) Sketch for measurement of vertical damping ratio of tractor tire; (b) Sketch for measurement of damping ratio of tire-soil system. 1. Steel tank; 2. Plate of link tire; 3. Acceleration sensor; 4. Air pressure sensor; 5. Tire valve

Nie *et al.* (2011) and Cuong *et al.* (2013) studied the variation of stiffness and damping coefficient of tire depending on tire inflation pressure, and concluded that the tire stiffness increased while the damping coefficient decreased relatively with increasing tire inflation pressure.

Soil has been treated as damping material. Soil damping is a function of many parameters including soil type, moisture content and temperature (Cautes and Nastac 2002). Cuong *et al.* (2014) concluded that the vertical equivalent damping ratio of tire-soil system strongly depended on soil depth, soil moisture content and tire structure while it slightly depended on tire inflation pressure. The vertical equivalent damping ratio of tire-soil significantly increased with the increase in soil depth and soil moisture content, but it decreased when soil moisture content was too high. It was found that there was a certain soil moisture content that would increase up to maximum of the damping ratio of the tire-soil. The vertical equivalent damping ratio of tire-soil significantly decreased as tire inflation pressure increased.

The tire-soil system has been modeled by few researchers. Bekker (1956, 1969) modeled tire-soil system which was the vertical elastic tire-soil mode. Rubinstein and Galili (1994) used the radial spring model to represent the soil-wheel interaction. Piotr (2006) modeled the pneumatic tired wheel-soil foundation system with elastic model and elastic-damping model. Emam *et al.* (2011) modeled tire-soil model as spring-mass-damper system.

Free-Vibration Logarithm Decay method has been usually used to determine tire vertical stiffness and damping characteristics (Taylor *et al.* 2000, Yong 2001). It has been used to determine equivalent stiffness and damping of sub-layer soil (GB50040 1997, Hou and Shi 2010) and also has been used to determine vertical equivalent damping ratio of tire-soil system (Cuong *et al.* 2014). In this study, Free-Vibration Logarithm Decay method was also used to get the vertical damping ratio of tire, tire-soil system and establish a damping model of tire-soil system referring to these results.

Table 1 Physical and mechanical properties of paddy soil

Soil Moisture Content (%)	Dry Bulk Density (g/cm^3)	Cone Index (KPa)
37.9	1.22	83
48.8	1.25	32
66.7	1.18	24
77.4	1.12	14

Table 2 Factors and levels of tire and soil

Tire code	Test code	Tire load, (kg)	Soil MC (%)	Distribution of tire inflation pressure, (kPa)				
7.50-20 R1	**T1**	364	-	60	90	120	150	180
6.00-12 R1	**T2**	246	-	60	90	120	150	180
7.50-20 R1	**TS1**	364	37.9	60	90	120	150	180
7.50-20 R1	**TS2**	364	48.8	60	90	120	150	180
7.50-20 R1	**TS3**	364	66.7	60	90	120	150	180
7.50-20 R1	**TS4**	364	77.4	60	90	120	150	180
6.00-12 R1	**TS5**	246	37.9	60	90	120	150	180
6.00-12 R1	**TS6**	246	48.8	60	90	120	150	180
6.00-12 R1	**TS7**	246	66.7	60	90	120	150	180
6.00-12 R1	**TS8**	246	77.4	60	90	120	150	180

2. Materials and methods

2.1 Instrumentation

The experiments were carried out at Vibration laboratory, College of Engineering, Nanjing Agricultural University, China. In this study, a single-direction accelerometer (CA-YD 185TNC with frequency bandwidths of 0.5-5000 Hz and maximum acceleration of 1000 m/s^2) was attached at wheel axle to measure vertical vibration amplitude of test tire; an air pressure sensor (NS-F/No. 21121268 with measuring range from 0 to 1.5 MPa) and an air compressor were used to measure and change tire inflation pressure; A LMS Test Xpress data acquisition and analysis system was used to collect the data signals during measurement process (sampled rate of 200 Hz). The signals were recorded by laptop computer. The test rigs are shown in Fig. 1.

2.2 Test conditions and treatments

Soil samples were taken from the paddy soil (farming layer) at the depth of 0-200 mm located at Jiangpu experimental farm, Nanjing City, China. Then the soil was shifted in iron containers of 1400×900×300mm (Length×Width×Height), and then covered with a polyethylene sheet to prevent evaporation. The water content and the dry bulk density of soil were determined by gravimetric method (Black 1965), while the soil penetration resistance was directly measured by a digital soil compaction meter (TJSD-750). Soil samples were also taken to determine its texture using Bouyoucos Hydrometer Method (Bouyoucos 1927). The paddy soil was comprised of 56% sand, 28% silt and 16% clay, i.e., sandy loam. Some main physical and mechanical properties of paddy soil at different moisture contents are represented in Table 1.

The two kinds of tractor tire (7.50-20 R1 and 6.00-12 R1) were selected as test tires, referring to Chinese standard GB 2979 (GB-2979 1992) to determine the tire loads. The vertical accelerations of the test tires were measured when the test tires were moving free decaying vibration for a total of 90 treatments: two kinds of tractor tire, five levels of tire inflation pressure, four soil moisture content and two soil depths. Each treatment was conducted three times. Distribution factor and levels of the experiments are given in Table 2.

2.3 The principle of testing

The tire and tire-soil system were modeled as an equivalent system by mechanism of linear parallel spring and damper in vertical with one end connected to tire axle and another one connected to hard layer. The equivalent mechanical model for tire-soil system and tire only are shown in Fig. 2(a).

On the basis of the theory of decay oscillation (Clarence 2007), the testing principle to determine vertical equivalent damping ratio of the tire-soil system is presented in Fig. 2.

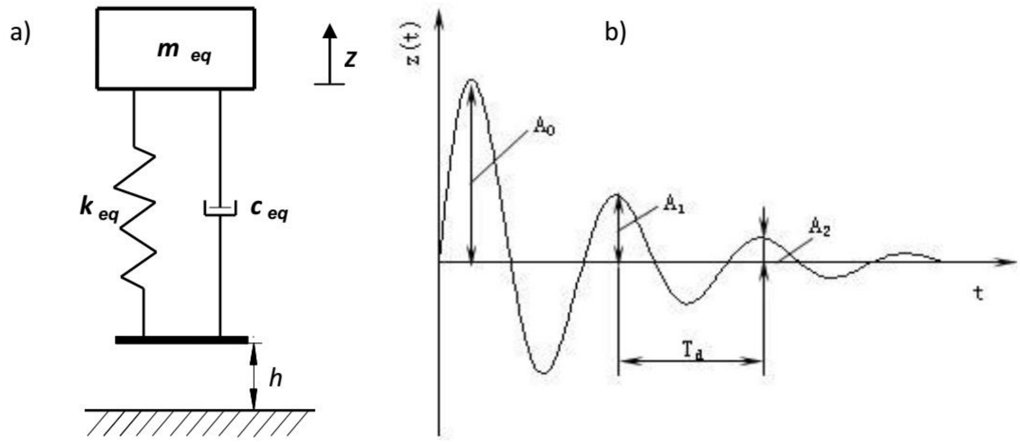


Fig. 2(a) Single degree of freedom system for measuring the vertical equivalent damping ratio of tire and tire-soil system; (b) The decay curve of the free vibration with damping

During the test, the wheel system performed a free decaying vibration. The vibration differential equation is

$$m_{eq}\ddot{z} + c_{eq}\dot{z} + k_{eq}z = 0 \quad (1)$$

which can be written as

$$\ddot{z} + 2\xi\omega_n\dot{z} + \omega_n^2 z = 0 \quad (2)$$

where, m_{eq} is tire load, it is static and dynamic load of tractor wheel, kg;
c_{eq} is the equivalent damping coefficient of tire-soil system, N.s/m;
k_{eq} is the equivalent stiffness constant of tire-soil system, N/m;
z is displacement of wheel axle, m;
ω_n is undamped natural angular frequency, radian/s.

where,

$$\omega_n{}^2 = \frac{k_{eq}}{m_{eq}} \tag{3}$$

and the damping ratio ξ is defined as

$$\xi = \frac{c_{eq}}{2\sqrt{m_{eq}k_{eq}}} \tag{4}$$

We only consider damped scenarios $\xi < 1$, in which case the solution of differential Eq. (2) is given by

$$z = Ae^{-\xi\omega_n t} \sin\left(\omega_n \sqrt{1-\xi^2}.t + \varphi\right) \tag{5}$$

where,

A, φ - Initial amplitude and initial phase.

In addition, let the damped natural angular frequency, ω_d, be defined by

$$\omega_d = \omega_n \sqrt{1-\xi^2} \tag{6}$$

The damped vibration period is defined as

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{2\pi}{\sqrt{\frac{k_{eq}}{m_{eq}}(1-\xi^2)}} \tag{7}$$

To set ratio of two successive amplitudes, η only depends on the damping ratio as

$$\eta = \frac{A_0}{A_1} = \frac{A_1}{A_2} = \ldots = \frac{A_i}{A_{i+1}} = \ldots = \frac{A_{m-1}}{A_m} = e^{\xi\omega_n T_d} \tag{8}$$

where, A_{i+1} is the amplitude after i cycles of oscillation, $i = 0, 1, 2,\ldots,m\text{-}1$ (Fig. 1(b)). Therefore,

$$\frac{A_0}{A_m} = \frac{A_0}{A_1}\frac{A_1}{A_2}\ldots\frac{A_{m-2}}{A_{m-1}}\frac{A_{m-1}}{A_m} \tag{9}$$

Using Eq. (8),

$$\frac{A_0}{A_m} = \left(\frac{A_0}{A_1}\right)^m \tag{10}$$

The logarithm of two amplitudes of peak displacement is known to get logarithmic decrement δ, that is

$$\delta = \ln \frac{A_0}{A_1} = \ln \eta = \xi \omega_n T_d \tag{11}$$

By taking natural logarithm of both sides of Eq. (10) with respect to the base *e*, and using the definition of logarithmic decrement in Eq. (11), we can obtain

$$\ln \frac{A_0}{A_m} = \ln \left(\frac{A_0}{A_1} \right)^m = m \ln \left(\frac{A_0}{A_1} \right) = m\delta \tag{12}$$

From which the logarithm δ can be easily computed as

$$\delta = \frac{1}{m} \ln \frac{A_0}{A_m} = \frac{1}{m} ln \frac{\ddot{z_0}}{\ddot{z_m}} \tag{13}$$

where A_0 and A_m are the reference amplitudes of corresponding displacement, $\ddot{z_0}$ and $\ddot{z_m}$ are reference amplitudes of corresponding acceleration.

Combining equations above allows us to write the damping ratio of tire-soil system as

$$\zeta_{eq} = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \tag{14}$$

Therefore, to obtain the cycles of the vibration acceleration and its amplitude using test results and by using the Eqs. (13) and (14), the vertical equivalent damping ratio of the tire-soil system was determined.

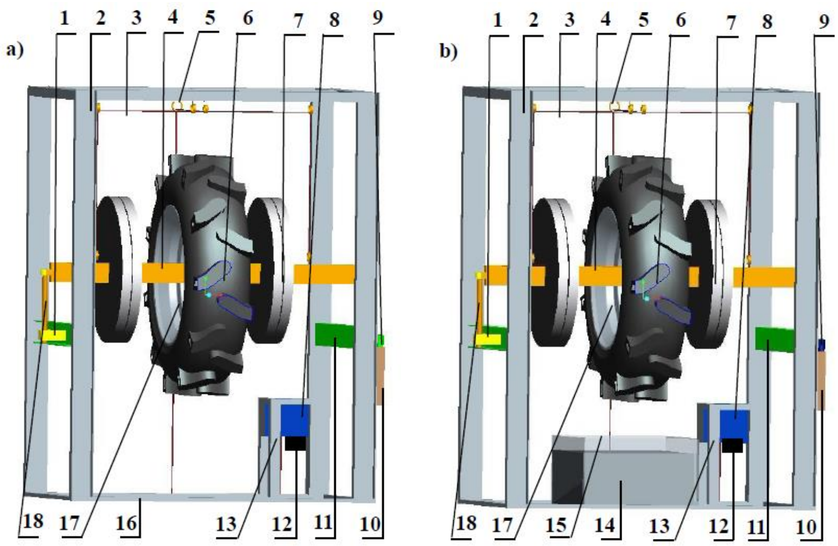


Fig. 3 The test rig: (a) For measuring the damping ratio of tires; (b) For measuring the damping ratio of tire-soil system 1. Linear bearing; 2. Support frame; 3. Steel wire rope; 4. Beam; 5. Hanging ring; 6. Test tire; 7. Load; 8. Electro-permanent lift magnet; 9. Displacement sensor; 10. Fixed bracket of displacement; 11. Beam of linear bearing; 12. Carrying plate; 13. Fixed bracket of electro-permanent lift magnet; 14. Paddy soil; 15. Steel tank; 16. Rigid plane; 17. Plate of link tire; 18. Guiding bar.

2.4 Test procedure

The test rig is shown in Fig. 3. It consists of a support frame, two linear bearings, steel wire rope, a beam, hanging rings, a test tire, loads, an electro-permanent lift magnet, a displacement sensor, a fixed bracket of displacement, left and right beam of linear bearing, a carrying plate, a fixed bracket of electro-permanent lift magnet, paddy soil, a steel tank, a plate of link tire, two guiding bars.

An accelerometer, an air pressure sensor, an air compressor, a LMS Test. Xpress data acquisition system and a computer were used in the tests.

The support frame includes upper and lower rectangular frames which are attached to the uprights by bolts, the test tire was fixed on the beam through plate of link tire. The loads were imposed on the beam via constant corresponding weights.

In addition, in order to ensure that the beam (test tire) moves only in vertical direction through the linear bearings the guiding bars were also used in this test. The steel wire rope was used to cornect the beam (test tire) and the carrying plate through the hanging rings which are attached to the frame, the carrying plate was adsorbed by magnetic force of the electro-permanent lift magnet.

A chain block was used to lift the test tire up to the height of about 200 mm (the height (h) should be determined to ensure that the measured acceleration signal has sufficient fluctuation), the electro-permanent lift magnet adsorbed carrying plate to keep the test tire at the height of 200 mm by the steel wire rope.

Suddenly controlling the electro-permanent lift magnet being unabsorbed with the carrying plate, the test tire free fell down to contact rigid plane in tire testing (Fig. 3(a)) and contacted soil layer in tire-soil system testing (Fig. 3(b)), because of the elastic and damping of tire only and tire-soil system, the test tire was moved up-down by sliding of the guiding bars on the linear bearings to perform a free decaying vibration. The vertical acceleration signal was measured by an accelerometer connecting with a LMS Test-Xpress data acquisition system and a computer. The sampling signal of acceleration and displacement decay curve are represented in Fig. 4.

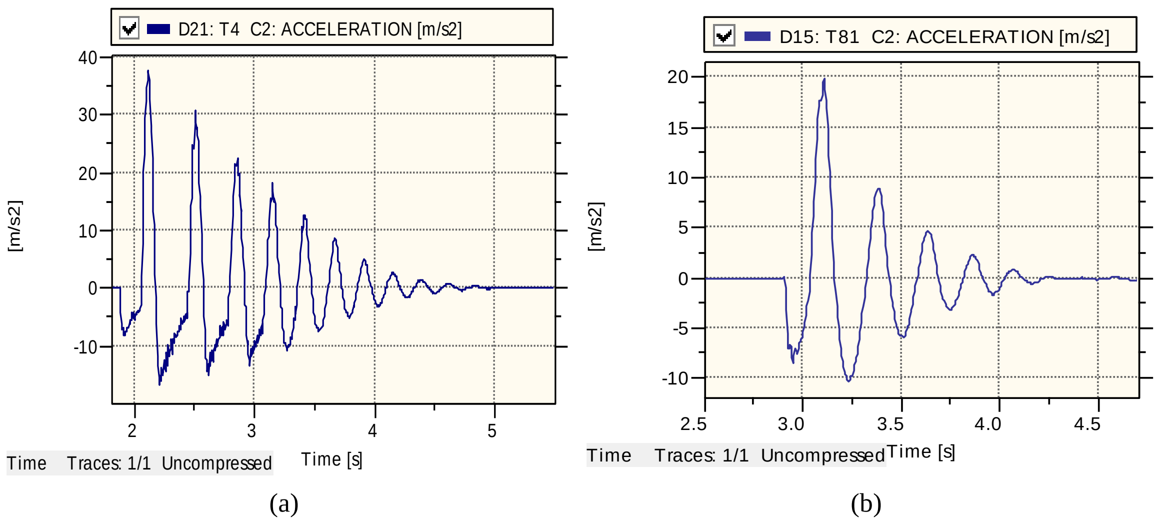
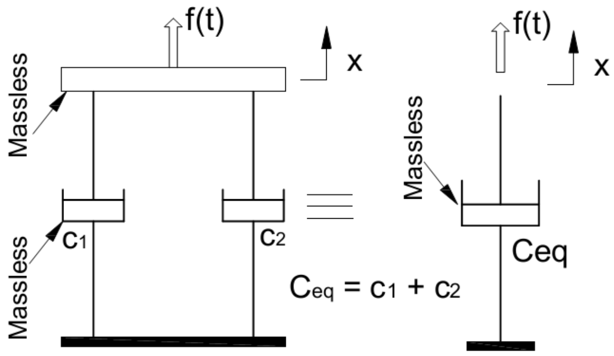


Fig. 4 The sampling signal of acceleration decay curve of tire T2 and TS5 at 150 kPa pressure and 120 mm soil depth (a) For tire; (b) For tire-soil system

2.5 Theory of dampers in parallel

According to vibration theory (Taylor 2000), considering a parallel combination of massless dampers as coefficients c_1 and c_2 (Fig. 5). An important point needs to be noted here is that both dampers being in parallel, have the same velocity. Here, c_{eq} is the equivalent damping constant, the system in Fig. 5(a) can be replaced by a system with only one damper with the coefficient c_{eq} as shown in Fig. 5(b), it is calculated by Eq. (15),

$$c_{eq} = c_1 + c_2 \tag{15}$$



(a) (b)

Fig. 5(a) A parallel combination of massless dampers; (b) Equivalent system with only one damper for a parallel combination (Taylor 2000)

3. Results and discussion

3.1 The vertical equivalent damping ratio of ***T1, TS1, TS2, TS3*** and ***TS4*** tests

Fig. 6 shows that the vertical damping ratio of tire **T1** decreased with the increase in tire inflation pressure. The vertical damping ratio of tire decreased in range from 0.121, 0.096, 0.084, and 0.093 to 0.077 when the tire inflation pressure increased from 60, 90, 120, 150 to 180 kPa, respectively.

For soil depth of 80 mm, the vertical equivalent damping ratio of tire-soil decreased when tire inflation pressure increased (Cuong *et al.* 2014). It decreased from 0.140 to 0.105 as tire inflation pressure increased from 60 to 180 kPa at 37.9% soil moisture content. Similar trends were observed at 48.8%, 66.7% and 77.4% soil moisture content (Fig. 6).

The vertical equivalent damping ratios of tire-soil system were not proportionally increased with soil moisture content. It increased from 0.140 to 0.163 to 0.257 then decreased to 0.228 at tire inflation pressure of 60 kPa, from 0.124 to 0.150 to 0.244 then decreased to 0.214 at tire inflation pressure of 90 kPa and from 0.118 to 0.134 to 0.235 then decreased to 0.202 at tire inflation pressure of 120 kPa when soil moisture content increased from 37.9 to 48.8 to 66.7 and 77.4%, respectively. Similar trend was observed in case of tire inflation pressure of 150 and 180 kPa.

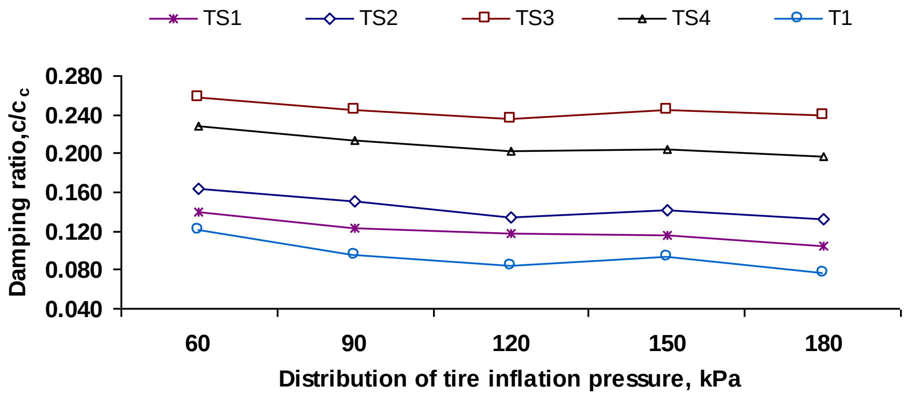


Fig. 6 The vertical equivalent damping ratios of **T1**, **TS1**, **TS2**, **TS3** and **TS4** test in soil depth of 80 mm

With soil depth of 120 mm, the vertical equivalent damping ratio of tire-soil decreased when tire inflation pressure increased (Cuong *et al.* 2014). It also decreased from 0.159 to 0.132 when tire inflation pressure increased from 60 to 180 kPa at 37.9% soil moisture content. Similar trends were also observed in case of 48.8%, 66.7% and 77.4% soil moisture content.

By changing soil moisture, the vertical equivalent damping ratios of tire-soil system increased from 0.159 to 0.189 to 0.298 then decreased to 0.253 at tire inflation pressure of 60 kPa and from 0.156 to 0.171 to 0.284 then decreased to 0.229 at tire inflation pressure of 90 kPa when soil moisture content increased from 37.9 to 48.8 to 66.7 and 77.4%, respectively. Similar trends were observed at tire inflation pressure of 120, 150 and 180 kPa (Fig. 7).

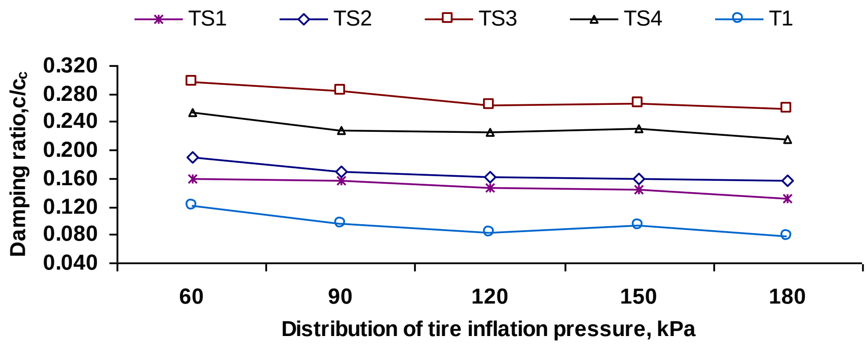


Fig. 7 The vertical equivalent damping ratios of **T1**, **TS1**, **TS2**, **TS3** and **TS4** tests in soil depth of 120 mm

In comparison with the vertical equivalent damping ratio of tire-soil system and tire only in whole tests with changing soil moisture content, the vertical equivalent damping ratio of tire-soil system was always higher than that of tire only (Figs. 6-7). At tire inflation pressure of 60 kPa, the damping ratio of tire was 0.121 while the damping ratio of tire-soil was 0.140, 1.63, 2.57 and 0.228 at 37.9%, 48.8%, 0.257 and 66.7% soil moisture content, respectively, in soil depth of 80 mm. In cases of tire inflation pressure of 90, 120, 150 and 180 kPa and soil depth of 120mm, all vertical equivalent damping ratios of tire-soil system were also higher than that of tire that of tire. As referring to vibration theory, this revealed that the vertical damping ratio of tire-soil system was sum of two individual components of the damping of tire and soil which consisted of two dampers in parallel model (Fig. 5).

*3.2 The vertical equivalent damping ratio of **T2**, **TS5**, **TS6**, **TS7** and **TS8** tests*

For **T2** test (tire only), the vertical damping ratio also decreased with the increase in tire inflation pressure. The damping ratio decreased in range from 0.118, 0.092, 0.078, 0.073 to 0.069 when tire inflation pressure increased from 60, 90, 120, 150 to 180 kPa, respectively (Fig. 8).

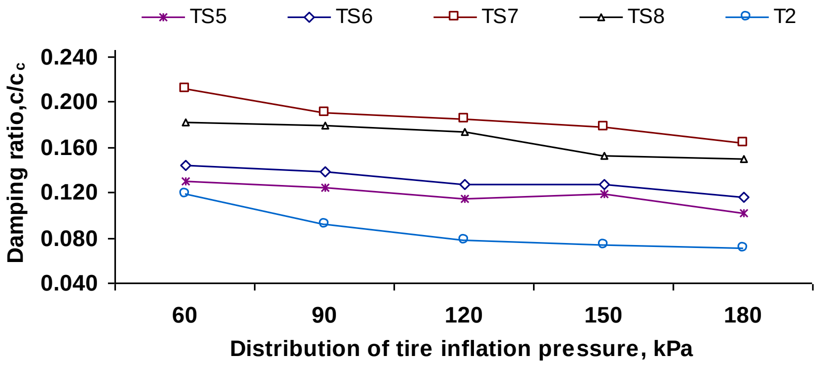


Fig. 8 The vertical equivalent damping ratio of **T2**, **TS5**, **TS6**, **TS7** and **TS8** tests in soil depth of 80 mm

The vertical equivalent damping ratios of tire-soil system changed by changing the tire inflation pressure, soil moisture content and soil depth (Cuong *et al.* 2014). It decreased from 0.130 to 0.102 at 37.9% soil moisture content by increasing tire inflation pressure from 60 to 180 kPa. Similar trends were also observed in case of 48.8%, 66.7% and 77.4% soil moisture content and in both soil depth of 80 and 120 mm (Figs. 8-9).

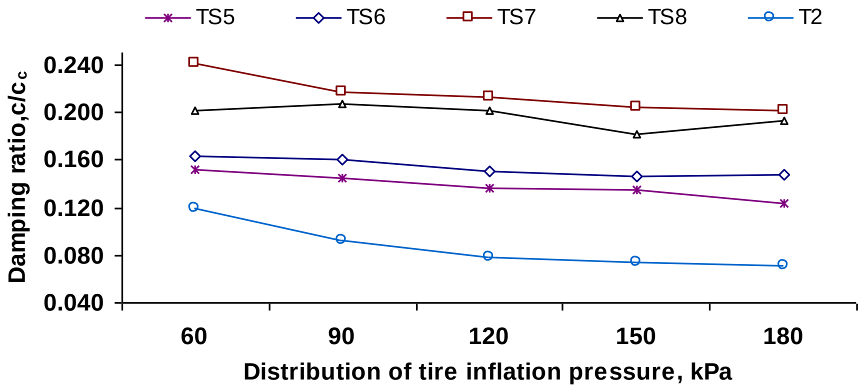


Fig. 9 The vertical equivalent damping ratio of **T2**, **TS5**, **TS6**, **TS7** and **TS8** tests in soil depth of 120 mm

Figs. 8 and 9 show that the vertical damping ratios of tire-soil system were always much higher than of tire only. At tire inflation pressure of 60 kPa, the vertical damping ratio of tire was 0.118 while the damping ratio of tire-soil was 0.130, 0.144, 0.212 and 0.182 at 37.9%, 48.8%, 66.7% and 77.4% soil moisture content, respectively, in soil depth of 80 mm (Fig. 8). The damping ratios of tire-soil system were also higher than that of tire which was observed in cases of tire inflation pressure of 60, 90, 120, 150 and 180 kPa and soil depth of 120 mm (Fig. 9).

Above results show that the vibration properties of tire-soil depended on many elements as tire inflation pressure, soil moisture content and soil depth…, this was confirmed by Cautes and Nastac (2002) and Cuong *et al.* (2014). Experimental results showed that the vertical equivalent damping ratios of tire-soil system were much higher than the damping ratio of tire only, especially in the case of the higher soil moisture content.

3.3 Establishment of a new tire-soil model

With the vibration theory (Fig. 5), a damping model with two dampers in parallel can be treated as a single damper with an equivalent damping (c_{eq}) being the sum of two damping components with coefficient of c_1 and c_2. Based upon these theories and above experiment results: a new damping model of tire-soil (see Fig. 10), in which the vertical equivalent damping coefficient of tire-soil system (c_{eq}) can be written as the sum of the equivalent damping of tire (c_t) and of the equivalent damping of paddy soil (c_s) was proposed.

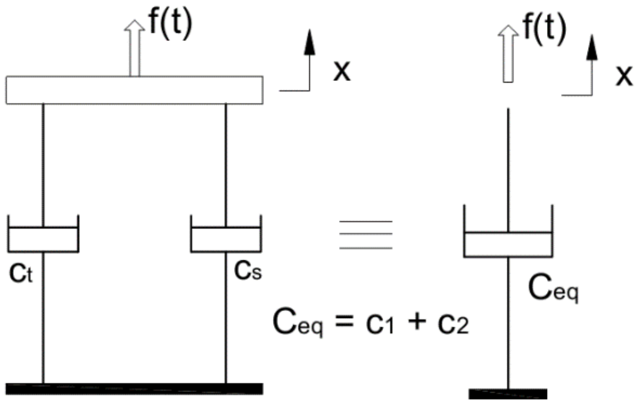


Fig. 10 New damping model of tire-soil system c_{eq} - equivalent damping coefficient of tire-soil system; c_t - damping coefficient of tire; c_s - damping coefficient of paddy soil

4. Conclusions

This research confirms that the vertical equivalent damping ratio of tire and tire-soil system depends on tire inflation pressure, soil moisture content and soil depth which indicated that these relationships should be regarded in the design of tractor suspension system.

According to these results, a new damping model of tire-soil system is established based on vibration theory and experiment results. This can be used in vibration simulation work to predict vibration characteristics of the tractor working on the paddy soil.

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