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Three-dimensional numerical simulation and cracking analysis of fiber-reinforced cement-based composites

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Abstract. Three-dimensional graphic objects created by MATLAB are exported to the AUTOCAD program through the MATLAB handle functions. The imported SAT format files are used to produce the finite element mesh for MSC.PATRAN. Based on the Monte-Carlo random sample principle, the material heterogeneity of cement composites with randomly distributed fibers is described by the WEIBULL distribution function. In this paper, a concept called "soft region" including micro-defects, micro-voids, etc. is put forward for the simulation of crack propagation in fiber-reinforced cement composites. The performance of the numerical model is demonstrated by several examples involving crack initiation and growth in the composites under three-dimensional stress conditions: tensile loading; compressive loading and crack growth along a bi-material interface.

Keywords: fiber-reinforced cement composites; stress-strain behavior; interfacial transition zone; finite element analysis (FEA); multi-scale modeling.

1 Introduction

Concrete is a brittle composite material characterized by a non-homogeneous internal structure. Under the influence of external loads the failure of composites can be caused by discontinuities created by the technological defects or local differences of mechanical properties of the concrete components. In the vicinity of discontinuities local stress concentrations occur, which can initiate defects (under tensile or compressive loading). The propagation of such defects may further degrade the material or structural element. A brittle fracture process starts when large defects start to grow in an unstable manner (Ruiz *et al*. 2000, Most and Bucher 2007, Remmers *et al.* 2008, Sancho *et al*. 2007a, 2007b, Oliver *et al*. 2004, Sadowski and Golewski 2008).

Concrete is a mixture of cement, water and aggregates. In terms of microstructure, besides the cement paste matrix and aggregate inclusions, there is a third phase, called the interfacial transition zone (ITZ). The ITZ is formed due to the wall effect and can be treated as a thin shell that randomly forms around each aggregate particle. Thus, concrete can be viewed as a bulk paste matrix containing composite inclusions (Sun *et al*. 2007). Leite *et al.* (2007) present a mathematical modeling and computational tool to simulate the fracture processes of cement composites at the

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meso-scopic level. Three-dimensional beam lattice models are extended and used to simulate the size effect on strength in 3-point bending fracture experiments. At the meso-level concrete is treated as a three-phase material, consisting of the aggregate particles, cement matrix and the ITZ, which separates aggregates and cement matrix (Man and Van Mier 2008). To date, the modeling of large multiple defects, micro-voids etc. in a matrix with three-dimensional randomly distributed properties has not yet been effectively carried out.

The simulation of damage and failure of short fiber reinforced cement composites is a challenging task. Due to the number of failure mechanisms introduced by the fibers on the meso-scale of the material, randomly oriented fibers, such as steel fibers, affect the macroscopic failure behavior in compression, tension, flexure, and shear. On the meso-scale the fibers introduce additional damage and failure mechanisms like fiber fracture, fiber bridging, fiber bending and matrix spalling for inclined fibers and fiber pull-out (Okabe *et al.* 2005, Sivakumar *et al.* 2008, Zairi *et al.* 2008, Cheng *et al.* 2004). However, the numerical simulation of three-dimensional crack failure in fiber-reinforced cement composites has rarely been reported in the literature. The internal structure of fiber-reinforced concrete is very complex as it varies with the number of fibers and changing mechanical properties during the loading process. Also, cracks along the interface between fiber and matrix propagate mainly into the cement matrix surrounding the fibers.

In this paper, a “soft-region” concept is presented that addresses the crack problem of fiber-reinforced cement composites. A soft region is a region that contains many micro-cracks and micro-voids which represent the initial damage state. Instead of individual discrete cracks, only the entire region with initial damage is considered. This region can be modeled with general finite elements for a numerical test, but such elements fail at a lower load level. The shape and size of defects are very difficult to determine, thus the influence of the shape and size on soft-region elements is ignored.

Under external load the original degree of damage of samples is represented by the soft-region volume ratio and the corresponding average strength. In the next sections, the implementation of fibers randomly distributed in a three-dimensional matrix is discussed, based on the Monte Carlo sampling principle. Thereafter, a mathematical model for the constitutive relationship and material properties is presented. The application of the model is illustrated through two numerical examples: (i) a tensile crack test and (ii) a compressive crack test.

2. Numerical models

2.1 Fiber-reinforced cement composite model

In short-fiber reinforced cement composites, large numbers of chopped fibers are randomly distributed in the matrix. Different types of fiber have been used in engineering applications, e.g. straight, hooked end, corrugated, etc. To simplify the study, the types of used in this paper are assumed to be straight and cylindrical. Fibers randomly distributed in the matrix can be simulated by the MATLAB program (Uzunoglu *et al.* 2003), a powerful programming language as well as an interactive computational tool. Files that contain codes in MATLAB language are called M-files. There are two types of M-files that we can write: scripts and functions. In this paper, the second one is selected. MATLAB provides a wide variety of techniques to display data graphically. Interactive tools can manipulate graphs to achieve the results that reveal the most information from the relevant data.

The cylinder function treats its first argument as a profile curve, and generates *x*-, *y*-, and *z*-coordinates of a unit cylinder. E.g. [*X*, *Y*, *Z*] = cylinder(*r*) returns the *x*-, *y*-, and *z*-coordinates of a cylinder using *r* to define a profile curve. The cylinder function treats each element in *r* as a radius at equally spaced heights along the unit height of the cylinder. The cylinder has 20 equally spaced points around its circumference. To ensure the randomness of fibers distributed in the matrix, the rand matrix function and rotate function are used. Of course, fibers produced through MATLAB must be restricted to the matrix.

There is a problem: no fiber should intersect with other fibers. So once one fiber is created, the next fiber not only should be restricted to the matrix but also have no points in common with the former fiber. The other fibers are treated by analogy. The case can be finished by "if" sentence and "dsearchn" function, i.e. if the distance between each point of one fiber and any points of other fibers is less than a given threshold value, it can be assumed that the two fibers intersect. So the next fiber must be created again.

In addition to fiber and concrete matrix, fiber-reinforced cement composites have a third phase (a thin interfacial layer), that exists between the fiber and matrix. To create the interfacial layer, the previously generated fibers are assumed to include the interfacial layer. In this method, randomly distributed fibers can also be created without including the interfacial layer. So a Boolean algorithm is used by MSC.PATRAN to create the interfacial layer.

In fact, MATLAB only exports the figure files instead of the object files. The figure files cannot

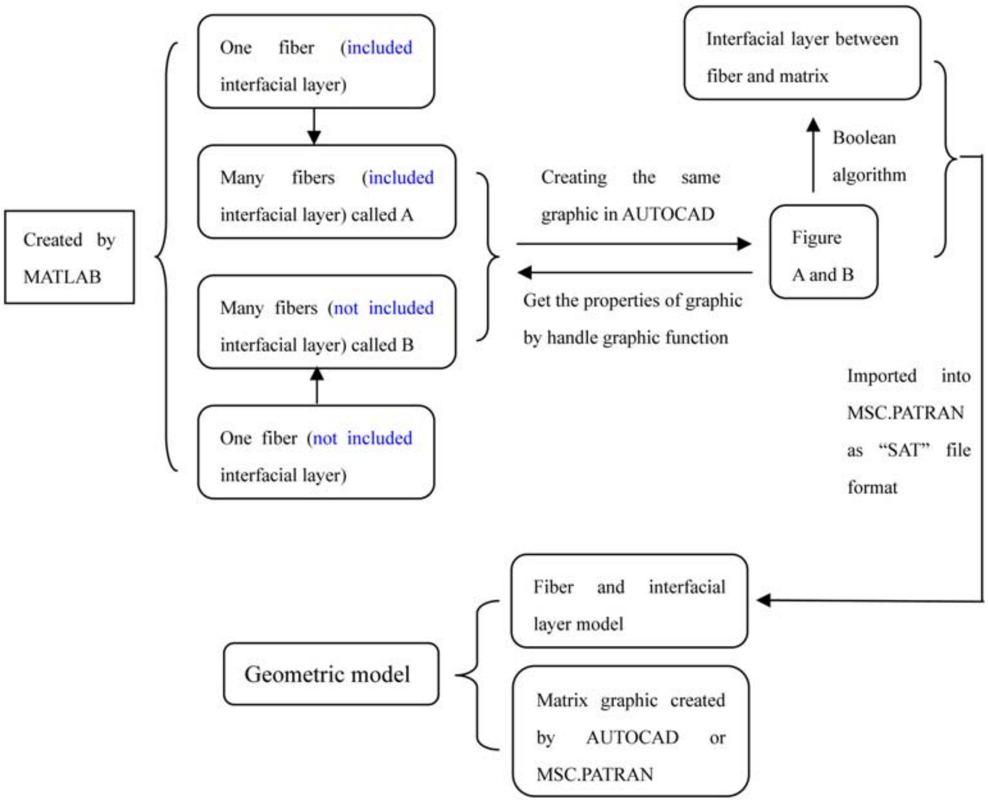


Fig. 1 Flow chart of geometric model created

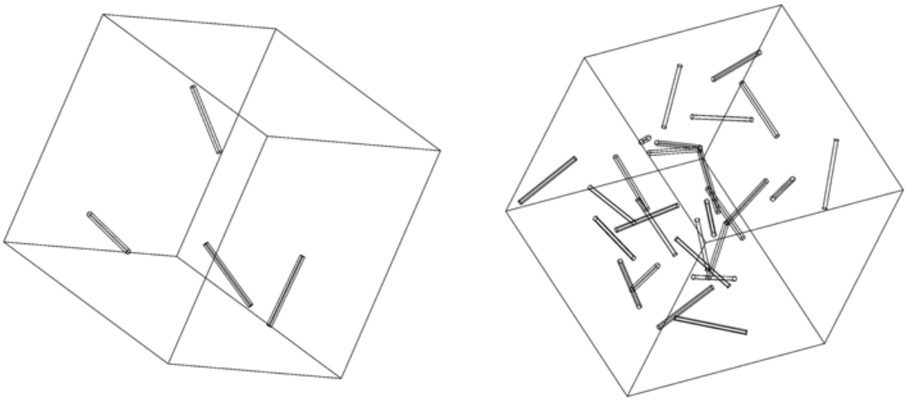


Fig. 2 Geometric model of fiber-reinforced cement composites

be imported into commercial finite element programs such as MSC.MARC or ABAQUS. However, the object files created by AUTOCAD can be imported. Using the handle graphic function of MATLAB, the figure files created by MATLAB can be successfully copied into AUTOCAD. Handle Graphics refers to a system of graphics objects that MATLAB uses to implement the graphic and visualization functions. Each object created has a fixed set of properties. We can use these properties to control the behavior and appearance of a given graph. MATLAB creates a graphics object, and it assigns an identifier (called a handle) to the object. This handle is used to access the object's properties with the "set" and "get" functions. For example, the following statements create a graph and return a handle to an object in h: h = mesh (X, Y, Z). In terms of the "get" function, some data of key points of graphic created by MATLAB is attained and used to produce the corresponding graphic of AUTOCAD as follows:

$X1$ = get(h, '$x$data')
$Y1$ = get(h, '$y$data')
$Z1$ = get(h, 'Zdata')

Afterwards, the graphic created by AUTOCAD is exported to the "SAT" file that then can be imported to commercial finite element programs (e.g. MSC.PATRAN). The flow chart is shown in Fig. 1. Using this procedure, a geometric model of the fiber-reinforced cement composite can be obtained (Fig. 2).

2.2 Analysis of mechanical properties

Considering the initial defects of materials, a soft region concept is presented to depict the initial micro-defects, micro-voids, etc. The cement-based materials are assumed to be two-phase composites consisting of the matrix phase and matrix soft region phase. The two phases may be of the same element type (hexahedron element) in the finite element model, but they have different mechanical properties. The number of matrix soft region elements controls the initial degree of damage of the cement-based composite. In this study, the percentage of soft region elements is fixed at 20%. In fact, the mechanical properties of cement composite components vary with the different local

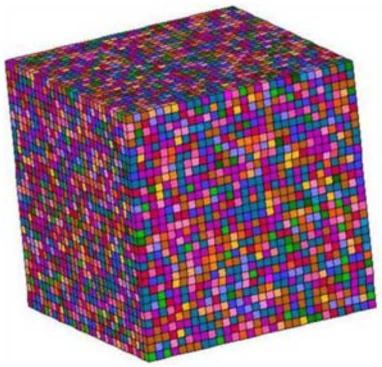


Fig. 3 Finite element model of cement based composite considering heterogeneous material

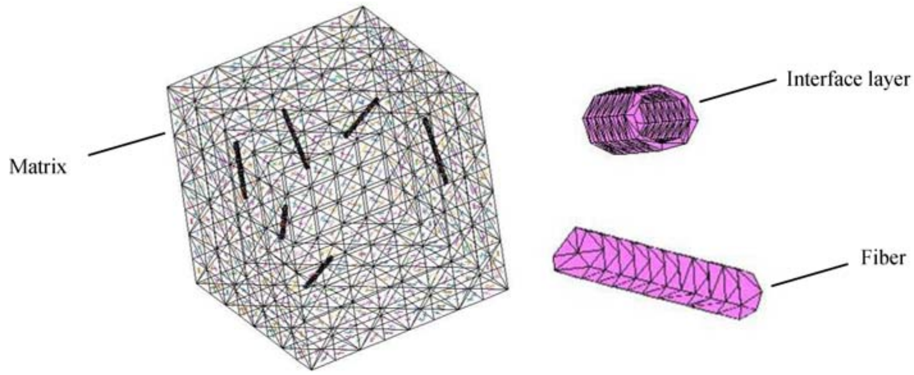


Fig. 4 Finite element model of fiber-reinforced cement composites with the mechanical properties differences

positions. This can be modeled by assigning different soft region elements different mechanical properties. Once the mechanical properties of each local region are distinguished, a finite element model of the cement composite can be obtained (Fig. 3). The model mesh can be created by MENTAT-MARC, but the cracking strength of each meso-element may be different. In Fig. 3 different colors denote different cracking strengths. Element cracking strength of each phase except for the fiber phase independently obeys the Weibull distribution.

To reduce the computational effort, it is assumed that cracks do not occur at the aggregate-paste interface but in the soft region, the effect of aggregate is ignored, and concrete is treated as a two-phase composite. For the same reason, the fiber-reinforced cement based composite is treated as a four-phase composite including the matrix, matrix soft region, fiber and interfacial layer between matrix and fiber. In the three-dimensional mode, the mesh subdivision has a little problem with the cavity created by the fiber and interfacial layer, but MSC.PATRAN (Patran 2001) can solve this instead of MENTAT-MARC. Obviously, enough mesh density is needed. Here the tetrahedron element will replace the hexahedron element. Similarly, elements in different local positions of the same material will have different mechanical properties (e.g. strength, elastic modulus, etc.) except for the fibers. A finite element model consisting of 6 fibers and corresponding interfaces is illustrated in Fig. 4.

Strength and elastic modulus are two important parameters of the composites, but the strength parameter is more crucial for the macro-mechanical properties of the composites than the elastic modulus (Huang and Jiang 2007). Now the strength properties of each phase except for the fibers are assumed to obey the Weibull distribution, a powerful modeling tool used in reliability analyses to predict failure rates and to provide a description of the failure of materials. This is necessitated by various factors such as the anisotropy, internal structure, and service environment of composites, which prevent designers from having a specific strength value to characterize their mechanical behavior. The Weibull distribution is defined by the 'shape' and 'scale' parameters. The two-parameter Weibull distribution function that models the fracture strength of a material is given by

$$\begin{aligned} f(\sigma) &= \lambda\alpha(\lambda\sigma)^{\alpha-1}e^{-(\lambda\sigma)\alpha} \\ F(\sigma) &= 1-e^{-(\lambda\sigma)\alpha} \end{aligned} \qquad (\sigma\geq 0,\ \alpha>0,\ \lambda>0) \tag{1}$$

where F is the fracture probability of the material under direct tensile stress σ, α is the shape parameter or Weibull modulus, and $1/\lambda$ is the scale parameter of the distribution. The parameters α and $1/\lambda$ are estimated from the relevant literature (Birgoren and Dirikolu 2004).

To obtain the strength parameters for the matrix, matrix soft region, interface layer obeying the Weibull distribution, firstly, a homogeneous pseudo-random number is created by a FORTRAN procedure, and secondly, a sample formula obeying the Weibull distribution can be obtained by the transform method (Muralidhar 2004).

$$\sigma = \frac{1}{\lambda}(-\ln u)^{\frac{1}{\alpha}} \qquad u \in (0,1) \tag{2}$$

where u is the pseudo-random number obeying the uniformity distribution.

Cement-based composites are materials of low tensile strength. It is assumed that they do not fail in compression. Relevant Weibull parameters of the tensile numerical model are listed as follows. Here homogeneity denotes the shape parameter, i.e. α in Eq. (2). Average denotes the average cracking strength of the corresponding element, i.e. $1/\lambda$ in Eq. (2). The critical crushing strain means that an element will be crushed if the element strain achieves a specific value (Table 1).

The selected numerical specimen is assumed to be of the size $200\times200\times200$ mm^3, and the length and radius of the fiber are taken as 30 mm and 1 mm, respectively. The fiber's elastic modulus is taken as 210 GPa and its Poisson's ratio as 0.25. To reduce the computational effort, only 6 fibers

Table 1 Weibull parameters

	Cracking strength		Critical crush strain
	Homogeneity	Average(Mpa)	
Homogeneous matrix	6	3	10^{10}
Matrix soft region	3	0.3	10^{10}
Fiber-matrix interface layer	3	1	10^{10}

Table 2 Geometric parameters of numerical model

Case	Length (mm)	Radius (mm)	Interface thickness (mm)		Fiber total number	Matrix soft region ratio(%)
			radial	axial		
Case 1	30	1	0.25	0.5	6	20
Case 2	Pure cement composites					

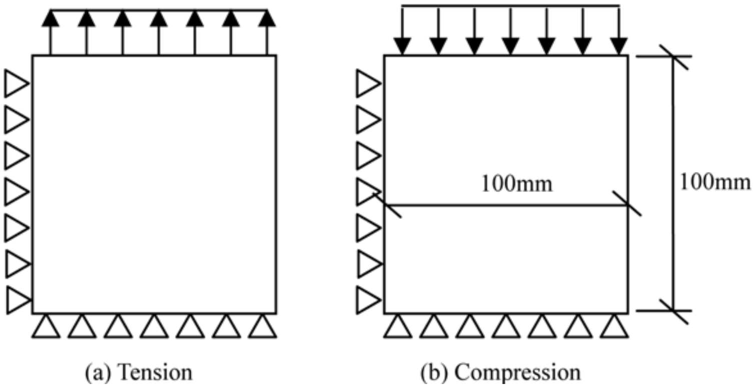


Fig. 5 Sketch diagram of mechanics model

and corresponding interfaces are considered in the fiber-reinforced cement composite model. Unreinforced cement composites are also analyzed for comparison with the fiber-reinforced composites. Relevant geometric parameters are listed in Table 2.

In this study, two numerical tests are simulated: tension and compression. Due to the symmetry of the numerical specimen, only one-eighth part of the model is analyzed, Fig. 5.

3. Numerical simulations

3.1 Tensile numerical test

Cement composites are of low tensile strength. They display nonlinear behavior on the macro-scale, but brittle behavior on the meso-scale. For the meso-scopic heterogeneity, an elastic brittle hypothesis of meso-elements is enough to describe the macro-mechanical properties. Under external load, the meso-element shows linear properties. A crack develops in the material perpendicular to the direction of the maximum principal stress if this exceeds a certain value. After an initial crack has formed at a material point, a second crack can form perpendicular to the first one. Likewise, a third crack can form perpendicular to the first two. The material loses all load-carrying capacity across the crack unless tension softening is included. If tension softening is included, the stress in the direction of maximum stress does not go immediately to zero; instead the material softens until there is no stress across the crack. The softening behavior is characterized by a descending branch in the tensile stress-strain diagram, which may depend on the element size. Now a series of numerical analyses of composites with different tension softening moduli are carried out to study their effects on the macro-mechanical properties. Four groups of numerical tests simulate the tensile behavior of the composites. In each group the meso-element has a different softening modulus for the descending branch of the stress-strain curve. Here σ_{cr} denotes the tensile strength of the meso-element, and E_{soft} denotes the softening modulus of the descending branch of the stress-strain curve of the meso-element (Fig. 6).

After a crack forms, the loading can be reversed, therefore, the opening crack width must be considered. In this case, the crack can close again, and partial recovery occurs, in which case it is assumed that the material regains full compressive strength and that shear stresses are transmitted across the crack, but with a reduced shear modulus. Actually, it is found that the reduced shear

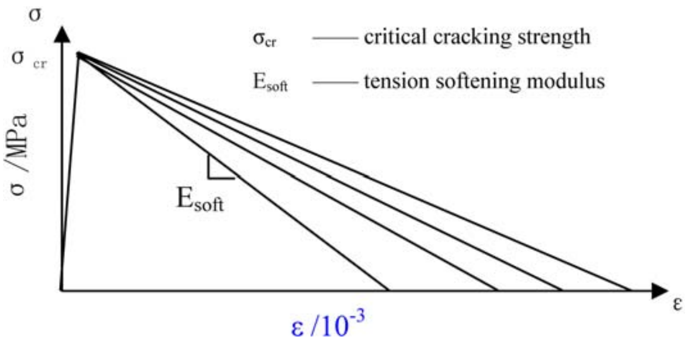


Fig. 6 Elastic brittle constitutive equation of meso-element with varied E_{soft}

modulus affects the macro-mechanical performance of composite materials. To compare the differences, four cases with different residual shear factors are considered.

It is assumed that cracking strains occur at a given integration point of an element, once cracking starts. The material strain is not equal to the element strain. The difference is defined as the cracking strain which refers to crack opening. The Newton-Raphson method is selected in the nonlinear analysis and as the convergence criterion the residual stress method is used. If at an integration point cracking occurs and the crack is not closed, the cracking strain can be calculated first. The relationship between strain increments is given as

$$\Delta\varepsilon = \Delta\varepsilon^{co} + \Delta\varepsilon^{cr} \tag{3}$$

where is the integration point strain related to the node displacement, ε^{co} is the material strain consisting of the elastic strain and the plastic strain, ε^{cr} is the cracking strain.

Firstly, the material is assumed to be isotropic without cracks ($\varepsilon^{cr} = 0$), and its behavior is depicted by an elastic-plastic relationship. The strain increment at a given point can then be calculated. Computing the new principal stress values, the transformation matrix is formed to ensure that stress and strain refer to the principal coordinate axes. If the stress and strain in the principal direction are denoted by S and e, we can get: $S = R\sigma$, $e = R\varepsilon^{co}$, so the constitutive equation in the element coordinate system can be transformed to the principal directions.

If the principal stress S in one direction exceeds the threshold value S^{cr}, cracking strain e^{cr} will be created in the direction θ_i ($i = 1, 2, 3$). The stress in the specified direction reduces to zero, thus the constitutive relationship can be considered in a concise system. For 2D plane stress conditions, one crack formation results in a one-dimensional stress-strain relationship. For the Poisson effect of elastic materials and the incompressibility of plastic materials, continuous deformation exists perpendicular to the cracking direction. For a solution, several items should be considered. To simulate the tensile softening, each cracking direction will be carried out using a uniaxial overlay. The constitutive relationship is given as

$$\Delta S_i = -E_{soft}\Delta e_i^{cr} \tag{4}$$

Here the stress in the cracking direction should be non-zero. Finally, the stress-strain relationship in the principal direction will be transformed to the element coordinates to obtain the stiffness relationship at each Gauss point.

The heterogeneity properties with failure strength of the same phase are considered, and the

Table 3 Mechanic parameters of cement composites (fiber-reinforced cement composites)

Case NO.		Homogeneous matrix softening modulus (MPa)	Matrix soft region's softening modulus (MPa)	Interfacial layer (MPa)	Shear residual factor
NO.1	NO.1.1	5000	4000	4500 (FRC)	0.5
	NO.1.2				0.1
	NO.1.3				0.05
	NO.1.4				0.01
NO.2	NO.2.1	1000	900	950 (FRC)	0.5
	NO.2.2				0.1
	NO.2.3				0.05
	NO.2.4				0.01
NO.3	NO.3.1	500	400	450 (FRC)	0.5
	NO.3.2				0.1
	NO.3.3				0.05
	NO.3.4				0.01
NO.4	NO.4.1	200	100	150 (FRC)	0.5
	NO.4.2				0.1
	NO.4.3				0.05
	NO.4.4				0.01

elastic modulus is given a fixed value. Values assumed for the elastic moduli of the homogeneous matrix, matrix soft region and interface layer are 30 GPa, 10 GPa, 20 GPa, respectively. All material Poisson's ratios are assumed to be 0.2. The softening moduli of the components (the homogeneous matrix, matrix soft region, fiber-matrix interface) are chosen according to reference (Tang and Zhu 2003). However, these parameters are not the same as introduced in (Tang and Zhu 2003). When these parameters are selected, all numerical test results must be based on real experimental data. Similarly, the "shear residual factor" must obey that principle too. To obtain a rational result, the analysis must be repeated many times, selecting different parameter values. To compare these cases, a series of parameter values are shown in Table 3.

3.1.1 Tension test results for cement composites (CC)

For cement-based composites consisting of two phases (homogeneous matrix and matrix soft region), each element may have a different cracking strength. When the external load increases to a certain value, a crack is formed in the element with the lowest strength. As the load increases further, more and more cracks occur. Finally, cracks develop throughout the model and result in the rupture of the specimen (Fig. 7).

Assuming that the elastic modulus or softening modulus of the matrix soft region is less than that of the corresponding homogeneous matrix, a series of numerical tests with different softening modulus are analyzed by MSC.MARC (Fig. 8(a)). The results show that the slope of the descending branch decreases with increasing softening modulus, while the tensile strength increases slightly. When the softening modulus of the homogeneous matrix or matrix soft region is taken separately as 5000 MPa or 4000 MPa, the whole stress-strain curve of the numerical specimen cannot be computed. For the same reason, relevant numerical tests using different shear residual factors are also analyzed

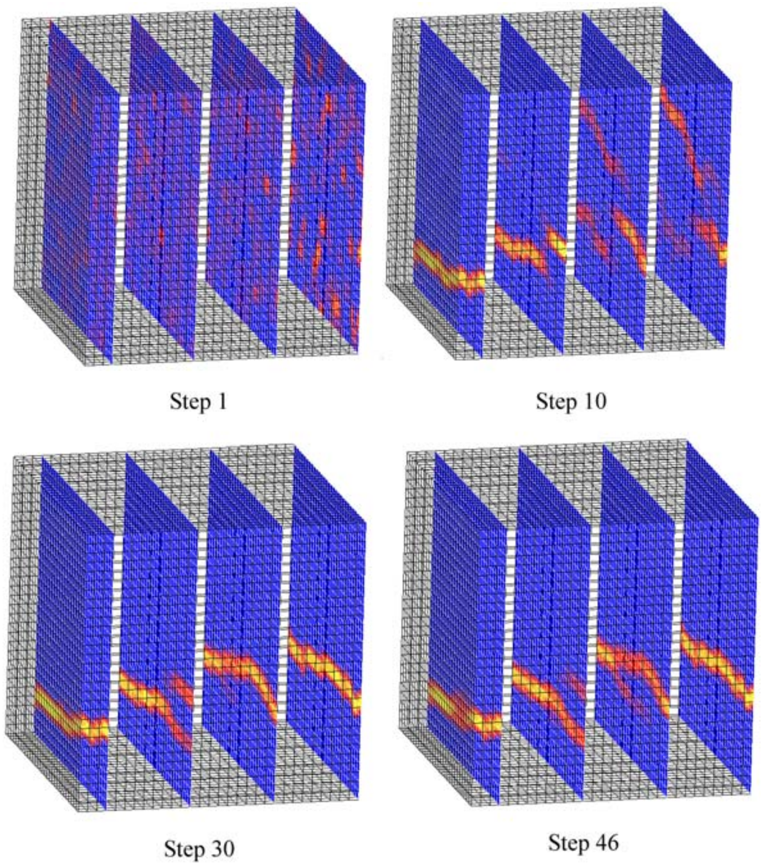
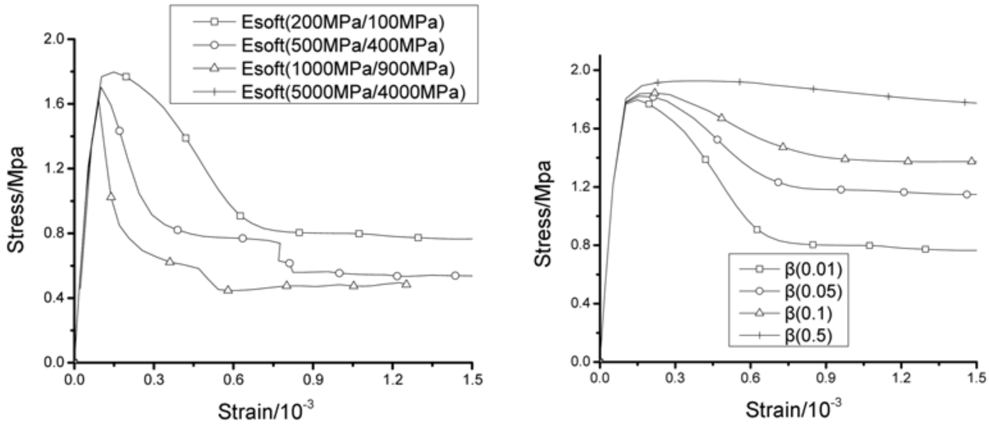


Fig. 7 Cracking strain cutting planes of CC



(a) Varied tension softening modulus

(b) Varied shear residual factor (β)

Fig. 8 Direct tensile stress-strain curve

(Fig. 8(b)). With increasing shear residual factor the material softening behavior becomes more and more pronounced. When the shear residual factor for all elements is taken as 0.5 (here the homogeneous matrix and matrix soft region have the same shear residual factor), the numerical results may be suspect. The shear residual factor has hardly any effect on the strength of CC.

3.1.2 Tension test results for fiber-reinforced cement composites (FRCC)

The addition of fibers makes the internal material structure more complex. The real mechanical behavior can be viewed when the FRCC contains large numbers of short fibers, but this can be computationally prohibitive. Therefore, a simple numerical model with six fibers was analyzed. To observe the initiation of cracks and their development, the element mesh of the fibers and interface layer should be refined. Still, the number of elements for the matrix soft region remains 20% of all matrix elements, and some valuable results can be obtained by a series of numerical analyses. As the external load increases, elements with low tensile strength begin to crack, e.g. in step 5. Also, some of the soft region elements and interface layer elements between fiber and matrix start to crack at the

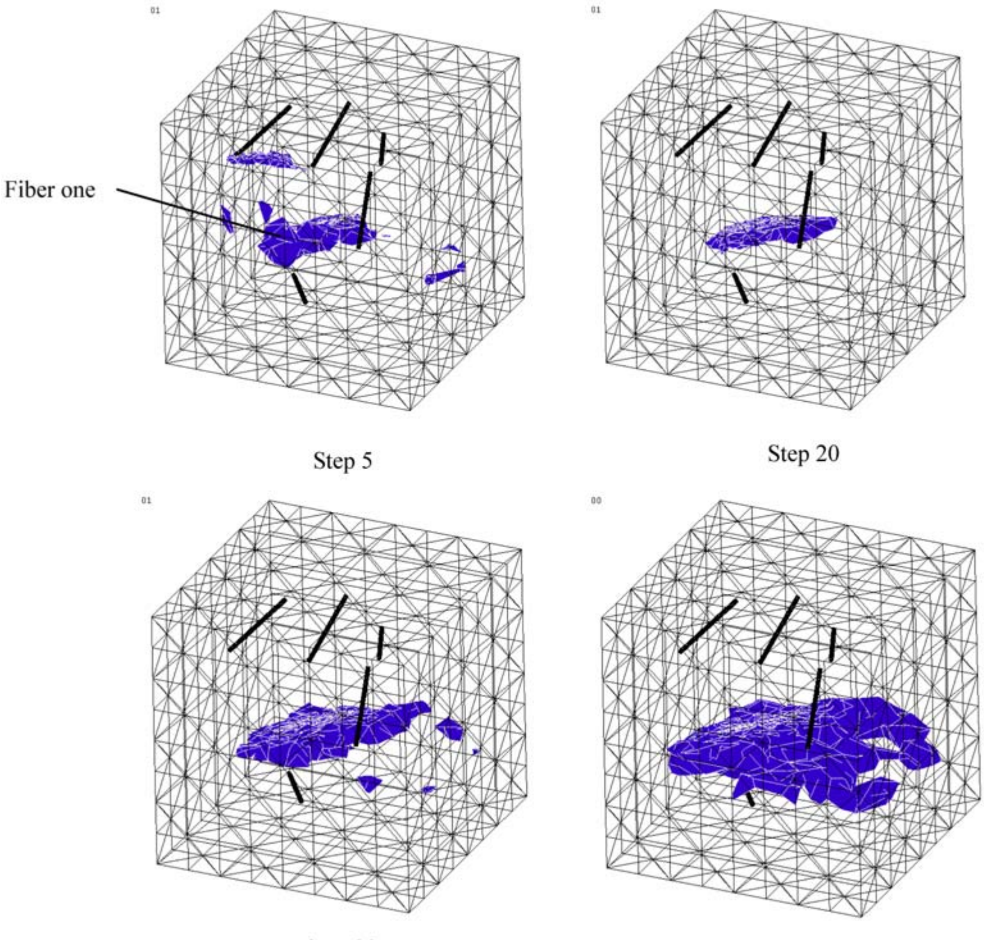
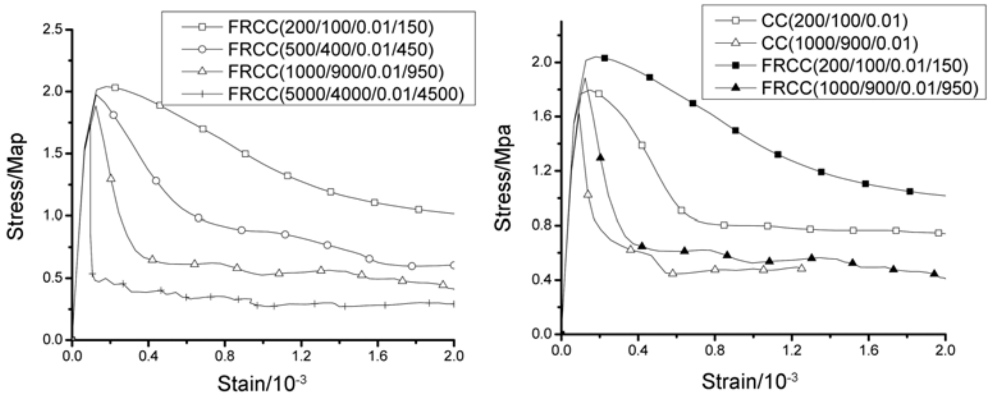


Fig. 9 Cracking strain isosurfaces of fiber-reinforced cement composites



(a) Varied tension softening modulus (b) Compared CC with FRCC

Fig. 10 Tensile stress-strain curve of CC and FRCC

same time. Upon further loading, more extensive cracks develop between fiber and matrix. Finally, the growing cracks cause specimen rupture, and the failure planes are readily identified (Fig. 9).

Varying the softening modulus, several stress-strain curves of FRCC are obtained (Fig. 10(a)). The legend in the brackets refers to the softening modulus of the homogeneous matrix, the softening modulus of the matrix soft region, the shear residual factor, and the softening modulus of the interface layer. The four stress-strain curves are similar to those of CC. In order to validate the fiber effect on the macro-mechanical properties of composites, two stress-strain curves are selected to compare FRCC with CC (Fig. 10(b)). In any case, adding fibers having high strength and high modulus can improve the mechanical properties of cement composites. Of course, if large numbers of fibers are mixed into matrix, the numerical simulation results will be found to be more correct.

Related tensile experiment data can be found in reference (Muralidhar 2004), although some mechanical parameters (including matrix and soft region average failure strength, etc.) are different, the numerical simulation results are very rational.

3.2 Compressive numerical simulations

3.2.1 Compression results of cement composites (CC)

Under external compressive load, for internal heterogeneity of CC, the stress state of each point in the material becomes more complex. Tension and shear of the internal structure may occur. Double failure criteria can be used to depict CC's cracking. When tensile stress in a meso-element occurs, the cracking strength criterion is adopted. If shear stress in the meso-element occurs, the von Mises criterion is adopted. On the meso-scale, the von Mises criterion is accurate enough compared with the Mohr-Coulomb criterion or the Drucker-Prager criterion. The cracking strength criterion is used as the principal failure criterion. The von Mises criterion assumes that failure (yielding) occurs when the octahedral shear stress reaches its critical value. Mathematically, the failure criterion can be expressed in the following form

$$f(J_2) = J_2 - k^2 = 0 \tag{2}$$

where J_2 is the second invariant of the stress deviator tensor, k is the failure (yielding) stress in pure shear. Now the yielding stresses for the matrix soft region and homogeneous matrix are separately

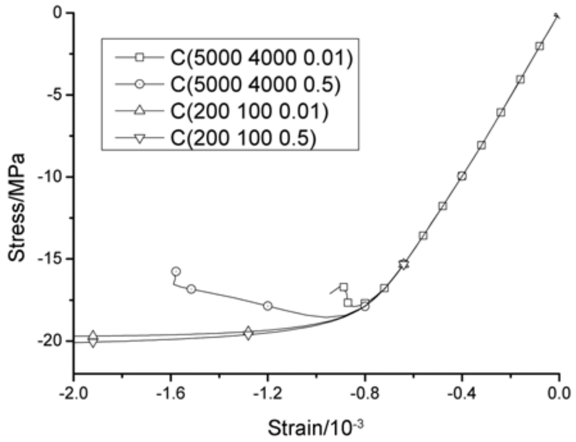


Fig. 11 compressive stress-strain curve of CC

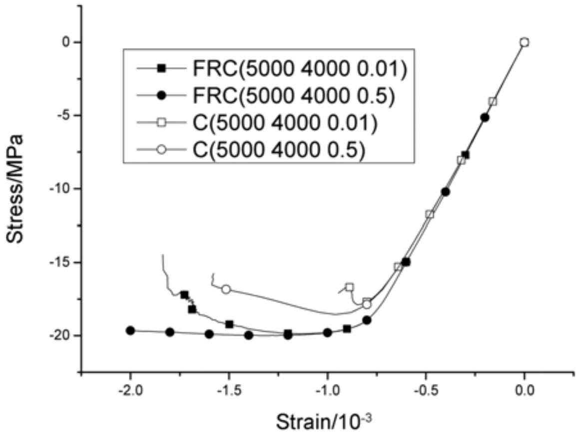


Fig. 12 compressive stress-strain curve of CC and FRCC

taken as 4 MPa and 24 MPa, and the other parameters refer to previous data. With these data, a compression test can be simulated. In step 62, computation is broken off because of rupture of CC and the material element loses its strength completely.

Several representative curves for different softening moduli and shear residual factors are drawn in Fig. 11. The legend in the brackets refers to the softening modulus of homogeneous matrix, softening modulus of matrix soft region and their shear residual factor. It is easy to see that CC has more distinct brittleness for a high softening modulus and low shear residual factor. In fact, when a lower softening modulus is selected, no failure phenomenon is observed.

Compared with the tensile stress-strain curves of Fig. 8(a), under compressive load CC's toughness is more obvious. So when analyzing the mechanical behavior of CC, selecting a lower softening modulus is improper and incorrect.

3.2.2 Compression results of fiber-reinforced cement composites (FRCC)

In the previous analysis, a lower softening modulus may cause irrational results, thus all numerical simulations for FRCC use a higher softening modulus. At the same time, two shear residual factor values are used to compare its effect on the mechanical properties of FRCC. From the representative cracking strain isosurfaces, when the external load is small, cracking only occurs in the matrix soft region and interface layer between fiber and matrix. With increasing load, those cracks begin to converge. Finally, the specimen loses its strength completely.

Actually, adding fibers has few effects on the strength of composites and a little effect on the toughness of composites (Fig. 12). However, when the shear residual factor value takes 0.5, the results are not credible. Therefore, when analyzing compression problems, a higher softening modulus and lower shear residual factor value may lead to more reasonable results. Of course, all numerical tests must agree with the corresponding experiments.

4. Conclusions

A concept called the soft region is introduced to simulate crack propagation in CC or FRCC.

Special attention is paid to the application of the Monte Carlo sampling principle. An important conclusion is that, in contrast to the nonlinear constitutive model, a simple elastic brittle constitutive equation of the meso-elements can depict the macro-mechanical behaviors of the composites. When the heterogeneity of each phase component is considered, the simulation becomes more credible. Additional studies show that the MATLAB program can be used to produce the three-dimensional graphic files. The files have been exported into the AUTOCAD program in the MATLAB handle function. In terms of MSC.PATRAN, a three-dimensional finite element model of fibers randomly distributed within cement composites has been implemented.

Under an external load the stress-strain curve descending branch is affected by two parameters (softening modulus and shear residual factor). Tensile and compressive numerical tests are analyzed with different corresponding parameters. It is clear that the composites display more brittleness in tension, and a lower softening modulus is unfit for the compressive mode. At the same time, the numerical simulations of FRCC show that high strength and high modulus fibers can increase the composites' tensile strength and improve their toughness.

To reduce the computational effort and complexity, the number of fibers or fiber volume in FRCC used in this study was small. As more fibers are added, the same numerical method can be adopted.

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References

Birgoren, B. and Dirikolu, M.H. (2004), "A computer simulation for estimating lower bound fracture strength of composites using Weibull distribution", *Compos. Part B*, **35**, 263-266.

Cheng, T.L., Qiao, Rui and Xia, Y.M. (2004), "A monte carlo simulation of damage and failure process with crack saturation for unidirectional fiber reinforced ceramic composites", *Comp. Sci. Technol*., **64**, 2251-2260.

Tang, C. and Zhu, W. (2003), *The numerical test of concrete in failure and damage analysis[M]*, Beijing, Science Press (in Chinese).

Huang, J. and Jiang, H.D. (2007), "Numerical analysis of short fiber reinforced cement-based composites in direct tension", *Yanshilixue Yu Gongcheng Xuebao*., **26**(9), 1913-1922.

Leite, J.P.B., Slowik, V. and Apel, J. (2007), "Computational model of mesoscopic structure of concrete for simulation of fracture processes", *Comput. Struct.*, **85**, 1293-1303.

Man, H.K. and Van Mier, J.G.M. (2008), "Influence of particle density on 3D size effects in the fracture of (numerical) concrete", *Mech. Mater*., **40**, 470-486.

Most, T. and Bucher, C. (2007), "Energy-based simulation of concrete cracking using an improved mixed-mode cohesive crack model within a meshless discretization", *Int. J. Numer. Anal. Met.*, **31**, 285-305.

Muralidhar, K. (2004), *Monte Carlo simulation*, Encyclopedia of Information Systems, Pages 193-201.

Okabe, T., Sekine, H., Ishii, K., Nishikawa, M. and Takeda, N. (2005), "Numerical method for failure simulation of unidirectional fiber-reinforced composites with spring element model", *Compos. Sci. Technol*., **65**, 921-933.

Oliver, J., Huespe, A.E., Samaniego, E. and Chaves, W.V. (2004), "Continuum approach to the numerical simulation of material failure in concrete", *Int. J. Numer. Anal. Met*., **28**, 609-632.

Patran, M.S.C. (2001), User's manual version 2001, MSC.Software Corporation.

Remmers, J.J.G., Borst, R.D. and Needleman, A. (2008), "The simulation of dynamic crack propagation using

the cohesive segments method", *J. Mech. Phys. Solids.*, **56**, 70-92.
Ruiz, G., Ortiz, M. and Pandolfi, A. (2000), "Three-dimensional finite-element simulation of the dynamic Brazilian tests on concrete cylinders", *Int. J. Numer. Meth. Eng.*, **48**, 963-994.
Sadowski, T. and Golewski, G. (2008), "Effect of aggregate kind and graining on modelling of plain concrete under compression", *Comp. Mater. Sci.*, **43**, 119-126.
Sancho, J.M., Planas, J., Fathy, A.M., Galvez, J.C. and Cendo, D.A. (2007a), "Three-dimensional simulation of concrete fracture using embedded crack elements without enforcing crack path continuity", *Int. J. Numer. Anal. Met.*, **31**, 173-187.
Sancho, J.M., Planas, J., Fathy, A.M., Gendon, D.A., Reyes, E. and Galvez, G.C. (2007b), "An embedded crack model for finite element analysis of concrete fracture", *Eng. Fract. Mech.*, **74**, 75-86.
Sivakumar Babu, G.L., Vasudevan, A.K. and Haldar, S. (2008), "Numerical simulation of fiber-reinforced sand behavior", *Geotext. Geomembranes*, **26**, 181-188.
Sun, Z.H., Garboczi, E.J. and Shah, S.P. (2007), "Modeling the elastic properties of concrete composites: experiment, differential effective medium theory and numerical simulation", *Cement Concrete Comp.*, **29**, 22-38.
Uzunoglu, M., Kizil, A. and Onar, O.C. (2003), Proficiency in MATLAB, 2nd ed., Turkmen Kitabevi, Istanbul. (in Turkish)
Zairi, F., Nait Abdelaziz, M., Gloaguen, J.M., Bouaziz, A. and Lefebvre, J.M. (2008), "Micromechanical modeling and simulation of chopped random fiber reinforced polymer composites with progressive debonding damage", *Int. J. Solids Struct.*, **45**, 5220-5236.

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Notations

$f(\sigma)$ = probability density function
$F(\sigma)$ = probability distribution function
σ = the failure strength
λ = the reciprocal of the scale parameter
α = the shape parameter
u = the pseudo-random number obeyed the uniformity distribution
ε = the integrate point strain
ε^{co} = the material strain consisting of elastic strain and plastic strain
ε^{cr} = the cracking strain
S = the stress in principal direction
e = the strain in principal direction
R = the converted matrix
S^{cr} = the stress threshold value
e^{cr} = the strain threshold value
θ = the direction angle
ΔS_i = the stress in cracking direction at the ith direction
Δe_i^{cr} = the cracking strain at the ith direction
E_{soft} = the soft modulus
$f(J_2)$ = the failure criteria
J_2 = the second invariants of the stress deviator tensor
k = the failure (yielding) stress in pure shear