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Importance of a rigorous evaluation of the cracking moment in RC beams and slabs

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Abstract. The service loads are often decisive in the design of concrete structures. The evaluation of the cracking moment, M_{cr}, is an important issue to predict the performance of the structure, such as, the deflections of the reinforced concrete beams and slabs. To neglect the steel bars of the section is a simplification that is normally used in the computation of the cracking moment. Such simplification leads to small errors in the value of this moment (typically less than 20%). However, these small errors can conduce to significant errors when the values of deflections need to be computed from M_{cr}. The article shows that an error of 10% on the evaluation of M_{cr} can lead to errors over 100% in the deformation values. When the deformation of the structure is the decisive design parameter, the exact computing of the cracking moment is obviously very important. Such rigorous computing might lead to important savings in the cost of the structure. With this article the authors wish to draw the attention of the technical community to this fact. A simple equation to evaluate the cracking moment, M_{cr}, is proposed for a rectangular cross-section. This equation leads to cracking moments higher than those obtained by neglecting the reinforcement bars and is a simple rule that can be included in Eurocode 2. To verify the accuracy of the developed model, the results of the proposed equation was compared with a rigorous computational procedure. The proposed equation corresponds to a good agreement when compared with the previous approach and, therefore, this model can be used as a practical aid for calculating an accurate value of the cracking moment.

Keywords: reinforced concrete; concrete beams; cracking; deformations; serviceability.

1. Introduction

The architectural needs of today's buildings lead to relatively high span to height ratios both for beams and slabs. Consequently, such elements could be very flexible. One solution to ensure the safety of these new structures consists of using materials of higher resistance. As a result of that, the structure will be able to tolerate higher stresses under the service load application. This situation gives importance to SLS (Serviceability Limit States) when compared with ULS (Ultimate Limit States), and either SLS or ULS could be the critical in the design process.

There is a growing application of high strength concretes. As known, high strength concretes have brittle nature when compared with normal strength concretes. Nevertheless, when used in structures, high strength concretes may have sufficient ductility. Recent studies show that it is possible to have reinforced concrete beams with acceptable bending ductility (Bernardo and Lopes 2004, Bernardo and Lopes 2009a, Carmo and Lopes 2005, Carmo and Lopes 2008, Lopes and Bernardo 2003). Even in

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beams under torsion, there could be found some ductility, as reported in recent articles (Bernardo and Lopes 2008, Bernardo and Lopes 2009b, Lopes and Bernardo 2009). These studies indicate that high strength concretes seem to have a satisfactory behaviour in reinforced concrete structures as far as Ultimate Limit States are concerned.

There has been not much research regarding SLS, when compared to ULS topics. Even so, some works deserve to be mentioned. Bazant and Oh concluded that the deflection of a beam, dependent on the bending stiffness, was significantly influenced by the opening and spacing of cracks (Bazant and Oh 1983). In general, the cracking moment is the parameter to be used for identifying the transition from uncracked to cracked states. As far as the cracking moment is concerned, an important aspect is the variability of the tensile strength of concrete. Ignoring this factor, the evaluation of the cracking moment has also an important influence, as will be discussed in the current study. Regarding to the design, it is common to make some simplifications, such as to neglect the presence of reinforcement bars in the computing of the cracking moment and to neglect the tensile strength of concrete when computing the internal forces of cracked sections. There are also some recent studies on modelling cracking in beams, shear walls and plates (Dufour *et al.* 2008, Dujc *et al.* 2009, Hutchinson and Wang 2009 Marzouk *et al*. 2009, Oh *et al.* 2007, Zanuy 2010 and Zanuy *et al.* 2010). With EC2 (CEN 2004) the deformation could be the decisive aspect of the design. As a consequence, if the above simplifications have some impact on estimation of the deformation, a more rigorous calculation can lead to substantial savings. This article studies the influence of such simplifications and on the subsequent deformation of RC members.

2. Common evaluation of the cracking moment

SLS checks include the evaluation of cracking, deformation, and the limitations of stresses and vibrations. Focusing on the evaluation of deformation, the safety check in the context of SLS is a relatively complex process, whose diversity criteria and possible options, can lead to some degree of variability of the results. This check is based on four essential criteria (Beeby 1995):

- Limit of deformation for a satisfactory behaviour;
- Actions for calculations;
- Properties of materials;
- Structural behaviour models.

With regard to the first criterion, EC2 proposes a limit of span/250 to beams or slabs under normal service conditions. For deformations that may occur shortly after construction, i.e. short-term deformations, such limit is reduced to half, i.e. span/500. Mayer and Rusch (Mayer and Rusch 1967) studied in detail these limits and concluded that values between span/300 and span/500 would be appropriate. Kidder recommended a limit to the deformation of span/360 (Kidder 1984).

In reference to the second criterion, the prediction of the deformation implies the knowledge of the actions that matter for the required combinations of actions. For SLS calculations, EC0 points to three possible combinations of actions: characteristic, frequent and quasi permanent combinations (CEN 2002). Although EC2 does not specify what combination of loads should be used to compute deformations, Beeby (Beeby 1995) suggested the use of the quasi permanent combination, since much of the final deformation is due to creep and shrinkage effects.

For the prediction of the deformation, the third criterion corresponds to the definition of properties of materials, which should be taken as average values with respect to SLS. While in ULS the

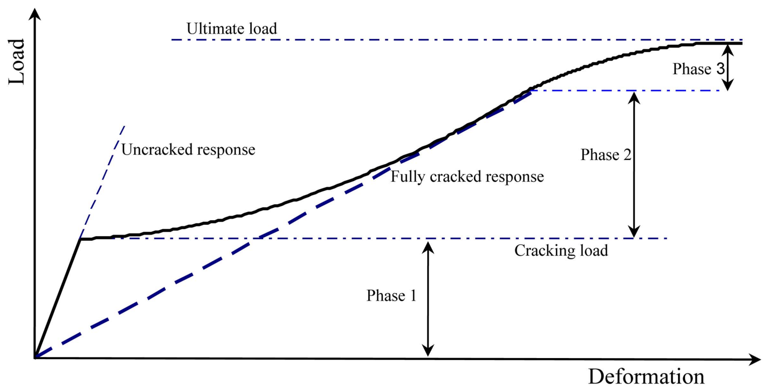

Fig. 1 Load-deformation relationship model (Beeby 1995)

strength properties of the materials are the most important, in SLS the elastic properties are more important both concerning short or long term conditions. They have a great influence on the evaluation of deformation. In particular the modulus of elasticity and the long term phenomena, such as, creep and shrinkage are of outmost importance. EC2 uses an effective elasticity module, to model creep and a shrinkage curvature to simulate the effects of concrete shrinkage.

The load-deformation relationship corresponding to the fourth criterion can be subdivided into 3 phases as presented in Fig. 1: uncracked phase, cracked phase and inelastic phase (Beeby 1995).

In the uncracked phase, the stresses in concrete are less than its tensile strength, and the behaviour is approximately linear elastic. In the cracked phase, the maximum tensile strength is exceeded, which leads to formation of cracks in the structure. At this stage, there is some contribution of the concrete between cracks for the tensile resistance of a segment of the RC member. This phenomenon is often named “tension stiffening effect” and such contribution decreases with the increase of the number of cracks. The inelastic phase occurs for a high level of stresses and is normally associated with the yielding of the reinforcement bars.

If the cracking moment is exceeded, normally it does not happen on the entire length of the member. It continues to show a Phase 1 behaviour in areas where the moments are lower than the cracking ones, and a Phase 2 behaviour in other areas. The loss of stiffness is very high when a member (or a segment of a member) passes from Phase 1 to Phase 2. This makes the correct evaluation of the cracking moment very important.

The cracking moment, M_{cr}, is often computed by considering only the concrete section (without reinforcement bars). The key condition is that the stresses in one of the extreme fibres are, at the most, equal to the mean value of axial tensile strength of concrete, f_{ctm}. Handbook of EC2 by Beeby (Beeby 1995) indicates the following equation

$$M_{cr} = W_c f_{ctm} \tag{1}$$

Where: W_c is the concrete gross section bending modulus; in particular case of rectangular cross sections $W_c = bh^2/6$; b is the width of the section; h is the height of the section. According to EC2, for normal concretes, $f_{ctm} = 0.30\, f_{ck}^{(2/3)}$ where f_{ck} is the characteristic (5%) cylinder compressive strength of concrete.

The manual “Cracking and Deformation” (CEB 1985) suggests, alternatively, the use of characteristic 5% value of axial tensile strength of concrete, $f_{ctk0.05}$, instead of f_{ctm}, i.e.,

$$M_{cr} = W_c f_{ctk0.05} \tag{2}$$

According to EC2, $f_{ctk0.05} = 0.70\, f_{ctm}$.

Equivalently, the book “Structural Concrete” (Fib 1999) suggests the use of the value of the mean flexural tensile strength of reinforced concrete members $f_{ctm,fl}$, i.e.,

$$M_{cr} = W_c f_{ctm,fl} \tag{3}$$

The procedure to compute the value of the mean flexural tensile strength of reinforced concrete members is also suggested in the book “Structural Concrete’, leading the values which are very similar to the following equation indicated in EC2:

$$f_{ctm,fl} = \max\{f_{ctm}(1.6 - h);\, f_{ctm}\} \tag{4}$$

where the total member depth, h, is in m. Consequently, for small beams, the cracking moment computed based on Eq. (3) will be higher than those of previous equations, since the adopted tensile strength is superior.

As stated above, the previous equations do not consider any effect resulting from the presence of reinforcement bars when the cracking moment is computed. These procedures do not consider the long term effects of concrete, such as creep and shrinkage, either.

Among other things, Cohn and Riva underline the reinforcing ratio as one of the most important governing factors in cracking control and/or spacing and width of cracks (Cohn and Riva 1992).

3. Properties of materials

3.1 Concrete

The short-term stress-strain relationship, for compressed concrete, can be represented in absolute values using the following equation indicated by EC2 and by Model Code 1990 (CEB-FIP 1993)

$$\sigma_c = f_{cm}\frac{k\eta - \eta^2}{1 + (k-2)\eta} \tag{5}$$

Where: $\eta = \varepsilon_c/\varepsilon_{c1}$; $k = 1.05\, E_c \varepsilon_{c1}/f_{cm}$; σ_c is the compressive strength of concrete; ε_c is the strain of compressed concrete; ε_{c1} is the strain correspondent to the maximum compressive stress; E_c is the modulus of elasticity of concrete ; f_{cm} is the mean value of the compressive strength of concrete.

As mentioned above, the characteristics of the compressed concrete do not remain constant with time due to the effects of creep and shrinkage. Gutierrez and Ochoa describe a stress-strain relationship for the compressed concrete (Gutierrez and Ochoa 2007), which was developed by Popovics and is presented below (Popovics 1970, 1973).

$$\sigma_c = \frac{f_{cm}\beta_c(\eta - \eta_{cs})}{\beta_c - 1 + (\eta - \eta_{cs})^{\beta_c}} \tag{6}$$

Where: $\beta_c = 1/[1 - f_{cm}/(E_c(t)\varepsilon_{c1}(t))]$; $\varepsilon_{c1}(t) = \varepsilon_{c1}(1+\varphi)$; $E_c(t) = E_c/(1+\chi\varphi)$; $\eta = \varepsilon_c/\varepsilon_{c1}(t)$; $\eta_{cs} = \varepsilon_{cs}/\varepsilon_{c1}(t)$; ε_{cs} is the free shrinkage strain; φ is the creep coefficient; χ is the creep reduction factor.

It should be emphasised that Eq. (6) can simulate both short and long term conditions. It is also possible to change Eq. (5) in order to include long term effects. The parameters k and η should be

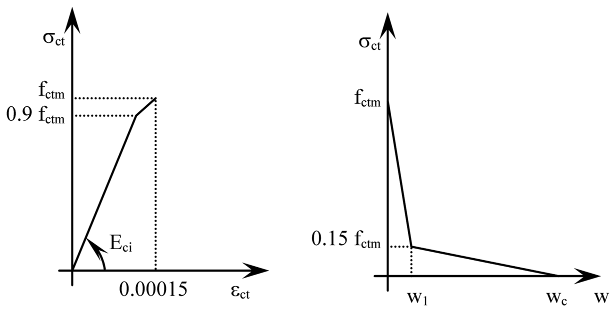

Fig. 2 σ-ε and σ-w for tensioned concrete according to MC90 (CEB-FIP 1993)

replaced by $k = 1.05E_c(t)\varepsilon_{c1}(t)/f_{cm}$ and $\eta = (\varepsilon_c - \varepsilon_{cs})/\varepsilon_{c1}(t)$ respectively.

The tensile behaviour of concrete is a phenomenon that is not generally considered in design. Not only because it is more unpredictable and difficult to express, but also because the tensile strength of concrete is often very low, normally less than 1/10 of compressive strength.

MC90 (CEB-FIP 1993) presents two diagrams to characterize the behaviour of tensioned concrete. The first intends to model the behaviour at State 1, i.e. before the formation of the first crack (Fig. 2). After cracking, the behaviour is modelled by a second diagram, in which the stress is not function of the strain, but of the cracks width, w, (Fig. 2). In Fig. 2, σ_{ct} represents the concrete stresses, ε_{ct} the tensile concrete strains, E_{ci} is the initial modulus of elasticity, w_1 is the crack width that corresponds to 15% of the mean value of the tensile strength of concrete, f_{ctm}, and w_c is the maximum crack width.

3.1.1 Steel

The main characteristics of the reinforcement bars used in reinforced concrete are the yield and ultimate stresses and strains. EC2 indicates an idealized behaviour diagram consisting of an initial branch defined by the modulus of elasticity, E_s, up to the yielding point. After this point, a more inclined straight is presented and goes up to an ultimate point. The inclination could be zero if the so-called hardening of steel is neglected. The stresses in compression are symmetrical to those in tension. The overall behaviour of a 400 MPa Steel Class used in this work is presented in Fig. 3. The adopted yield stress of steel was 400 MPa, the tensile maximum stress was 10% higher, i.e.

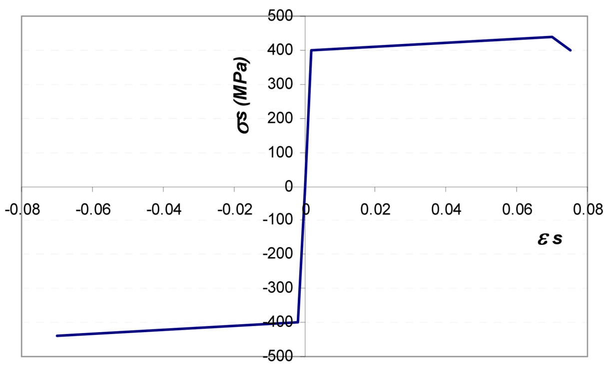

Fig. 3 Stress-strain relationship for a 400 MPa Steel

440 MPa, the tensile strain of the peak stress was 7.0%, and the ultimate strain was 7.5%. Despite the plastic deformation not to be important for this purpose, the authors have chosen a S400 steel grade just to show an example. However, any other steel grade could be used without any influence on the conclusions of this article (the modulus of elasticity is the important parameter).

4. Methodology for computing M_{cr}

The following methodology is fully developed for rectangular cross sections, as those ilustrated in Fig. 4, but it can be adapted to any other shape.

The cracking moment, M_{cr}, and the depth of the neutral axis, x, may be computed by taking into consideration the contribution of reinforcement bars. Since the level of stresses is relatively low in SLS, the stress-strain behaviour for concrete can be considered linear, both in tension and in compression. Plane sections are supposed to remain plane after deformations. Balancing the internal forces and moments, the following equations can be developed

$$\varepsilon_c = \varepsilon_{ct} x/(h-x) \tag{7}$$

$$\varepsilon_{ct} = f_{ct}/E_c \tag{8}$$

$$\varepsilon_s = \varepsilon_c (d-x)/x \tag{9}$$

$$\varepsilon_s' = \varepsilon_c (x-d')/x \tag{10}$$

$$\sigma_c = E_c \varepsilon_c \tag{11}$$

$$\sigma_{ct} = E_c \varepsilon_{ct} \tag{12}$$

$$\sigma_s = E_s \varepsilon_s \tag{13}$$

$$\sigma_s' = E_s \varepsilon_s' \tag{14}$$

$$F_c = \sigma_c b x/2 \tag{15}$$

$$F_{ct} = \sigma_{ct} b(h-x)/2 \tag{16}$$

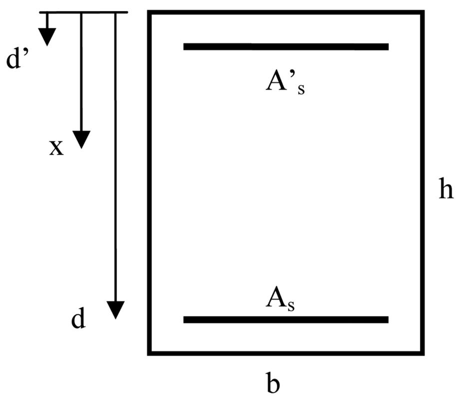

Fig. 4 A rectangular cross section

$$F_s = A_s \sigma_s \tag{17}$$

$$F_s' = A_s' \sigma_s' \tag{18}$$

$$F_c + F_s' = F_{ct} + F_s \tag{19}$$

$$M_{cr} = \frac{2}{3} F_c x + F_s'(x-d') + \frac{2}{3} F_{ct}(h-x) + F_s(d-x) \tag{20}$$

Where: x is the depth of neutral axis; d is the effective height of the section; d' is the distance between the top of the section and the axis of compressive reinforcement bar; ε_s is the strain of the tensioned steel; ε_s' is the strain of the compressed steel; E_s is the modulus of elasticity of steel; σ_s is the stress of the tensioned steel; σ_s' is the stress of the compressed steel; F_c is the compressive force in concrete; F_{ct} is the tensile force in concrete; F_s is the tensile force in steel; F_s' is the compressive force in steel; f_{ct} is the maximum tensile stress of concrete.

The calculation procedure can be implemented as follows. In Eqs. (7)-(10), the extensions in steel and concrete are based on simple geometric relationships of the deformed section and on the ε_{ct} values, computed from Eq. (8). In this equation, either f_{ctm} or any other value of the tensile strength can be used, whichever considered the most appropriate. The stresses of Eqs. (11)-(14) are determined by considering linear elastic behaviour for the materials. Then, the internal forces are computed from Eqs. (15) to (18). Finally, by ensuring the internal equilibrium of forces expressed by Eq. (19), the position of neutral axis of the section is determined, and consequently, the cracking moment, M_{cr}, of the section is known Eq. (20). This moment takes the contribution of reinforcement bars into account.

In order to generalize the previous procedure, a relative cracking moment, μ_{cr}, can be used.

$$\mu_{cr} = M_{cr}/(bh^2 f_{ct}) \tag{21}$$

In addition, some known relationships are used to compute the value of μ_{cr} as follows

$$\rho = A_s/bh \tag{22}$$

$$\alpha = E_s/E_c \tag{23}$$

$$\theta = A_s'/A_s \tag{24}$$

$$d = 0.9h \tag{25}$$

$$d' = 0.1h \tag{26}$$

$$\mu_{cr} = \frac{0.1667 + \alpha\rho(0.4867 + \theta(0.4867 + 1.28\alpha\rho))}{1 + \alpha(0.2 + 1.8\theta)\rho} \tag{27}$$

This equation gives the value of μ_{cr} as a function of ρ, θ and α. The amount of steel is reflected by the parameters ρ and θ, while the characteristics of deformable steel and concrete are introduced by parameter α. The increasing in value of any of these three variables implies an increasing of μ_{cr}.

If ρ is equal to zero, the value of μ_{cr} comes equal to 0.1667, i.e., 1/6. Replacing this value in Eq. (21) and considering for the concrete tensile strength the value f_{ctm}, one falls into Eq. (1).

Given the complexity of Eq. (27), whose practical applicability could be difficult if it is necessary to estimate the reinforcement ratio for a known moment, the authors developed a simplified equation, which could be easily applied in design and takes into account the existence of steel.

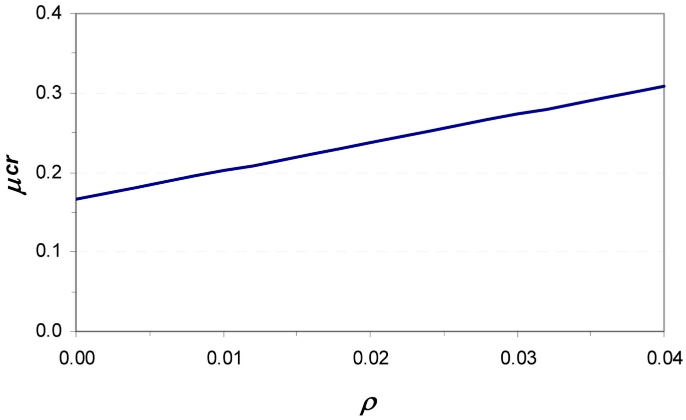

Fig. 5 μ_{cr} for $\theta = 0.4$ and $\alpha = 6.67$

By fixing the values of θ and α, it is possible to establish a relationship between μ_{cr} and ρ. Fig. 5 presents such relationship computed from Eq. (27), considering $\theta = 0.4$ and $\alpha = 6.67$. As can be seen, μ_{cr} is 20% higher when compared the case of $\rho = 0$ with that of $\rho = 0.01$.

It can also be seen that the $\mu_{cr} - \rho$ relationship is almost linear. Therefore, a straight line seems to be a good approximation. For practical applications the range of the ρ values will be fixed between the two limits, $\rho = 0.003$ and $\rho = 0.020$. The equation to be found is Eq. (28), where k_1 and k_2 are constants to be defined.

$$\mu_{cr} = k_1 + k_2\rho \tag{28}$$

By fixing ρ an equation of μ_{cr} as a function of θ and α is obtained (Fig. 5). In this figure, $\rho = 0.003$. The percentage of compressive reinforcement, θ is assumed to vary from 0.1 to 1.0 (for construction purposes, there is always some bars in the compression zone). The coefficient α ($\alpha = E_s/E_c$) was considered to vary between a minimum value of about 5.0, for a 60 MPa concrete strength under short term loading, and a maximum value of about 20.0, for a 20 MPa concrete strength under long term loading (creep effects have a great influence on the modulus of elasticity). It should be noted that the influence of the percentage of compressive reinforcement in the

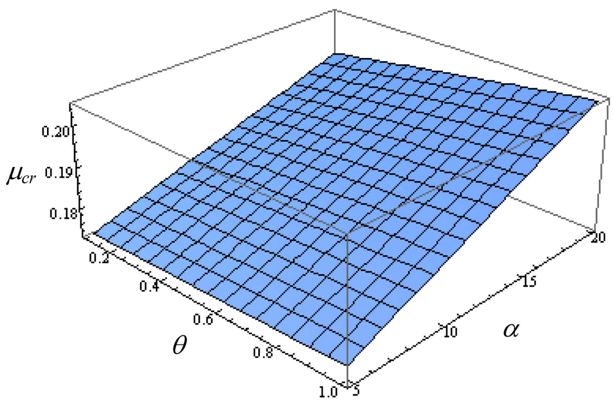

Fig. 6 μ_{cr} as a function of θ and α ($\rho = 0.003$)

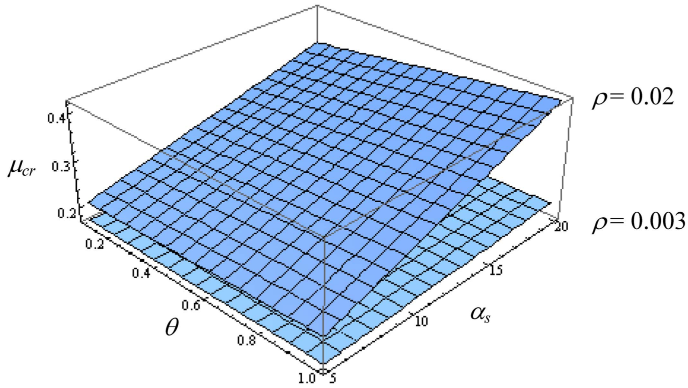

Fig. 7 μ_{cr} as a function of θ and α ($\rho = 0.003$ and $\rho = 0.02$)

evaluation of the cracking moment is negligible, if low values of α are to be considered, i.e., for the stiffest concretes. For more deformable concrete, i.e., weak or with significant creeping effects, the percentage of compressive reinforcement will have some importance.

In Fig. 6, for $\alpha = 20$, the cracking moment is incremented by 6%, when θ varies from $\theta = 0$ to $\theta = 1$. The influence of the concrete deformability is also high. For instance, with $\theta = 1$, the cracking moment is incremented by 9%, when α varies from $\alpha = 6.67$ to $\alpha = 15$. These effects are more obvious in the case of a heavily reinforced section. For example, for $\rho = 0.01$ (instead of $\rho = 0.003$ of the graph of Fig. 6) the above referred increments on the cracking moment of 6% and 9% would be 16% and 25% respectively. The influence of the reinforcement ratio ρ on cracking moment will be detailed in Section 6.

Returning to the objective of simplifying Eq. (27), it is possible to observe that the graph of Fig. 6 represents approximately a slightly distorted plan. Therefore, the four vertices of the "plan" could define a simplified equation of the type

$$\mu_{cr} = k_3 + k_4\theta + k_5\alpha + k_6\theta\alpha \tag{29}$$

Where: k_3, k_4, k_5, k_6 are the constants to be determined.

If, instead of $\rho = 0.003$, the graph was plotted by considering $\rho = 0.02$, then the values of the constants $k_{i,\ i=3\ldots6}$, in Eq. (29), would obviously be different. This is visible in Fig. 7. However, bearing in mind that the relationship between μ_{cr} and ρ observed in Fig. 5 is linear Eq. (28) it is possible to compute the intermediate values of μ_{cr} by linear interpolation between these two "planes" (that correspond to $\rho = 0.003$ and $\rho = 0.020$).

5. Proposal for M_{cr}

Based on the analyses of the last section, a simplified equation to predict the value of μ_{cr} can be developed (see Eq. 30). This equation is useful for design proposes, particularly, when the designer needs to know the reinforcement ratio for a given M_{SLS}.

$$\mu_{cr} = c_1 + c_2\alpha + c_3\theta - c_2\alpha\theta + (c_4 + c_5\alpha - c_4\theta + c_6\alpha\theta)\rho \tag{30}$$

Table 1 Constants of the proposed equation for μ_{cr}

	c_1	c_2	c_3	c_4	c_5	c_6
$d = 0.9h$ $d' = 0.1h$	0.1664	7.203×10^{-5}	2.781×10^{-4}	0.1094	0.4251	0.2149
$d = 0.8h$ $d' = 0.2h$	0.1662	1.161×10^{-4}	4.410×10^{-4}	0.1755	0.2341	0.1259

Where:
$$\rho = \frac{A_s}{bh};\ \alpha = \frac{E_s}{E_c};\ \theta = \frac{A_s'}{A_s} \tag{31}$$

A_s is the tensile reinforcement bar; A_s' is the compression reinforcement bar; constants c_1 to c_6 are shown on Table 1. These constants were evaluated in 8 points ($\rho = 0.03$ and $\rho = 0.02$) × ($\theta = 0.1$ and $\theta = 1$) × ($\alpha = 5$ and $\alpha = 20$). This table presents the values of constants for two cases: Case 1- $d = 0.9h$; $d' = 0.1h$; Case 2- $d = 0.8h$; $d' = 0.2h$. Case 1 would be used for beams, and Case 2 for slabs. Linear interpolation can be used for other values of d and d'.

The cracking moment, M_{cr}, of a rectangular section, where the contribution of reinforcement is taken into account, can then be obtained from the following equation

$$M_{cr} = \mu_{cr} b h^2 f_{ct} \tag{32}$$

This procedure can be easily programmed on a worksheet. It should be noted that this proposed procedure accounts for the effects of creep, but not for those of shrinkage.

6. Computational evaluation of M_{cr}

The computational evaluation of the cracking moment, M_{cr}, is carried out using a non-linear program, NLP, which identifies the moment-curvature curve for a RC cross-section. This model was initially developed by Lopes (2005), and further developed for this work, with the introduction of other possibilities to model the materials and with the consideration of long-term effects, including creep and shrinkage. Plane sections were assumed to remain plane after deformations.

Based on the moment-curvature curve, it is also possible to determine the value of the cracking moment more accurately. The evolution of the tensile stresses in concrete was taking into account by using the models found in bibliography (as explained before). Creep effects were also considered.

Fig. 7 shows the evolution of the cracking moment with the reinforcement ratio, ρ, for a rectangular cross-section made with a 20 MPa concrete strength. In Fig. 8 “Proposed Eq.” refers to the values obtained from the simplified equation that was proposed by the authors in the previous section, and “NLP-Exact” refers to computational results from the non linear procedure described above. The word “exact” is exaggerated, but since the procedure reflects a rigorous computation, the authors decided to use such label to enhance the implicit “message” of the graph. The more complicated equation Eq. (27) from which the proposed equation was derived was also added to the graphs to show how close are the results of the proposed equation to Eq. (27).

In Fig. 8, the value of M_{cr}, obtained by the Proposed Eq. for $\rho = 0$ is 6.6 kN.m (the same as that given by Eq. 1). The presence of reinforcement bars originates, as expected, an increase of M_{cr}, i.e., the higher the percentage of steel, the higher the value of M_{cr}. This trend is similar to that of NLP-Exact, but the values that correspond to this procedure are higher than those of Proposed Eq.. The differences are mainly due to the way the tensile behaviour of concrete was considered. A descending

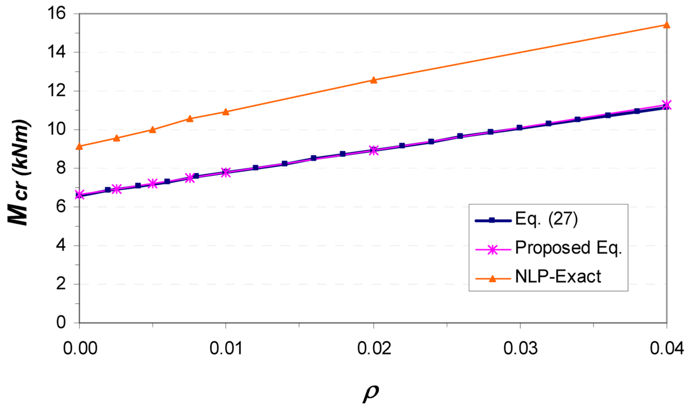

Fig. 8 Evolution of M_{cr} with ρ ($b=0.20$; $h=0.30$; $d=0.9h$; $\theta=0$; $\alpha=6.67$)

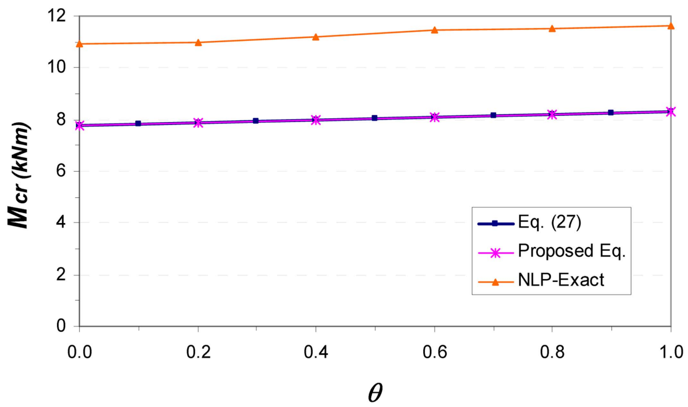

Fig. 9 Evolution of M_{cr} with θ ($b=0.20$; $h=0.30$; $d=0.9h$; $\alpha=6.67$; $\rho=0.01$)

branch of the curve was considered for NLP-Exact (see Fig. 2) but not for the proposed equation. The descending branch reaches relatively high levels of strains and this fact introduces some differences between the two procedures.

Fig. 9 shows the evolution of the cracking moment with the percentage of compressive reinforcement, for a rectangular cross-section with $\rho=0.01$ and a 20 MPa concrete strength. As for Fig. 8, the cracking moment of NLP-Exact is higher than that obtained by the proposed equation.

Looking at this figure, one may conclude that the variation of the cracking moment is not very significant. For example, in the case of Fig. 8, the presence of a percentage of tensile reinforcement of $\rho=0.01$ in comparison with zero reinforcement, led to an increase of 17.6% in M_{cr}, whereas in Fig. 9, the existence of a compression reinforcement equal to tensile reinforcement ($\theta=1$; $\rho'=\rho=0.01$) in comparison with zero compressed reinforcement just implies an increase of 6.6% in M_{cr}.

In the proposed equation, the effect of creep may be taken into account within the modulus of elasticity of concrete $E_c(t)$. The same can also be done when using the computer programme (NLP-Exact). Fig. 10 shows the evolution of the cracking moment with α, for a rectangular cross-section with $\rho=0.01$ and a 20 MPa concrete strength. The effect of creep is considered within the range of

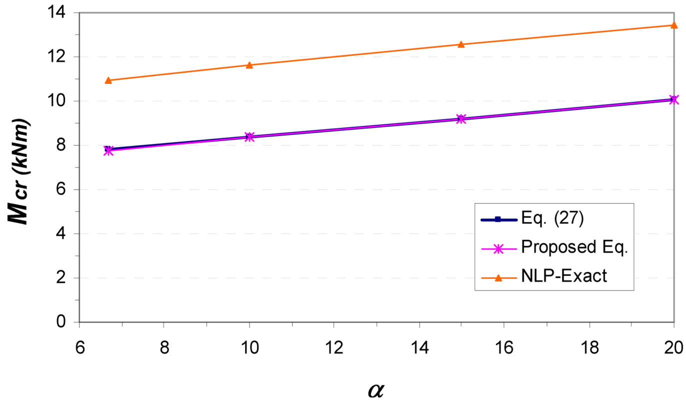

Fig. 10 Evolution of M_{cr} with α - creep effect ($b = 0.20$; $h = 0.30$; $d = 0.9h$; $\rho = 0.01$; $\theta = 0$)

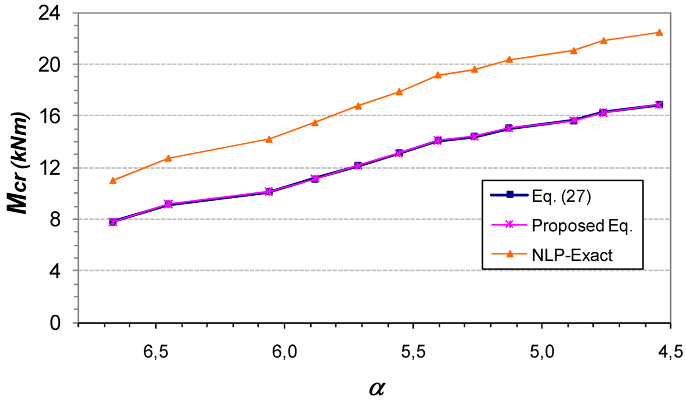

Fig. 11 Evolution of M_{cr} with α - concrete strength effect ($b = 0.20$; $h = 0.30$; $d = 0.9h$; $\rho = 0.01$; $\theta = 0$)

α chosen for the graph. As can be seen, the effect of creep leads to a significant increase in the cracking moment. For example, for $\alpha = 20$, corresponding to $E_c(t) = 10$ GPa, the increasing on the M_{cr} value is 29.4% when compared to the short-term value.

The variation of α may be also concerned to the variation of the concrete strength when the different concrete classes proposed, for instance, by EC2, are considered. Fig. 11 shows the variation of M_{cr} with α for different classes of concrete strength (20 MPa to 90 MPa). This graph is only for short time behaviour. This graph shows that the cracking moment decreases with the increase of α. Since α decreases as the concrete class increases, higher concrete classes correspond to higher values of M_{cr}.

The variation of M_{cr} is not linear because the variation of characteristics of various classes of concrete indicated in EC2 is not constant either. Once again, the trend of Proposed Eq. is similar to that of NLP-Exact, but the values that correspond to NLP-Exact are higher than those of Proposed Eq.. The differences are slightly sharper in the case of high strength concretes, because the area of the descendent branch of tensile σ-ε diagram is bigger in HSC members. NLP-Exact can lead to values 40% higher than those of the proposed equation (Eq. 30 of this work).

In order to compare the proposed evaluation procedure of M_{cr} with experimental results, three works were considered (Desayi and Rao 1987, Makhlouf and Malhas 1996, and Yoon, Cook and

Table 2 Evaluated cracking moments

Reference	$M_{cr,Exp}$ (kNm)	$M_{cr,Effet}$ (kNm)	EC2 (kNm)	Eq. 27 (kNm)	Proposed Eq. (kNm)	NLP (kNm)
Desayi	12.3	16.1	9.07	10.05	10.00	14.66
Makhlouf G2	86	88.7	42.13	48.17	46.64	64.8
Makhlouf G3	124	126.7	42.13	50.96	48.73	68.5
Yoon N	215	231.3	106.98	151.46	146.89	209.6
Yoon M	311	326.9	175.83	236.79	230.78	281.8
Yoon H	334	350.6	213.60	282.20	275.60	315.9

Mitchell 1996). Table 2 presents the results of 6 cases from those 3 references.

It is important to understand that the cracking moments from experimental research, that are presented in the literature, are values recorded when the researchers first spot a crack. This moment is usually not exactly when the crack actually appears, but some time after because the crack is formed and it stays some time not visible. Therefore, these values are normally higher than the real ones.

According to each reference, the first value of Table 2, $M_{cr,Exp}$, is the experimental value referred in the literature. The second one is the effective moment, $M_{cr,Effect}$, when the dead load of the beam is considered. The third value is determined by the Eq. (1) according to EC2, the fourth by the Eq. (27), the fifth by the proposed Eq. (30), and the last one is the NLP value.

The comparison is made in Table 3. In first comparison column, the results from the proposed equation are compared with the theoretical Eq. (27). Despite Eq. (27) not covering $d/h = 0.9$ cases, the results show a very good agreement. The biggest difference was obtained for the worst quotient $d/h = 0.84$. In the following column, the authors compare Eq. (30) with EC2 procedure. The predicted values of M_{cr} are higher when Eq. (30) is used in comparison with EC2. The column presents the excess, in percentage, of the values of M_{cr} when evaluated by Eq. (30) in comparison with EC2. For some of the beams, the difference between the two procedures is not neglectable and Eq. (30) could lead to important cost savings in design.

The following column presents a sort of “safety factor” for Eq. (30) if NPL values are taken as the exact values. As can be seen in the last column, the values of NPL are not very far away from the recorded values of the test beams, the experimental value $M_{cr,Effet}$. As explained before, such recorded values are normally somehow higher than the real cracking moment, since it is very difficult to catch

Table 3 Comparing cracking moments

Reference	$\frac{Eq.27\text{-}Eq.30}{Eq.30}$	$\frac{Eq.30-EC2}{Eq.30}$	$\frac{NLP}{Eq.30}$	$\frac{M_{cr,Effect}-NLP}{NLP}$
Desayi	0.5%	9.3%	1.47	10.1%
Makhlouf G2	3.3%	9.7%	1.39	36.9%
Makhlouf G3	4.6%	13.6%	1.41	85.0%
Yoon N	3.1%	27.2%	1.43	10.3%
Yoon M	2.6%	23.8%	1.22	16.0%
Yoon H	2.4%	22.5%	1.15	11.0%

the exact moment when the beam cracks. Taken this into account, the approximations to Desayi and to Yoon values are remarkable. This is not the case of the approximation to Makhlouf report. However, the first crack widths recorded in this work were too high (0.095 mm and 0.14 mm) and probably the cracks were already open for a while when the first recordings took place. Considering that 0.05 mm would be a typical value for a first reading of the crack width, the values of 37% and 85% would be reduced such values to 18.4% and 16%, respectively, if a linear extrapolation is done. These values are consistent with those by Desayi and Yoon and would restrict the whole values to the range of 10-20% and attest the adequacy of NPL in evaluating the exact value of the cracking moment.

7. Significance of an error on the value of M_{cr} when computing the deflection

In order to evaluate the significance of an error on the value of M_{cr}, when the deflections of a beam/slab are to be computed, a relative deflection will be used: $c\delta$. As shown before, if the reinforcement steel is considered for the calculation of M_{cr}, the values are higher than those computed without steel. Those first moments will be further referred by M_{cr2}. In opposition, M_{cr1}, will be the moment which is obtained from EC2 rules. In the study presented in this section, the authors considered 3 levels of increasing for such cracking moments: 10%, 20% and 40%, (this decision was meant to reflect different situations of reinforcement ratio, concrete strength and other parameters). The relative error, e, is obtained from the following equation

$$e = \frac{c\delta_{Mcr1} - c\delta_{Mcr2}}{c\delta_{Mcr2}} \tag{33}$$

Where

$$c\delta_i = \frac{\delta_i}{\delta_1} \tag{34}$$

where δ_i represents the deflection determined by procedure i, and δ_1 represents the elastic deflection, i.e., that computed for uncracked conditions.

Figs. 11 to 13 show the results obtained for simply supported beam under a distributed load. The

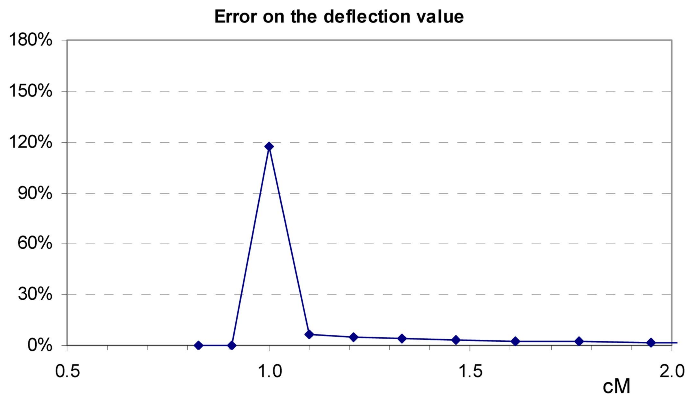

Fig. 12 Error of evaluation of δ provoked by a 10% error on M_{cr}

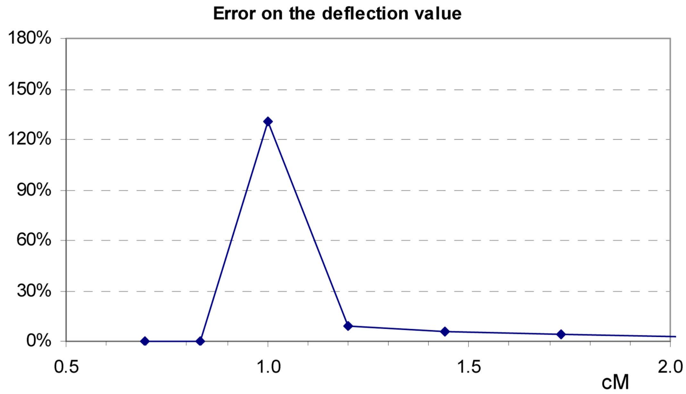

Fig. 13 Error of evaluation of δ provoked by a 20% error on M_{cr}

Fig. 14 Error of evaluation of δ provoked by a 40% error on M_{cr}

deflection of the cracked part was evaluated considering that the cracked stiffness is one third of the elastic bending stiffness, i.e. $EI_{cr} = EI_{elastic}/3$. In these figures, the abscissas represent the coefficient of moment cM, defined by

$$cM = \frac{M_{SLS}}{M_{cr}} \tag{35}$$

Where M_{SLS} is the Service Limit State Moment, SLS Moment.

The results show that, in the zone where $M_{SLS} \sim M_{cr}$, relatively small errors in the evaluation of M_{cr} could provoke significant errors on the evaluation of the deflections that are normally higher than 100%, and may even reach values as high as 150%. Outside that zone, the errors associated with the evaluation of the cracking moment have smaller impacts on the value of deflections.

8. Conclusions

In this article the authors proposed a new procedure to evaluate the value of the cracking moment, M_{cr}, for rectangular cross-sections of reinforced concrete. As it stands, EC2 leads to simplified computing procedures which normally does not account for the presence of steel bars in the evaluation of the cracking moment. A more general method that would take the reinforcement steel into account might be found excessively laborious by designers. Therefore, the proposal presented by the authors is very simple to be adopted for practical applications and conduces to higher values of M_{cr}, when compared to the procedure that does not take the reinforcement into account. Therefore, the proposed procedure is, in general, more economical.

The authors have also used a general numerical procedure implemented on a computer programme to know the moment-curvature relationships. Such programme can take the long-term effects into account. They have compared the cracking moments obtained through the computer programme with those obtained through the proposed equation. The comparison of the results confirmed the good performance of the proposed equation. The values of M_{cr} obtained through the computer programme were always higher in about 40% than those from the proposal. This difference was due to the descending branch of the stress-strain diagram of the tensioned concrete, which was considered in computer programme, but not for the proposed equation. To propose a simplified equation, it would be very difficult to take such descending branch of the curve into account. Nevertheless, the simplified proposed equation is an improvement when compared with the current computing procedure. For proposing a new equation, the authors have analysed the influence of some parameters and the main conclusions are as follows:

- The cracking moment increases with the increase of reinforcement ratio;
- The presence of compressive steel has an insignificant influence on the increase of the cracking moment;
- The effect of creep leads to a significant increase in the cracking moment;
- For the same cross-section, the cracking moment increases as the concrete strength increases.

Finally, the authors showed that a misevaluation of the value of M_{cr} of only 10% could lead to errors over 100% in the prediction of the deflection. Since the control of deformations is sometimes the critical parameter in design, a procedure such the proposed one could be very useful, since M_{cr} comes greater than that obtained through commonly used Eq. (1). If the value of M_{cr} obtained through the proposed equation is not sufficient to pass the maximum deformation fixed by EC2, than a general method, such as that implemented in the computer programme used for this study, could be tried.

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