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Predicting the flexural capacity of RC beam with partially unbonded steel reinforcement

Xiao-Hui Wang† and Xi-La Liu

Department of Civil Engineering, Shanghai Jiaotong University, Minhang, Shanghai, 200240, P. R. China

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Abstract. Due to the reduction of bond strength resulting from the high corrosion level of reinforcing bars, influence of this reduction on flexural capacity of reinforced concrete (RC) beam should be considered. An extreme case is considered, where bond strength is complete lost and/or the tensile steel are exposed due to heavy corrosion over a fraction of the beam length. A compatibility condition of deformations of the RC beam with partially unbonded length is proposed. Flexural capacity of this kind of RC beam is predicted by combining the proposed compatibility condition of deformations with equilibrium condition of forces. Comparison between the model's predictions with the experimental results published in the literature shows the practicability of the proposed model. Finally, influence of some parameters on the flexural capacity of RC beam with partially unbonded length is discussed. It is concluded that the flexural capacity of the beam may not be influenced by the completely loss of bond of the whole beam span as long as the tensile steel can yield; whether or not the reduction of the flexural capacity of the beam resulting from the loss of bond over certain length may occur depends on the detailed parameters of the given beam.

Keywords: flexural capacity; partial; unbonded length; exposed steel; compatibility condition of deformations

1. Introduction

Steel reinforcement corrosion is a major cause of deterioration of concrete structures in aggressive environment. Steel corrosion in concrete leads to cracking, spalling of the cover concrete, reduction of bond strength between reinforcing bar and concrete, reduction of steel cross section and change of steel mechanical properties. Mechanical behavior and load capacity of reinforced concrete (RC) structural elements in their service and ultimate states are then influenced by the effect of these changes. Therefore, many research works were carried out to study the effect of steel corrosion on mechanical characteristics and load carrying capacity of corroded RC structural elements. The majority of these works were experimentally focused on the load carrying capacity of RC elements with corroded reinforcing bar along the whole length of the RC structural elements (Al-Sulaimani *et al.* 1990, Almusallam *et al.* 1996, Rodriguez 1997, Mangat and Elgarf 1999, Torres-Acosta 2007); and in order to numerically investigate the load carrying capacity of this kind of corroded RC elements, the nonlinear finite element analyses were widely used (Dekoster *et al.* 2003, Coronell and Gambarova 2004).

† Ph. D., Corresponding Author, E-mail address: w_xiaoh@163.com

Meanwhile, Long-term mechanical behavior of the corroded RC beams kept in a confined salt fog was also studied to take into account the actual conditions of the RC structures (Castel *et al.* 2000, Vidal *et al.* 2007). In order to simulate the case where only a fraction of the beam length is corroded in field situations, the term partial length corrosion was used to avoid confusion with localized corrosion and pitting corrosion (Torres-Acosta *et al.* 2007) and the mechanical behavior of the RC beam with partial length corrosion was experimentally carried out recently (Torres-Acosta *et al.* 2004, EI Maaddawy *et al.* 2005, Du *et al.* 2007). In order to obtain the load capacity of the RC structural element with partial length corrosion, a conservative assessment was adopted by assuming the complete loss of bond over the corroded region of the beam length and/or exposure of the tension reinforcements. The behavior of RC structural element with complete loss of bond and/or exposure of the tension reinforcements over partial length of element was mainly studied (Cairns and Zhao 1993, Raoof and Lin 1997, Wang *et al.* 2001, Li 2005, Nokhasteh *et al.* 1992, Eyre and Nokhasteh 1992, Zhang and Raoof 1995). In analytical research work about the behavior of this kind of RC element, a simplified numerical model was developed by assuming the plane-section behaviour of the concrete section (Cairns and Zhao1993); the non-linear finite element analysis was used to obtain the load-central deflection curves (Nokhasteh *et al.* 1992); an algebraic formulation was presented to predict the flexural strength (Eyre and Nokhasteh 1992); and an iteration procedure was proposed to calculate the ultimate moment of the RC beam with exposed reinforcement on the basis of plane-section bending (Zhang and Raoof 1995).

However, as pointed out by Eyre and Nokhasteh (1992), any loss of bond following reinforcement corrosion means that the code equations for ultimate moment of resistance, which are dependent on strain compatibility at all sections, might become invalid. Due to the loss of bond over corroded region of the beam length or exposure of the tension reinforcements over this length, the compatibility condition of deformations the RC beam will shift to a new one and the corresponding structural action is also changed.

In the present paper, considering the heavy level of reinforcing bar corrosion and the assumption of the complete loss of bond within the corroded beam length, the term partially unbonded length is introduced to describe this situation. By using the concept of the equivalent plastic region length of the unbonded prestressed beam and considering the compatibility condition of deformations at the level of tension reinforcement, a compatibility condition of deformations of the RC beam with partially unbonded length is proposed. The principles of equilibrium of forces and this proposed compatibility of deformations of the RC beam are used to calculate the flexural capacity of this kind of RC beam, where the symmetrical arrangement of partially unbonded length and loads is assumed. Then, the practicability of the proposed model is examined by comparing the model's predictions with the corresponding experimental results; and some factors which may affect the flexural capacity of the RC beam with partially unbonded length are also discussed.

2. Material modeling

2.1. Concrete

The stress-strain relationship of ordinary concrete in compression, which was proposed by Hognestad (Park and Paulay 1975), is given by

$$\sigma_c = \begin{cases} f_c' \cdot \left[\dfrac{2\varepsilon_c}{\varepsilon_0} - \left(\dfrac{\varepsilon_c}{\varepsilon_0}\right)^2\right] & \varepsilon_c \le \varepsilon_0 \\ f_c' \cdot \left[1 - 0.15\left(\dfrac{\varepsilon_c - \varepsilon_0}{\varepsilon_{cu} - \varepsilon_0}\right)\right] & \varepsilon_0 < \varepsilon_c \le \varepsilon_{cu} \end{cases} \tag{1}$$

where ε_0 is taken as ε_0=0.002; and ε_{cu} is the ultimate strain of concrete in compression; f_c' is the cylinder compressive strength of concrete.

2.2. Steel reinforcement

The stress-strain relationship of steel reinforcement is idealized to be elastic perfectly plastic, and can be expressed as follows

$$\sigma_s = \begin{cases} E_s\varepsilon_s & \varepsilon_s \le \varepsilon_y \\ f_y & \varepsilon_s > \varepsilon_y \end{cases} \tag{2}$$

where ε_y is the yield strain of steel reinforcement, $\varepsilon_y = f_y/E_s$; E_s is the modulus of steel reinforcement; f_y is the yield strength of steel reinforcement.

3. Flexural capacity modeling

A simply supported RC beam with two-point loading and partially unbonded length symmetrically arranged about the mid-span is considered. The case where the partially unbonded length is not symmetrically arranged about the mid-span will be considered in the subsequent research work. Due to the flexural capacity of the RC beam with partially unbonded length is mainly studied, the precedence of the flexural failure over shear failure and anchorage failure is assumed.

Conditions of equilibrium of forces and compatibility of deformations must be satisfied, whether or not reinforcement is bonded to the concrete (Cairns and Zhao 1993). Then, the following equations can be obtained

$$\text{equilibrium of forces: } \sum F = 0 \tag{3}$$

$$\text{compatibility of deformations: } \int^L \varepsilon_{st} dl = \int^L \varepsilon_{cd} dl \tag{4}$$

where Eq. (3) means that the net horizontal force at any section of a beam in pure bending must be zero; Eq. (4) means that deformations of concrete and reinforcement must be compatible. ε_{st} is the strain of longitudinal tensile steel; ε_{cd} is the strain in concrete at the level of tension reinforcement; L is length of beam span.

3.1. Compatibility condition of deformations

In field situations, corrosion of the reinforcing bar is located in a fraction of the beam length of the RC beam. When the corrosion level is very high, the bond strength between the corroded reinforcing bar and the surrounding concrete decreases greatly and the complete loss of bond can be assumed within the partial length. Consider the RC beam with complete loss of bond over partial length L_{ub} in Fig. 1, the code equations for the ultimate flexural capacity of RC beam with perfect bond become invalid and it is difficult to describe the equation of compatibility of deformations of this kind of RC beam.

In order to solve the problem mentioned above simply and practicably, a special case is firstly considered, where the whole length of the beam span except for the two beam-ends is corroded. Therefore, RC beam with complete loss of bond over the whole beam span is assumed. For an unbonded concrete beam having the same geometries and form of loading as the beam in Fig. 1a, where L_{ub}=L, the strain of the unbonded steel depends on the deformations of the whole member and it can not be determined from the analysis of the cross section alone as in the case of bonded steel. However, the methodology used in the study of the unbounded prestressed concrete beam can be adopted. Under this case, the unbonded steel can be treated as the prestressing steel with zero effective prestress.

For the unbonded prestressed concrete beams (see Fig. 2a) reinforced with or without bonded reinforcements, due to the lengthening of the concrete at steel level in the elastic zone is negligible compared with the lengthening in the plastic zone, the concept of "equivalent plastic region length L_{eq}" was proposed (Pannell 1969); and it was assumed that the lengthening of the concrete at steel level was mainly due to the plastic deformations occurring within L_{eq} in the vicinity of applied load (Fig. 2b). Then, the total elongation of the unbonded prestressing steel between the end anchorages can equate to the plastics deformation of concrete within the equivalent plastic region length L_{eq}. It was found that the ratio φ of L_{eq} to neutral axis depth x_c at ultimate, namely $\varphi=L_{eq}/x_c$, was a constant value for prestressed concrete beam with unbonded steel even for different span-depth ratio (Pannell 1969). Basing on this methodology, detailed study was carried out and the suggested value of the ratio φ is φ=9.3 (Au and Du 2004).

Then, for the unbonded RC beam with L_{ub}=L, where the effective prestress in the unbonded steel is zero, $\int^L \varepsilon_{cd} dl$ in the right hand of Eq. (4) can be given as follows by using the concept of the equivalent plastic region length L_{eq}

$$\int^L \varepsilon_{cd} dl = L_{eq} \cdot \varepsilon_{cu} \cdot \frac{h_0 - x_c^{ub}}{x_c^{ub}} \tag{5}$$

where x_c^{ub} is the depth of the compression zone at the critical section within the equivalent plastic region length L_{eq}, and L_{eq} is given by $L_{eq} = 9.3 \cdot x_c^{ub}$; h_0 is the effective depth of the unbonded RC beam. The superscript "ub" denotes "unbonded". Within L_{eq}, the concrete compression strain in extreme fiber reaches the limit strain of the concrete ε_{cu}.

While $\int^L \varepsilon_s dl$ in the left hand of Eq. (4) for this RC beam having L_{ub}=L can be given by

$$\int^L \varepsilon_s dl = L \cdot \varepsilon_s^{ub} \tag{6}$$

where ε_s^{ub} is the strain of tensile steel over L and $\varepsilon_s^{ub} = \sigma_s^{ub}/E_s$; σ_s^{ub} is the corresponding stress and satisfies $\sigma_s^{ub} \le f_y$.

The total elongation of the unbonded steel between the end anchorages equating to the plastics deformation of concrete within the equivalent plastic region length L_{eq} gives

$$L \cdot \varepsilon_s^{ub} = L_{eq} \cdot \varepsilon_{cu} \cdot \frac{h_0 - x_c^{ub}}{x_c^{ub}} \tag{7}$$

Eq. (7) can be further expressed as follows

$$\varepsilon_s^{ub} = \frac{L_{eq}}{L} \cdot \varepsilon_{cu} \cdot \frac{h_0 - x_c^{ub}}{x_c^{ub}} \tag{8}$$

Secondly, considering the perfectly bonded RC beam corresponding to the unbonded RC beam, the following equation can easily be obtained for this perfectly bonded beam according to the hypothesis of the plane sections remaining plane

$$\varepsilon_s^{b} = \varepsilon_c^{b} \cdot \frac{h_0 - x_c^{b}}{x_c^{b}} \tag{9}$$

where ε_s^b is the strain of tensile steel of the perfectly bonded RC beam and satisfies $\varepsilon_s^b \le \varepsilon_y$; ε_c^b is the concrete compression strain in extreme fiber and satisfies $\varepsilon_s^b \le \varepsilon_{cu}$; x_c^b is the depth of the compression zone; where the superscript "b" denotes "bonded".

Finally, the RC beam with partially unbonded length L_{ub} in Fig. 1a is considered. For the perfectly bonded RC beam, which can be considered as unbonded RC beam with partial length L_{ub}=0, the structural action is described as pure beam behaviour and the equation of compatibility of deformations is given by Eq. (9); with the increase of the partial length L_{ub} and finally reaches the whole beam span L, the structural action shifts from pure beam behaviour to tied arch behaviour. When the partial length L_{ub} reaches L, where enough anchorages at the beam-ends are assumed, the equation of compatibility of deformations is given by Eq. (8). Thus, it can be concluded that the structural action of the RC beam with partially unbonded length L_{ub} lies in between the pure beam behaviour and tied arch behaviour and can be described as "beam-arch" behaviour. Noticing that the equations of compatibility of deformations corresponding to the minimum and maximum partial length L_{ub} are already known; and it is difficult to describe the exact equation of compatibility of deformations of RC beam with L_{ub}, an interpolating function $g(L_{ub})$ of this partial length L_{ub}, obtained by linear interpolation, is introduced to describe the shifting of the structural action from pure beam behaviour to tied arch behaviour

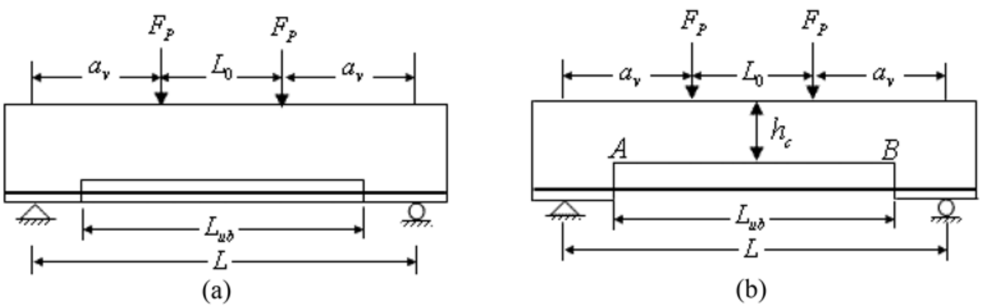

Fig. 1 RC beam with complete loss of bond over partial length L_{ub}: (a) with concrete cover; (b) without concrete cover and with exposed steel

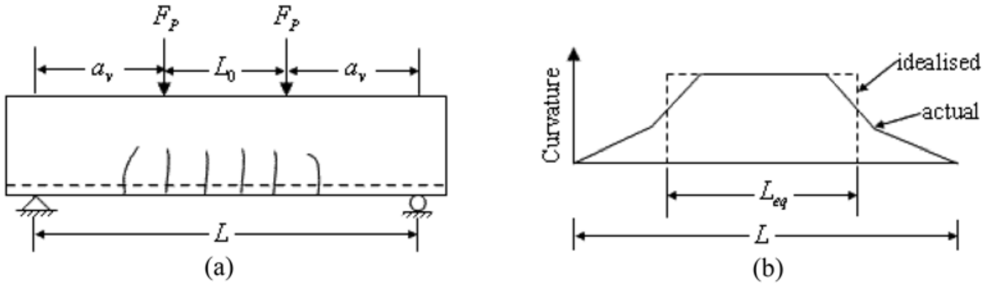

Fig. 2 A simply supported prestressed concrete beam with unbonded steel: (a) arrangement of loading; (b) actual and idealized curvature distribution along the beam (Au and Du 2004)

$$g(L_{ub}) = 1 - \frac{L_{ub} \cdot (L - L_{eq})}{L^2} \tag{10}$$

where the range of L_{ub} is $0 \leq L_{ub} \leq L$. Correspondingly, the relationship of the strain ε_s^{pub} of tensile steel, the concrete compression strain ε_c^{pub} in extreme fiber and the depth of compression zone x_c^{pub} of the RC beam with L_{ub} can be given by

$$\varepsilon_s^{pub} = g(L_{ub}) \cdot \varepsilon_c^{pub} \cdot \frac{h_0^{pub} - x_c^{pub}}{x_c^{pub}} \tag{11}$$

where strain $\varepsilon_s^{pub} = \sigma_s^{pub}/E_s$ and satisfies $\varepsilon_s^{pub} \leq \varepsilon_y$, σ_s^{pub} is the corresponding stress of tensile steel at the critical section and satisfies $\sigma_s^{pub} \leq f_y$; strain ε_c^{pub} satisfies $\varepsilon_c^{pub} \leq \varepsilon_{cu}$; h_0^{pub} is the effective depth of the RC beam with L_{ub}, for the RC beam in Fig. 1a, $h_0^{pub} = h_0$. The superscript "pub" denotes "partly unbonded".

Now, consider the beam in Fig. 1b, where cover concrete in tensile zone is delaminated and the tensile bars are completely exposed. Assuming the depth of concrete where reinforcement is exposed is h_c, see Fig. 1b, influence of removal of concrete cover on the flexural capacity of the RC beam must be considered.

When the RC beam in Fig. 1b is loaded, due to the absence of the confinement of the tensile concrete, the exposed steel over L_{ub} will be tightened and move vertically to the bottom section of the concrete within L_{ub} (Zhang and Raoof 1995); on the other hand, the portion of the beam within L_{ub} has vertical displacement downwards. Thus, it is difficult to determine the exact effective depth at the critical section at the failure of the RC beam. A conservative assumption that the exposed reinforcements finally approach the bottom section AB (Fig. 1b) is used. As a result, the effective depth h_0^{pub} of the RC beam in Fig. 1b, which is the distance between the center of the tensile steel reinforcements and the edge of compression zone, can be modify as $h_0^{pub} = h_c + d/2$, where d is diameter of tensile steel.

3.2. Equilibrium condition of forces

In the ultimate state of the RC beam with partially unbonded length, the schematic drawing of the critical section of the RC beam is shown in Fig. 3; and the failure of the RC beam is defined as the load at which the concrete compression strain reaches the limiting compressive strain of concrete ε_{cu}, and ε_{cu} is taken as 0.003 as specified in the ACI Building Code (American Concrete Institute

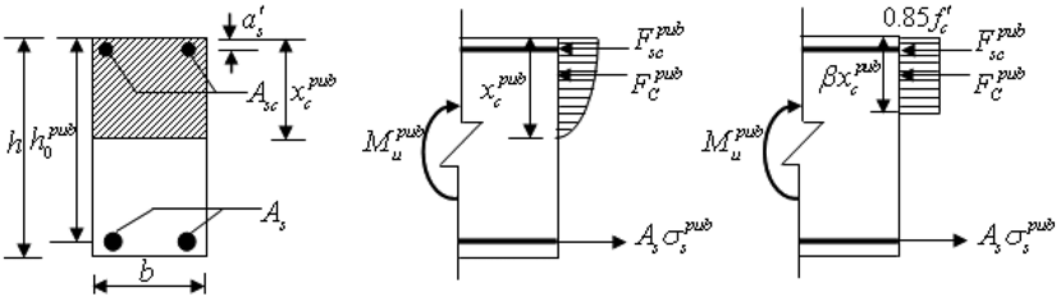

Fig. 3 Schematic drawing of the critical section of the RC beam in ultimate state: (a) cross section of the beam; (b) stress; (c) equivalent rectangular stress

1983). Then, the flexural strength M_u^{pub} of the RC beam with partially unbonded length can be calculated from Eqs. (12)-(18). For RC beam with "T" cross section, Eqs. (12)-(18) can also be modified to calculate the M_u^{pub}.

$$\varepsilon_s^{pub} = \left[1 - \frac{L_{ub}\cdot(L-L_{eq})}{L^2}\right]\cdot\varepsilon_{cu}\cdot\frac{h_0^{pub}-x_c^{pub}}{x_c^{pub}} \text{ with } \varepsilon_s^{pub}\le\varepsilon_y \tag{12}$$

$$A_s\cdot\sigma_s^{pub} = F_C^{pub}+F_{sc}^{pub} \tag{13}$$

$$M_u^{pub} = F_C^{pub}\cdot(h_0^{pub}-0.5\beta\cdot x_c^{pub})+F_{sc}^{pub}\cdot(h_0^{pub}-a_s') \tag{14}$$

where:

$$\sigma_s^{pub} = E_s\cdot\varepsilon_s^{pub}\text{, with } \sigma_s^{pub}\le f_y \tag{15}$$

$$F_C^{pub} = 0.85b\cdot f_c'\cdot\beta\cdot x_c^{pub} \tag{16}$$

$$\beta=\begin{cases} 0.85 & f_c'<28MPa \\ 0.85-0.5(f_c'-28)/7 & 28MPa\le f_c'\le 56MPa \\ 0.65 & f_c'>56MPa \end{cases} \tag{17}$$

$$F_{sc}^{pub} = E_{sc}\cdot\varepsilon_{cu}\cdot\frac{x_c^{pub}-a_s'}{x_c^{pub}}\cdot A_{sc}\le A_{sc}\cdot f_{yc} \tag{18}$$

where A_s, A_{sc} are the cross section area of tensile and compressive steel, respectively; b is the beam width for rectangular cross section RC beam; E_{sc}, f_{yc} are the modulus and yield strength of the compressive steel reinforcements, respectively; F_C^{pub}, F_{sc}^{pub}, are the total force of compressive concrete and steel at critical section of RC beam with L_{ub}, respectively; a_s' the distance from the extreme compression fiber to the centroid of the compression steel; β is the stress block factor.

It should be pointed out that the present model does not include the possibility of change in strength and cross section of the longitudinal steel due to metal loss during corrosion. Actually, for RC beams with heavy corrosion of the longitudinal steel, if complete loss of bond can be assumed within the corroded length and corrosion of the reinforcement is not so localized, Eqs. (12)-(18) can be easily used to take into account the possible change in strength and reduction in cross section of the longitudinal steel due to corrosion. However, when corrosion of the reinforcement is so localized that the metal loss may reach losses of almost 70% of the original diameter (Torres-Acosta *et al.* 2007), the corroded RC beam may fail in the rupture of the tensile longitudinal steel resulting from the heavy localized area loss. This case is not included in the present model.

3.3. Critical partially-unbonded length L_{ub}^{cr}

For the convenience of discussion, variable L_{ub}^{cr} is introduced, where L_{ub}^{cr} is the critical partially-unbonded length of the RC beam. For a given RC beam with L_{ub}^{cr}, under the ultimate state of the beam, the tensile steel reinforcements can exactly yield; beyond length L_{ub}^{cr}, the tensile steel reinforcements can not yield. In other word, for a given RC beam with different partially unbonded lengths, if $L_{ub} \leq L_{ub}^{cr}$, the tensile steel reinforcements can yield and the flexural capacity of the RC beam is not influenced by the completely loss of bond over length L_{ub}; if $L_{ub} > L_{ub}^{cr}$, the tensile steel reinforcements can not yield and the flexural capacity of the RC beam is decreased. Thus, for a given RC beam, L_{ub}^{cr} can be calculated by substituting $\sigma_s^{pub} = f_y$ and $\varepsilon_s^{pub} = \varepsilon_y$ into Eqs. (12)-(13) as follows

$$f_y / E_s = \left[1 - \frac{L_{ub}^{cr} \cdot (L - L_{eq})}{L^2} \right] \cdot \varepsilon_{cu} \cdot \frac{h_0^{pub} - x_c^{pub}}{x_c^{pub}} \tag{19}$$

$$A_s \cdot f_y = F_C^{pub} + F_{sc}^{pub} \tag{20}$$

If neglecting the compressive force F_{sc}^{pub} in Eq. (20), L_{ub}^{cr} is given by

$$L_{ub}^{cr} = \frac{L^2}{L - L_{eq}} \left(1 - \frac{f_y}{E_s \cdot \varepsilon_{cu}} \cdot \frac{1}{h_0^{pub} / x_c^{pub} - 1} \right) \quad x_c^{pub} = \frac{A_s \cdot f_y}{0.85 f_c' b \beta} \tag{21}$$

Eq. (21) can rewritten as follows

$$L_{ub}^{cr} = \frac{L^2}{L - L_{eq}} \left(1 - \frac{f_y}{E_s \cdot \varepsilon_{cu}} \cdot \frac{1}{0.85 f_c' \beta / (\rho_s \cdot f_y) - 1} \right) \tag{22}$$

where ρ_s is the nominal percentage of tensile steel of RC beam with L_{ub} and defined as $\rho_s = A_s / (b h_0^{pub})$; for the RC beam with exposed tensile bars, modifying of h_0^{pub} is considered. It can be seen from Eq. (22) that, for RC beam with identical geometries and loading form, different material parameters, such as ρ_s, f_y and f_c', may result in different value of L_{ub}^{cr}.

Due to the neglecting of compressive force F_{sc}^{pub}, depth of compression zone x_x^{pub} increases and the corresponding L_{ub}^{cr} decreases. It means that a more conservative value of L_{ub}^{cr} is obtained.

4. Examination of model accuracy

In the study of Nokhasteh *et al.* (1992), Cairns and Zhao (1993), Wang *et al.* (2001) and Li (2005), tests were conducted on simply supported RC beams with two-point loading and complete loss of bond over partial length symmetrically arranged about the mid-span, only the RC beams of Wang *et al.* (2001) had no exposed bottom tensile reinforcements. Based on the proposed model, flexural capacity of the mentioned RC beams are calculated and compared with the corresponding test results, see Tables 1~4. In Table 1, all test beams had L=2000 mm. In Table 2, beam W1 and

Table 1 Comparison with experimental results of Nokhasteh *et al.* (1992)

Details of the test RC beam								Ultimate load (kN)		$\frac{P_u^{pub}}{P_{u,\exp}^{pub}}$	P_u^b of bonded RC beam (kN)	$\frac{P_u^{pub}}{P_u^b}$
beam No.	b (mm)	h_0 (mm)	h_c (mm)	L_{ub} (mm)	f_{cu} (MPa)	f_y (MPa)	A_s (mm^2)	$P_{u,\exp}^{pub}$	P_u^{pub}			
B1/25	130	167	159	500	34.9	365	201(1ϕ16)	25.2	24.09	0.964	26.86	0.897
B1/85	130	167	159	1700	36.7	365	201(1ϕ16)	23.7	24.19	1.021	26.96	0.897
B2/85	130	167	159	1700	29.9	365	402(2ϕ16)	32.0	39.97	1.249	48.22	0.829
Mean value										1.078		
Standard deviation										0.151		

Table 2 Comparison with experimental results of Cairns and Zhao (1993)

Details of the test RC beam									Failure moment (kN·m)		$\frac{M_u^{pub}}{M_{u,\exp}^{pub}}$	M_u^b of bonded RC beam (kN·m)	$\frac{M_u^{pub}}{M_u^b}$
Beam No.	b/b_w (mm)	h_0 (mm)	h_c (mm)	L_{ub} (mm)	f_{cu} (MPa)	No. of bras	d (mm)	f_y (MPa)	$M_{u,\exp}^{pub}$	M_u^{pub}			
S2	225	372	350	2500	25.0	2	20	529	96.5	105.21	1.090	109.20	0.963
S3	225	380	340	1700	31.2	2	20	529	113.0	104.76	0.927	114.73	0.913
S4	230	253	225	2520	38.2	2	20	524	61.4	59.33	0.966	74.22	0.799
S4B	221	225	205	2520	24.9	2	20	525	46.1	40.81	0.885	59.68	0.684
S5	230	195	155	2540	35.4	2	20	524	28.9	30.05	1.039	54.41	0.552
S7	228	358	340	2320	30.3	3	20	524	135.5	146.89	1.084	150.84	0.974
S8	150	340	320	2320	29.6	2	25	487	92.1	105.84	1.149	124.70	0.849
S9	230	350	340	2560	32.4	2	16	529	69.2	69.56	1.005	69.99	0.994
S10	230	200	180	2550	29.9	3	12	517	27.4	23.48	0.857	31.79	0.739
S11	230	200	180	1620	34.9	3	12	517	30.0	29.81	0.994	32.26	0.924
W1	210/160	245	225	2460	25.5	2	25	487	58.3	60.85	1.044	85.75	0.710
W2	210/160	250	225	1880	25.4	2	25	487	67.2	66.16	0.985	88.02	0.751
Mean value											1.002		
Standard deviation											0.086		

W2 had "T" cross section, and b and b_w were the width of compression flange and web of concrete section, respectively; all test RC beams had L=2700 mm. In Table 3, all test RC beams had L=2100 mm. In Table 4, the test RC beams had L=2100 mm and L_{ub}=1900 mm; the 28-day cube compressive strength of concrete f_{cu} was, on average, f_{cu}=33.88 MPa; all beams carried two 10 mm-diameter Grade steel bars as top steel.

The accuracy of the proposed model is examined, at the bottom of the Tables 1~4, by means of the calculated -to-experimental ratio ($P_u^{pub}/P_{u,\exp}^{pub}$ and $M_u^{pub}/M_{u,\exp}^{pub}$) mean value and standard deviation, where $P_{u,\exp}^{pub}$ and P_u^{pub} are the experimental and calculated ultimate load of the RC beam with partially unbonded length, respectively; $M_{u,\exp}^{pub}$ and M_u^{pub} are the experimental and calculated failure moment of the RC beam with partially unbonded length, respectively. It can be seen from the tables that the proposed model provided good predictions.

The practicability of the proposed model is also examined in Tables 1~4, by means of the ratio of the P_u^{pub}/P_u^b and M_u^{pub}/M_u^b, where P_u^b, M_u^b are the calculated ultimate load and failure moment of the corresponding perfectly bonded RC beam respectively. In order to check the trend of the

Table 3 Comparison with experimental results of Wang *et al.* (2001)

Details of the test RC beam							Failure moment (kN·m)		$\frac{M_u^{pub}}{M_{u,\exp}^{pub}}$	M_u^b of bonded RC beam (kN·m)	$\frac{M_u^{pub}}{M_u^b}$
Beam No.	b (mm)	h_0 (mm)	L_{ub} (mm)	f_{cu} (MPa)	A_s (mm^2)	f_y (MPa)	$M_{u,\exp}^{pub}$	M_u^{pub}			
L-1	105	160	2100	22.8	113(1ϕ12)	321.2	5.515	5.402	0.980	5.402	1.0
L-2	102	167	1400	22.8	113(1ϕ12)	298.3	5.761	5.269	0.915	5.269	1.0
L-3	103	165	700	22.8	113(1ϕ12)	298.3	6.149	5.205	0.846	5.205	1.0
L-7	101	164	0	22.8	113(1ϕ12)	321.2	5.555	5.531	0.996	5.531	1.0
L-8	101	161	2100	22.8	226(2ϕ12)	321.2	8.972	9.309	1.038	10.004	0.931
Mean value									0.955		
Standard deviation									0.075		

Table 4 Comparison with experimental results of Li (2005)

Details of the test RC beam								Failure moment (kN·m)		$\frac{M_u^{pub}}{M_{u,\exp}^{pub}}$	M_u^b of bonded RC beam (kN·m)	$\frac{M_u^{pub}}{M_u^b}$
Beam No.	b (mm)	h_0 (mm)	h_c (mm)	L_0 (mm)	A_s (mm^2)	$E_s\times10^5$ (MPa)	f_y (MPa)	$M_{u,\exp}^{pub}$	M_u^{pub}			
L-1a	119.33	164.67	144.67	700	101(2ϕ8)	2.27	433.4	7.15	6.344	0.887	6.859	0.925
L-1b	121.33	163	143	700	101(2ϕ8)	2.27	433.4	6.6	6.287	0.953	6.792	0.926
L-2a	118	167.33	147.33	170	226(2ϕ12)	2.1	327.1	11.8	10.320	0.875	11.355	0.909
L-2b	121	166.33	146.33	170	226(2ϕ12)	2.06	319.2	10.8	10.052	0.931	11.062	0.909
L-3a	120	167	147	700	402(2ϕ16)	1.85	314.8	18.6	17.006	0.914	18.525	0.918
L-3b	121	165	145	170	402(2ϕ16)	2.18	343.1	17.1	18.137	1.061	19.782	0.917
L-4a	120	172.67	152.67	700	509(2ϕ18)	2.01	279.1	19.5	19.825	1.017	21.391	0.927
L-4b	120	172.33	152.33	700	509(2ϕ18)	2.06	247.5	19.6	17.745	0.905	19.130	0.928
L-5a	119.67	177.67	157.67	170	628(2ϕ20)	2.06	240.6	20.0	21.814	1.091	23.325	0.935
L-6a	120.33	173.67	153.67	700	760(2ϕ22)	1.75	289.4	26.0	26.596	1.023	30.903	0.861
L-6b	120.33	173.33	153.33	700	760(2ϕ22)	1.88	310.4	24.2	27.216	1.125	32.473	0.838
L-7	123.33	222.67	202.67	170	402(2ϕ16)	2.0	298	25.4	22.899	0.902	24.337	0.941
L-8	118	106.33	86.33	170	402(2ϕ16)	2.1	329.7	7.7	7.396	0.961	11.268	0.656
Mean value										0.973		
Standard deviation										0.082		

reduction of the flexural capacity of RC beam with partially unbonded length relative to the corresponding perfectly bonded RC beam, the balanced percentage of tensile reinforcement ρ_{sb} is introduced, where ρ_{sb} is given by (Park and Paulay 1975)

$$\rho_{sb} = \frac{0.85 f_c' \beta}{f_y} \cdot \frac{\varepsilon_{cu} E_s}{\varepsilon_{cu} E_s + f_y} \tag{23}$$

where for the perfectly bonded RC beam with ρ_{sb}, when the tensile steel yields, the extreme fiber concrete compression strain reaches ε_{cu} simultaneously.

Then, ρ_s and the corresponding ρ_{sb} are calculated to observe the trend of the reduction of the RC beams in Tables 1~4. For B1/85 and B2/85 in Table 1, which have the same L_{ub}, greater reduction is shown by B2/85 with a relatively higher ρ_s=2.04852%; this trend is also observed in the beam L-8 of Wang *et al.* (2001) and L-6a, L-6b, L-8 of Li (2005). While for the RC beams with the similar

ρ_s, such as beams S10 and S11, W1 and W2 of Cairns and Zhao (1993), greater reduction in flexural capacity is shown by S10 and W1 with longer L_{ub}. On the other hand, for the beam specimen S9 of Cairns and Zhao (1993) with the smallest ρ_s, little reduction is shown; when the concrete cover of the tensile steel is available and the ρ_s is relatively low, no reduction is shown by beam L-1 of Wang *et al.* (2001) even the complete loss of bond over the whole beam span. Those conclusions agree with the experimental results published in the literature (Cairns and Zhao1993, Raoof and Lin 1997).

The assumption of the modifying of h_0^{pub} for RC beam in Fig. 1b is also checked. For example, for the beam S5 in Cairns and Zhao (1993), which had the maximum depth of removal of concrete, the calculated failure moment agrees the experimental failure moment very well. If taking $h_0^{pub} = h_0$, the calculated failure moment of S5 is M_u^{pub}=39.43 kN·m, this value is 36.4% larger than $M_{u,\exp}^{pub}$=28.9 kN·m of S5. Under heavy level of corrosion of the tensile steel, complete destroy of concrete may occur on both sides of the tensile steel. The maximum depth of removal of concrete might be only twice as thick as the depth of the clear concrete cover of the tensile steel and the majority of the concrete cross section is still available. In this case, the modifying of h_0^{pub} is effective in conservatively calculating the flexural capacity of RC beam with L_{ub}. However, further research work is still needed.

5. Discussion

In this section, keeping the other parameters constant, the effects of ρ_s, f_y, L_{ub}, f_c' (or f_{cu}) and h_c on the ratio of M_u^{pub}/M_u^b are discussed respectively by using the numerical model. Typical test specimens in Cairns and Zhao (1993) are mainly used in the following theoretical analysis.

5.1. Effect of nominal percentage of tensile steel ρ_s

The effect of ρ_s on ratio M_u^{pub}/M_u^b is examined in Fig. 4. Numerical simulation results are

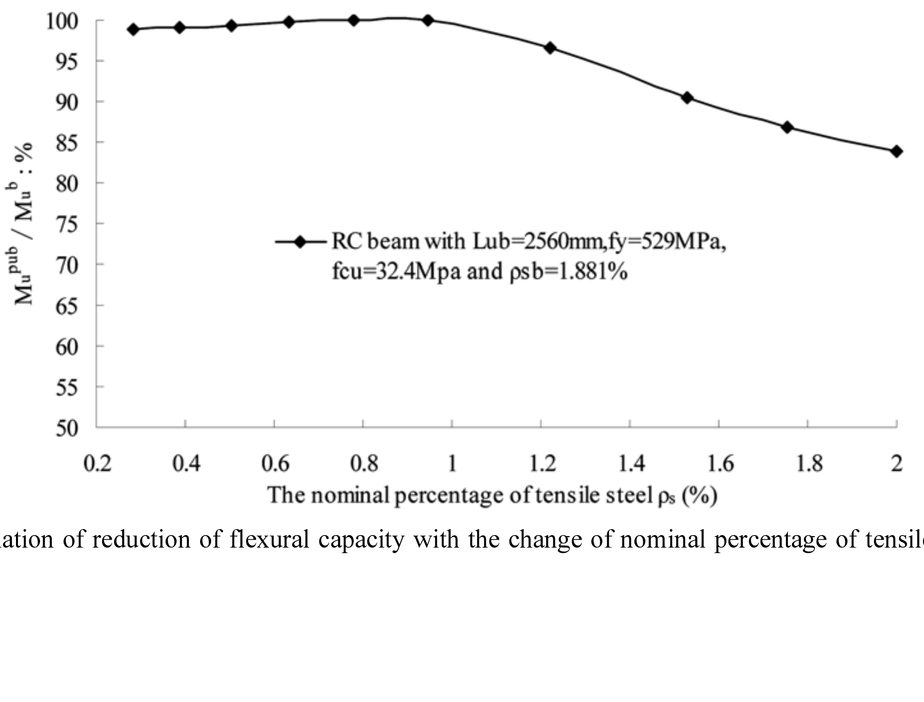

Fig. 4 Variation of reduction of flexural capacity with the change of nominal percentage of tensile steel ρ_s

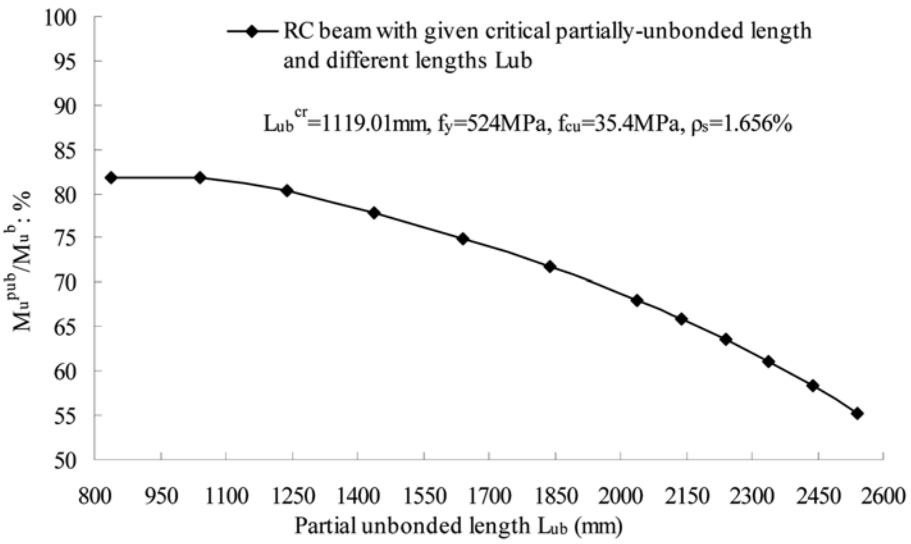

Fig. 5 Variation of reduction of flexural capacity with the change of partially unbonded length L_{ub}

obtained for a beam of identical cross section to beam S9 in Cairns and Zhao (1993), with the same other parameters except for variable A_s. A_s is changed by the change of diameter of tensile steel d and d=12, 14, 16, 18, 20, 22, 25, 28, 30, 32 are chosen. For given f_y and f_{cu} of beam S9, the corresponding ρ_{sb}=1.881%. It is shown that, when ρ_s is low, the tensile steel can yield in the ultimate state and the reduction of the flexural capacity mainly results from the removal of the concrete cover; when ρ_s is larger than 1%, the tensile steel can not yield. As a result, with the increase of ρ_s and approaching ρ_{sb}, reduction of the flexural capacity of RC beam with partially unbonded length increases greatly.

5.2. Effect of partially unbonded length L_{ub}

The effect of L_{ub} on M_u^{pub}/M_u^b is showed in Fig. 5. Numerical simulation results are obtained for a beam of identical cross section to beam S5 in Cairns and Zhao (1993), with the same other parameters except for variable L_{ub}. For given other parameters, the corresponding calculated L_{ub}^{cr} of beam S5 is L_{ub}^{cr}=1119.01 mm. It can be seen from Fig. 5 that, for the beam with $L_{ub} \le L_{ub}^{cr}$, only 18.15% loss of flexural capacity is shown. Due to the yielding of the tensile steel, this reduction mainly results from the removal of the concrete cover; while for the beam with $L_{ub} > L_{ub}^{cr}$, with the increase of the partially unbonded lengths, the higher levels of loss of flexural capacity is observed.

5.3. Effect of yield strength of tensile steel f_y

Raoof and Lin (1997) pointed out that the yield strength of the steel used in Cairns and Zhao (1993) was very high and the losses of strength were likely to increase with increasing yield strength of the main reinforcement. Then, the effect of f_y on M_u^{pub}/M_u^b is theoretically discussed and showed in Fig. 6, where a beam of identical cross section to beam S5 in Cairns and Zhao (1993), with the same other parameters except for variable f_y is used in the numerical simulation. It is shown that, for the RC beam with low f_y, the tensile steel of the RC beam with given L_{ub} can easily

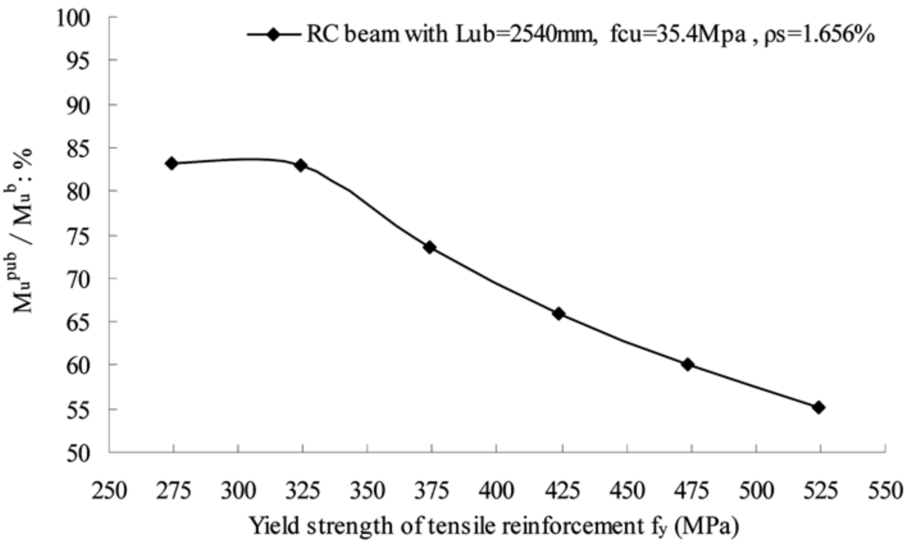

Fig. 6 Variation of reduction of flexural capacity with the change of yield strength of tensile steel f_y

yield in the ultimate state, and the reduction of the flexural capacity mainly results from the removal of the concrete cover; with the increase of f_y, the tensile steel can not yield and larger loss of flexural capacity of RC beam with partially unbonded length relative to perfectly bonded beam is shown. Therefore, loss of flexural capacity of RC beam with partially unbonded length increases with higher f_y.

5.4. Effect of compression strength of concrete f_c' (f_{cu})

The effect of f_c' (f_{cu}) on M_u^{pub}/M_u^b is theoretically discussed and showed in Fig. 7, where a beam of identical cross section to beam S5 in Cairns and Zhao (1993), with the same other parameters except for variable f_c' (f_{cu}) is used in the numerical simulation. It is shown that, under given L_{ub}, with the increase of the f_c' (f_{cu}), the tensile stress of the tensile steel increase correspondingly in the ultimate state of the RC beam. Thus, with the increase of compression strength of concrete, lower loss of flexural capacity of RC beam with partially unbonded length relative to perfectly bonded beam is obtained. The similar results were also obtained from the test results of Cairns and Zhao (1993).

5.5. Effect of exposed concrete depth h_c

The effect of exposed concrete depth h_c on M_u^{pub}/M_u^b is showed in Fig. 8, where a beam of identical cross section to beam S5 in Cairns and Zhao (1993), with the same other parameters except for variable h_c is used in the numerical simulation. It is shown that, with the increase of the h_c, the tensile stress of the tensile steel increase correspondingly in the ultimate state. Thus, the smaller exposed concrete depth h_c or the larger depth of removal of concrete in the RC beam, the greater loss of flexural capacity of RC beam with partially unbonded length relative to perfectly bonded beam is shown.

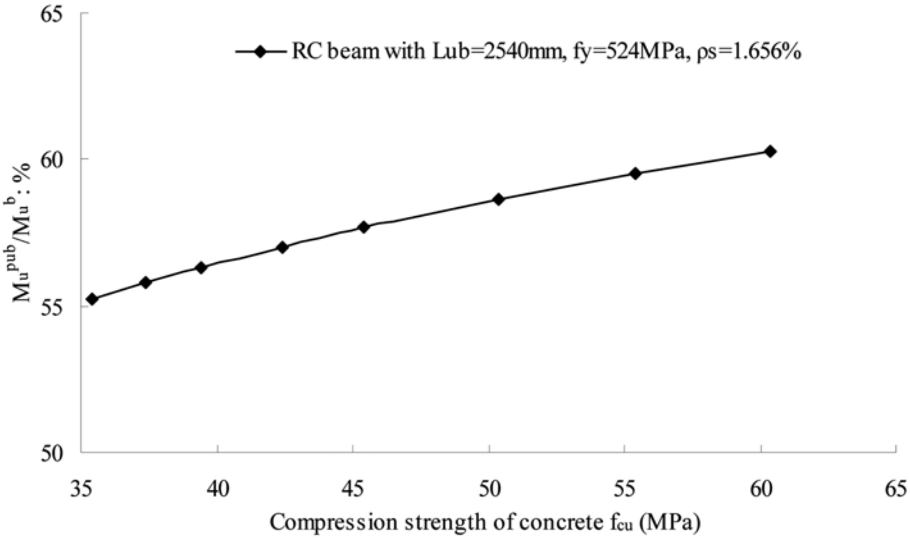

Fig. 7 Variation of reduction of flexural capacity with the change of compression strength of concrete f_{cu}

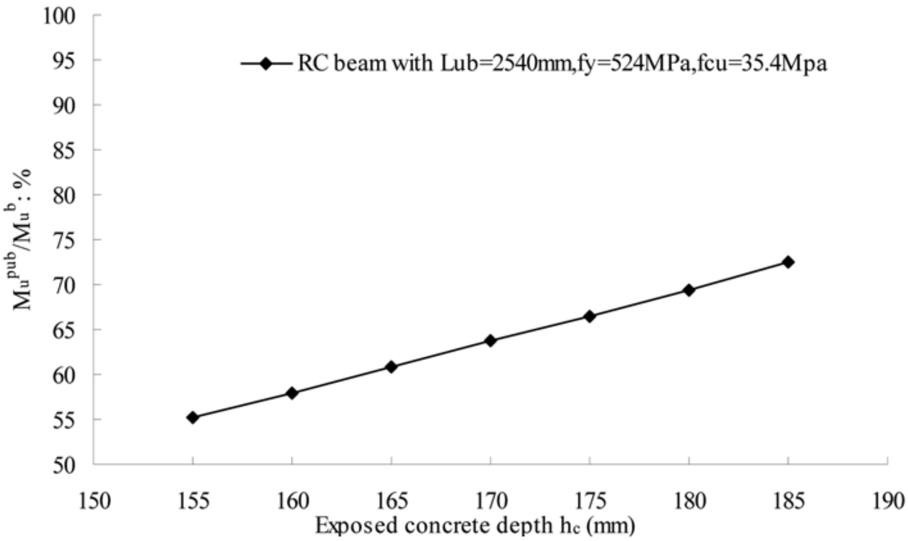

Fig. 8 Variation of reduction of flexural capacity with the change of exposed concrete depth h_c

6. Conclusions

Compatibility condition of deformations of the RC beams with partially unbonded length is proposed. This proposed compatibility condition of deformations and the equilibrium of forces are used to calculate the flexural capacity of RC beam with partially unbonded length. The accuracy of the proposed model is examined by comparing the model's predictions with the results of experimental study carried out by Nokhasteh *et al.* (1992), Cairns and Zhao (1993), Wang *et al.* (2001) and Li (2005). Good agreement is shown between the predicted results and the corresponding test results. Then, the factors which may affect the flexural capacity of RC beam with partially unbonded length, including ρ_s, L_{ub}, f_y, f_c' (or f_{cu}) and h_c, are discussed. The following conclusions are obtained.

1) For the RC beam with given L_{ub} and available concrete cover in tension zone, as long as the tensile steel can yield in the ultimate state, the flexural capacity of the RC beam may not be influenced by the completely loss of bond of the whole beam span;

2) For the RC beam with given L_{ub}, smaller ρ_s, lower f_y, higher f_c' (or f_{cu}) and larger h_c, lower loss of flexural capacity of the RC beam with partially unbonded length relative to the corresponding perfectly bonded RC beam is shown;

3) For the RC beam with given ρ_s, f_y and f_c' (or f_{cu}), the longer partially unbonded length L_{ub}, the higher levels of loss of flexural capacity is observed.

It is concluded that for given RC beam with different material parameters, such as ρ_s, f_y, and f_c' (or f_{cu}), the corresponding critical partially-unbonded lengths L_{ub}^{cr} are quite different. The so-called critical partially-unbonded length obtained from certain experimental results of Li (2005), therefore, is not suitable for all the RC beams with different beam parameters and partially unbonded length. The determining of L_{ub}^{cr} for a given RC beam with L_{ub} should base on the specified geometries, loading form and material parameters.

For the RC beam with given material parameters, partially unbonded length L_{ub} and available concrete cover in tension zone, if $L_{ub} \leq L_{ub}^{cr}$, the tensile steel can yield in the ultimate state and the flexural capacity of the RC beam with L_{ub} is not influenced. The same value of flexural capacity can also be obtained by using the code equations. This is why good agreement between the test results and the calculated results obtained by using the code equations is still showed (Wang *et al.* 2001). However, in the case of $L_{ub} > L_{ub}^{cr}$ and flexural capacity of the RC beam is influenced by the partially unbonded length, code equations are no longer valid.

Due to neglecting the effect of the bond strength between the corroded reinforcing bar and the surrounding concrete, flexural capacity M_u^{pub} of the RC beam with partially unbonded length is the conservative result of the flexural capacity of the identical RC beam with partially length corrosion. In predicting the residual capacity of the existing deteriorated structures in field situation, M_u^{pub} can be used as a low limit of the flexural capacity of the identical RC beam with partially length corrosion.

The propose model can be used to predict the flexural capacity of the RC beam with two-point loading and partially unbonded length symmetrically arranged about the mid-span. The present work is a primary step forward to the further study of the load capacity and failure mode of RC beam with symmetrical partially-unbonded length, where the shear failure of the beam and anchorage failure of the bars near the ends of the breakout zone may occur. Also, only two-point loading is considered. The influence of the form of loading should be considered in the further study.

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Notations

A_s, A_{sc}	cross section area of tensile and compressive steel, respectively;
a_s'	distance from the extreme compression fiber to the centroid of the compression steel
b	beam width of rectangular cross section or width of compression flange of "T" cross section
b_w	width of web of "T" concrete cross section
d	diameter of tensile steel
E_s	modulus of tensile steel reinforcement
E_{sc}	modulus of compressive steel reinforcements
F_c^{pub}	total force of compressive concrete at critical section of RC beam with L_{ub}
F_{sc}^{pub}	total force of compressive steel at critical section of RC beam with L_{ub}
f_c'	cylinder compressive strength of concrete
f_{cu}	cube compressive strength of concrete
f_y	yield strength of the tensile steel reinforcement
f_{yc}	yield strength of compressive steel reinforcements
$g(\mathrm{L_{ub}})$	interpolating function of partial unbonded length L_{ub}
h_0	effective depth of unbonded and bonded RC beam
h_0^{pub}	effective depth of RC beam with L_{ub}
h_c	depth of concrete where reinforcement is exposed
L	length of the beam span
L_{eq}	equivalent plastic region length of unbonded RC beam
L_{ub}	length of complete loss of bond along the RC beam
L_{ub}^{cr}	critical partially-unbonded length of RC beam with L_{ub}
M_u^b	calculated ultimate flexural moment of perfectly bonded RC beam
M_u^{pub}	calculated ultimate flexural moment of RC beam with L_{ub}
$M_{u,\exp}^{pub}$	experimental failure moment of RC beam with L_{ub}
P_u^b	calculated ultimate load of perfectly bonded RC beam
P_u^{pub}	calculated ultimate load of RC beam with L_{ub}
$P_{u,\exp}^{pub}$	experimental ultimate load of RC beam with L_{ub}
x_c^b	depth of the compression zone at the critical section of perfectly bonded RC beam
x_c^{pub}	depth of the compression zone at the critical section of RC beam with L_{ub}
x_c^{ub}	depth of the compression zone at the critical section of unbonded RC beam
β	stress block factor
φ	ratio of L_{eq} to neutral axis depth x_c at ultimate
ρ_s	nominal percentage of tensile steel of RC beam with L_{ub}

ρ_{sb}	balanced percentage of tensile reinforcement of perfectly bonded RC beam
ε_{cd}	strain in concrete at the level of tension reinforcement
ε_{cu}	ultimate strain of concrete in compression
ε_c^b	concrete compression strain in extreme fiber of perfectly bonded RC beam
ε_c^{pub}	concrete compression strain in extreme fiber of RC beam with L_{ub}
ε_s^b	strain of the tensile steel of perfectly bonded RC beam
ε_s^{pub}	strain of tensile steel of RC beam with L_{ub}
ε_s^{ub}	strain of the tensile steel of unbonded RC beam
ε_{st}	strain of longitudinal tensile steel
ε_y	yield strain of steel reinforcement
σ_s^{pub}	stress of tensile steel of RC beam with L_{ub}
σ_s^{ub}	stress of tensile steel of unbonded RC beam