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Technical Note

Mapping method for elliptical shape loadings on finite element mesh for airport pavement anaysis

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1. Introduction

Structural analysis of concrete pavement in airfield application under heavy wheel loading is taking greater importance as the airplane gear loadings become heavier and more complicated, hence the engineers requires more specific information about the behavior of the pavement system for their pavement design. Since the earliest effort for the structural analysis of concrete pavement, engineers now can achieve much more detailed investigation of the structural behavior of pavement systems using full three-dimensional finite element analysis (FEA) technology (Patil *et al*. 2010). In the language of FEA for airport pavement system, an ellipse is a meaningful shape since it accurately represents the physical tire footprints of airplane landing gears. When an elliptical shape loading is applied to a finite element mesh (Fig. 1), the loading region covers the element mesh in two different manners: (1) fully loaded elements (2) partially loaded elements. For the fully loaded elements, the nodal force vector can be readily calculated by the traditional way to compute the work equivalent nodal force vector (Cook *et al.* 2002). However, for the partially loaded element, a special treatment should be considered for a precise evaluation of the nodal force vectors.

Hjelmstad *et al.* (Hjelmstad, Zuo and Kim 1997) utilized the two-dimensional midpoint rule for the numerical integration. The partially loaded element was transformed into $\xi\eta$-coordinate system, and the loaded surface of the element was subdivided into square segments as shown in Fig. 2. For each segment, the integrand was evaluated at the center location of the segment and multiplied by the area of the square segment. The results were summed up to the equivalent nodal force vector. This study presents here an alternative way to evaluate the integrals by means of a mapping method, which can effectively evaluate the integral near the curved domain.

2. Numerical integration – mapping method

The special treatment for partially loaded element is not from the definition of the work

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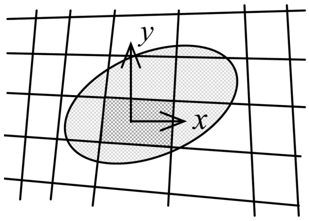

Fig. 1 Fully and partially loaded elements due to elliptical shape loading

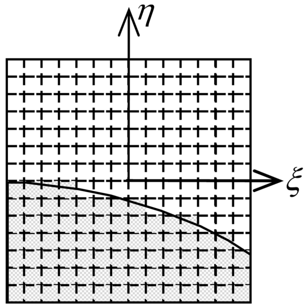

Fig. 2 Midpoint rule for the partially loaded element (Hjelmstad, Zuo and Kim 1997)

equivalent nodal force but from the way how to evaluate the integral. Hence the precision of the integral depends only on the task, "How precisely to approximate the curved shape of ellipse." The key idea of the mapping method is as simple as shown in the schematic in Fig. 3. The partially loaded domain Ω in the original xy-coordinate system (Fig. 3c) is mapped into the domain Γ in $\xi\eta$-coordinate system. However, if we properly transform the partially loaded region Γ in $\xi\eta$-coordinates into a regulated (square or triangular) region $\bar{\Gamma}$ (Fig. 3a) in a new coordinate $\bar{\xi}\bar{\eta}$-system, we can utilize the "efficient" Gaussian quadrature rule.

The only attacking point at this stage is to find out the mapping function which transforms the curved region Γ into a regulated region $\bar{\Gamma}$. To find out the exact mapping function for this transformation is not a trivial problem since the curved segment of the domain Γ is part of ellipse transformed from xy-coordinate system into $\xi\eta$-coordinate system. Hence, instead of finding the exact mapping function, approximated polynomial based mapping functions were considered for this study.

Actually, using a polynomial based mapping function is the same method used for meshing geometry in iso-parametric finite element formulation. High order shape functions may be preferred for an accurate approximation therefore quadrilateral shape functions were employed for this study.

3. Implementation

The mapping method was implemented only for 8-node cubic element. We present here how to choose the nodal points that define the approximated loading region in $\xi\eta$-coordinate system. Fig. 4(a) shows an element and a partially loaded domain (shaded region with a polynomial shape) in $\xi\eta$-coordinate system (Lee 2007).

Fig. 4 shows a step-by-step procedure for choosing the nodal coordinates that defines sub-domains

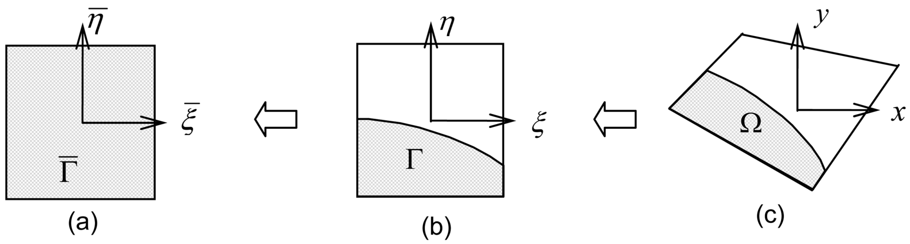

Fig. 3 Partially loaded region on element in different coordinate systems

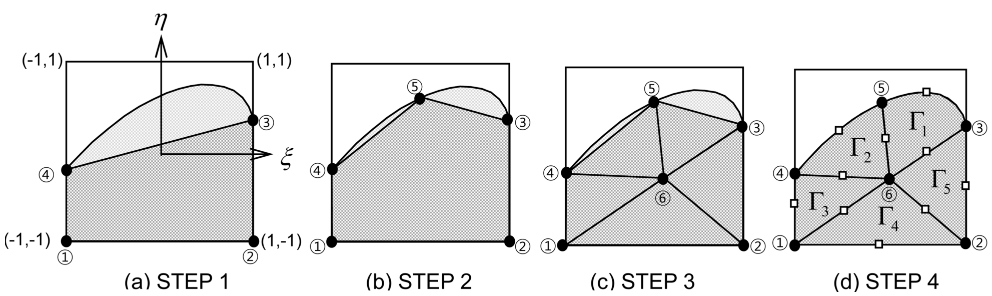

Fig. 4 Choosing nodal coordinates defining approximated domain

approximating the original loaded domain, Γ. The detailed procedure is given below.

STEP1: The first step is to extract the polygon ①-②-③-④, i.e. identifying the coordinates of the nodes, ①, ②, ③ and ④. STEP 2: The polygon ①-②-③-④, obtained in STEP 1, may be used as an approximated domain for the partially loaded domain, Γ. However, as seen in the figure, a significant error can be formed. In order to increase the accuracy of the geometric fitting, we added a node on the curved segment, ③ ~ ④ , into the polygon. Now the polygon ① - ② - ③ - ⑤ - ④ represents the approximated domain for the partially loaded domain, Γ. The more nodes for the curved segment, the better accuracy of the geometric fitting can be achieved. STEP 3: The next step is to find the centroid (node ⑥) of the polygon and divide the polygon into triangular sub-region. The polygon we have is a convex one and the formula for the centroid can be found in other place (Bourke 2009). STEP 4: The final step is to generate curved triangular domain using the sub-triangles obtained from STEP 3. In Fig. 4(d), the curved triangular domains Γ_1 and Γ_2 has a curved region, hence we need to the same procedure shown in STEP 2 in order to determine the edge node of the curved triangular region. On the other hand, the triangular domains, Γ_3, Γ_4 and Γ_5, have only straight edges hence the mid-points of the edges are enough for the edge nodes.

4. Performance test and conclusions

For a performance test, an ellipse was placed at the center of a unit size square as shown in Fig. 5. The ellipse has the semi-axes with a length of 1.0 and 0.5, and the rotational angle of θ. The rotational angle was changed from 0 to 45° with 11.25° step increase (5 different rotational angles). For each angle case, finite element meshes were generated so that the number of grid in x- and y- axis becomes 4 to 48 with various step increases (9 different grid sizes).

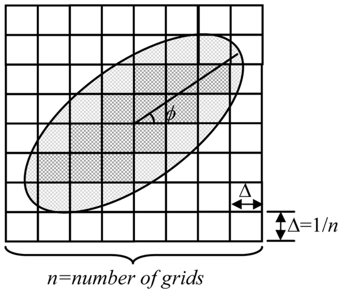

Fig. 5 Configuration for Performance Test

The computational time and error for each case were evaluated. The fully loaded elements (darkly shaded elements) were integrated using a traditional way. The partially loaded elements (lightly shaded elements) were evaluated by using both the midpoint rule and the mapping method for comparison.

For the midpoint rule, each partially loaded element was subdivided into 100 by 100 square regions. For each square region, the integrand was evaluated at the center location of the square and multiplied by the area of the square. The results were summed up to the equivalent nodal force vector. As reported in the work by Hjelmstad *et al.*, the

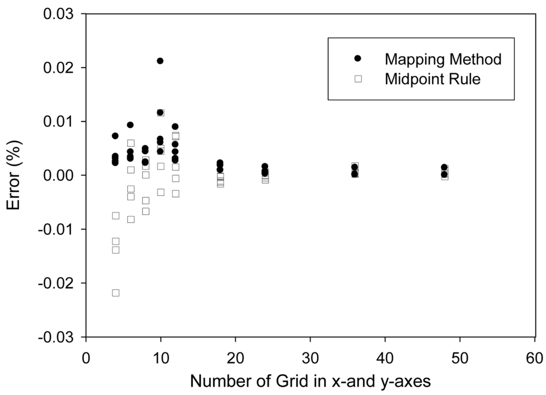

Fig. 6. Error Estimation

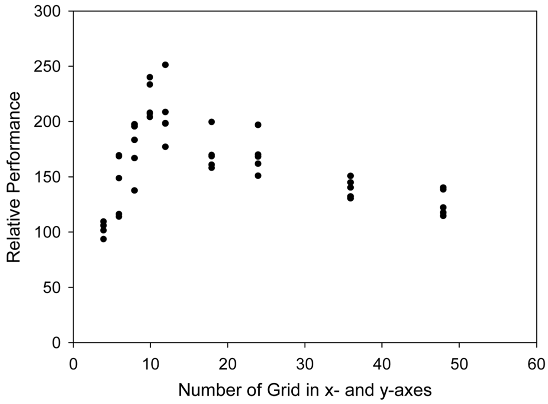

Fig. 7. Relative Performances

error yielded less than 0.1% in terms of total applied load.

In case of the mapping method, the accuracy is dependent on the errors arise near curved segments. In order to increase the accuracy, one may divide the sub-domain containing a curved segment into more numbers of sub-domains. For a reasonable accuracy and computational efficiency, the curved segment was divided so that the total error became less than 0.05% in terms total applied load.

Fig. 6 shows the errors arisen from the two different methods. The maximum error of about 0.1% for both methods arose when the number of grid are small. As the number of grid increased, the errors are getting decreased. Note that the data points in the figure hold the cases of 5 different rotational angles and various grid sizes.

Fig. 7 shows the relative performance, which was defined as the computational time of the midpoint rule divided by that of the mapping method. The mapping method is about 100 times faster than the midpoint rule, and shows a peak value of about 250 with the number of grid of 12. This better performance is pretty much expectable since the mapping method required much smaller number of calculations of the integrand although the integrand has a bit more complicated form than that of the midpoint.

Through the performance test, we successfully demonstrated that the mapping method shows, for a given degree of precision, a better performance compared to the midpoint rule in terms of computational time. The computational time of the mapping method was about 100 times faster than that of the midpoint rule. However, it is restricted for 8-node cubical element at this point. The method may be generalized for other types of element in future including higher-order elements.

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