Computers and Concrete, Vol. 8, No. 6 (2011) 631-645

DOI: http://dx.doi.org/10.12989/cac.2011.8.6.631

Reliability based calibration of the capacity design rule of reinforced concrete beam-column joints

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(Received January 17, 2010, Revised October 30, 2010, Accepted November 19, 2010)

Abstract. The capacity design rule for beam-column joints, as adopted by the EC8, forces the formation of the plastic hinges to be developed in beams rather than in columns. This is achieved by deriving the design moments of the columns of a joint from equilibrium conditions, assuming that plastic hinges with their possible overstrengths have been developed in the adjacent beams of the joint. In this equilibrium the parameters (dimensions, material properties, axial forces etc) are, in general, random variables. Hence, the capacity design is associated with a probability of non-compliance (probability of failure). In the present study the probability of non-compliance of the capacity design rule of joints is being calculated by assuming the basic variables as random variables. Parameters affecting this probability are examined and a modification of the capacity design rule for beam-column joints is proposed, in order to achieve uniformity of the safety level.

Keywords: reliability; capacity design; partial safety factors; concrete structures.

1. Introduction

EC8 2004, as most of the modern seismic codes for the design of earthquake resistant structures, focuses on the ability of structures to dissipate energy through large inelastic cyclic deformation without substantial reduction of their resistance. In order for this to be achieved the whole structure needs to exhibit a ductile behavior. Brittle failures must be avoided while the structure must maintain its ability to transfer the vertical loads to the ground. These two demands are satisfied with the shear capacity design rules and the beam-column joints capacity design rules, respectively. The shear capacity design rules intend to prevent the brittle shear failure of the building elements (beams, columns and walls) and the beam-column joints capacity design rule intends to prevent the formation of storey mechanism (beam failure before column failure).

Results for the efficiency of the beam-column joints capacity design rule derive from various experimental investigations (e.g. Chalioris *et al.* 2008, Benavent 2005, Calvi *et al.* 2002). In the present study the beam-column joints capacity design rule is examined from a probabilistic point of view. For the beam-column joints of structures, the formation of the plastic hinges is forced to develop in beams rather than in columns. This is achieved by designing the columns in bending by using the resisting moments of the beams framing to the joint. For the satisfaction of this criterion, partial safety factors are used which give the desired overstrength to columns. The parameters in the

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above calculations (dimensions, material properties, axial forces etc) are, in general, random variables. In that sense, the capacity design is associated with a probability of non-compliance (probability of failure). Failure of the capacity design rule for beam-column joints is considered as the event in which "the sum of beam resisting moments is greater than the sum of column resisting moments in the joint under consideration".

The aim of this study is to investigate the safety level of the capacity design rule for beam-column joints. The safety level is quantified with the probability of non-compliance (probability of failure). Parameters affecting this probability are determined and a modification of the partial safety factor of the capacity design is proposed, in order to achieve uniformity of the safety level.

2. Methodology

2.1 Capacity design of the beam-column joints

According to EC8, the following condition should be satisfied at all joints of primary or secondary seismic beams with primary seismic columns (Relation (4.29), EC8 2004)

$$\sum M_{Rc} \geq \gamma_{Rd} \cdot \sum M_{Rb} \tag{1}$$

where, $\sum M_{Rc}$ is the sum of the design values of the resisting moments of the columns framing to the joint, $\sum M_{Rb}$ is the sum of the design values of the resisting moments of the beams framing the joint and γ_{Rd} is the partial safety factor accounting for possible overstrength of the resisting moments of the beams. In EC8 2004 the γ_{Rd} value is taken equal to 1.3.

The resisting moments of the relationship (1) are affected from parameters such as concrete strength, steel strength, dimensions etc. which are random variables. There is, therefore, a probability for the sum of the beam resisting moments to be greater than the sum of the column resisting moments although relationship (1) has been used for designing the members of the joint. This probability can be expressed as

$$p_f = P(M_{rc,1} + M_{rc,2} < M_{rb,1} + M_{rb,2}) \tag{2}$$

where, $M_{rc,1}$, $M_{rc,2}$ are random variables that represent the resisting moments of the columns of the joint and $M_{rb,1}$, $M_{rb,2}$ are random variables that represent the resisting moments of the beams of the joint. Relationship 2 can be used for calculating the probability of failure of a joint the columns of which have been designed according to the capacity design. In this case the resultant probability is the probability of failure of the capacity design.

For calculating the probability of failure from the relationship (2), the Monte Carlo simulation can be used. Assuming that the probability distribution of the resisting moments may be approximated by the normal distribution (Trezos 1998), the probability of failure can be directly related to the safety index β

$$p_f = P(M_{rc,1} + M_{rc,2} < M_{rb,1} + M_{rb,2}) = PHI(-\beta) \tag{3}$$

Where, PHI( ) is the cumulative distribution function of the standard normal distribution.

In the following, the safety index β is used instead of the probability of failure p_f, for presentation purposes.

In order to examine the influence of basic variables to the probability of failure of beam-column

Table 1 Probabilistic models of the random variables

Variable	Distribution	Mean value	Coefficient of variation
Unconfined Concrete:	f_{co}: normal	$1.33f_{co,k}$	0.15
Compressive strength: $f_c = f_{co} \cdot Y_1$	Y_1: lognormal	1	0.06
Modulus of elasticity: $E_c = 10.5 \cdot f_c^{1/3} \cdot Y_2$	Y_2: lognormal	1	0.15
Ultimate strain: $\varepsilon_{cu} = 6 \cdot 10^{-3} \cdot f_c^{-1/6} \cdot Y_3$	Y_3: lognormal	1	0.15
Confined Concrete:			
Compressive strength: $f_c^* = Y_{conf,1} \cdot f_c \cdot \begin{cases} 1+2.5 \cdot \alpha \cdot \omega_w, for \ldots \omega_w \le 0.1/\alpha \\ 1.125+1.25 \cdot \alpha \cdot \omega_w, for \ldots \omega_w > 0.1/\alpha \end{cases}$	$Y_{conf,2}$: lognormal	1	0.15
Strain at strength: $\varepsilon_{co}^* = Y_{conf,2} \cdot 0.002 \cdot (f_c^*/f_c)^2$	$Y_{conf,2}$: lognormal	1	0.10
Ultimate strain: $\varepsilon_{cu}^* = Y_{conf,3} \cdot (\varepsilon_{cu} + 0.1 \cdot \alpha \cdot \omega_w)$	$Y_{conf,3}$: lognormal	1	0.50
Steel properties:			
Yield stress: f_{sy}	Normal	$1.09f_{sy,k}$	0.05
Ultimate strain: ε_{su}	Normal	0.05	0.10
Dimensions:			
Cross-sectional dimensions: X	Normal	Nominal	(4 mm+6‰ $X_{,nom})/X_{,nom}$
Area of re-bars: A_s	Normal	Nominal	0.02

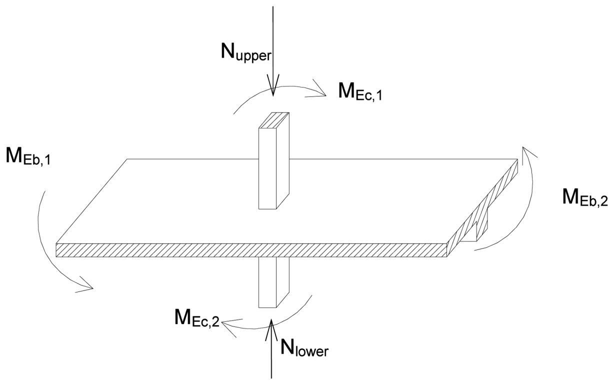

Fig. 1 Interior joint. Moments under seismic action

joints criterion, a set of fictitious individual joints was examined covering a large range of the basic variable potentially affecting the safety. More precisely the parameters used were:

- column dimensions 0.30 m×0.30 m, 0.40 m×0.40 m and 0.50 m×0.50 m
- geometric ratio of total reinforcement of the columns ($\rho_{c,tot} = A_{s,tot}/A_c$) varied from 0.01 to 0.04
- geometric ratio of tensile reinforcement of beams ($\rho_{b,tens} = A_{s,t}/A_{beam}$) varied from 0.004 to 0.015
- beam dimensions 0.25 m×0.40 m, 0.30 m×0.45 m, 0.25 m×0.50 m, 0.25 m×0.55 m, 0.20 m×0.60 m, 0.30 m×0.60 m, 0.30 m×0.65 m and 0.35 m×0.65 m
- reduced axial force $\nu = \dfrac{N}{b_{col} \cdot h_{col} \cdot f_{cd}}$ from 0.00 to 0.40 (positive for compression) for the upper

Table 2 Examined joint configurations

Columns					Beams		Materials			Number of joints
Dimensions (m)	Axial force (ν) Up	Axial force (ν) Down	Confinement $\alpha\omega_w$	Total reinf. $\rho_{c,tot}$ (%)	Dimensions (m)	Tensile reinf. $\rho_{b,tens}$ (%)	b_{eff} (m)	Steel	Concrete	
0.30/0.30	0.00	0.00	0.0	(four values) 1.0… 4.0	0.25/0.40	0.8 1.4	2.00	S500	f_{ck}=20MPa f_{ck}=30MPa	768
	0.14	0.28	0.1		0.25/0.50	0.6 1.2				
	0.28	0.56	0.2		0.25/0.55	0.4 0.7				
	0.40	0.60	0.3							
0.40/0.40	0.00	0.00	0.0	(four values) 1.0… 4.0	0.20/0.60	0.9 1.3	2.00	S500	f_{ck}=20 MPa f_{ck}=30 MPa	768
	0.08	0.16	0.1		0.30/0.60	0.4 0.7				
	0.16	0.31	0.2		0.30/0.45	0.8 1.5				
	0.23	0.34	0.3							
0.50/0.50	0.00	0.00	0.0	(four values) 1.0… 4.0	0.30/0.45	0.8 1.5	2.00	S500	f_{ck}=20 MPa f_{ck}=30 MPa	768
	0.05	0.10	0.1		0.30/0.65	0.4 0.6				
	0.10	0.20	0.2		0.35/0.65	0.3 0.5				
	0.15	0.22	0.3							
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columns of the joints and from 0.00 to 0.60 for the lower columns of the joints.

- confinement ratio $\alpha\omega_w$ varied from 0.0 to 0.3
- flange width of T-beams b_{eff} 2.00 m and flange thickness h_f 0.15 m (for the beams subjected to positive moment: flange in compression)
- Concrete with 5% characteristic value of the compressive strength f_{ck} = 20 MPa and f_{ck} = 30 MPa. Steel grade S500

The joint configurations under consideration are summarized in Table 2.

γ_{Rd} factor is calculated directly from the data of Table 2, using the design values of the resisting moments of the columns and the beams of each joint

$$\gamma_{rd} = \sum M_{Rc} / \sum M_{Rb} \tag{4}$$

When relation (4) is applied to the joints of Table 2, γ_{Rd} factor gets values between 0.3 and 4,0.

Fig. 2 shows the calculation steps for calculating the index β for the examined joints. The resisting moments are calculated according to EC2 using the material properties shown in Fig. 3. For normal design, EC2 permits the assumption, for the reinforcing steel, of a horizontal top branch without the need to check the limit strain. In the present study the limit strain was limited to 20‰ as it was a current European practice. Nevertheless the resisting moment is not affected by this stain

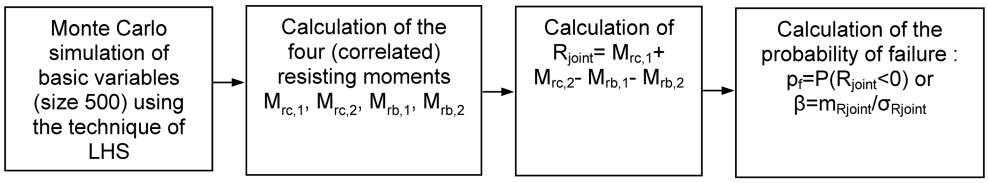

Fig. 2 Methodology for calculating the capacity design probability of failure of beam-column joints

limitation, and thus the results would be the same.

The random variable simulation is implemented using the Latin Hypercube Sampling technique (LHS) (Ayyub and Lai 1989, Iman and Conover 1980, McKay *et al.* 1979, Nowak and Collins 2000). The LHS is a selective sample technique by which, for a desirable accuracy level, the sample size is significantly smaller than that of the direct Monte Carlo simulation. LHS provides a constrained sampling scheme instead of random sampling. In LHS, the region between 0 and 1 (which is used for generating random numbers and then used for generating random variables according to the prescribed distribution function) is uniformly divided into non-overlapping intervals.

The random numbers used for generating After testing the simulation with various sampling sizes, it was found that a sample size of 500 analyses offers an adequate accuracy level, as the results for the index β did not change from simulation to simulation.

3. Random variables

Many probabilistic models for the random variables are given in the international literature (Ditlevsen and Madsen 1996, Melcher 1999, Joint Committee on Structural Safety 2001, Gardoni *et al.* 2002, Epaarachchi and Stewart 2004, Lu *et al.* 2005). In the present study the considerations of random variables are based on probabilistic models that have been thoroughly investigated at Thomos and Trezos (2006). In the following, details of the assumed distributions are presented. The distributions are presented synoptically in Table 1.

3.1 Materials

The stress-strain diagrams of the materials are shown in Fig. 3. For the conventional design, the left column σ-ε diagrams of Fig. 3 were used, while, as for the simulation the right column diagrams of Fig. 3.

3.1.1 Unconfined concrete

The models of concrete properties for a particular element are (see Fig. 3)

Compressive strength: $f_c = f_{co} \cdot Y_1$ (5)

Modulus of elasticity: $E_c = 10.5 \cdot f_c^{1/3} \cdot Y_2$ (6)

Ultimate strain: $\varepsilon_{cu} = 6 \cdot 10^{-3} \cdot f_c^{-1/6} \cdot Y_3$ (7)

Where, f_{co} a normal random variable with mean value related to the 5% characteristic value of the compressive strength $f_{co,k}$: $E[f_{co}] = f_{co,k}/(1 - 1.64 \cdot \mathrm{Cov}[f_{co}])$ and coefficient of variation: $\mathrm{Cov}[f_{co}] = 0.15$

Y_1: log-normal variables reflecting floor to floor variation of casting conditions with mean value 1

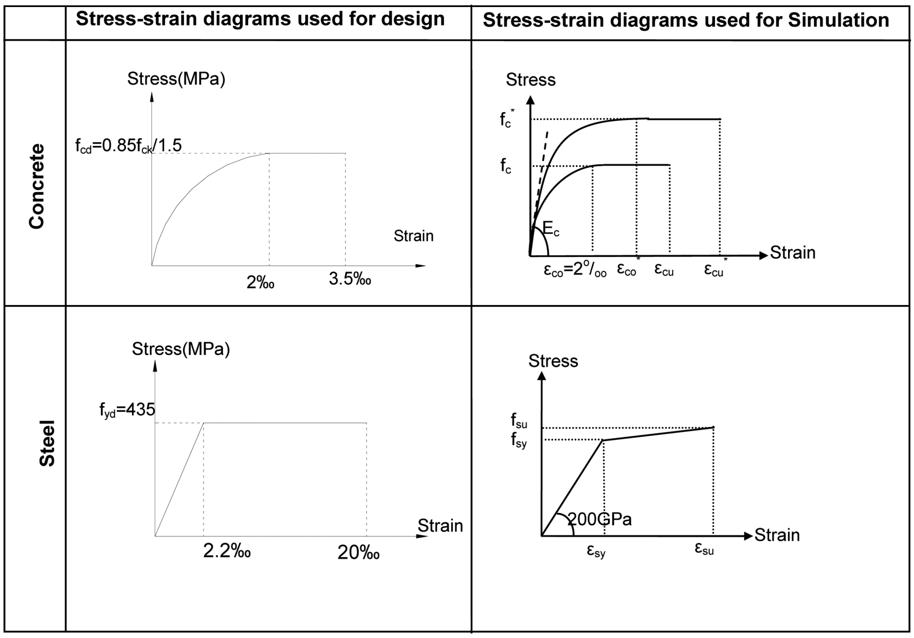

Fig. 3 Stress-strain diagrams of concrete and steel

and coefficient of variation 0.06.

Y_2, Y_3: lognormal variables reflecting factors not well accounted for by concrete compressive strength (e.g. gravel type and size, chemical composition of cement and other ingredients, climatic conditions) with mean value 1 and coefficient of variation 0.15.

3.1.2 Confined concrete

The deterministic model for the confinement, proposed by Tassios and Lefas (1986) adopted in CEB-FIP Model Code 1990 (1993), has been used for simulating the confined concrete. This model has been converted to a probabilistic model by introducing random variables $Y_{conf,1}$, $Y_{conf,2}$, $Y_{conf,3}$, taking into account the uncertainties of the model (Eqs. (8)-(10)).

$$f_c^* = Y_{conf,1} \cdot f_c \cdot \begin{cases} 1 + 2.5 \cdot \alpha \cdot \omega_w, \; for \ldots \omega_w \leq 0.1/\alpha \\ 1.125 + 1.25 \cdot \alpha \cdot \omega_w, \;\; for \ldots \omega_w > 0.1/\alpha \end{cases} \tag{8}$$

$$\varepsilon_{co}^* = Y_{conf,2} \cdot 0.002 \cdot (f_c^*/f_c)^2 \tag{9}$$

$$\varepsilon_{co}^* = Y_{conf,3} \cdot (\varepsilon_{cu} + 0.1 \cdot \alpha \cdot \omega_w) \tag{10}$$

Where:

ε_{co} : 0.002, deterministic value corresponding to the maximum stress (strength) of the unconfined concrete

α : confinement effectiveness factor (Tassios and Lefas 1986)

ω_w : mechanical volumetric ratio of the transverse reinforcement

$Y_{conf,1}$, $Y_{conf,2}$, $Y_{conf,3}$: lognormal random variables representing model uncertainties with a mean value of 1 and coefficients of variation 0.15, 0.10 and 0.50 respectively.

3.1.3 Steel properties

Yield stress f_{sy} (see Fig. 3) : normal variable with mean value related to the 5% characteristic value of the yield stress: $E[f_{sy}] = f_{sy,k}/(1 - 1.64 \cdot \mathrm{Cov}[f_{sy}])$ and $\mathrm{Cov}[f_{sy}] = 0.05$

Ultimate stress f_{su} : perfectly correlated to the yield stress, $f_{su} = 1.15 \cdot f_{sy,i}$

Ultimate strain: normal variable with mean value $E[\varepsilon_{su}] = 0.05$ and a coefficient of variation of $\mathrm{Cov}[\varepsilon_{su}] = 0.1$

3.2 Dimensions

The cross-sectional dimensions are modeled as random variables that follow a normal distribution with mean values equal to the nominal values $E[X] = X_{nom}$ and standard deviations $\sigma_X = 4$ mm + $0.006 \cdot X_{nom}$.

Areas of re-bars are assumed to be independent normal random variables with mean values equal to the nominal values $E[A_s] = A_{s,nom}$ and coefficient of variation $\mathrm{Cov}[A_s] = 0.02$.

For an interior joint with four framing elements, the four resting moments are not independent random variables, since they have some variables in common. Thus, assuming the two beams and the lower column are cast simultaneously, the concrete strengths of these elements are correlated: variables Y_1, Y_2, Y_3, have the same value for calculating the concrete properties of these three members.

4. Results

In Fig. 4 the safety indices β for the 2304 joints of Table 2 are shown as a function of the partial safety factor γ_{Rd} used in the relationship (1). The results have been derived by applying the methodology of Fig. 2 to the joints of Table 2.

For increasing values of γ_{Rd}, an increasing scatter of β is observed. For the code value of $\gamma_{Rd} = 1.3$ the variation of the safety index β is significant as the minimum calculated value of the safety index β was $\beta_{min} = 5$ and the maximum was $\beta_{max} = 9$.

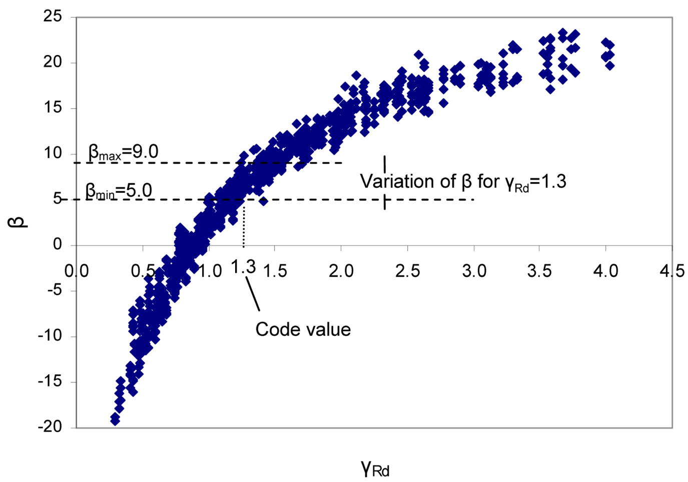

Fig. 4 Safety index β as a function of γ_{Rd} (for the 2304 individual joints)

In order to achieve a uniform safety level or a constant value of the safety index β, equal to a β_{target}, it is necessary to modify the partial safety factor γ_{Rd}. So, a relationship that gives the safety index β as a function of the partial safety factor γ_{Rd} and the other parameters of the beam-column joints is needed. A simplified relationship between β and γ_{Rd}, considering that resisting moments follow the normal distribution and ignoring for the moment the correlations between the four resisting moments can be derived from Eq. (3) as follows

$$p_f = P(M_{rc,1}+M_{rc,2}<M_{rb,1}+M_{rb,2}) = Erf(-\beta) \Rightarrow$$

$$\beta = \frac{(M_{rc,1})_m+(M_{rc,2})_m-(M_{rb,1})_m-(M_{rb,2})_m}{\sqrt{\sigma_{rc,1}^2+\sigma_{rc,2}^2+\sigma_{rb,1}^2+\sigma_{rb,2}^2}} \tag{11}$$

Where $(M_{rc,1})_m$, $(M_{rc,2})_m$, $(M_{rb,1})_m$, $(M_{rb,2})_m$ are the mean values of the resisting moments of the framing elements and $\sigma_{rc,1}$, $\sigma_{rc,2}$, $\sigma_{rb,1}$, $\sigma_{rb,2}$ are the corresponding standard deviations.

From the relationships (1) and (11) can be concluded that

$$\beta = \frac{a\cdot\gamma_{Rd}-b}{\sqrt{c\cdot\gamma_{Rd}^2+d}} \tag{12}$$

Where,

$$a = \frac{(M_{rc,1})_m+(M_{rc,2})_m}{M_{Rd,c,1}+M_{Rd,c,2}},\ b = \frac{(M_{rb,1})_m+(M_{rb,2})_m}{M_{Rd,b,1}+M_{Rd,b,2}},\ c \approx \frac{(\sigma_{r,c,1}+\sigma_{r,c,2})^2}{2\cdot(M_{Rd,c,1}+M_{Rd,c,2})^2},\ d \approx \frac{(\sigma_{r,b,1}+\sigma_{r,b,2})^2}{2\cdot(M_{Rd,b,1}+M_{Rd,b,2})^2}$$

The factors a, b, c and d depend on the joint parameters (axial force, dimensions, reinforcement ratio etc). Furthermore, the factors are also influenced by the correlations between the four elements, which have been ignored in the equations above.

Inspired from Eq. (12), an equation that gives the expected value of the safety index β as a function of the partial safety factor γ_{Rd} and the joint parameters is proposed. Relationship (13) has been derived by applying regression analysis to the data of Table 2 and the results of the simulations

$$R = \frac{a\cdot\gamma_{Rd}-25.00}{\sqrt{\gamma_{Rd}^2+0.96}},\ [R^2 = 0.98,\ \text{Standard error} = 1.12] \tag{13}$$

with $a = 26.05 - 8.87\cdot\nu_m^2+8.53\cdot\nu_m+6.08\cdot\alpha\cdot\omega_w-0.04\cdot\rho_{c,tot,m}/\rho_{b,tens,m}$ (13.1)

where:

ν_m : the average axial force of the two columns of the joint [$\nu_m = 0.5(\nu_{upper}+\nu_{lower})$]

$\rho_{c,tot,m}$: the average geometric reinforcement ratio of the columns [$\rho_{c,tot,m} = 0.5(\rho_{column,up,tot}+\rho_{column,lower,tot})$]

$\rho_{b,tens,m}$: the average geometric tensile reinforcement ratio of the beams [$\rho_{b,tens,m} = 0.5(\rho_{beam,left,t}+\rho_{beam,right,t})$]

α : confinement effectiveness factor

ω_w : mechanical volumetric ratio of confining reinforcement

From the above relations (13) and (13.1) it can be seen that β increases proportionally (through factor "a") to the confinement, ($\alpha\omega_w$), as it is expected since increased values of ($\alpha\omega_w$) result to increased concrete strength and thus increased resisting moments of columns. Furthermore, the influence of axial force, ν, is more complicated, increasing the β values for ν up to $\nu = 0.48$, and then decreasing. This is due to the fact that the "distance" of the mean resisting moment, M_{Rm}, from the design moment, M_{Rd}, measured in terms of standard deviation, σ_R,: $(M_{Rm}-M_{Rd})/\sigma_R$ it is not

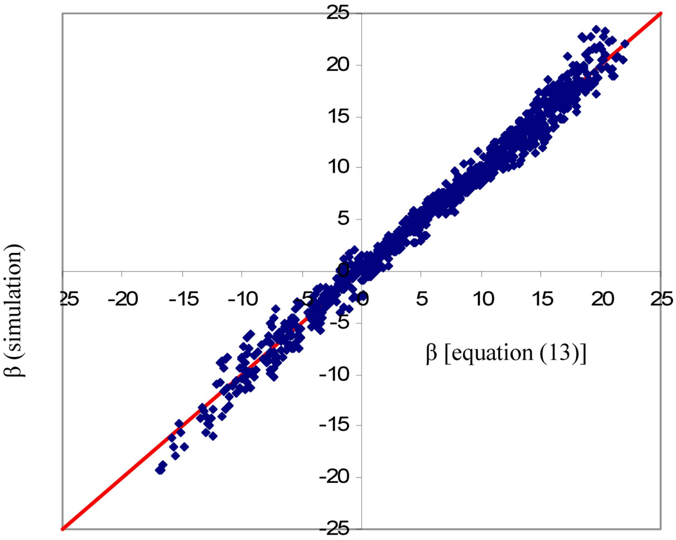

Fig. 5 Values of β calculated from Eq. (13) and simulation

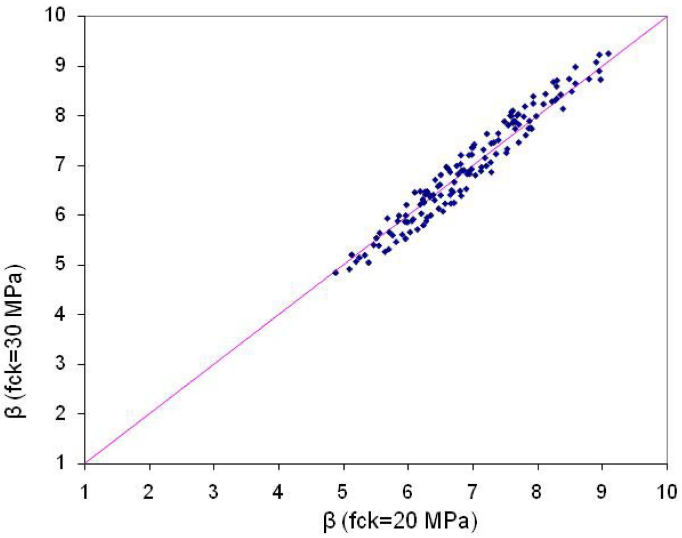

Fig. 6 Comparison of the safety index β of joints designed with concrete with f_{ck} = 20 MPa and with f_{ck} = 30. The results refer to the joints of Table 2 that have been designed with safety factor γ_{Rd} = 1.30

constant depending on the axial force, ν.

In the diagram of Fig. 5, the values of β calculated from the simulation are compared to the values of β derived from relationship (13).

The results showed that concrete grade does not influence the safety index β significantly (see Fig. 6). Therefore, concrete grade is not included in relationship 13. It was also found that the dimensions of beams and columns have no effect on the safety index β, so they also are excluded from relationship 13.

In the design practice, the reinforcement of the columns is usually calculated using relationship (1) after the calculation of the beams' reinforcement. Relationship (13) gives the safety index as a

Table 3 Example of joint-capacity design with desirable β

Given parameters of the joint				
COLUMNS				
Dimensions	ν_{up}	ν_{down}	$\rho_{c,tot,m}$	$\alpha\omega_w$
0.30/0.30	0.1	0.2	0.021 (12Φ14)	0.10
BEAMS				
Dimensions	$\rho_{b,tens,m}$			
0.25/0.45(b_{eff}= 2.00 m)	0.0041 (4Φ12)			

function of the columns' reinforcement. Therefore, although it provides the means for predicting the safety level of the capacity design of joints, it is not easy to be implemented in the design practice as it contains the reinforcement of the columns, which is usually the result of the capacity design. To bypass this difficulty, in relationship (14) the parameter $\rho_{c,tot,m}/\rho_{b,tens,m}$ has been neglected. This relationship is less accurate than relationship (13), as the safety index β is actually influenced by the reinforcement of members, but it is more useful as it does not include the reinforcement of the members

$$\beta = \frac{a \cdot \gamma_{Rd} - 25.00}{\sqrt{\gamma_{Rd}^2 + 0.96}}, \ [R^2 = 0.97, \text{Standard error} = 1.14] \tag{14}$$

with $a = 25.84 - 10.51 \cdot \nu_m^2 + 9.10 \cdot \nu_m + 6.08 \cdot \alpha \cdot \omega_w$.

Using Eq. (14) is possible to calculate the required γ_{Rd} for a given (desirable) safety index β. For example, from the parameters of the joint that are shown in Table 3, the factor "a" of the Eq. (14) is calculated

$$a = 25.84 - 10.51 \cdot 0.15^2 + 9.10 \cdot 0.15 + 6.08 \cdot 0.10 = 27.57 \text{ and thus } \beta = \frac{27.57 \cdot \gamma_{Rd} - 25.00}{\sqrt{\gamma_{Rd}^2 + 0.96}}$$

If the code value of γ_{Rd} = 1.30 is used in the capacity design, then the resulting safety index is β = 6.67. If a higher safety index value is desired (for example β = 9.78) then the required value for the partial safety factor γ_{Rd} is 1.56. The resulting difference in the column reinforcement is significant: with γ_{Rd} = 1.30 the columns must be reinforced with 12 bars of 14 mm diameter, while when γ_{Rd} = 1.56 is used the columns must be reinforced with 12 bars of 16 mm diameter (30% increase).

Relationship (14) seems to be suitable to be used for modifying the capacity design of joints. The modified design would lead all joints to result to a safety index β equal to the safety index of a "standard joint". For example the standard joint could be the case of the joint of Table 3 which results to a safety index equal to 6.67 (when designed with γ_{Rd} = 1.30).

The safety index β of the joints of Table 2 that have been designed using γ_{Rd} = 1.30 is presented in Fig. 7 related with the average axial force of the columns and the confinement factors, $\alpha\omega_w$. In the same figure, relationship (14) is used for presenting the safety index β as a function of the average axial force, ν, for three different values of $\alpha\omega_w$.

As shown in Fig. 7, index β is a function of the axial force and the confinement of the columns. If the value of γ_{Rd} which derives from relationship (14) with β equal to 6.67 was used, instead of using γ_{Rd} = 1.30 for each joint of the Table 2, then more uniform values of β could be achieved. In

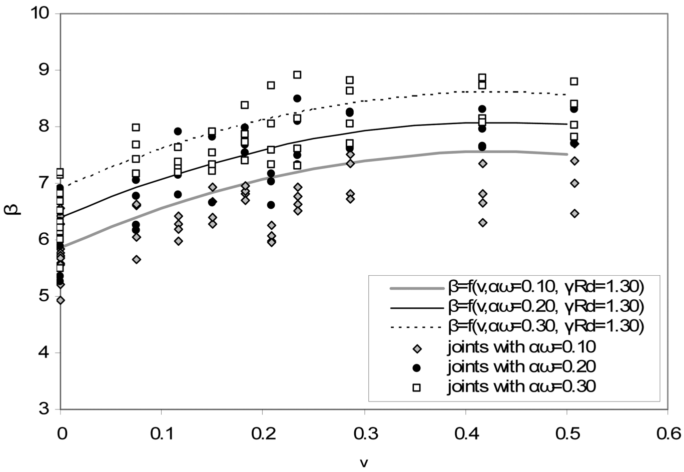

Fig. 7 Safety index β for the joints of Table 2 with γ_{Rd} = 1.30. Relationship 12 as a function of v

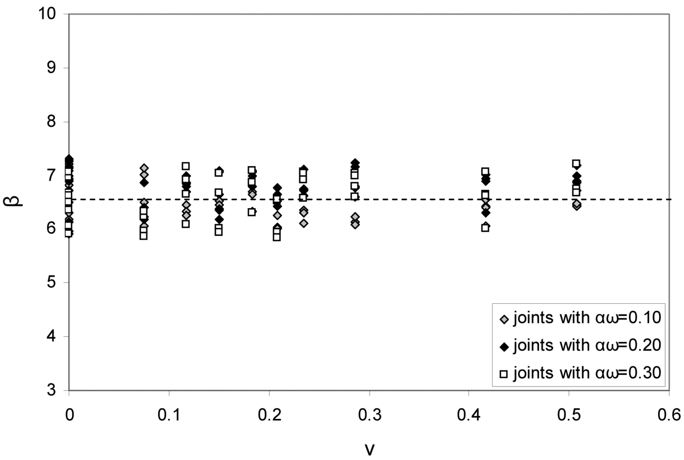

Fig. 8 Safety index β for the joints of Table 2 using for γ_{Rd} safety factor the values derived from relationship (12) for β = 6.67

Fig. 8 the safety index β of the re-designed joints of Table 2 [with individual values of γ_{Rd} for each joint so as: $6.67 = (a\ \gamma_{Rd} - 25.00)/(\gamma_{Rd}^2 + 0.96)^{0.5}$] is shown. The values of β have been calculated by using the methodology of Fig. 2. The safety index varies from 6.0 to 7.2 and a relationship between β and v_m can not be found, contrary to the results of the standard case of capacity design (see Fig. 7) where the variance of β is larger and β is correlated with the average axial force v.

Table 4 presents an alternative use of relationship (14). It contains the values of the safety factor γ_{Rd} that should be used in any case of joint, in order for the safety level to be common. These values of γ_{Rd} have been calculated using relationship 14 and considering as "standard" case of joint, the joint of Table 3. Any joint that is designed using the values of Table 4 for the safety factor γ_{Rd}, results to safety index β equal to the safety index β of the joint of Table 3.

The value of each region of Table 4 is the value of γ_{Rd} that corresponds to the worst case of this region. For example the value 1.27 is the value of γ_{Rd} that must be used for a case of joint for

Table 4 γ_{Rd} values for achieving uniformity of the safety level. $\beta_{target} = 6.67$

$\alpha\omega_w$ \ v	$v \le 0.20$	$0.20 \le v \le 0.40$	$0.40 \le v \le 0.65$
$\alpha\omega_w \le 0.10$	1.43	1.33	1.32
$0.10 \le \alpha\omega_w \le 0.20$	1.36	1.27	1.26
$0.20 \le \alpha\omega_w \le 0.30$	1.32	1.24	1.23
$0.30 \le \alpha\omega_w \le 0.40$	1.28	1.21	1.17

Table 5 Ratios of γ_{Rd} values $\gamma_{Rd,\ \alpha\omega w,\ v}/\gamma_{Rd,\ ref}$

$\alpha\omega_w$ \ v	$v \le 0.20$	$0.20 \le v \le 0.40$	$0.40 \le v \le 0.65$
$\alpha\omega_w \le 0.10$	1.13	1.05	1.04
$0.10 \le \alpha\omega_w \le 0.20$	1.07	1.00	0.99
$0.20 \le \alpha\omega_w \le 0.30$	1.04	0.98	0.97
$0.30 \le \alpha\omega_w \le 0.40$	1.01	0.95	0.92

Table 6 Modified γ_{Rd} values for achieving more uniform safety level (code value $\gamma_{Rd} = 1.3$)

$\alpha\omega_w$ \ v	$v \le 0.20$	$0.20 \le v \le 0.40$	$0.40 \le v \le 0.65$
$\alpha\omega_w \le 0.10$	1.46	1.36	1.35
$0.10 \le \alpha\omega_w \le 0.20$	1.39	1.30	1.29
$0.20 \le \alpha\omega_w \le 0.30$	1.35	1.27	1.26
$0.30 \le \alpha\omega_w \le 0.40$	1.31	1.24	1.20

which the columns have an average value of $a\omega_w$ equal to 0.1 and v equal to 0.2. If the value 1.27 is used for a different joint that belong σ to this region, larger values of β would result.

The resulting difference in the capacity design caused by the use of Table 4, can be shown in the example that follows: For the joint of the Table 3, if the code value of γ_{Rd}= 1.3 is used in the capacity design, the required reinforcement ratio of columns is **0.021** (A_s = 18.46 cm^2) for $v_m = 0.15$ ($v_{upper} = 0.10$, $v_{lower} = 0.20$) and **0.020** (A_s = 17.68 cm^2) for $v_m = 0.40$ ($v_{upper} = 0.35$, $v_{lower} = 0.45$) (regardless of the confinement parameters). If Table 4 is used for choosing the value of γ_{Rd}, then with $v_m = 0.15$ ($v_{upper} = 0.10$, $v_{lower} = 0.20$) and $a\omega_w = 0.10$ (γ_{Rd}= 1.43) the required reinforcement ratio of columns is **0.024** (A_s = 21.20 cm^2) [14.3% increase compared to the code requirement (0.021)] while with v_m = 0.40 (v_{upper} = 0.35, v_{lower} = 0.45) and $a\omega_w = 0.35$ (γ_{Rd}= 1.17) the required reinforcement ratio of columns is **0.017** (A_s = 14.96 cm^2) [15.0% decrease compared to the code requirement (0.020)].

In the above, the value β = 6.67, has been used as an example. In the case of Code Makers, the value of β will be selected depending on general criteria (social, economic etc). A different value of β will result in different γ_{Rd} values. But, regardless the β-value that has been chosen, it can be shown that the ratios $\gamma_{Rd,\alpha\omega w,\ v}/\gamma_{Rd,\ ref}$ are almost independent on the β-value (see Table 5). $\gamma_{Rd,\ ref}$, is the value that corresponds to: $0.10 \le \alpha\omega_w \le 0.20$ and $0.20 \le v \le 0.40$. Even in the case where an explicit β-value is not chosen, nevertheless more rational γ_{Rd} values can be applied using Table 5 to modify the γ_{Rd} value given in the code (γ_{Rd} = 1.3) taking into account the axial force, v, and the confinement, $\alpha\omega_w$. The modified values are given in Table 6.

5. Conclusions

In the present study, the safety level of the capacity design of the beam-column joints has been examined. The basic variables have been considered as random and the probability of failure has been calculated for various combinations of basic variables.

The safety index β of the capacity design of joints that are designed according to EC8 varies from 5 to 10. This variation mainly depends on the axial force since the axial force affects the probabilistic characteristics of the resistant moment (see paragraph 4) and the confinement of the columns, since the confinement is not taken into account in the design. On the other hand, the concrete grade, the structural dimensions and the reinforcement ratio of the columns and beams do not have a significant influence (since their role is taken into account in the design).

In order for the safety level to be uniform for all cases of joints, a modification of the partial safety factor γ_{Rd} is needed. The proposed relationships (or the proposed values of Table 4) could be used effectively for calibrating of the capacity design of the beam - column joints. The implementation of this "modified" capacity design needs the use of a specific value of γ_{Rd} for each case of joint, instead of the standard value of 1.3, and leads to different reinforcement requirements for columns compared to EC8 (e.g. from +14.3% to -15% change of reinforcement for the example that presented at the end of paragraph 4).

This modification of the capacity design of joints leads to a more uniform safety level. Nevertheless, this uniformity may be altered in unsystematic way because of the implementation of other code demands such as minimum reinforcement ratio, maximum rebars spacing.

References

Ayyub, B.M. and Lai, K.L. (1989), "Structural reliability assessment using latin hypercube sampling", *Proceeding of the 5th international conference on structural safety and reliability*, Vol. 2, p. 1177-84, San Francisco, August.

Benavent, C.A. (2005), "Shaking table tests of reinforced concrete wide beam-column connections", *Earthq. Eng. Struct. Dyn.*, **34**, 1833-1839.

Calvi, G.M., Magenes, G. and Pampanin, S. (2002), "Relevance of beam-column joint damage and collapse in rc frame assessment", *J. Earthq. Eng.*, **6**(1), 75-100.

Chalioris, C.E., Favvata, M.J. and Karayannis, C.G. (2008), "Reinforced concrete beam-column joints with crossed inclined bars under cyclic deformations", *Earthq. Eng. Struct. Dyn.*, **37**, 881-897.

Ditlevsen, O. and Madsen, H.O. (1996), *Structural reliability methods*, Chichester, John Wiley & Sons.

EC 2 (2004), *Design of concrete structure, Part 1-1: general rules and rules for buildings*, European standard EN 1992-1-1, European Committee for Standardization (CEN), Brussels.

EC 8-1 (2004), *Design of structures for earthquake resistance, Part 1: general rules, seismic actions and rules for buildings*, European standard EN 1998-1, European Committee for Standardization (CEN), Brussels.

Epaarachchi, D.C. and Stewart, M.G. (2004), "Human error and reliability of multistory reinforced-concrete building construction", *J. Perform. Constr. Fac.*, **18**, 12-20.

Euro-International Committee for Concrete (CEB) (1993), *CEB-FIP model code 1990*, London, Thomas Telford.

Gardoni, P. Kiureghian, A.D. and Mosalam, K.M. (2002), "Probabilistic capacity models and fragility estimates for reinforced concrete columns based on experimental observations", *J. Eng. Mech.*, **128**, 1024-1038.

Iman, R.L. and Conover, W.J. (1980), "Small sample sensitivity analysis techniques for computer models, with an application to risk assessment", *Commun. Stat. Theor. M.*, **A9**(17), 1749-842.

Joint Committee on Structural Safety (2001), *Probabilistic model code, Internet publication*, http://www.jcss.ethz.ch.

Lu, Y., Gu X., and Guan, J. (2005), "Probabilistic drift limits and performance evaluation of reinforced concrete columns", *J. Struct. Eng.,* **131**, 966-978.

Nowak, A.S. and Collins, K.R. (2000), *Reliability of structures*, McGraw-Hill.
McKay, M.D., Conover, W.J. and Beckman, R.J. (1979), "A comparison of three methods for selecting values of input variables in the analysis of output from a computer code", *Technometrics*, **21**, 239-245.
Melcher, R.E. (1999), *Structural reliability analysis and prediction*, Chichester, John Wiley & Sons.
Tassios, T.P. and Lefas, J. (1984), "Ductility of concrete columns due to confinement", *Scientific Papers of the Faculty of Civ. Eng.*, NTUA, **8**(1-4).
Thomos, G.C. and Trezos, C.G. (2006), "Examination of the probabilistic response of reinforced concrete structures under static non-linear analysis", *Eng. Struct.*, **28**, 120-133.
Trezos, C.G. (1998), "Reliability consideration of the structural response of reinforced concrete structures under seismic conditions", *11th European Conf. on Earthquake Engin.*, Paris.

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Notations

The following symbols are used in this paper:

A_s = random variable representing the area of re-bar
$A_{s,nom}$ = nominal value for the area of re-bar
b_{eff} = flange width of T-beams
E_c = random variable representing the modulus of elasticity of unconfined concrete
PHI( ) = cumulative distribution function of the standard normal distribution
f_c = random variable representing the compressive strength of unconfined concrete
f_c^* = random variable representing the compressive strength of confined concrete
f_{cd} = design value of the compressive strength
f_{ck} = 5% characteristic value of the compressive strength
f_{co} = random variable with mean value $E[f_{co}] = f_{co,k}/(1 - 1.64 \cdot \mathrm{Cov}[f_{co}])$ and coefficient of variation: $\mathrm{Cov}[f_{co}]=0.15$
f_{su} = random variable representing the ultimate stress of steel
f_{sy} = random variable representing the yield stress of steel
$f_{sy,k}$ = 5% characteristic value of the yield stress of steel
h_f = flange thickness of beam
$M_{Eb,1}$, $M_{Eb,2}$ = moments of beams under seismic action
$M_{Ec,1}$, $M_{Ec,2}$ = moments of columns under seismic action
$M_{rb,1}$, $M_{rb,2}$ = random variables that represent the resisting moments of the beams of the joint
$M_{rc,1}$, $M_{rc,2}$ = random variables that represent the resisting moments of the columns of the joint
v = reduced axial force
X = random variable representing the dimension of an element
$X_{,nom}$ = nominal value of dimension of an element
Y_1 = log-normal variables with mean value 1 and coefficient of variation 0.06
Y_2, Y_3 = lognormal variables with mean value 1 and coefficient of variation 0.15
$Y_{conf,1}$, $Y_{conf,2}$, $Y_{conf,3}$= lognormal random with a mean value of 1 and coefficients of variation 0.15, 0.10 and 0.50 respectively
α = confinement effectiveness factor
β = safety index
γ_{Rd} = partial safety factor accounting for possible overstrength of the resisting moments of the

beams

ε_{co} = random variable representing the strain corresponding to the maximum stress of the unconfined concrete

ε_{co}^{*} = random variable representing the strain corresponding to the maximum stress (strength) of the confined concrete

ε_{cu} = random variable representing the ultimate strain of unconfined concrete

ε_{cu}^{*} = random variable representing the ultimate strain of confined concrete

ε_{su} = random variable representing the ultimate strain of steel

ν_m = the average axial force of the two columns of the joint

$\rho_{b,tens}$ = geometric ratio of tensile reinforcement of beams

$\rho_{b,tens,m}$ = the average geometric tensile reinforcement ratio of the beams

$\rho_{c,tot}$ = geometric ratio of total reinforcement of the columns

$\rho_{c,tot,m}$ = the average geometric reinforcement ratio of the column

$\sum M_{Rb}$ = of the design values of the resisting moments of the beams framing the joint

$\sum M_{Rc}$ = sum of the design values of the resisting moments of the columns framing to the joint

ω_w = mechanical volumetric ratio of the transverse reinforcement