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Deflection prediction for reinforced concrete deep beams

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Abstract A simplified method, developed from the softened strut-and-tie model, for determining the mid-span deflection of deep beams at ultimate state is proposed. The mid-span deflection and shear strength predictions of the proposed model are compared with the experimental data collected from 70 simply supported reinforced concrete deep beams, loaded with concentrated loads located at a distance a from an end reaction. The comparison shows that the proposed model can accurately predict the mid-span deflection and shear strength of deep beams with different shear span-to-depth ratios, different concrete strengths, and different horizontal and vertical hoops.

Keywords: deep beams; reinforced concrete; deflection; shear strength.

1. Introduction

Because of their geometric proportions, the strength of reinforced concrete deep beams is usually controlled by shear, rather than by flexure if normal amount of longitudinal reinforcement is used. The shear action in the beam web leads to diagonal compression and tension in a direction perpendicular thereto. The deep beams do not fail immediately due to the formation of diagonal cracks. After diagonal cracking, the concrete between the diagonal cracks can serve as a concrete compression strut. The external shear is assumed to be transferred by the concrete compression strut. By detailing the end anchorage of longitudinal bars and bearing zones of deep beams, premature failures such as shear tension failure (due to insufficient anchorage of reinforcing bars) and bearing failure can be effectively avoided. The usual failure mode of deep beams is crushing of the concrete strut as shown in Fig. 1.

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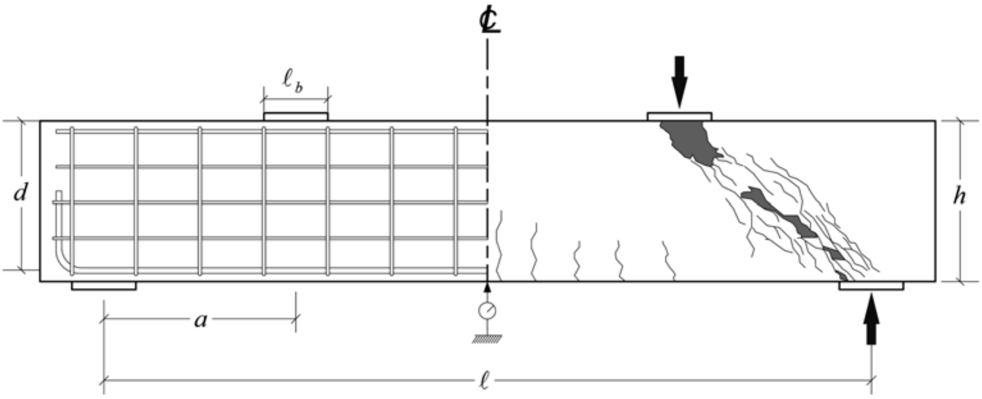


Fig. 1 Typical failure of deep beams

The shear strength of reinforced concrete deep beams had been accurately predicted by Tang and Tan (2004); Russo *et al.* (2005); and Hwang *et al.* (2000), but there are very few, if any, theoretical models for predicting the deflection of deep beams. The model proposed by Hwang *et al.* (2000), termed as the softened strut-and-tie (SST) model, is developed from the strut-and-tie concept and is derived to satisfy equilibrium, compatibility, and constitutive law of cracked reinforced concrete. The strength analysis of the extent of softening involves five unknowns. With the five equations given by the strain compatibility and the constitutive lows of concrete and steel, the solution to these unknowns can be obtained through iteration procedures (Hwang and Lee 2002). The solution procedures of the SST model (Hwang *et al*. 2000) are tedious; hence, estimation of the softening effect has been further simplified (Hwang and Lee 2002). The load-deflection responses of squat walls have also been computed using the SST model (Tu 2005, Bali and Hwang 2007). While the lateral deflection of squat walls is due to flexure, shear and slip, the deflection of deep beams is due to flexure and shear. Deflection prediction for deep beams is performed in this study.

This paper proposes an analytical model for determining the mid-span deflection of deep beams at ultimate state. The mid-span deflection at ultimate state in this study is defined as the mid-span deflection at peak loads. According to the available experimental data, the applicability of the proposed model to deep beams for predicting the mid-span deflection at ultimate state and shear strength is examined.

2. Research significance

This paper proposes an analytical model for predicting the mid-span deflection of deep beams. A total of 70 deep beams of various parameters were used in this study. The proposed model can adequately predict the mid-span deflection at ultimate state and shear strength of deep beams of different compressive strengths of concrete, shear span-to-depth ratios, as well as horizontal and vertical hoops.

3. Softened strut-and-tie model

Fig. 2 shows the loads acting on a deep beam and the force transferring mechanisms in view of

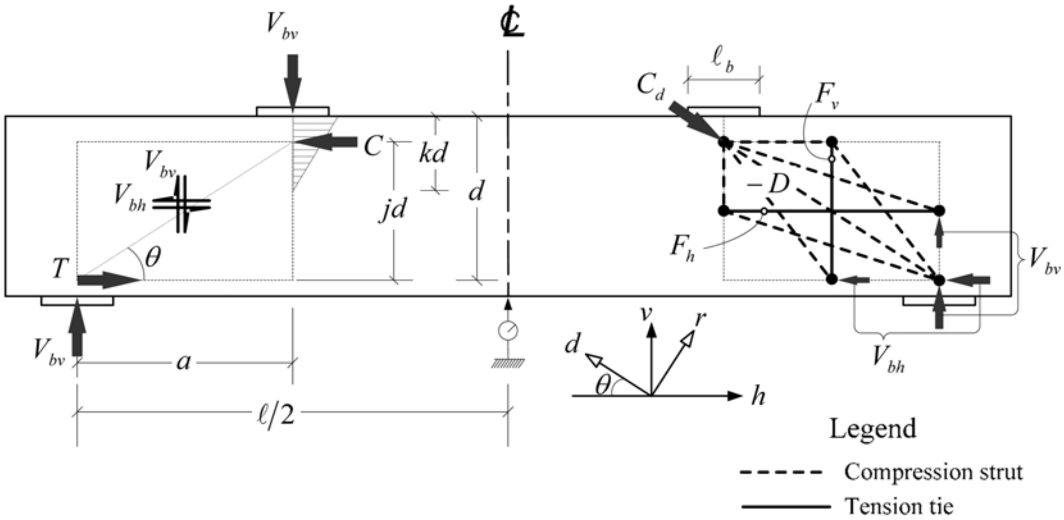


Fig. 2 SST model for internal forces

the proposed SST model. By considering the distances between force couples (Fig. 2), it will be sufficiently accurate to express the following relationship between vertical and horizontal shears.

$$\frac{V_{bv}}{V_{bh}} \approx \frac{jd}{a} \tag{1}$$

where V_{bv} is the vertical shear force, V_{bh} is the horizontal shear force, jd is the length of the lever arm from the resultant compressive force to the centroid of the flexural reinforcement, and a is the shear span measured center-to-center from load to support. According to the linear bending theory, the lever arm jd can be estimated as

$$jd = d - kd/3 \tag{2}$$

where d is the effective depth of the deep beam, kd is the depth of compression zone at the section and coefficient k can be derived as

$$k = \sqrt{[n\rho + (n-1)\rho']^2 + 2[n\rho + (n-1)\rho' d'/d]} - [n\rho + (n-1)\rho'] \tag{3}$$

where n is the modular ratio of elasticity, ρ is the ratio of tension reinforcement, ρ' is the ratio of compression reinforcement and d' is the distance from the extreme compression fiber to the centroid of the compression reinforcement.

Fig. 2 shows the proposed SST model, which comprises the diagonal, horizontal and vertical mechanisms (Hwang *et al.* 2000, Hwang and Lee 2002). The diagonal mechanism is a diagonal compression strut whose angle of inclination θ is taken as (Hwang *et al.* 2000)

$$\theta = \tan^{-1}\left(\frac{jd}{a}\right) \tag{4}$$

The effective area of the diagonal strut (A_{str}) can be estimated as

$$A_{str} = t_s \times b_s \tag{5}$$

where t_s is the thickness of the diagonal strut and b_s is the width of the diagonal strut which can be taken as the width of the beam web.

The thickness of the diagonal strut (t_s) depends on its end condition, which is provided by the compression zone at the section and the bearing plate (Hwang *et al.* 2000). It is intuitively assumed that

$$t_s = \sqrt{(kd)^2 + l_b^2} \tag{6}$$

where l_b is the width of the bearing plate, measured parallel to the axis of the beams.

The horizontal mechanism consists of one horizontal tie and two flat struts (Hwang *et al.* 2000, Hwang and Lee 2002). The horizontal tie is made up of horizontal hoops. When computing the area of the horizontal tie (A_{th}), it is roughly assumed that the horizontal hoops within the center half of the height are fully effective, and the rest at 50% effectiveness (Hwang *et al.* 2000, Hwang and Lee 2002). If the horizontal hoops are uniformly distributed within the length of the lever arm, then $A_{th} = 0.75A_h$, where A_h is the area of horizontal hoops. The vertical mechanism consists of one vertical tie and two steep struts (Hwang *et al.* 2000, Hwang and Lee 2002). The vertical tie is made up of vertical hoops. The area of the vertical tie (A_{tv}) is computed in the same way as that of the horizontal tie. If the vertical hoops are uniformly distributed within the shear span, then $A_{tv} = 0.75A_v$; in which, A_v is the area of the vertical hoops within the shear span.

3.1. Evaluation of shear strength

According to Hwang and Lee (2002), the shear strength of deep beams can be estimated as follows:

$$V_{bv,calc} = (K_h + K_v - 1)\zeta f_c' A_{str} \sin\theta \tag{7}$$

where $V_{bv,calc}$ is the predicted shear strength, K_h is the horizontal tie index (Hwang and Lee 2002), K_v is the vertical tie index (Hwang and Lee 2002), f_c' is the compressive strength of concrete and ζ is the softening coefficient of concrete.

The horizontal tie index can be estimated as follows (Hwang and Lee 2002):

$$K_h = 1 + (\overline{K}_h - 1)\frac{A_{th} f_{yh}}{\overline{F}_h} \le \overline{K}_h \tag{8}$$

where

$$\overline{K}_h \approx \frac{1}{1 - 0.2(\gamma_h + \gamma_h^2)} \tag{9}$$

$$\gamma_h = \frac{2\tan\theta - 1}{3}, \text{ but } 0 \le \gamma_h \le 1 \tag{10}$$

$$\overline{F}_h = \gamma_h \times (\overline{K}_h \zeta f_c' A_{str}) \times \cos\theta \tag{11}$$

$$\zeta = \frac{40}{\sqrt{f_c'}} \le 0.52 \text{ ...(psi)} \tag{12}$$

$$\zeta = \frac{3.35}{\sqrt{f_c'}} \le 0.52 \text{ ...(MPa)}$$

here $\overline{K}_h$ is the horizontal tie index with sufficient horizontal hoops, f_{yh} is the yield stress of horizontal hoops, γ_h is the fraction of horizontal shear transferred by the horizontal tie in the absence of the vertical tie and $\overline{F}_h$ is the balance amount of horizontal tie force.

The vertical tie index can be estimated as follows (Hwang and Lee 2002):

$$K_v = 1+(\overline{K}_v-1)\frac{A_{tv}f_{yv}}{\overline{F}_v} \leq \overline{K}_v \tag{13}$$

where

$$\overline{K}_v \approx \frac{1}{1-0.2(\gamma_v+\gamma_v^2)} \tag{14}$$

$$\gamma_v = \frac{2\cot\theta-1}{3}, \text{ but } 0 \leq \gamma_v \leq 1 \tag{15}$$

$$\overline{F}_v = \gamma_v \times (\overline{K}_v \zeta f_c' A_{str}) \times \sin\theta \tag{16}$$

where $\overline{K}_v$ is the vertical tie index with sufficient vertical hoops, f_{yv} is the yield stress of vertical hoops, γ_v is the fraction of vertical shear transferred by the vertical tie in the absence of the horizontal tie and $\overline{F}_v$ is the balance amount of vertical tie force.

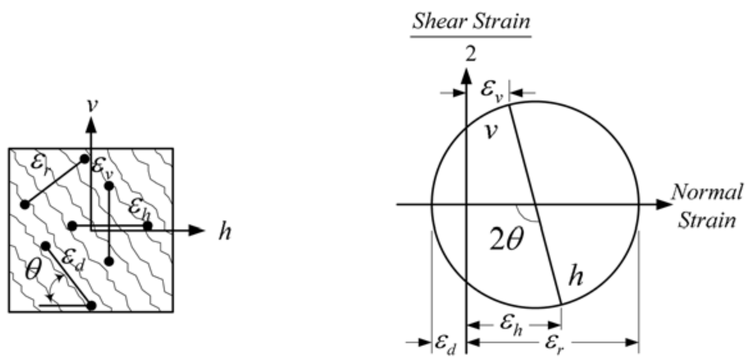
3.2. Evaluation of mid-span deflection at ultimate state

The deflection of simply supported deep beams at ultimate state is the sum of both shear and flexural deflections, and that is

$$\Delta = \Delta_s + \Delta_f \tag{17}$$

where Δ is the vertical mid-span deflection of deep beams at ultimate state, Δ_s is the vertical mid-span deflection of deep beams at ultimate due to shear, and Δ_f is the vertical mid-span deflection of deep beam at ultimate state due to flexure.

Assuming that each shear span of the deep beam is subjected to uniform shear strain, the mid-span deflection of deep beam at ultimate state due to shear can be estimated as



(a) Stains averaged over an area containing many cracks (b) Mohr's circle of average strains

Fig. 3 Compatibility conditions for diagonally cracked concrete

$$\Delta_s = \gamma_{hv} a \tag{18}$$

where γ_{hv} is the average shear strains in the shear span of deep beams at ultimate state.

It is assumed that the average shear strains (γ_{hv}) in the shear span of deep beams at ultimate state should satisfy Mohr's circle for strain (Fig. 3), which gives

$$\gamma_{hv} = 2(\varepsilon_r - \varepsilon_d)\sin\theta\cos\theta \tag{19}$$

where ε_r denotes the average normal strains in r-direction (positive for tension), and ε_d denotes the average normal strains in d-direction (positive for tension). The d-direction is the direction of the diagonal concrete strut which is the assumed direction of principal compressive stress of concrete (Fig. 2). The r-direction is the direction perpendicular to d-direction which is the assumed direction of principal tensile stress.

When the average principal stress of concrete reaches the capacity of the softened concrete, the average normal strains in d-directions of the deep beams can be estimated as (Zhang and Hsu 1998)

$$\varepsilon_d = -\zeta\varepsilon_0 \tag{20}$$

where ε_o is the strain at peak stress of standard concrete cylinder.

As shown in Fig. 3, the average normal strains in r-direction can be determined by the following compatibility equation (Hwang *et al.* 2000)

$$\varepsilon_r = \varepsilon_h + \varepsilon_v - \varepsilon_d \tag{21}$$

where ε_h denotes the average normal strains in h-direction (positive for tension) and ε_v denotes the average normal strains in v-direction (positive for tension). The value of ε_h varies with the magnitude of tension force in the horizontal tie. In order not to overestimate the softening effect of concrete, the value of ε_h should be limited by the yielding strain of reinforcement (Hwang *et al.* 2000, Vecchio and Collins 1993). It is estimated as

$$\varepsilon_h = \frac{F_h}{A_{th}E_s} \le \frac{f_{yh}}{E_s} \tag{22}$$

where F_h is the tension force in the horizontal tie (positive for tension), and E_s is the modulus of elasticity of reinforcement. It is noted that the value of ε_h is set to a yielding strain of 0.002 for the deep beams not provided with horizontal hoops.

Similarly, the average normal strains in v-direction can be estimated as

$$\varepsilon_v = \frac{F_v}{A_{tv}E_s} \le \frac{f_{yv}}{E_s} \tag{23}$$

where F_v is the tension force in the vertical tie (positive for tension). The value of ε_v is set to a yielding strain of 0.002 for the deep beams not provided with vertical hoops.

The vertical mid-span deflection of deep beam at ultimate state due to flexure Δ_f can be calculated using the moment area method, which has been developed by Gere and Timoshenko (1997). Although the 70 tested deep beams collected in this study all failed in shear, when employing the elastic moment area method to calculate Δ_f some minor errors might be introduced due to the inelastic behavior of deep beams.

$$\Delta_f = \frac{V_{bv,calc}\,a}{6E_cI}\left(\frac{3}{4}l^2 - a^2\right) \tag{24}$$

$$\theta \quad f'_c \quad f_{yh} \quad f_{yv} \quad A_{str} \quad A_{th} \quad A_{tv}$$

$$\gamma_h = \frac{2\tan\theta - 1}{3} \text{ where } 0 \le \gamma_h \le 1 \text{ and } \gamma_h = 0 \text{ for } A_{th} = 0$$

$$\gamma_v = \frac{2\cot\theta - 1}{3} \text{ where } 0 \le \gamma_v \le 1 \text{ and } \gamma_v = 0 \text{ for } A_{tv} = 0$$

$$\overline{K}_h = \frac{1}{1 - 0.2(\gamma_h + \gamma_h^2)} \quad ; \quad \overline{K}_v = \frac{1}{1 - 0.2(\gamma_v + \gamma_v^2)}$$

$$\zeta = \frac{40}{\sqrt{f'_c}} \le 0.52 \ldots \text{(psi)}; \quad \zeta = \frac{3.35}{\sqrt{f'_c}} \le 0.52 \ldots \text{(MPa)}$$

$$\overline{F}_h = \gamma_h \overline{K}_h \zeta f'_c A_{str} \cos\theta \quad ; \quad \overline{F}_v = \gamma_v \overline{K}_v \zeta f'_c A_{str} \sin\theta$$

$$K_h = 1 + (\overline{K}_h - 1)\frac{A_{th} f_{yh}}{\overline{F}_h} \le \overline{K}_h \,; \, K_v = 1 + (\overline{K}_v - 1)\frac{A_{tv} f_{yv}}{\overline{F}_v} \le \overline{K}_v$$

$$V_{bv,calc} = (K_h + K_v - 1)\zeta f'_c A_{str} \sin\theta$$

$$R_h = \frac{\gamma_h(1 - \gamma_v)}{1 - \gamma_h\gamma_v}; \quad R_v = \frac{\gamma_v(1 - \gamma_h)}{1 - \gamma_h\gamma_v}$$

$$F_h = R_h V_{bv,calc} / \tan\theta; \quad F_v = R_v V_{bv,calc}$$

$$\varepsilon_h = \frac{F_h}{A_{th}E_s} \le \frac{f_{yh}}{E_s}; \quad \varepsilon_v = \frac{F_v}{A_{tv}E_s} \le \frac{f_{yv}}{E_s}$$

$$\varepsilon_h = 0.002 \text{ for } A_{th} = 0; \quad \varepsilon_v = 0.002 \text{ for } A_{tv} = 0$$

$$-\varepsilon_d = \zeta\varepsilon_o$$

$$\varepsilon_r = \varepsilon_h + \varepsilon_v - \varepsilon_d$$

$$\gamma_{hv} = 2(\varepsilon_r - \varepsilon_d)\sin\theta\cos\theta$$

$$\Delta = \gamma_{hv} a + \frac{V_{bv,calc}\, a}{6E_c I}\left(\frac{3}{4}\ell^2 - a^2\right)$$

END

Fig. 4 Flow chart showing solution procedures where l is the span length of the deep beam (Fig. 1), E_c is the modulus of elasticity of concrete and I is the moment of inertia of the section about the centroidal axis.

According to the ACI 318-08 Code (2008), the modulus of elasticity of concrete can be estimated as,

$$E_c = 57000\sqrt{f_c'} \ \text{...(psi)} \tag{25}$$
$$E_c = 4700\sqrt{f_c'} \ \text{...(MPa)}$$

According to the provisions on the magnified moments of the ACI 318-08 Code (2008), the cracked moment of inertia of a beam in a frame can be estimated as,

$$I = 0.35 I_g \tag{26}$$

where I_g is the moment of inertia of the gross concrete section about the centroidal axis, with reinforcement neglected.

However, the term "0.35" in Eq. (26) includes a stability reduction factor of 0.875 (ACI 318-08 Code 2008). For an isolated deep beam, the reduction for stability is not needed when calculating the cracked moment of inertia. The number of 0.35 divided by 0.875 is equal to 0.4. Thus the cracked moment of inertia for an isolated deep beam can be estimated as,

$$I = 0.4 I_g \tag{27}$$

The solution algorithm for shear strength and mid-span deflection of deep beam at ultimate state is summarized in Fig. 4.

4. Experimental verification

A total of 70 test specimens of simply supported deep beams and their results were employed to verify the proposed model. These are the test results presented by Smith and Vantsiotis (1982); Tan *et al.* (1995), Tan *et al.* (1997a), Tan *et al.* (1997b); and Aguilar *et al.* (2002) that are listed in Table 1 in chronological order for easy reference. In selecting these data, the test specimens satisfying the following conditions were considered:

1. Failure due to crushing of strut, shear-compression, not shear-tension, bearing, flexure, or diagonal splitting.
2. Mid-span deflection present.
3. Simply supported.
4. Usage of bearing plates.
5. Shear span-to-depth ratio less than 2.5.

Accuracy for the proposed model on deflection prediction is evaluated in terms of deflection ratio, which is defined as the ratio of the measured mid-span deflection at ultimate state (Δ_{test}) to the predicted mid-span deflection at ultimate state (Δ_{calc}). Accuracy for the proposed model on shear strength prediction is evaluated in terms of strength ratio, which is defined as the ratio of the measured shear strength ($V_{bv,test}$) to the predicted shear strength ($V_{bv,calc}$). In this study, the proposed model reproduced the 70 test results with reasonable accuracy (Table 1). The mean of the measured-to-predicted mid-span deflection ratio is 1.24 with a coefficient of variation of 0.16, the mean of the measured-to-predicted shear strength ratio is 1.19 with a coefficient of variation of 0.10 (Table 1). The test-to-theory comparisons in this paper use parametric studies to further assess the applicability

Table 1 Experimental verification

Author	Specimen	b		a/d	d		f_c'		ρ (%)	ρ' (%)	d'		ρ_h (%)	ρ_v (%)	$\rho_h f_{yh}$		$\rho_v f_{yv}$		$V_{bv,test}$		Δ_{test}		$\frac{\Delta_{test}}{\Delta_{calc}}$	$\frac{V_{bv,test}}{V_{bv,calc}}$
		in.	mm		in.	mm	psi	MPa			in.	mm			psi	MPa	psi	MPa	kips	kN	in.	mm		
Smith and Vantsiotis 1982	0A0-44	4	101.6	1.00	12	304.8	2973	20.5	1.94	0.10	1	25.4	0	0	0	0	0	0	31.4	139.5	0.115	2.9	1.13	1.17
	0A0-48	4	101.6	1.00	12	304.8	3031	20.9	1.94	0.10	1	25.4	0	0	0	0	0	0	30.6	136.1	0.113	2.9	1.11	1.12
	0B0-49	4	101.6	1.21	12	304.8	3147	21.7	1.94	0.10	1	25.4	0	0	0	0	0	0	33.5	149.0	0.173	4.4	1.38	1.34
	0C0-50	4	101.6	1.50	12	304.8	3002	20.7	1.94	0.10	1	25.4	0	0	0	0	0	0	26.0	115.7	0.207	5.3	1.33	1.27
	0D0-47	4	101.6	2.08	12	304.8	2828	19.5	1.94	0.10	1	25.4	0	0	0	0	0	0	16.5	73.4	0.225	5.7	1.06	1.11
	1A1-10	4	101.6	1.00	12	304.8	2712	18.7	1.94	0.10	1	25.4	0.23	0.28	153	1.06	187	1.29	36.2	161.2	0.154	3.9	1.39	1.33
	1A3-11	4	101.6	1.00	12	304.8	2611	18.0	1.94	0.10	1	25.4	0.45	0.28	300	2.07	187	1.29	33.3	148.3	0.132	3.4	1.29	1.24
	1A4-12	4	101.6	1.00	12	304.8	2335	16.1	1.94	0.10	1	25.4	0.68	0.28	454	3.13	187	1.29	31.7	141.2	0.133	3.4	1.42	1.29
	1A4-51	4	101.6	1.00	12	304.8	2988	20.6	1.94	0.10	1	25.4	0.68	0.28	454	3.13	187	1.29	38.4	170.9	0.140	3.6	1.40	1.27
	1A6-37	4	101.6	1.00	12	304.8	3060	21.1	1.94	0.10	1	25.4	0.91	0.28	607	4.19	187	1.29	41.4	184.1	0.140	3.6	1.44	1.33
	2A1-38	4	101.6	1.00	12	304.8	3147	21.7	1.94	0.10	1	25.4	0.23	0.63	153	1.06	420	2.90	39.2	174.5	0.145	3.7	1.29	1.28
	2A3-39	4	101.6	1.00	12	304.8	2872	19.8	1.94	0.10	1	25.4	0.45	0.63	300	2.07	420	2.90	38.4	170.6	0.123	3.1	1.16	1.23
	2A4-40	4	101.6	1.00	12	304.8	2944	20.3	1.94	0.10	1	25.4	0.68	0.63	454	3.13	420	2.90	38.6	171.9	0.119	3.0	1.18	1.21
	2A6-41	4	101.6	1.00	12	304.8	2770	19.1	1.94	0.10	1	25.4	0.91	0.63	607	4.19	420	2.90	36.4	161.9	0.121	3.1	1.28	1.20
	3A1-42	4	101.6	1.00	12	304.8	2669	18.4	1.94	0.10	1	25.4	0.23	1.25	153	1.06	834	5.75	36.2	161.0	0.125	3.2	1.16	1.27
	3A3-43	4	101.6	1.00	12	304.8	2785	19.2	1.94	0.10	1	25.4	0.45	1.25	300	2.07	834	5.75	38.8	172.7	0.122	3.1	1.16	1.28
	3A4-45	4	101.6	1.00	12	304.8	3017	20.8	1.94	0.10	1	25.4	0.68	1.25	454	3.13	834	5.75	40.1	178.5	0.137	3.5	1.35	1.23
	3A6-46	4	101.6	1.00	12	304.8	2886	19.9	1.94	0.10	1	25.4	0.91	1.25	607	4.19	834	5.75	37.8	168.1	0.133	3.4	1.39	1.20
	1B1-01	4	101.6	1.21	12	304.8	3205	22.1	1.94	0.10	1	25.4	0.23	0.24	153	1.06	160	1.10	33.2	147.5	0.139	3.5	1.07	1.18
	1B3-29	4	101.6	1.21	12	304.8	2915	20.1	1.94	0.10	1	25.4	0.45	0.24	300	2.07	160	1.10	32.3	143.6	0.148	3.8	1.27	1.23
	1B4-30	4	101.6	1.21	12	304.8	3017	20.8	1.94	0.10	1	25.4	0.68	0.24	454	3.13	160	1.10	31.5	140.3	0.143	3.6	1.26	1.17
	1B6-31	4	101.6	1.21	12	304.8	2828	19.5	1.94	0.10	1	25.4	0.91	0.24	607	4.19	160	1.10	34.5	153.3	0.142	3.6	1.28	1.35
	2B1-05	4	101.6	1.21	12	304.8	2785	19.2	1.94	0.10	1	25.4	0.23	0.42	153	1.06	280	1.93	29.0	129.0	0.139	3.5	1.07	1.07
	2B3-06	4	101.6	1.21	12	304.8	2756	19	1.94	0.10	1	25.4	0.45	0.42	300	2.07	280	1.93	29.5	131.2	0.127	3.2	1.06	1.09
	2B4-07	4	101.6	1.21	12	304.8	2538	17.5	1.94	0.10	1	25.4	0.68	0.42	454	3.13	280	1.93	28.3	126.1	0.144	3.7	1.26	1.11
	2B4-52	4	101.6	1.21	12	304.8	3162	21.8	1.94	0.10	1	25.4	0.68	0.42	454	3.13	280	1.93	33.7	149.9	0.145	3.7	1.22	1.12
	2B6-32	4	101.6	1.21	12	304.8	2872	19.8	1.94	0.10	1	25.4	0.91	0.42	607	4.19	280	1.93	32.6	145.2	0.146	3.7	1.27	1.17
	3B1-08	4	101.6	1.21	12	304.8	2350	16.2	1.94	0.10	1	25.4	0.23	0.63	153	1.06	420	2.90	29.4	130.8	0.155	3.9	1.24	1.23
	3B1-36	4	101.6	1.21	12	304.8	2959	20.4	1.94	0.10	1	25.4	0.23	0.77	153	1.06	514	3.54	35.7	159.0	0.148	3.8	1.12	1.26
	3B3-33	4	101.6	1.21	12	304.8	2756	19	1.94	0.10	1	25.4	0.45	0.77	300	2.07	514	3.54	35.6	158.4	0.153	3.9	1.28	1.32
	3B4-34	4	101.6	1.21	12	304.8	2785	19.2	1.94	0.10	1	25.4	0.68	0.77	454	3.13	514	3.54	34.8	155	0.159	4.0	1.37	1.28
	3B6-35	4	101.6	1.21	12	304.8	3002	20.7	1.94	0.10	1	25.4	0.91	0.77	607	4.19	514	3.54	37.3	166.1	0.163	4.1	1.41	1.29
	4B1-09	4	101.6	1.21	12	304.8	2480	17.1	1.94	0.10	1	25.4	0.23	1.25	153	1.06	834	5.75	34.5	153.5	0.172	4.4	1.37	1.39

Table 1 Continued

Author	Specimen	b		a/d	d		f_c'		ρ (%)	ρ' (%)	d'		ρ_h (%)	ρ_v (%)	$\rho_h f_{yh}$		$\rho_v f_{yv}$		$V_{bv,test}$		Δ_{test}		$\frac{\Delta_{test}}{\Delta_{calc}}$	$\frac{V_{bv,test}}{V_{bv,calc}}$
		in.	mm		in.	mm	psi	MPa			in.	mm			psi	MPa	psi	MPa	kips	kN	in.	mm		
Smith and Vantsiotis 1982	1C1-14	4	101.6	1.50	12	304.8	2785	19.2	1.94	0.10	1	25.4	0.23	0.18	153	1.06	120	0.83	26.8	119.0	0.184	4.7	1.33	1.23
	1C3-02	4	101.6	1.50	12	304.8	3176	21.9	1.94	0.10	1	25.4	0.45	0.18	300	2.07	120	0.83	27.7	123.4	0.169	4.3	1.20	1.14
	1C4-15	4	101.6	1.50	12	304.8	3292	22.7	1.94	0.10	1	25.4	0.68	0.18	454	3.13	120	0.83	29.5	131.0	0.164	4.2	1.17	1.18
	1C6-16	4	101.6	1.50	12	304.8	3162	21.8	1.94	0.10	1	25.4	0.91	0.18	607	4.19	120	0.83	27.5	122.3	0.168	4.3	1.21	1.14
	2C1-17	4	101.6	1.50	12	304.8	2886	19.9	1.94	0.10	1	25.4	0.23	0.31	153	1.06	207	1.43	27.9	124.1	0.177	4.5	1.21	1.12
	2C3-03	4	101.6	1.50	12	304.8	2785	19.2	1.94	0.10	1	25.4	0.45	0.31	300	2.07	207	1.43	23.3	103.6	0.165	4.2	1.14	0.96
	2C3-27	4	101.6	1.50	12	304.8	2799	19.3	1.94	0.10	1	25.4	0.45	0.31	300	2.07	207	1.43	25.9	115.3	0.157	4.0	1.09	1.07
	2C4-18	4	101.6	1.50	12	304.8	2959	20.4	1.94	0.10	1	25.4	0.68	0.31	454	3.13	207	1.43	28.0	124.5	0.165	4.2	1.14	1.11
	2C6-19	4	101.6	1.50	12	304.8	3017	20.8	1.94	0.10	1	25.4	0.91	0.31	607	4.19	207	1.43	27.9	124.1	0.175	4.4	1.21	1.09
	3C1-20	4	101.6	1.50	12	304.8	3046	21.0	1.94	0.10	1	25.4	0.23	0.56	153	1.06	374	2.58	31.7	140.8	0.210	5.3	1.42	1.23
	3C3-21	4	101.6	1.50	12	304.8	2408	16.6	1.94	0.10	1	25.4	0.45	0.56	300	2.07	374	2.58	28.1	125.0	0.180	4.6	1.27	1.29
	3C4-22	4	101.6	1.50	12	304.8	2654	18.3	1.94	0.10	1	25.4	0.68	0.56	454	3.13	374	2.58	28.7	127.7	0.183	4.6	1.28	1.23
	3C6-23	4	101.6	1.50	12	304.8	2756	19.0	1.94	0.10	1	25.4	0.91	0.56	607	4.19	374	2.58	30.8	137.2	0.193	4.9	1.35	1.29
	4C1-24	4	101.6	1.50	12	304.8	2843	19.6	1.94	0.10	1	25.4	0.23	0.77	153	1.06	514	3.54	33.0	146.6	0.220	5.6	1.43	1.22
	4C3-04	4	101.6	1.50	12	304.8	2698	18.6	1.94	0.10	1	25.4	0.45	0.63	300	2.07	420	2.90	28.9	128.6	0.203	5.2	1.34	1.11
	4C3-28	4	101.6	1.50	12	304.8	2785	19.2	1.94	0.10	1	25.4	0.45	0.77	300	2.07	514	3.54	34.2	152.3	0.218	5.5	1.43	1.29
	4C4-25	4	101.6	1.50	12	304.8	2683	18.5	1.94	0.10	1	25.4	0.68	0.77	454	3.13	514	3.54	34.3	152.6	0.210	5.3	1.39	1.32
	4C6-26	4	101.6	1.50	12	304.8	3075	21.2	1.94	0.10	1	25.4	0.91	0.77	607	4.19	514	3.54	35.9	159.5	0.215	5.5	1.41	1.26
	4D1-13	4	101.6	2.08	12	304.8	2335	16.1	1.94	0.10	1	25.4	0.23	0.42	153	1.06	280	1.93	19.6	87.4	0.236	6.0	0.99	0.96
Tan *et al.* 1995.	A-0.27-5.38	4.3	110	0.27	18.2	463.0	8312	57.3	1.23	0.10	1.18	30.0	0	0.48	0	0	260	1.78	141.6	630	0.205	5.2	1.40	1.14
	B-0.54-5.38	4.3	110	0.54	18.2	463.0	7685	53.0	1.23	0.10	1.18	30.0	0	0.48	0	0	260	1.78	107.9	480	0.248	6.3	0.91	1.00
Tan *et al* 1997a.	1-2N/0.75	4.3	110	0.85	17.4	442.5	8151	56.2	2.58	0.32	0.79	20	0	2.86	0	0	1465	10.1	170.9	760	0.291	7.4	1.22	1.49
	1-3/0.75	4.3	110	0.85	17.4	442.5	8586	59.2	2.58	0.32	0.79	20	1.59	0	815	5.6	0	0	125.9	560	0.244	6.2	0.98	1.07
	1-4/0.75	4.3	110	0.85	17.4	442.5	9253	63.8	2.58	0.32	0.79	20	1.59	0	1030	7.1	0	0	130.4	580	0.185	4.7	0.74	1.07
	1-5/0.75	4.3	110	0.85	17.4	442.5	8354	57.6	2.58	0.32	0.79	20	3.17	0	2054	14.2	0	0	174.2	775	0.256	6.5	1.06	1.49
	2-6N/1.00	4.3	110	1.13	17.4	442.5	10921	75.3	2.58	0.32	0.79	20	1.59	1.43	1030	7.1	732	5.1	150.6	670	0.406	10.3	1.18	1.26
	3-2S/1.50	4.3	110	1.69	17.4	442.5	11255	77.6	2.58	0.32	0.79	20	0	1.43	0	0	927	6.4	89.9	400	0.591	15.0	0.87	0.81
	3-6N/1.50	4.3	110	1.69	17.4	442.5	11443	78.9	2.58	0.32	0.79	20	1.59	1.43	1030	7.1	732	5.1	103.4	460	0.583	14.8	1.00	0.99

Table 1 Continued

Author	Specimen	b		a/d	d		f_c'		ρ	ρ'	d'		ρ_h	ρ_v	$\rho_h f_{yh}$		$\rho_v f_{yv}$		$V_{bv,test}$		Δ_{test}		$\frac{\Delta_{test}}{\Delta_{calc}}$	$\frac{V_{bv,test}}{V_{bv,calc}}$
		in.	mm		in.	mm	psi	MPa	(%)	(%)	in.	mm	(%)	(%)	psi	MPa	psi	MPa	kips	kN	in.	mm		
Tan *et al.* 1997b.	1-2.00/0.75	4.3	110	0.84	17.6	446	10326	71.2	2.00	0.32	0.79	20	0	0.48	0	0	268	1.85	122.5	545	0.299	7.6	1.26	1.06
	1-2.00/1.00	4.3	110	1.12	17.6	446	10326	71.2	2.00	0.32	0.79	20	0	0.48	0	0	268	1.85	112.4	500	0.366	9.3	1.09	1.13
	2-2.58/0.75	4.3	110	0.85	17.4	441	9369	64.6	2.58	0.32	0.79	20	0	0.48	0	0	247	1.70	119.2	530	0.307	7.8	1.26	1.04
	3-4.08/0.75	4.3	110	0.89	16.6	421	9369	64.6	4.08	0.34	0.79	20	0	0.48	0	0	247	1.70	150.6	670	0.303	7.7	1.23	1.28
	3-4.08/1.00	4.3	110	1.19	16.5	420	9877	68.1	4.08	0.34	0.79	20	0	0.48	0	0	247	1.70	116.9	520	0.311	7.9	0.92	1.14
	4-5.80/0.75	4.3	110	0.94	15.7	399	10326	71.2	5.80	0.36	0.79	20	0	0.48	0	0	268	1.85	157.4	700	0.331	8.4	1.33	1.29
	4-5.80/1.00	4.3	110	1.26	15.6	397	10326	71.2	5.80	0.36	0.79	20	0	0.48	0	0	268	1.85	119.2	530	0.331	8.4	0.98	1.16
Aguilar *et al.* 2002.	STM-H	12	305	1.14	31.5	800	4061	28.0	1.25	0.42	4.00	102	0.06	0.31	38	0.26	200	1.38	289	1286	1.32	33.5	1.96	1.05
	STM-M	12	305	1.14	31.5	800	4061	28.0	1.25	0.42	4.00	102	0	0.31	0	0	200	1.38	287	1277	1.27	32.3	2.00	1.11
	Total																					AVG	1.24	1.19
	70																					COV	0.16	0.10

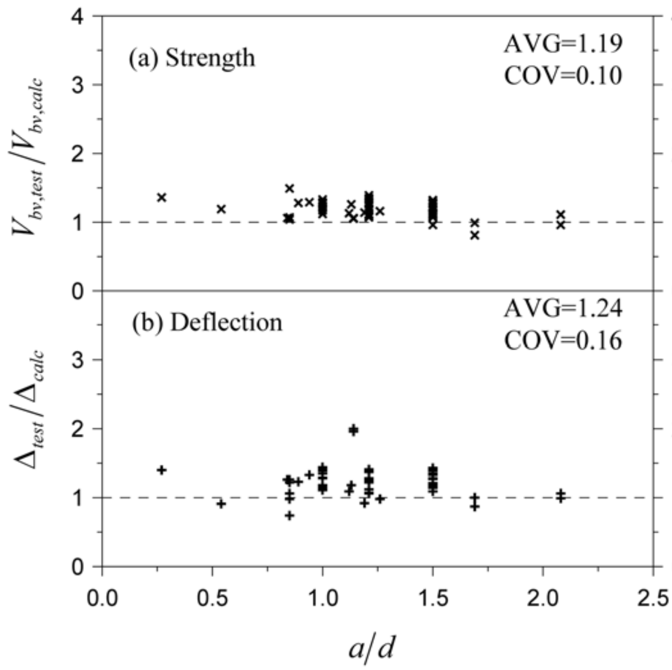


Fig. 5 Effect of shear span-to-depth ratio on shear strength and deflection predictions

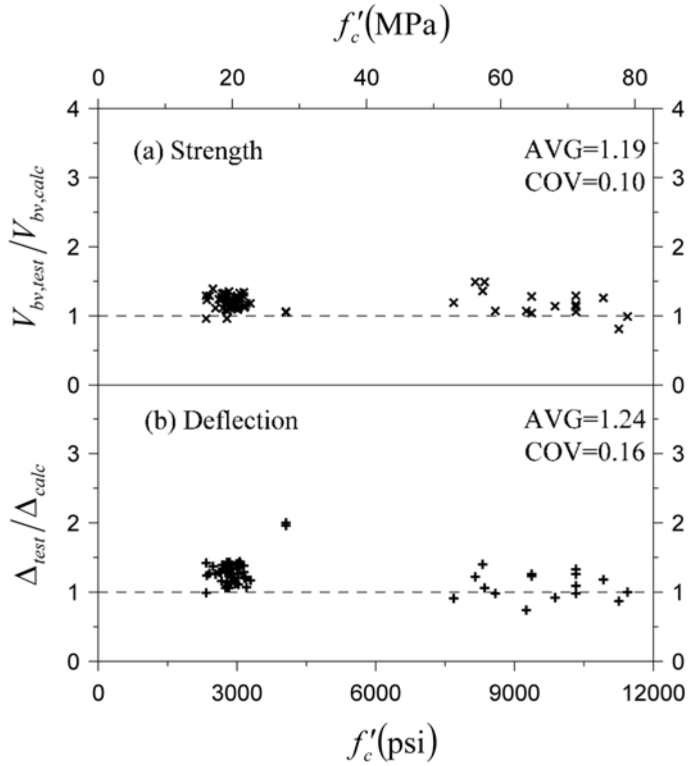


Fig. 6 Effect of concrete strength on shear strength and deflection predictions

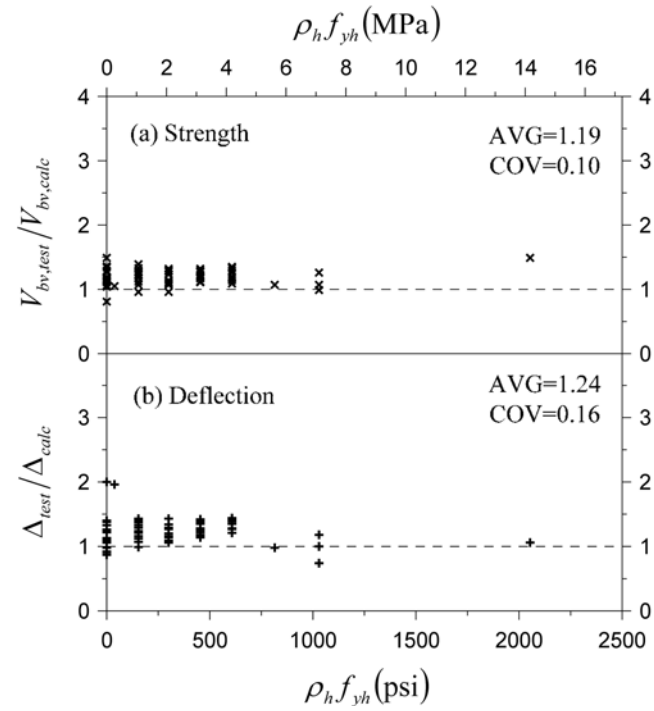


Fig. 7 Effect of horizontal hoops on shear strength and deflection predictions

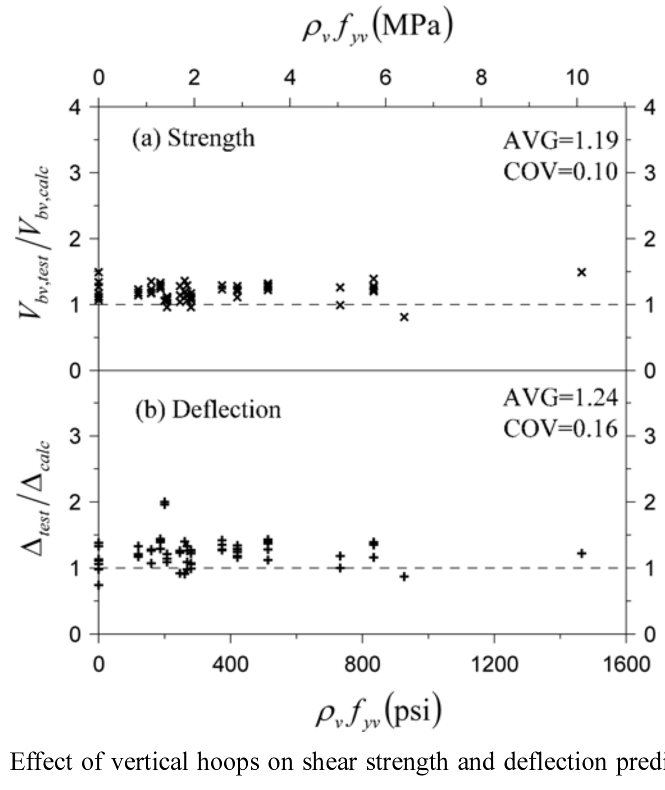


Fig. 8 Effect of vertical hoops on shear strength and deflection predictions

of the proposed model for deep beams. The major factors influencing deflection and shear strength of deep beams, such as the shear span-to-depth ratio, the compressive strength of concrete, and the horizontal and vertical hoops are explored in the following comparison study.

Fig. 5 shows the effect of shear span-to-depth ratios on deflection and shear strength predictions. The deflection and shear strength predictions of the proposed model are consistent for shear span-to-depth ratios ranging from 0.27 to 2.08 (Fig. 5).

Fig. 6 shows the effect of compressive strength of concrete on deflection and shear strength predictions. The deflection and shear strength predictions of the proposed model are consistent for compressive strength of concrete ranging from 2330 psi to 11443 psi (16.1 MPa to 78.9 MPa) (Fig. 6).

Fig. 7 shows the effect of horizontal hoops on deflection and shear strength predictions. The proposed model expresses reasonably well the functions of horizontal hoops on the deflection and shear strength of the deep beam. The deflection and shear strength predictions of the proposed model are consistent for horizontal hoops ranging from 0 to 2054 psi (0 to 14.16 MPa) (Fig. 7).

Fig.8 shows the effect of vertical hoops on deflection and shear strength predictions. The proposed model expresses reasonably well the functions of vertical hoops on the deflection and shear strength of the deep beam. The deflection and shear strength predictions of the proposed model are consistent for vertical hoops ranging from 0 to 1465 psi (0 to 10.10 MPa) (Fig. 8).

5. Conclusions

A simplified model for determining the mid-span deflection and shear strength of deep beams at ultimate state was proposed in this study. Comparisons with the available test results in the literature reveal that the proposed model can accurately predict the mid-span deflection and shear strength of simply supported deep beams at ultimate state for a wide range of shear span-to-depth ratios, compressive strengths of concrete, as well as horizontal and vertical hoops.

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Notations

a = shear span measured center-to-center from load to support
a/d = shear span-to-depth ratio
A_h = area of the horizontal hoops
A_{str} = effective area of the diagonal strut
A_{th} = area of the horizontal tie
A_{tv} = area of the vertical tie
A_v = area of the vertical hoops within the shear span
b = width of the deep beams
b_s = width of the diagonal strut
C_d = diagonal compression
d = effective depth of the deep beam
= direction of the diagonal concrete strut which is the assumed direction of principal compressive stress of concrete (Fig. 2)
D = compression force in the diagonal strut (negative for compression)
d' = distance from the extreme compression fiber to the centroid of the compression reinforcement
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of reinforcement
f_c' = compressive strength of concrete
F_h = tension force in the horizontal tie (positive for tension)
$\overline{F}_h$ = balanced amount of horizontal tie force
F_v = tension force in the vertical tie (positive for tension)

$\overline{F}_v$ = balanced amount of vertical tie force
f_{yh} = yield stress of horizontal hoops
f_{yv} = yield stress of vertical hoops
I = moment of inertia of section about centroidal axis.
I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement
jd = length of the lever arm from the resultant compressive force to the centeroid of the flexural reinforcement
k = coefficient
kd = depth of compression zone at the section
K_h = horizontal tie index
$\overline{K}_h$ = horizontal tie index with sufficient horizontal hoops
K_v = vertical tie index
$\overline{K}_v$ = vertical tie index with sufficient vertical hoops
l = span length of deep beam (Fig. 1)
l_b = width of the bearing plate, measured parallel to the axis of the beams
n = modular ratio of elasticity
r = direction perpendicular to d-direction
= assumed direction of principal tensile stress (Fig. 2)
R_h = beam shear ratio carried by the horizontal mechanism
R_v = beam shear ratio carried by the vertical mechanism
t_s = thickness of the diagonal strut
V_{bh} = horizontal shear force
V_{bv} = vertical shear force
γ_h = fraction of horizontal shear transferred by horizontal tie
γ_v = fraction of vertical shear transferred by vertical tie
γ_{hv} = average shear strain in the shear span of deep beams at ultimate state
ε_d = average normal strains in d-direction (positive for tension strain)
ε_o = average normal strains in h-direction (positive for tension strain)
ε_r = strain at peak stress of standard concrete cylinder
ε_v = average normal strains in r-direction (positive for tension strain)
θ = average normal strains in v-direction (positive for tension strain)
ρ = angle of inclination of the diagonal compression strut
ρ' = ratio of the tension reinforcement
ρ_h = ratio of compression reinforcement
= ratio of horizontal hoops
= A_h/bd
ρ_v = ratio of vertical hoops
= A_v/ba
ζ = softening coefficient of concrete
Δ = vertical mid-span deflection of deep beam at ultimate state
Δ_s = vertical mid-span deflection of deep beam at ultimate state due to shear
Δ_f = vertical mid-span deflection of deep beam at ultimate state due to flexure