

A multiscale creep model as basis for simulation of early-age concrete behavior

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Abstract A previously published multiscale model for early-age cement-based materials [Pichler, *et al.* 2007. "A multiscale micromechanics model for the autogenous-shrinkage deformation of early-age cement-based materials." *Engineering Fracture Mechanics*, 74, 34-58] is extended towards upscaling of viscoelastic properties. The obtained model links macroscopic behavior, i.e., creep compliance of concrete samples, to the composition of concrete at finer scales and the (supposedly) intrinsic material properties of distinct phases at these scales. Whereas finer-scale composition (and its history) is accessible through recently developed hydration models for the main clinker phases in ordinary Portland cement (OPC), viscous properties of the creep active constituent at finer scales, i.e., calcium-silicate-hydrates (CSH) are identified from macroscopic creep tests using the proposed multiscale model. The proposed multiscale model is assessed by different concrete creep tests reported in the open literature. Moreover, the model prediction is compared to a commonly used macroscopic creep model, the so-called B3 model.

Keywords: concrete; calcium-silicate-hydrates; early-age concrete; multiscale modeling; logarithmic creep; continuum micromechanics; Laplace-Carson transformation.

1. Origin of creep of cement-based materials

Modeling of creep of cement-based materials and the identification of the underlying physical processes in the calcium-silicate-hydrates (CSH)¹ at finer observation scales remain intensively discussed topics within the scientific community. Bažant and coworkers (see, e.g., Bažant, *et al.* (1997)) have developed the, to date, most widely used model for creep of early-age cement-based materials. According to Bažant, *et al.* (1997), the source of creep is the relaxation of *micro-prestress*, which is generated as a reaction to the disjoining pressure in micropores (intra-hydrate pores). Increased stresses at these so-called creep sites increase the ease of bond breakage and, hence, increase the creep rate. Hereby, recently formed hydration products contain a high density of creep sites, whereas continuous microprestress relaxation reduces the creep potential of CSH.

Recently, Jennings pointed out that "... *there are fundamental chemical and thermodynamic arguments against the idea that large internal stresses are formed during hydration. CSH forms under*

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near-equilibrium conditions with respect to the aqueous phase, making it unlikely that high internal stresses that could act as creep sites would develop ..." (Thomas and Jennings 2006). Jennings and coworkers have developed a structural model for CSH at the nanometer-scale accounting for the colloidal nature of CSH. Hereby, CSH is described as an aggregation of precipitated, colloid-sized particles (Jennings 2000, 2004) with the "basic building blocks" (radius of 1.1 nm) aggregating into "spherical globules" (radius of 2.8 nm). The latter aggregate further into (i) low-density CSH (CSH-LD) or (ii) high-density CSH (CSH-HD), depending on w/c -ratio, age, and environmental conditions (Jennings 2000, 2004). According to Jennings (2004), the origin of viscous deformations is explained by the re-arrangement (dislocation) of globules under shear stress (or drying).

In both microscopic (Pichler and Lackner 2008) and macroscopic creep tests, a logarithmic-type behavior is encountered, i.e., the creep compliance is proportional to $\ln(1 + t/\tau^v)$, where τ^v denotes the characteristic time of the creep process. According to Nabarro (2001), the two mechanisms leading to logarithmic creep in crystalline solids are either

- *work hardening*: dislocations move forward under the applied stress by overcoming potential barriers, while successively raising the height of the potential barriers or
- *exhaustion*: while neglecting work hardening, the barriers to dislocation motion do not have equal activation energies; those with relatively small activation energies are overcome faster than those with relatively large activation energies; if each barrier which is overcome contributes an equal increment of strain, the total strain increases linearly with the increasing activation energy, where the latter is a logarithmic function of time.

The mentioned dislocation-based mechanism is consistent with the creep mechanism proposed in Jennings (2004).

Within the thermodynamic framework of chemically reactive porous media (Ulm and Coussy 1995, 1996), and also supported by experimental evidence (Lura, *et al.* 2003), the short-term creep process (microdiffusion) and the stress state in the solid skeleton [influenced in the (macroscopically) unloaded state by the stress state in the capillary pores] are coupled. This coupling has been postulated (Bažant 1988) and recently been considered in the modeling of creep and shrinkage of concrete in the context of the effective stress concept (Gawin, *et al.* 2007, 2006)².

In this paper, the logarithmic-type creep behavior encountered during nanoindentation tests is transferred to the macroscale by developing a multiscale model for basic creep of early-age cement-based materials. With the multiscale model at hand, viscous properties of the creep-active constituent at finer scales, i.e., CSH, can be assessed by means of results from macroscopic creep tests. The focus of this paper is the top-down experimental identification, i.e., based on macroscopic experiment, of CSH creep properties suitable for multiscale modeling. At finer scales, the respective properties of CSH are assessed by means of nanoindentation, providing the constitutive behavior and the processes associated with creep of CSH (bond breakage, ...) in a homogenized manner.

The paper is structured as follows: A review of multiscale modeling attempts for concrete is given in Section 2. In Sections 3 and 4 the previously developed multiscale model (Pichler, *et al.* 2007) and

¹Standard cement chemistry abbreviations are used throughout this paper: C=CaO, S=SiO₂, A=Al₂O₃, F=Fe₂O₃, $\bar{S}$ =SO₃, H=H₂O.

²In this paper, this coupling will be considered by the time constant of creep of CSH being a function of the capillary stress state.

upscaling of elastic properties are reviewed. Section 5 deals with upscaling of the viscoelastic compliance. The presented upscaling scheme is used in Section 6 for determination of creep parameters of CSH through a top-down application of the multiscale model starting from macroscopic creep tests.

2. Multiscale modeling attempts for concrete

The choice of the appropriate homogenization method employed for upscaling of the material response from one scale to the next higher one strongly depends on the type of microstructure encountered at the respective scale of observation:

- When the material can be described by a combination of local fluctuations at the level of the elementary (unit) cell and a drift at the level of the macroscopic structure, the periodic media theory is commonly employed.
- Cement-based materials are characterized by randomly distributed heterogeneities, which are well-described by the effective media theory. Within this theory mean-field methods characterized by the application of displacements (strains) or forces (stresses) at the boundary of the representative volume element (RVE) have been developed. Basically, there are two approaches for determination of the response of the composite material:
 - numerical techniques employing, e.g., the finite element method, the boundary element method, ... (see, e.g. Cusatic and Cedolin 2007, Grondin, *et al.* 2007, Zohdi 2004, Füssl, *et al.* 2008), and
 - analytical techniques based on continuum micromechanics (see, e.g. Suquet 1997, Bernard, *et al.* 2003, Stora, *et al.* 2006, Sanahuja, *et al.* 2007). Hereby, the so-called localization tensor, relating the local (strain) quantities within the RVE to the homogeneous (strain) field imposed at the boundary of the RVE, is estimated by means of the well-known solution for ellipsoidal inclusions by Eshelby (1957).

3. Proposed multiscale model

The multiscale model proposed in Pichler, *et al.* (2007) for upscaling of viscoelastic properties and autogenous-shrinkage of early-age cement-based materials comprises four length scales, which are identified as (see Fig. 1)³:

- Scale I comprises the four clinker phases, high-density CSH (CSH-HD) and low-density CSH (CSH-LD), and the water and air phase. The four clinker phases, which do not exhibit time-dependent behavior, are condensed into one material phase (Scale Ia). The constituents showing time-dependent behavior, on the other hand, are combined at Scale Ib-1, where CSH-HD is located in the space confined by the previously formed CSH-LD. At the porous CSH scale (Scale Ib-2), water and air are considered as inclusions in a matrix constituted by the homogenized material of Scale Ib-1.
- At Scale II (*cement-paste scale*), anhydrous cement (homogenized material of Scale Ia), gypsum $\text{C}\bar{\text{S}}\text{H}_2$, portlandite CH, and reaction products from C_3A and C_4AF hydration form inclusions in a matrix constituted by the homogenized material of Scale Ib-2.

³Morphological investigations at lower scales of observation can be found in, e.g., Bentz (1997) [anhydrous cement], Diamond (2004) [cement paste], and Neubauer and Jennings (2000) [CSH].

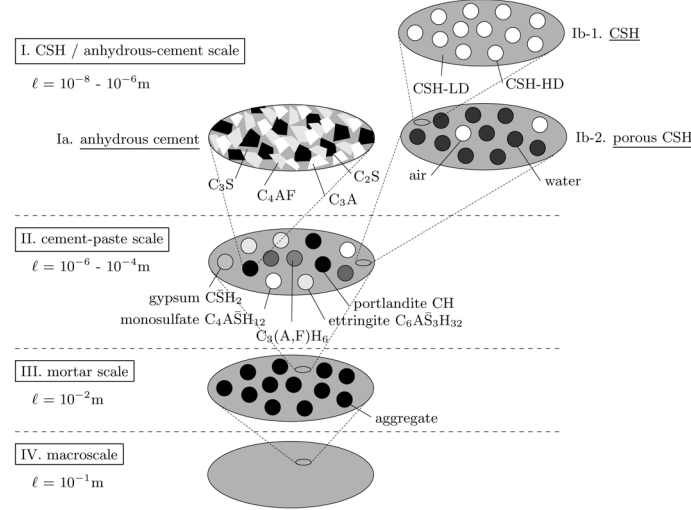
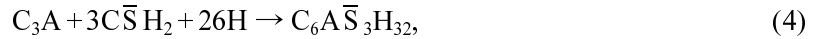
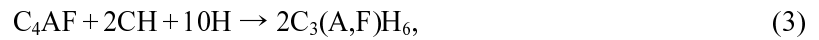
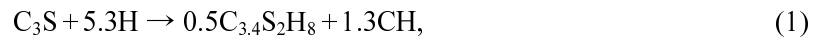


Fig. 1 Scales of observation for upscaling of properties of cement-based materials [ℓ =size of representative volume element (RVE)]

- At Scale III (*mortar or concrete scale*), aggregates are represented as inclusions in the cement paste (homogenized material of Scale II). In addition to aggregates and cement paste, the interface transition zone (ITZ) may be introduced at Scale II. Since the ITZ mainly influences strength and transport properties of concrete rather than viscous deformations, it is not considered in the present model.
- Finally, at Scale IV (*macroscale*), concrete is treated as a continuum.

In order to determine the volume fractions of the different phases in the respective RVEs, the following set of stoichiometric reactions is employed for the four main clinker phases of ordinary Portland cement (OPC) (Tennis and Jennings 2000):



Eqs. (4) to (6) describe the formation of calcium aluminate hydrates from C₃A in the presence of gypsum, which is added to prevent rapid setting of C₃A.

The hydration extent is described by the degree of hydration of the clinker phases, ξ_x with $x \in \{\text{C}_3\text{S}, \text{C}_2\text{S}, \text{C}_4\text{AF}\}$. A recently developed (Bernard, *et al.* 2003) and refined (Pichler 2007) kinetics model is used to determine the hydration history $\xi_x(t)$. Input parameters for the kinetics model comprise the Blaine (grinding) fineness ϕ of the employed Portland cement, the medium initial radius of the clinker grains R , the mass fractions of the clinker phases, and the water/cement-ratio

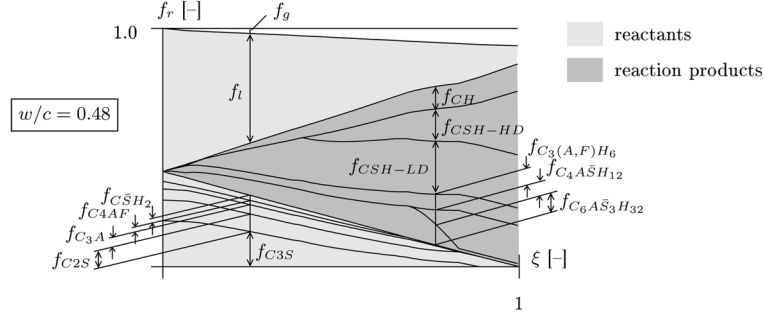


Fig. 2 Variation of the volume fractions at the cement-paste scale as a function of the overall degree of hydration ξ for shotcrete employed at the Lainzer tunnel (see Table 2)

w/c . Based on the stoichiometric reactions given in Eqs. (1) to (6), and the molar masses $\mathcal{M}$ and densities ρ of the different material phases (see, e.g. Tennis and Jennings 2000), the volume fractions of the different phases (at the cement-paste scale) can be determined as a function of the hydration degrees ξ_x and the mass fractions of the clinker phases m_x . This is shown exemplarily for the volume fraction of $C_{3,4}S_2H_8$ appearing in Eqs. (1) and (2):

$$f_{C_{3,4}S_2H_8}(t) = \left[\xi_{C_3S}(t) \frac{m_{C_3S}}{\mathcal{M}_{C_3S}} \frac{0.5\mathcal{M}_{C_{3,4}S_2H_8}}{\rho_{C_{3,4}S_2H_8}} + \xi_{C_2S}(t) \frac{m_{C_2S}}{\mathcal{M}_{C_2S}} \frac{0.5\mathcal{M}_{C_{3,4}S_2H_8}}{\rho_{C_{3,4}S_2H_8}} \right] \bar{\rho}, \quad (7)$$

with

$$\bar{\rho} = \frac{1}{\sum_x \frac{m_x}{\rho_x} + \frac{w/c}{\rho_H} \sum_x m_x}. \quad (8)$$

The overall degree of hydration, ξ , is computed as

$$\xi = \frac{\sum_x m_x \xi_x}{\sum_x m_x}, \quad (9)$$

where $x \in \{C_3S, C_2S, C_3A, C_4AF, C\bar{S}H_2\}$ and m_x represents the mass fractions of the four clinker phases and gypsum. Hereby, $\xi_{C\bar{S}H_2}$ is linearly coupled to ξ_{C_3A} [see Eq. (4)]. Fig. 2 shows the evolution of the volume fractions at the cement-paste scale as a function of ξ for shotcrete with a w/c of 0.48 (see Table 2).

4. Upscaling of elastic properties

The four length scales introduced in the previous section obey the separability of scale condition, i.e., they are separated one from each other by at least one order of magnitude. With volume fractions of the different phases at the respective observation scales at hand, continuum micromechanics is employed to estimate effective elastic properties. For homogenization at Scale Ia, the self-consistent (SC) scheme, suitable for a polycrystalline microstructure, is used (Kroener 1958,

Hershey 1954). During homogenization at Scales Ib to III, the matrix-inclusion type morphology is taken into account by the Mori-Tanaka (MT) scheme (1973). Homogenization schemes based on continuum micromechanics consider a RVE subjected to a homogeneous strain $\mathbf{E}$ at its boundary. These schemes depart from the definition of the so-called strain-localization tensor $\mathbb{A}$, linking the effective strain tensor $\mathbf{E}$ with the local strain tensor $\boldsymbol{\varepsilon}$ at the location $\mathbf{x}$:

$$\boldsymbol{\varepsilon}(\mathbf{x}) = \mathbb{A}(\mathbf{x}) : \mathbf{E} \quad (10)$$

The effective strain tensor $\mathbf{E}$ represents the volume average of the local strain tensor $\boldsymbol{\varepsilon}$:

$$\mathbf{E} = \langle \boldsymbol{\varepsilon}(\mathbf{x}) \rangle_V = \frac{1}{V} \int_V \boldsymbol{\varepsilon}(\mathbf{x}) dV. \quad (11)$$

Inserting Eq. (10) into Eq. (11), one gets $\mathbf{E} = \langle \mathbb{A}(\mathbf{x}) \rangle_V : \mathbf{E}$ and, thus, $\langle \mathbb{A}(\mathbf{x}) \rangle_V = \mathbb{I}$. Considering an ellipsoidal inclusion i embedded in a reference medium characterized by the material tensor $\mathbf{c}_0$, the strain-localization tensor $\mathbb{A}$ within the domain i is constant and given by (Eshelby 1957)

$$\mathbb{A}_i = [\mathbb{I} + \mathbb{S}_i : (\mathbf{c}_0^{-1} : \mathbf{c}_i - \mathbb{I})]^{-1} : \left\{ \sum_{r=0,i} f_r [\mathbb{I} + \mathbb{S}_r : (\mathbf{c}_0^{-1} : \mathbf{c}_r - \mathbb{I})]^{-1} \right\}^{-1} = \text{const.}, \quad (12)$$

with $\mathbf{c}_i$ as the material tensor of the inclusion i and $\mathbf{c}_0$ as the material tensor of the reference medium. $\mathbb{S}_i$ denotes the Eshelby tensor, conditioned by the geometric properties of the inclusion and the elastic properties of the reference medium.

The volume average of the local stress tensor $\boldsymbol{\sigma}(\mathbf{x})$ determines the effective stress tensor $\boldsymbol{\Sigma}$:

$$\boldsymbol{\Sigma} = \langle \boldsymbol{\sigma}(\mathbf{x}) \rangle_V = \frac{1}{V} \int_V \boldsymbol{\sigma}(\mathbf{x}) dV. \quad (13)$$

Considering a linear-elastic constitutive law for the r -th material phase, linking the local strain tensor with the local stress tensor,

$$\boldsymbol{\sigma}_r(\mathbf{x}) = \mathbf{c}_r : \boldsymbol{\varepsilon}_r(\mathbf{x}) \quad (14)$$

and Eq. (10) in Eq. (13) one gets

$$\boldsymbol{\Sigma} = \langle \mathbf{c}(\mathbf{x}) : \mathbb{A}(\mathbf{x}) \rangle_V : \mathbf{E} \quad (15)$$

Comparison with $(\boldsymbol{\Sigma} = \mathbb{C}_{\text{eff}} : \mathbf{E})^4$ gives access to the effective material tensor $\mathbb{C}_{\text{eff}}$:

$$\mathbb{C}_{\text{eff}} = \langle \mathbf{c}(\mathbf{x}) : \mathbb{A}(\mathbf{x}) \rangle_V \quad (16)$$

Considering the morphology of the composite material, the unknown strain localization tensor $\mathbb{A}$, which so far is available for a single inclusion (see Eq. (12)), can be estimated based on the choice of $\mathbf{c}_0$:

- In case the microstructure is characterized by a distinct matrix/inclusion-type morphology, $\mathbf{c}_0$ is set equal to the material tensor of the matrix material $\mathbf{c}_m$. This estimation leads to the Mori-Tanaka (MT) scheme (Mori and Tanaka 1973).
- For a polycrystalline microstructure, i.e., the material phases are equally dispersed, and none of them forms a matrix, $\mathbf{c}_0$ is replaced by the effective material tensor $\mathbb{C}_{\text{eff}}$. The obtained implicit method is referred to as self-consistent (SC) scheme (Hershey 1954, Kroener 1958).

⁴Levin's theorem states that the effective state equation is of the same form as the local state equation (Zaoui 1997).

Using

$$\langle \mathbb{A}(\mathbf{x}) \rangle_V = \frac{V_0}{V} \langle \mathbb{A}(\mathbf{x}) \rangle_{V_0} + \frac{V_i}{V} \langle \mathbb{A}(\mathbf{x}) \rangle_{V_i} = \mathbb{I} \rightarrow f_0 \langle \mathbb{A}(\mathbf{x}) \rangle_{V_0} = \mathbb{I} - f_i \mathbb{A}_i, \quad (17)$$

where $\langle \mathbb{A}(\mathbf{x}) \rangle_{V_i} = \mathbb{A}_i = \text{const.}$ was used and f_i and f_0 denote the volume fractions of the inclusion and reference medium, respectively, and Eq. (12), one gets the volume average of the localization tensor over the reference medium as⁵

$$\langle \mathbb{A}(\mathbf{x}) \rangle_{V_0} = \left\{ \sum_{r \in 0, i} f_r [\mathbb{I} + \mathbb{S}_r : (\mathbf{c}_0^{-1} : \mathbf{c}_r - \mathbb{I})]^{-1} \right\}^{-1}. \quad (19)$$

Considering Eqs. (19) and (12) in Eq. (16) gives access to the effective material tensor

$$\begin{aligned} \mathbb{C}_{\text{eff}} &= f_0 \mathbf{c}_0 : \langle \mathbb{A}(\mathbf{x}) \rangle_{V_0} + f_i \mathbf{c}_i : \mathbb{A}_i \\ &= \left\{ \sum_{r \in 0, i} f_r \mathbf{c}_r [\mathbb{I} + \mathbb{S}_r : (\mathbf{c}_0^{-1} : \mathbf{c}_r - \mathbb{I})]^{-1} \right\} : \left\{ \sum_{r \in 0, i} f_r [\mathbb{I} + \mathbb{S}_r : (\mathbf{c}_0^{-1} : \mathbf{c}_r - \mathbb{I})]^{-1} \right\}^{-1}. \end{aligned} \quad (20)$$

Eq. (20) can be extended to multiple types of inclusions, reading

$$\mathbb{C}_{\text{eff}} = \left\{ \sum_r f_r \mathbf{c}_r : [\mathbb{I} + \mathbb{S}_r : (\mathbf{c}_0^{-1} : \mathbf{c}_r - \mathbb{I})]^{-1} \right\} : \left\{ \sum_r f_r [\mathbb{I} + \mathbb{S}_r : (\mathbf{c}_0^{-1} : \mathbf{c}_r - \mathbb{I})]^{-1} \right\}^{-1}, \quad (21)$$

with $r \in \{\text{matrix material} = \text{reference medium } 0, \text{inclusion } 1, \text{inclusion } 2, \dots\}$ for the case of the MT scheme and $r \in \{\text{material phases}\}$ in case of the SC scheme with the material tensor of the reference medium $\mathbf{c}_0$ set equal to the effective material tensor $\mathbb{C}_{\text{eff}}$. In the following, Eq. (21) is specialized for the application to cement-based materials, characterized by isotropic material behavior. Thus, Eq. (21) can be reduced to the specification of the effective shear and bulk modulus, μ_{eff} and k_{eff} , respectively:

$$\begin{aligned} {}^5 f_0 \langle \mathbb{A}(\mathbf{x}) \rangle_{V_0} &= \mathbb{I} - f_i [\square]^{-1} : \{\bullet\}^{-1} \\ &= \{\bullet\} : \{\bullet\}^{-1} - f_i [\square]^{-1} : \{\bullet\}^{-1} \\ &= (\{\bullet\} - f_i [\square]^{-1}) : \{\bullet\}^{-1} \\ &= (f_0 [\mathbb{I} + \mathbb{S}_0 : (\mathbf{c}_0^{-1} : \mathbf{c}_0 - \mathbb{I})]^{-1} + f_i [\square]^{-1} - f_i [\square]^{-1}) : \{\bullet\}^{-1} \\ &= f_0 \{\bullet\}^{-1} \end{aligned} \quad (18)$$

with the abbreviations

$$\begin{aligned} \{\bullet\} &= \left\{ \sum_{r=0, i} f_r [\mathbb{I} + \mathbb{S}_r : (\mathbf{c}_0^{-1} : \mathbf{c}_r - \mathbb{I})]^{-1} \right\} \text{ and} \\ [\square] &= [\mathbb{I} + \mathbb{S}_i : (\mathbf{c}_0^{-1} : \mathbf{c}_i - \mathbb{I})]. \end{aligned}$$

- For the SC scheme,

$$\mu_{\text{eff}} = \frac{\sum_r f_r \mu_r \left[1 + \beta \left(\frac{\mu_r}{\mu_{\text{eff}}} - 1 \right) \right]^{-1}}{\sum_r f_r \left[1 + \beta \left(\frac{\mu_r}{\mu_{\text{eff}}} - 1 \right) \right]^{-1}} \quad \text{and} \quad k_{\text{eff}} = \frac{\sum_r f_r k_r \left[1 + \alpha \left(\frac{k_r}{k_{\text{eff}}} - 1 \right) \right]^{-1}}{\sum_r f_r \left[1 + \alpha \left(\frac{k_r}{k_{\text{eff}}} - 1 \right) \right]^{-1}}, \quad (22)$$

where α and β represent the volumetric and deviatoric part of the Eshelby tensor $\mathbb{S}$ specialized for spherical inclusions, reading

$$\alpha = \frac{3k_{\text{eff}}}{3k_{\text{eff}} + 4\mu_{\text{eff}}} \quad \text{and} \quad \beta = \frac{6(k_{\text{eff}} + 2\mu_{\text{eff}})}{5(3k_{\text{eff}} + 4\mu_{\text{eff}})}. \quad (23)$$

- For the MT scheme,

$$\mu_{\text{eff}} = \frac{\sum_r f_r \mu_r \left[1 + \beta \left(\frac{\mu_r}{\mu_m} - 1 \right) \right]^{-1}}{\sum_r f_r \left[1 + \beta \left(\frac{\mu_r}{\mu_m} - 1 \right) \right]^{-1}} \quad \text{and} \quad k_{\text{eff}} = \frac{\sum_r f_r k_r \left[1 + \alpha \left(\frac{k_r}{k_m} - 1 \right) \right]^{-1}}{\sum_r f_r \left[1 + \alpha \left(\frac{k_r}{k_m} - 1 \right) \right]^{-1}}, \quad (24)$$

with

$$\alpha = \frac{3k_m}{3k_m + 4\mu_m} \quad \text{and} \quad \beta = \frac{6(k_m + 2\mu_m)}{5(3k_m + 4\mu_m)}. \quad (25)$$

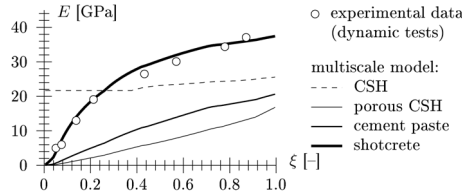


Fig. 3 Comparison between test results (Lackner, *et al.* 2002) and multiscale model for upscaling of elastic properties for shotcrete employed at the Lainzer tunnel (see Table 2) for isothermal conditions ($T = \text{const.} = 11^\circ\text{C}$)

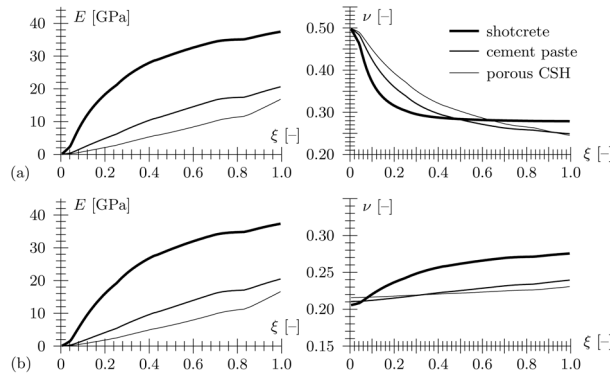


Fig. 4 Result from upscaling of elastic properties for shotcrete employed at the Lainzer tunnel (see Table 2) for isothermal conditions ($T = \text{const.} = 20^\circ\text{C}$) considering (a) $k_w = 2.3$ GPa and (b) $k_w = 0$

Applying the SC scheme to Scale Ia, the different material phases are given by $r \in \{\text{clinker phases } C_3S, C_2S, C_3A, C_4AF\}$. The MT scheme is applied at Scales Ib.1, Ib.2, II, and III with the material phases given by $r \in \{\text{matrix material } m \text{ and multiple inclusions, such as, e.g., water and air at Scale Ib.2}\}$. In Eqs. (24) and (25), the material matrix m is represented by low-density CSH at Scale Ib.1, the homogenized material determined at Scale Ib.1 at Scale Ib.2, the homogenized material determined at Scale Ib.2 at Scale II, and the homogenized material determined at Scale II at Scale III. Fig. 3 shows a comparison of the elastic properties predicted by the multiscale model and test results for shotcrete, using the finer-scale input listed in Table 2 (see Appendix A). Hereby, the effective Poisson's ratio, ν_{eff} , is continuously decreasing in case the bulk modulus of water is set to $k_w = 2.3 \text{ GPa}$, starting from an initial value of 0.5. For $k_w = 0$, on the other hand, ν_{eff} is continuously increasing (see Fig. 4).

5. Upscaling of creep properties

Viscous material response is characterized by (i) an increase of deformation during constant loading (creep) and (ii) a decrease of stress for constraint deformation (relaxation). The viscous response is commonly described by the creep compliance $J \text{ [Pa}^{-1}\text{]}$ and the relaxation modulus $R \text{ [Pa]}$, both dependent on time. The creep compliance associated with uniaxial loading is determined as

$$J(t) = \frac{\varepsilon(t)}{\sigma_0}, \quad (26)$$

with $\varepsilon(t)$ denoting the measured strain, and σ_0 representing the applied constant stress. The relaxation modulus, on the other hand, is determined from the measured stress decrease $\sigma(t)$ in consequence of a constant strain ε_0 as

$$R(t) = \frac{\sigma(t)}{\varepsilon_0}. \quad (27)$$

Introducing the Boltzmann convolution integral, Eqs. (26) and (27) can be expanded towards variable (non-constant) stress or strain histories, respectively:

$$\varepsilon(t) = \int_0^t J(t-\tau) \frac{\partial \sigma(\tau)}{\partial \tau} d\tau \quad \text{and} \quad \sigma(t) = \int_0^t R(t-\tau) \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \quad (28)$$

where τ denotes the time instant of loading. Applying the Laplace transform⁶ to Eqs. (28) gives

$$\hat{\varepsilon}(p) = p \hat{J}(p) \hat{\sigma}(p) \quad \text{and} \quad \hat{\sigma}(p) = p \hat{R}(p) \hat{\varepsilon}(p), \quad \text{with} \quad p \hat{J}(p) = \frac{1}{p \hat{R}(p)} \quad (32)$$

⁶Whereas the Laplace transformation of $f(t)$ is defined as

$$\mathcal{L}[f(t)] = \hat{f}(p) = \int_0^\infty f(t) e^{-pt} dt, \quad (29)$$

with p as the complex variable, the Laplace-Carson transformation of $f(t)$ is given as

$$\mathcal{LC}[f(t)] = f^*(p) = p \int_0^\infty f(t) e^{-pt} dt. \quad (30)$$

Hence, $f^*(p) = p \hat{f}(p)$. The inverse Laplace-Carson transformation is defined in the complex plane as

$$\mathcal{LC}^{-1}[f^*(p)] = f(t) = \frac{1}{2i\pi} \int_\Omega \frac{f^*(p)}{p} e^{pt} dp, \quad (31)$$

where Ω is a parallel to the imaginary axis having all poles of $f^*(p)$ to the left.

considering that the Laplace transform of the convolution integral becomes a multiplication and $\partial \varepsilon / \partial \tau$ turns into $p \hat{\varepsilon}$. Considering the definition of the Laplace-Carson transformation as $f^* = p \hat{f}$ in Eqs. (32) yields

$$\varepsilon^*(p) = J^*(p) \sigma^*(p) \text{ and } \sigma^*(p) = R^*(p) \varepsilon^*(p), \text{ with } J^*(p) = \frac{1}{R^*(p)}. \quad (33)$$

The analogous form of $\sigma^* = R^* \varepsilon$ in Eq. (33) and the elastic constitutive law $\sigma = E \varepsilon$ is the basis for the “correspondence principle” (Lee 1955, Mandel 1966, Laws and McLaughlin 1978). According to this principle, viscoelastic problems are solved using the respective solution of the elastic problem in the Laplace-Carson domain.

The Laplace-Carson transform method (Lee 1955) for the solution of linear viscoelastic boundary value problems (BVPs) is characterized by the elimination of the time dependence by applying the Laplace-Carson transform to the field equation (which contains the time dependent moduli) as well as the boundary conditions,⁷ and solving the “corresponding” elastic problem in the Laplace-Carson domain. The application of the Laplace-Carson transform method is restricted to BVPs with the location of the boundary conditions in tractions and displacements fixed in time. Hence, the method can be adopted for derivation of viscoelastic homogenization schemes, which are based on an inclusion embedded in an (infinite) matrix with homogeneous boundary conditions, as is the case in the framework of continuum micromechanics. Applying this method, the elastic material parameters, e.g., the shear compliance $J^{\text{dev}} = 1/\mu$, where μ is the shear modulus, are replaced by the Laplace-Carson transform of the respective viscoelastic material parameters, e.g., the Laplace-Carson transform of the creep compliance associated with deviatoric creep $J^{\text{dev}*}$. The solution of the viscoelastic problem in the time domain is obtained by inverse Laplace-Carson transformation. E.g., application of the correspondence principle to the MT scheme introduced for upscaling of elastic properties in Section 4 gives access to the effective creep compliance of matrix-inclusion type composites (see, e.g. Beurthey and Zaoui (2000) for an application of the correspondence principle to the self-consistent scheme).

As outlined in the Section 1, viscoelastic behavior of cement-based materials originates from dislocation-like processes within CSH. Hence, as for homogenization at Scale Ib2, viscoelastic material behavior is assigned to the matrix material (CSH), while the inclusions (water, air) exhibit elastic deformations only. Motivated by the macroscopic observation that, after a period of rapid decrease, the compliance rate of cement-based materials follows $\dot{J} \sim 1/t$ (Ulm, *et al.* 1999) (see, e.g. experimental results in Acker and Ulm 2001), a logarithmic-type deviatoric creep law is employed to describe creep of CSH:

$$J_{\text{CSH}}^{\text{dev}}(t - \tau) = \frac{1}{\mu_{\text{CSH}}} + J_{\text{CSH}}^{\nu, \text{dev}} \ln \left[1 + \frac{t - \tau}{\tau_{\text{CSH}}^{\nu, \text{dev}}} \right]. \quad (34)$$

Volumetric creep of CSH is, in the absence of a suitable experimental test method at the nano-to micrometer scale of observation, omitted. In addition to observations at the macroscale, this type of creep law was also found at the μm -scale of observation by means of nanoindentation tests (Pichler and Lackner 2008).

The Laplace-Carson transform of the deviatoric creep compliance of CSH given in Eq. (34) reads

$$J_{\text{CSH}}^{\text{dev}*} = \mathcal{LC}[J_{\text{CSH}}^{\text{dev}}(t - \tau)] = \frac{1}{\mu_{\text{CSH}}} + J_{\text{CSH}}^{\nu, \text{dev}} \exp[p \tau_{\text{CSH}}^{\nu, \text{dev}}] \Gamma[0, p \tau_{\text{CSH}}^{\nu, \text{dev}}], \quad (35)$$

with Γ denoting the incomplete gamma-function.⁸ Applying the correspondence principle to the

⁷Hence, this method is restricted to BVPs with boundary conditions admitting such an operation.

effective creep compliance obtained by the MT scheme [Eq. (24.1)], the Laplace-Carson transform of the effective deviatoric creep compliance at Scale Ib.2 is obtained as

$$J_{\text{eff}}^{\text{dev}*} = \frac{\sum_r f_r \left[1 + \beta^* \left(\frac{J_m^{\text{dev}*}}{J_r^{\text{dev}*}} - 1 \right) \right]^{-1}}{\sum_r \frac{f_r}{J_r^{\text{dev}*}} \left[1 + \beta^* \left(\frac{J_m^{\text{dev}*}}{J_r^{\text{dev}*}} - 1 \right) \right]^{-1}}, \quad (37)$$

with $r \in \{m = \text{CSH, air, water}\}$. In Eq. (37),

$$\beta^* = \frac{6(1/J_m^{\text{vol}*} + 2/J_m^{\text{dev}*})}{5(3/J_m^{\text{vol}*} + 4/J_m^{\text{dev}*})} \quad (38)$$

represents the deviatoric part of the (Laplace-Carson transformed) Eshelby tensor specialized for spherical inclusions. In Eqs. (37) and (38), $J_r^{\text{dev}*} = 1/\mu_r$ and $J_m^{\text{vol}*} = 1/k_m$ for the case of elastic material response. Inserting Eq. (35) into Eq. (37) and performing the inverse Laplace-Carson transformation gives access to the effective deviatoric creep compliance, $J_{\text{eff}}^{\text{dev}}(t) = \mathcal{LC}^{-1}[J_{\text{eff}}^{\text{dev}*}]$. Hereby, the inverse transformation was performed in a pointwise manner (for discrete values of $t > 0$) by applying the Gaver-Stehfest algorithm (Stehfest 1970). Implying an affine form of the creep compliance of CSH, $J_{\text{CSH}}^{\text{dev}}$ [see Eq. (34)], and the effective creep compliance, $J_{\text{eff}}^{\text{dev}}(t)$, respectively, the discrete points from inverse transformation are approximated by

$$J_{\text{eff}}^{\text{dev}}(t - \tau) = \frac{1}{\mu_{\text{eff}}[\xi(\tau)]} + J_{\text{eff}}^{\nu, \text{dev}}[\xi(\tau)] \ln \left(1 + \frac{t - \tau}{\tau_{\text{eff}}^{\nu, \text{dev}}[\xi(\tau)]} \right), \quad (39)$$

giving access to the effective creep parameters $J_{\text{eff}}^{\nu, \text{dev}}[\xi(\tau)]$ and $\tau_{\text{eff}}^{\nu, \text{dev}}[\xi(\tau)]$ [see Fig. 5(a)]. In Eq. (39), $\mu_{\text{eff}}[\xi(\tau)]$ is the effective shear modulus determined according to Eq. (24.1). Examining the volumetric part of the Eshelby tensor specialized for spherical inclusions, with

$$\alpha^* = \frac{3/J_m^{\text{vol}*}}{3/J_m^{\text{vol}*} + 4/J_m^{\text{dev}*}}, \quad (40)$$

deviatoric creep in the matrix material (CSH) triggers (effective) volumetric creep deformations at Scale Ib.2. The Laplace-Carson transform of the effective volumetric creep compliance is obtained as

$$J_{\text{eff}}^{\text{vol}*} = \frac{\sum_r f_r \left[1 + \alpha^* \left(\frac{J_m^{\text{vol}*}}{J_r^{\text{vol}*}} - 1 \right) \right]^{-1}}{\sum_r \frac{f_r}{J_r^{\text{vol}*}} \left[1 + \alpha^* \left(\frac{J_m^{\text{vol}*}}{J_r^{\text{vol}*}} - 1 \right) \right]^{-1}}, \quad (41)$$

⁸The incomplete gamma function $\Gamma[a, z]$ satisfies

$$\Gamma[a, z] = \int_z^\infty t^{a-1} e^{-t} dt. \quad (36)$$

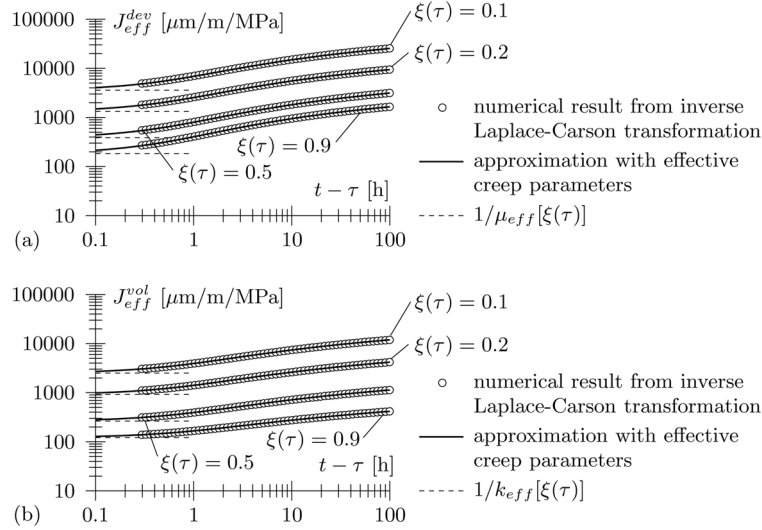


Fig. 5 Approximation of effective creep compliance obtained from inverse Laplace-Carson transformation at Scale Ib.2 for different hydration extents (using $J_{CSH}^{v,dev} = 0.175 \text{ GPa}^{-1}$ and $\tau_{CSH}^{v,dev} = 1 \text{ h}$)

with $r \in \{m = \text{CSH, air, water}\}$, where $J_r^{vol*} = 1/k_r$ for the case of elastic material response. After applying the inverse Laplace-Carson transformation (Stehfest 1970) to Eq. (41), the discrete points were approximated by

$$J_{eff}^{vol}(t-\tau) = \frac{1}{k_{eff}[\xi(\tau)]} + J_{eff}^{v,vol}[\xi(\tau)] \ln\left(1 + \frac{t-\tau}{\tau_{eff}^{v,vol}[\xi(\tau)]}\right) \quad (42)$$

[see Fig. 5(b)]. Hereby, $k_{eff}[\xi(\tau)]$ is the effective bulk modulus determined according to Eq. (24.2). The effective creep parameters $J_{eff}^{v,dev}[\xi(\tau)]$, $J_{eff}^{v,vol}[\xi(\tau)]$, $\tau_{eff}^{v,dev}[\xi(\tau)]$, and $\tau_{eff}^{v,vol}[\xi(\tau)]$ [see Eqs. (39) and (42)] define the matrix behavior at the next higher scale of observation (Scale II). At this scale, the homogenization procedure described above [Eqs. (37) to (42)] is applied accordingly, with the material phases $r \in \{\text{matrix } m \text{ as the effective material determined at Scale Ib.2; inclusions: anhydrous cement, gypsum, monosulfate, ettringite, portlandite, } C_3(A,F)H_6\}$. The effective creep parameters determined at Scale II serve as input for material matrix at Scale III, where the material phases $r \in \{\text{matrix } m \text{ and aggregates}\}$ and the homogenization procedure is applied once more. Hereby, inverse Laplace-Carson transformation is performed, equally to Scale Ib.2, in a point-wise manner, with subsequent approximation according to Eqs. (39) and (42) (see Fig. 6). Fig. 7 illustrates the described upscaling procedure for shotcrete already considered in Fig. 4 (see Table 2 in Appendix A), giving access to the effective creep parameters $J_{eff}^{v,dev}(\xi)$, $J_{eff}^{v,vol}(\xi)$, $\tau_{eff}^{v,dev}(\xi)$, and $\tau_{eff}^{v,vol}(\xi)$.

Early-age cement-based materials are characterized by a continuously changing microstructure, i.e., the volume fractions of the material phases at the respective scale of observation change with the hydration extent. E.g., at Scale Ib.2, the volume fraction of creep-active CSH increases, while the water fraction is continuously decreasing. The dependence of the creep parameters on the hydration extent and, thus, on the time is considered by the reformulation of the Boltzmann convolution integral [Eq. (28)] in terms of the creep-compliance rate $\mathbb{J}$, giving the viscous part of the strain tensor in the form

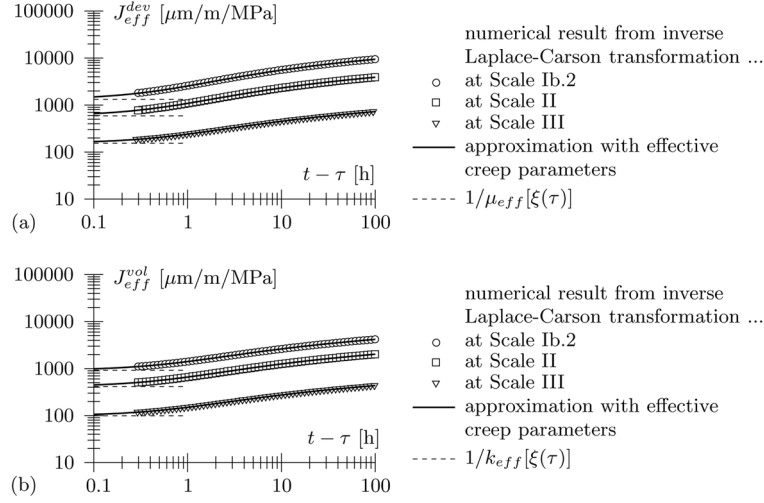


Fig. 6 Approximation of effective creep compliance obtained from inverse Laplace-Carson transformation for $\xi(\tau)=0.2$ (using $J_{\text{CSH}}^{\nu, \text{dev}} = 0.175 \text{ GPa}^{-1}$ and $\tau_{\text{CSH}}^{\nu, \text{dev}} = 1 \text{ h}$).

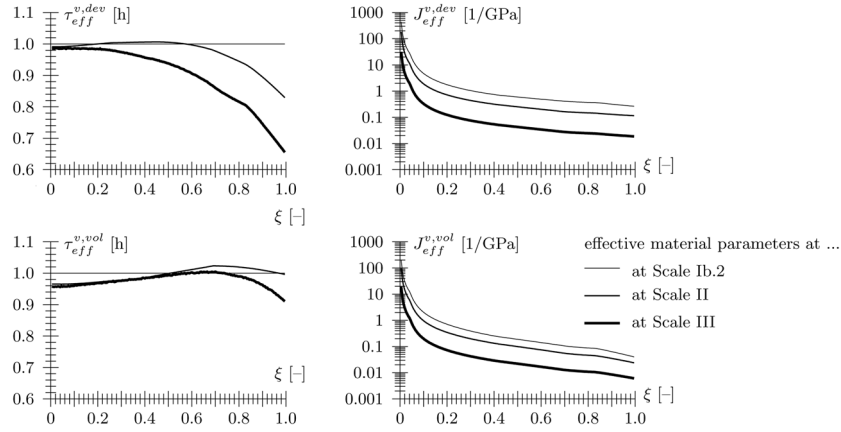


Fig. 7 Effective creep parameters for shotcrete employed at the Lainzer tunnel [see Table 2] (using $J_{\text{CSH}}^{\nu, \text{dev}} = 0.175 \text{ GPa}^{-1}$ and $\tau_{\text{CSH}}^{\nu, \text{dev}} = 1 \text{ h}$)

$$\varepsilon^{\nu}(t) = \int_0^t \left\{ \int_{\tau}^t \dot{\mathbb{J}}^{\nu, \text{age}}[\hat{t} - \tau, \xi(\hat{t})] d\hat{t} \right\} : \frac{\partial \sigma(\tau)}{\partial \tau} d\tau, \quad (43)$$

where $\dot{\mathbb{J}}^{\nu, \text{age}}$ denotes the aging creep-compliance rate, with

$$\dot{\mathbb{J}}^{\nu, \text{age}}[\hat{t} - \tau, \xi(\hat{t})] = \frac{1}{3} \mathbb{I}^{\text{vol}} \frac{J_{\text{eff}}^{\nu, \text{vol}}[\xi(\hat{t})]}{\hat{t} - \tau + \tau_{\text{eff}}^{\nu, \text{dev}}[\xi(\hat{t})]} + \frac{1}{2} \mathbb{I}^{\text{dev}} \frac{J_{\text{eff}}^{\nu, \text{dev}}[\xi(\hat{t})]}{\hat{t} - \tau + \tau_{\text{eff}}^{\nu, \text{dev}}[\xi(\hat{t})]}. \quad (44)$$

In Eq. (44), $\mathbb{I}^{\text{vol}}$ and $\mathbb{I}^{\text{dev}}$ are defined as

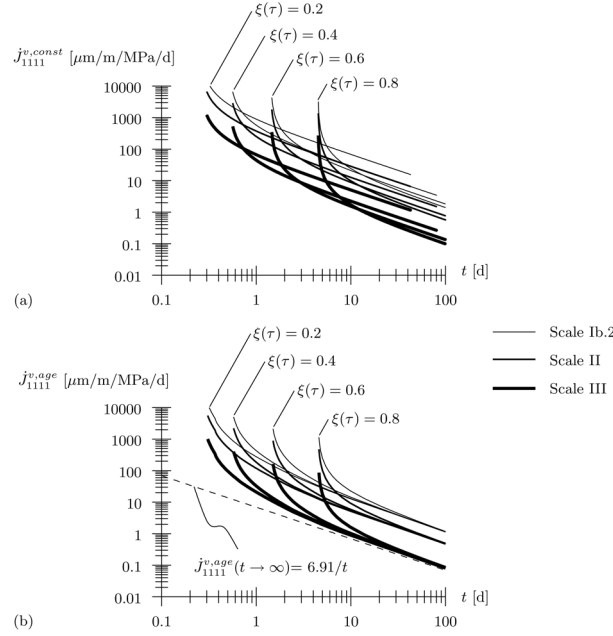


Fig. 8 Comparison between (a) uniaxial *non-aging* creep-compliance rate [see Eq. (47)] and (b) uniaxial *aging* creep-compliance rate [see Eq. (44)] ($J_{\text{CSH}}^{\nu,\text{dev}} = 0.175 \text{ GPa}^{-1}$ and $\tau_{\text{CSH}}^{\nu,\text{dev}} = 1 \text{ h}$)

$$\mathbb{I}^{\text{vol}} = \begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (45)$$

and

$$\mathbb{I}^{\text{dev}} = \begin{bmatrix} 2/3 & -1/3 & -1/3 & 0 & 0 & 0 \\ -1/3 & 2/3 & -1/3 & 0 & 0 & 0 \\ -1/3 & -1/3 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (46)$$

Fig. 8 compares the *non-aging* creep-compliance rate⁹ with the *aging* creep-compliance rate

⁹The *non-aging* creep-compliance rate is determined (for sake of comparison) as

$$\dot{\mathbb{J}}^{\nu,\text{non aging}}(t-\tau) = \frac{1}{3} \mathbb{I}^{\text{vol}} \dot{J}_{\text{eff}}^{\nu,\text{vol}}(t-\tau) + \frac{1}{2} \mathbb{I}^{\text{dev}} \dot{J}_{\text{eff}}^{\nu,\text{dev}}(t-\tau) = \frac{1}{3} \mathbb{I}^{\text{vol}} \frac{J_{\text{eff}}^{\nu,\text{vol}}[\xi(\tau)]}{t-\tau + \tau_{\text{eff}}^{\nu,\text{vol}}[\xi(\tau)]} + \frac{1}{2} \mathbb{I}^{\text{dev}} \frac{J_{\text{eff}}^{\nu,\text{dev}}[\xi(\tau)]}{t-\tau + \tau_{\text{eff}}^{\nu,\text{dev}}[\xi(\tau)]}, \quad (47)$$

where τ is the time instant of loading.

according to Eq. (44) for uniaxial loading and different time instants of loading τ , corresponding to $\xi(\tau)=0.2, 0.4, 0.6$, and 0.8 .

During hydration of concrete, new hydration products are formed in a state free of microstress (Bažant 1979), and loaded exclusively by stresses applied *after* formation. This situation is accounted for by an incremental stress-strain law for the elastic part of the strain reading

$$d\boldsymbol{\varepsilon}^e(\tau) = \mathbb{C}^{-1}[\xi(\tau)] : d\boldsymbol{\sigma}(\tau) = \left\{ \frac{1}{3} \mathbb{I}^{\text{vol}} \frac{1}{k_{\text{eff}}[\xi(\tau)]} + \frac{1}{2} \mathbb{I}^{\text{dev}} \frac{1}{\mu_{\text{eff}}[\xi(\tau)]} \right\} : d\boldsymbol{\sigma}(\tau), \quad (48)$$

giving the total strain tensor as

$$\boldsymbol{\varepsilon}(t) = \boldsymbol{\varepsilon}^e(t) + \boldsymbol{\varepsilon}^v(t) = \int_0^t \left\{ \mathbb{C}^{-1}[\xi(\tau)] + \int_{\tau}^t \mathbb{J}^{\text{v,age}}[\hat{t}-\tau, \xi(\hat{t})] d\hat{t} \right\} : \frac{\partial \boldsymbol{\sigma}(\tau)}{\partial \tau} d\tau. \quad (49)$$

6. Identification of creep parameters

6.1. Stress states considered in standard creep tests

The uniaxial viscoelastic compliance function in consequence of a uniaxial step load, with $\sigma_{11} = -\Delta p$ applied at $t = \tau$, follows from Eqs. (43) to (48) as

$$\begin{aligned} J_{1111}^{\text{ve,age}}(t-\tau) &= \frac{\varepsilon_{11}(t)}{\sigma_{11}} = -\frac{\varepsilon_{11}(t)}{\Delta p} = \frac{1}{9} \frac{1}{k_{\text{eff}}[\xi(\tau)]} + \frac{1}{3} \frac{1}{\mu_{\text{eff}}[\xi(\tau)]} \\ &+ \int_{\tau}^t \left\{ \frac{1}{9} \frac{J_{\text{eff}}^{\text{v,vol}}[\xi(\hat{t})]}{\hat{t}-\tau + \tau_{\text{eff}}^{\text{v,vol}}[\xi(\hat{t})]} + \frac{1}{3} \frac{J_{\text{eff}}^{\text{v,dev}}[\xi(\hat{t})]}{\hat{t}-\tau + \tau_{\text{eff}}^{\text{v,dev}}[\xi(\hat{t})]} \right\} d\hat{t}. \end{aligned} \quad (50)$$

In triaxial testing characterized by a chamber pressure p applied at $t = \tau_1$ ($\boldsymbol{\sigma} = [-p, -p, -p, 0, 0, 0]^T$) and a uniaxial overload Δp applied at $t = \tau_2$

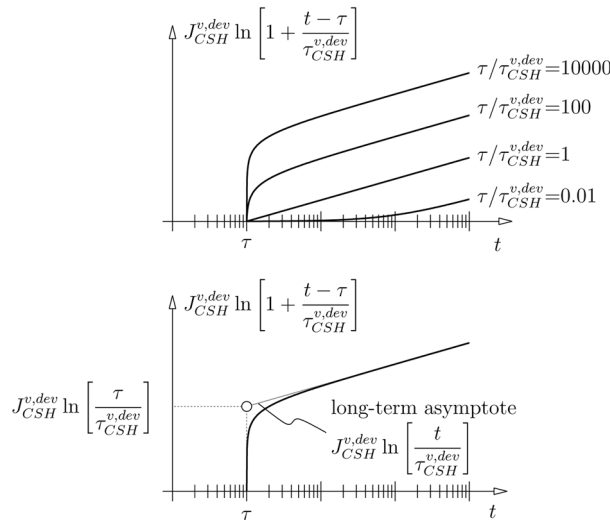


Fig. 9 Illustration of time-dependent part of $J_{\text{CSH}}^{\text{v,dev}}(t-\tau)$ (see Eq. (34)) for different time instants of loading τ (given by $\tau/\tau_{\text{CSH}}^{\text{v,dev}}$)

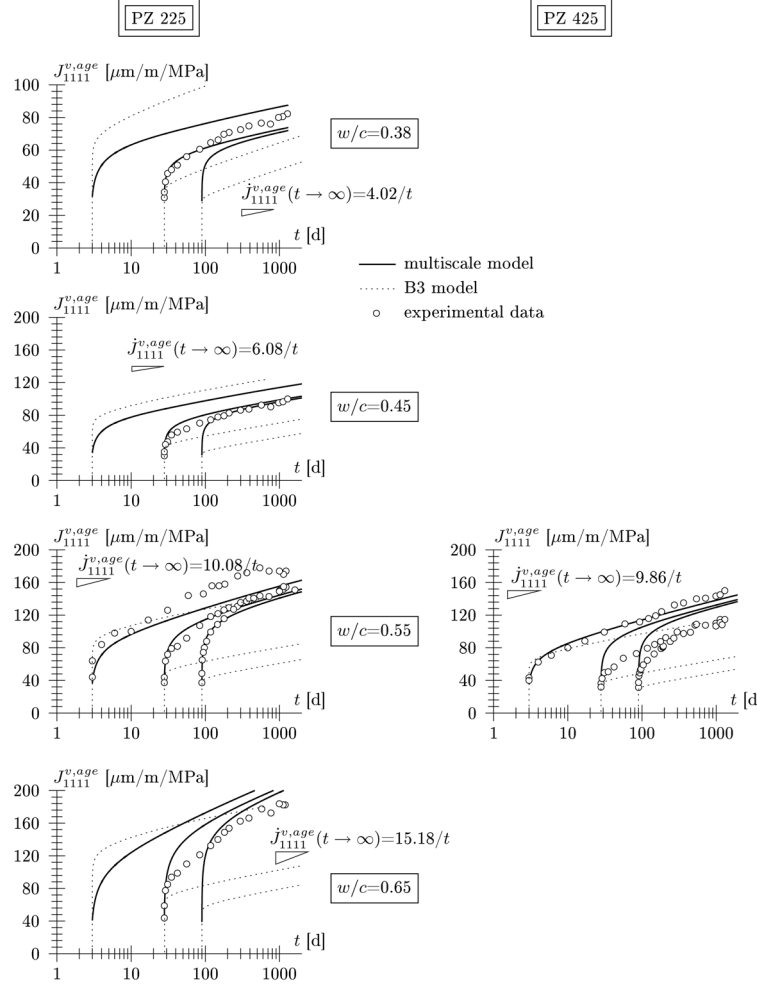


Fig. 10 Back calculation of macroscopic creep tests (Hummel, *et al.* 1962) with multiscale material model using finer-scale input parameters given in Table 4 and creep properties of CSH as $J_{\text{CSH}}^{\nu, \text{dev}} = 0.175 \text{ GPa}^{-1}$, $\tau_{\text{CSH}} = 3 \text{ h}$; macroscopic compliance predicted by the B3 model (Bažant and Baweja 1997)

($\sigma = [-p, -\Delta p, -p, -p, 0, 0, 0]^T$), the strain in direction of the applied uniaxial load is given as

$$\varepsilon_{11} = -[J_{1111}^{\nu e, \text{age}}(t - \tau_1)p + 2J_{1122}^{\nu e, \text{age}}(t - \tau_1)p + J_{1111}^{\nu e, \text{age}}(t - \tau_2)\Delta p], \quad (51)$$

with $J_{1111}^{\nu e, \text{age}}$ according to Eq. (50) and

$$J_{1122}^{\nu e, \text{age}}(t - \tau) = \frac{1}{9} \frac{1}{k_{\text{eff}}[\xi(\tau)]} - \frac{1}{6} \frac{1}{\mu_{\text{eff}}[\xi(\tau)]} + \int_{\tau}^t \left\{ \frac{1}{9} \frac{J_{\text{eff}}^{\nu, \text{vol}}[\xi(\hat{t})]}{\hat{t} - \tau + \tau_{\text{eff}}^{\nu, \text{vol}}[\xi(\hat{t})]} - \frac{1}{6} \frac{J_{\text{eff}}^{\nu, \text{dev}}[\xi(\hat{t})]}{\hat{t} - \tau + \tau_{\text{eff}}^{\nu, \text{dev}}[\xi(\hat{t})]} \right\} d\hat{t}. \quad (52)$$

6.2. Identification of $J_{\text{CSH}}^{\nu, \text{dev}}$ and $\tau_{\text{CSH}}^{\nu, \text{dev}}$

In order to determine the intrinsic creep parameters of CSH, $J_{\text{CSH}}^{\nu, \text{dev}}$ and $\tau_{\text{CSH}}^{\nu, \text{dev}}$, macroscopic creep tests are used in conjunction with the described multiscale model. For this purpose, a top-down

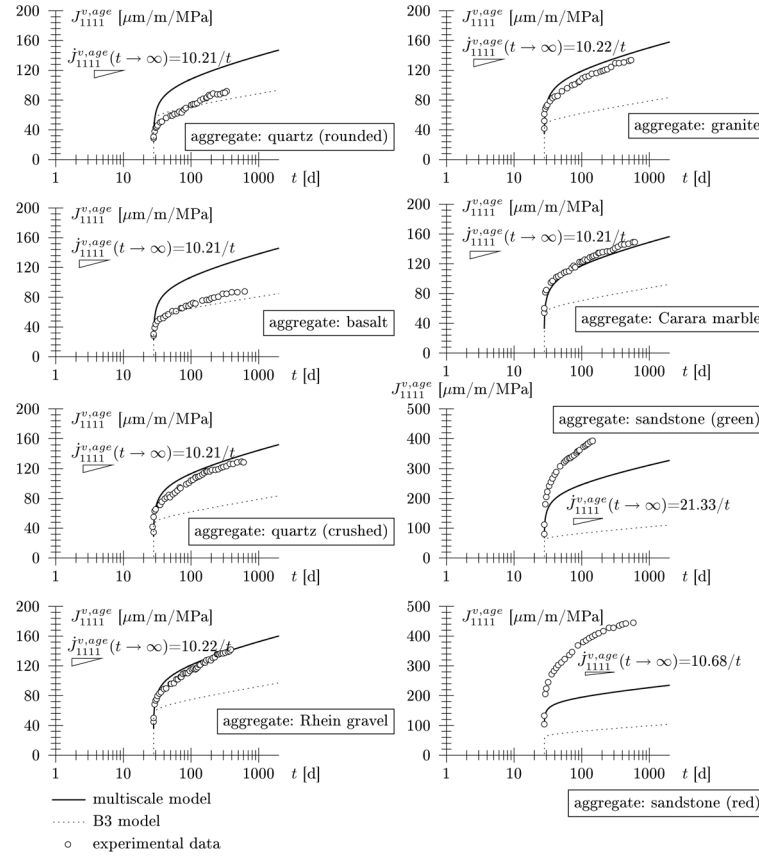


Fig. 11 Backcalculation of macroscopic creep tests (Hummel, *et al.* 1962) with multiscale material model using finer-scale input parameters given in Tables 5 and 6 and creep properties of CSH as $J_{\text{CSH}}^{\nu, \text{dev}} = 0.175 \text{ GPa}^{-1}$, $\tau_{\text{CSH}}^{\nu, \text{dev}} = 3 \text{ h}$; macroscopic compliance predicted by the B3 model (Bažant and Baweja 1997)

application of the presented multiscale model is applied by adapting the finer-scale intrinsic input parameters aiming at a correct prediction of the macroscopic creep data. Fig. 9 shows the time-dependent part of the employed deviatoric creep compliance for CSH, $J_{\text{CSH}}^{\nu, \text{dev}} \ln[1 + (t - \tau)/\tau_{\text{CSH}}^{\nu, \text{dev}}]$ [see Eq. (34)] for different time instants of loading τ , given by $\tau/\tau_{\text{CSH}}^{\nu, \text{dev}}$. Taking the derivative of Eq. (34), giving

$$\dot{J}_{\text{CSH}}^{\text{dev}} = \frac{J_{\text{CSH}}^{\nu, \text{dev}}}{t - \tau + \tau_{\text{CSH}}^{\nu, \text{dev}}}, \quad (53)$$

and specializing Eq. (53) for $(t - \tau) \gg \tau_{\text{CSH}}^{\nu, \text{dev}}$ gives access to the long-term asymptote of the creep-compliance rate as

$$\dot{J}_{\text{CSH}}^{\text{dev}} \Big|_{(t - \tau) \gg \tau_{\text{CSH}}^{\nu, \text{dev}}} = \frac{J_{\text{CSH}}^{\nu, \text{dev}}}{t - \tau}. \quad (54)$$

Hence, the long-term creep-compliance rate is solely controlled by $J_{\text{CSH}}^{\nu, \text{dev}}$, while the short-term creep-compliance rate is controlled by both parameters, with

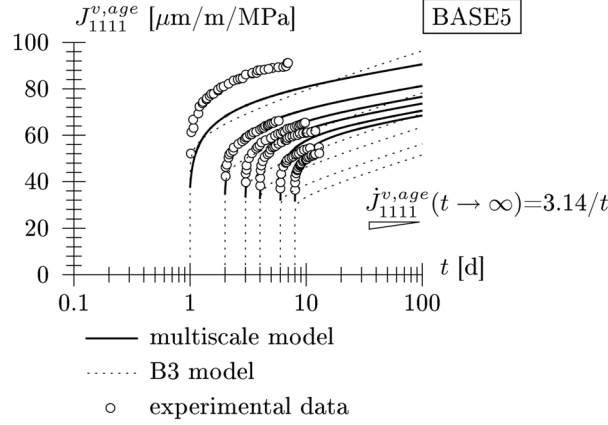


Fig. 12 Backcalculation of macroscopic creep tests (Athrusi 2003) with multiscale material model using finer-scale input parameters given in Table 7 and creep properties of CSH as $J_{\text{CSH}}^{\nu, \text{dev}} = 0.105 \text{ GPa}^{-1}$, $\tau_{\text{CSH}}^{\nu, \text{dev}} = 3/4 \text{ h}$; macroscopic compliance predicted by the B3 model (Bažant and Baweja 1997)

$$J_{\text{CSH}}^{\text{dev}}|_{(t-\tau)=0} = \frac{J_{\text{CSH}}^{\nu, \text{dev}}}{\tau_{\text{CSH}}^{\nu, \text{dev}}} . \quad (55)$$

Accordingly, $J_{\text{CSH}}^{\nu, \text{dev}}$ may be adapted in order to fit the long-term response of macroscopic test data, while $\tau_{\text{CSH}}^{\nu, \text{dev}}$ is adjusted to match the creep-compliance rate directly after application of the load. Experimental results given in Hummel, *et al.* (1962) and Athrusi (2003) are employed to identify $J_{\text{CSH}}^{\nu, \text{dev}}$ and $\tau_{\text{CSH}}^{\nu, \text{dev}}$. Whereas the best agreement between multiscale model and experimental data reported in Hummel, *et al.* (1962) is obtained for $J_{\text{CSH}}^{\nu, \text{dev}} = 0.175 \text{ GPa}^{-1}$ and $\tau_{\text{CSH}}^{\nu, \text{dev}} = 3 \text{ h}$ (see Figs. 10 and 11), $J_{\text{CSH}}^{\nu, \text{dev}} = 0.105 \text{ GPa}^{-1}$ and $\tau_{\text{CSH}}^{\nu, \text{dev}} = 3/4 \text{ h}$ give the best agreement when using experimental data from Athrusi (2003) (see Fig. 12).

7. Discussion

The variation of $J_{\text{CSH}}^{\nu, \text{dev}}$ highlighted in the previous section may be explained by the different environmental conditions present at the creep tests. Whereas both test series were conducted under isothermal conditions ($T = 20^\circ\text{C}$), the specimens tested in Hummel, *et al.* (1962) were subjected to a relative humidity h of 100% for the age $< 7 \text{ d}$ and $h = 65\%$ for an age $> 7 \text{ d}$.¹⁰ The tests reported in Athrusi (2003), on the other hand, are characterized by $h = 50\%$. Hence, the larger the humidity h , the larger $J_{\text{CSH}}^{\nu, \text{dev}}$ and, consequently, the larger the macroscopically-observed long-term compliance rate. The variation in the identified values for $\tau_{\text{CSH}}^{\nu, \text{dev}}$ suggests a dependence of $\tau_{\text{CSH}}^{\nu, \text{dev}}$ on the degree of hydration. According to Ruetz (1966); Wittmann (1982); Ulm (1998), the short-term creep behavior is associated with stress-induced microdiffusion of water in the capillary pores. Hereby, the underlying diffusion process may depend on the capillary depression p_c and the permeability of the

¹⁰This so-imposed drying regime over some period of the reported creep tests would preclude the results from use in a basic creep analysis (drying creep, Pickett's paradoxon, see, e.g., Acker and Ulm (2001)). The data was, however, by reason of the small number of experimental results for OPC-based concrete in the open literature, used for back-calculation of creep properties.

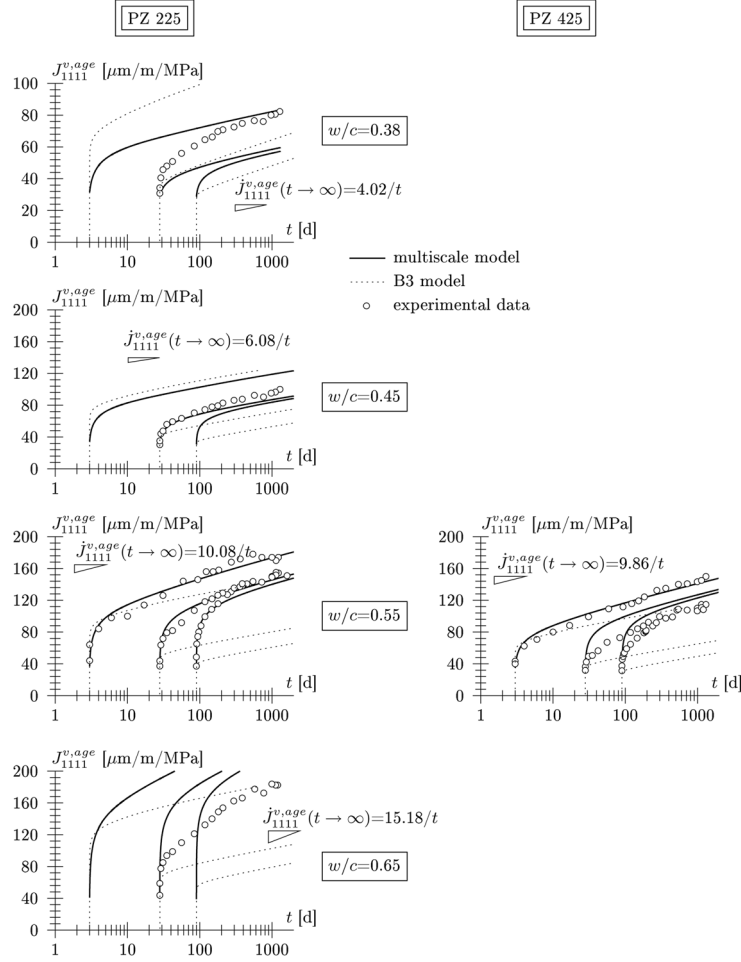


Fig. 13 Backcalculation of macroscopic creep tests (Hummel, *et al.* 1962) with multiscale material model using finer-scale input parameters given in Table 4 and creep properties of CSH as $J_{\text{CSH}}^{\nu, \text{dev}} = 0.175 \text{ GPa}^{-1}$, $\tau_{\text{CSH}, \infty}^{\nu, \text{dev}} = 4 \text{ h}$; macroscopic compliance predicted by the B3 model (Bažant and Baweja 1997)

solid skeleton, both of them depending on the hydration extent. Based on the capillary depression in the water phase $p_c(\xi)$ (see Pichler, *et al.* (2007) and Appendix B), a dependence of $\tau_{\text{CSH}}^{\nu, \text{dev}}$ on the hydration extent is proposed as

$$\tau_{\text{CSH}}^{\nu, \text{dev}}(\xi) = \tau_{\text{CSH}, \infty}^{\nu, \text{dev}} \frac{p_c(\xi)}{p_c(h_{\text{exp}})}, \quad (56)$$

with $p_c(h_{\text{exp}})$ as the capillary depression related to the relative humidity h_{exp} of the medium surrounding the sample during the experiment via Kelvin's equation.¹¹ Figs. 13, 14(a), and 15 show

¹¹Kelvin's equation expresses the liquid-vapor equilibrium under atmospheric conditions:

$$p_c = -\frac{\rho_H \mathcal{R} T}{\mathcal{M}_H} \ln h, \quad (57)$$

where $\rho_H = 998 \text{ kg/m}^3$ is the density of water, $\mathcal{R} = 8.3144 \text{ J/(mol K)}$ is the universal gas constant, T is the absolute temperature, $\mathcal{M}_H = 0.018 \text{ kg/mol}$ is the molar mass of water, and h is the relative humidity.

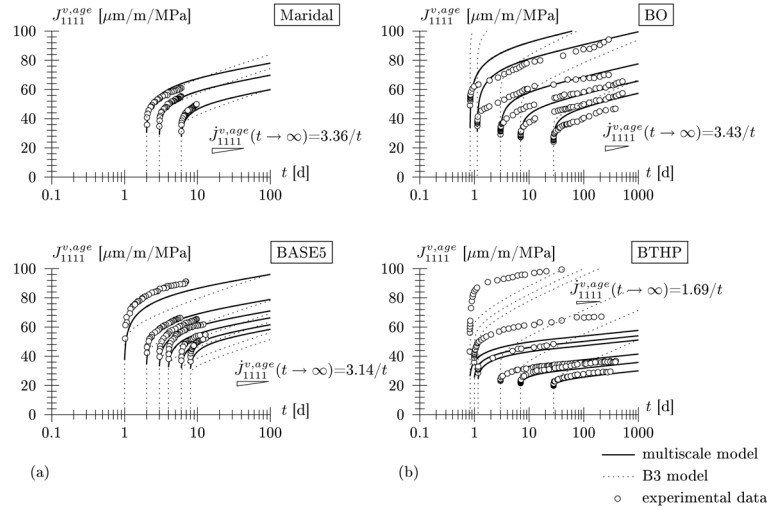


Fig. 14 Backcalculation of macroscopic creep tests taken from (a) Athrushi (2003) with multiscale material model using finer-scale input parameters given in Table 7 and creep properties of CSH as $J_{\text{CSH}}^{\nu, \text{dev}} = 0.105 \text{ GPa}^{-1}$, $\tau_{\text{CSH}, \infty}^{\nu, \text{dev}} = 4 \text{ h}$; (b) Laplante (1993) with multiscale material model using finer-scale input parameters given in Table 3 and creep properties of CSH as $J_{\text{CSH}}^{\nu, \text{dev}} = 0.075 \text{ GPa}^{-1}$, $\tau_{\text{CSH}, \infty}^{\nu, \text{dev}} = 4 \text{ h}$; macroscopic compliance predicted by the B3 model (Bažant and Baweja 1997).

the comparison between the creep compliance predicted by the multiscale model and experimental data (Hummel, *et al.* 1962; Athrushi 2003) for $\tau_{\text{CSH}}^{\nu, \text{dev}}$ according to Eq. (56) using $\tau_{\text{CSH}, \infty}^{\nu, \text{dev}} = 4 \text{ d}$, with $h_{\text{exp}} = 0.65$ for the experiments given in Hummel, *et al.* (1962) and $h_{\text{exp}} = 0.5$ characterizing the experimental results given in Athrushi (2003).

The uniaxial creep tests given in Laplante (1993) have frequently been used to calibrate macroscopic creep formulations (see, e.g., Sercombe, *et al.* (2000), Cervera, *et al.* (1999)). These tests are characterized by $h_{\text{exp}} = 50\%$, with the input parameters for the multiscale model being summarized in Table 3. A comparison between the creep compliance obtained from the multiscale model and the test data (Laplante 1993) is given in Fig. 14(b), showing good agreement between model and experimental data for $J_{\text{CSH}}^{\nu, \text{dev}} = 0.075 \text{ GPa}^{-1}$.

When using parts of the data sets reported in Hummel, *et al.* (1962), Athrushi (2003), Laplante (1993) characterized by a certain w/c -ratio for top-down identification of $J_{\text{CSH}}^{\nu, \text{dev}}$, a dependency of $J_{\text{CSH}}^{\nu, \text{dev}}$ on h_{exp} and the w/c -ratio is observed (see Figs. 16 and 17), with higher values of $J_{\text{CSH}}^{\nu, \text{dev}}$ for increasing h_{exp} and decreasing w/c -ratio. Consequently, CSH formed at a lower w/c -ratio has a greater creep compliance, which may be explained by more pronounced “intrinsic” microcracking/damage in CSH formed at lower w/c -ratios, yielding higher compliance under sustained loading¹² (see Fig. 18).

The back-calculation of nanoindentation creep tests on well-hydrated cement-paste samples (Pichler and Lackner 2008, Pichler, *et al.* 2008) reveal a logarithmic-type creep behavior with a mean value for $J^{\nu, \text{dev}}$ of approximately 0.04 GPa^{-1} for $w/c = 0.4$ and $h_{\text{exp}} = 50\%$. This value is lower

¹²Though not directly related, the study in Neubauer and Jennings (2000) contains a microstructural investigation of shrinking cement paste during drying. Hereby, large local deformation is associated with the collapse of the colloidal network of CSH. The observed damage (fracture) is more pronounced in stiffer samples, i.e., in samples with lower w/c -ratio.

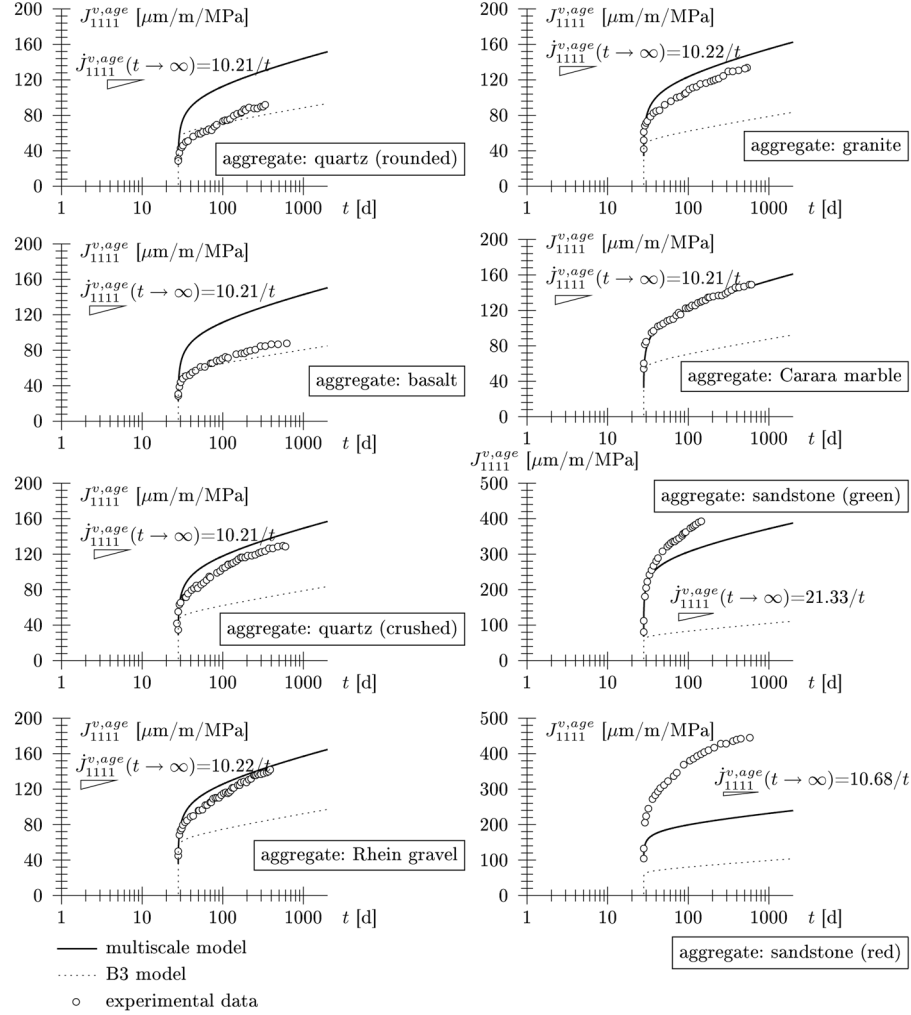


Fig. 15 Backcalculation of macroscopic creep tests (Hummel, *et al.* 1962) with multiscale material model using finer-scale input parameters given in Tables 5 and 6 and creep properties of CSH as $J_{\text{CSH}}^{\nu, \text{dev}} = 0.175 \text{ GPa}^{-1}$, $\tau_{\text{CSH}, \infty}^{\nu, \text{dev}} = 4 \text{ h}$; macroscopic compliance predicted by the B3 model (Bažant and Baweja 1997)

than the respective value obtained from top-down identification of $J_{\text{CSH}}^{\nu, \text{dev}}$ as a function of w/c and h_{exp} , giving approximately 0.09 GPa^{-1} (see Fig. 18), which is explained by the nanoindentation penetration depth of approximately 250 nm and the corresponding contact area of $(1.4 \text{ μm})^2 \pi / 4 = 1.5 \text{ μm}^2$, lying within Scales I and II (see Fig. 1).

8. Concluding remarks and engineering model

In this paper a multiscale model for upscaling of the creep properties of early-age cement-based materials was proposed. The input parameters include basic mix properties, cement characteristics,

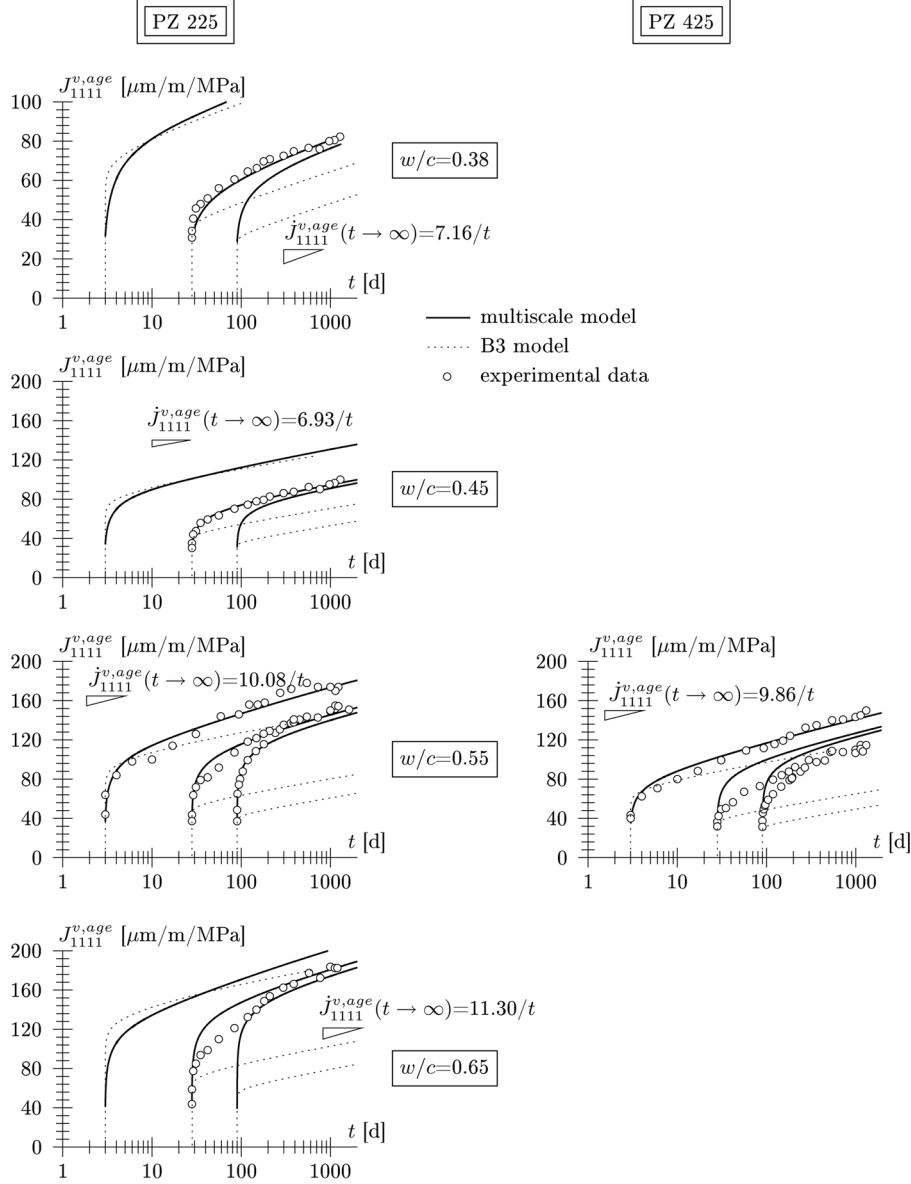


Fig. 16 Backcalculation of macroscopic creep tests (Hummel, *et al.* 1962) with multiscale material model using finer-scale input parameters given in Table 4 and creep properties of CSH as $J_{\text{CSH}}^{v,\text{dev}}(w/c=0.38)=0.315 \text{ GPa}^{-1}$, $J_{\text{CSH}}^{v,\text{dev}}(w/c=0.45)=0.200 \text{ GPa}^{-1}$, $J_{\text{CSH}}^{v,\text{dev}}(w/c=0.55)=0.175 \text{ GPa}^{-1}$, $J_{\text{CSH}}^{v,\text{dev}}(w/c=0.65)=0.130 \text{ GPa}^{-1}$, $\tau_{\text{CSH},\infty}=4 \text{ d}$; macroscopic compliance predicted by the B3 model (Bažant and Baweja 1997)

elastic properties of the aggregates, and viscous properties of CSH. The latter, which are essential for the creep behavior at the different scales of concrete, were determined through a top-down application of the multiscale model. Hereby, the long-term compliance of CSH was found to depend on (i) the relative humidity and (ii) the w/c -ratio considered in the mix design.

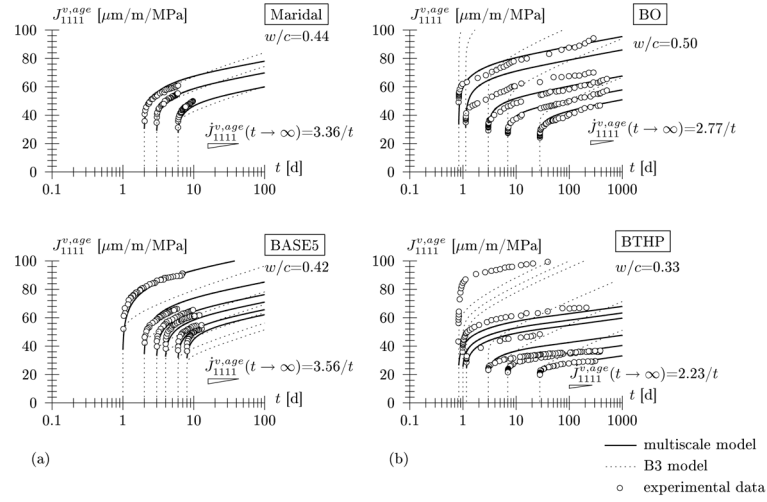


Fig. 17 Backcalculation of macroscopic creep tests taken from (a) Athrushi (2003) with multiscale material model using finer-scale input parameters given in Table 7 and creep properties of CSH as $J_{\text{CSH}}^{\nu,\text{dev}}$ (BASE5)=0.120 GPa⁻¹, $J_{\text{CSH}}^{\nu,\text{dev}}$ (Maridal)=0.105 GPa⁻¹, $\tau_{\text{CSH},\infty}^{\nu,\text{dev}}$ =4 d; (b) Laplante (1993) with multiscale material model using finer-scale input parameters given in Table 3 and creep properties of CSH as $J_{\text{CSH}}^{\nu,\text{dev}}$ (BO)=0.060 GPa⁻¹, $J_{\text{CSH}}^{\nu,\text{dev}}$ (BTHP)=0.100 GPa⁻¹, $\tau_{\text{CSH},\infty}^{\nu,\text{dev}}$ =4 d; macroscopic compliance predicted by the B3 model (Bažant and Baweja 1997)

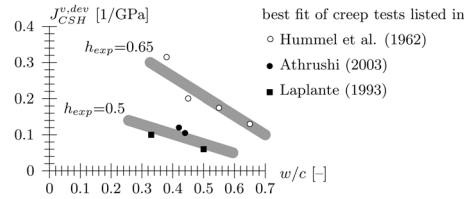


Fig. 18 Top-down identification of $J_{\text{CSH}}^{\nu,\text{dev}}$ being a function of w/c and h_{exp}

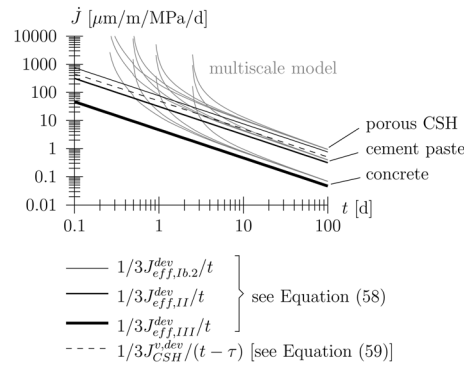


Fig. 19 Comparison of the long-term creep-compliance rate for (i) the multiscale model outlined in Section 5 and (ii) the engineering model using Eq. (58) for shotcrete with finer-scale input parameters listed in Table 2 (curing temperature 20°C=constant, $k_w=0$, $J_{\text{CSH}}^{\nu,\text{dev}}=0.150$ GPa⁻¹, $\tau_{\text{CSH},\infty}^{\nu,\text{dev}}=0.278$ h)

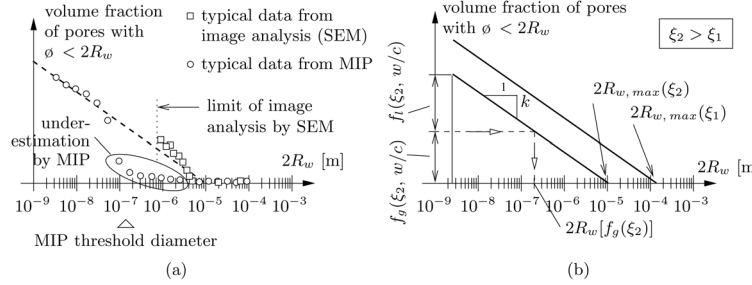


Fig. 20 Cumulative pore-size distributions obtained from MIP and image analysis: (a) typical experimental data, (b) line, fitting experimental data

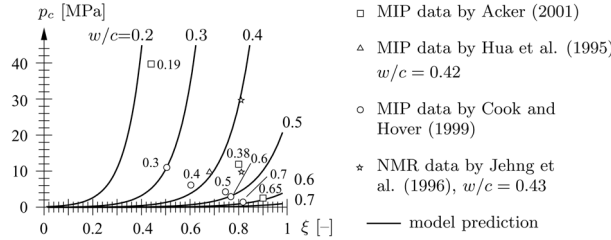


Fig. 21 Capillary pressure as a function of the w/c -ratio and ξ

Finally, the upscaling scheme presented in this paper, obtained from application of the correspondence principle to results of continuum micromechanics, may be significantly simplified by considering (i) a simple microstructure (pores and rigid inclusion in an incompressible matrix¹³) and (ii) volume fractions of the constituents corresponding to the state of complete hydration, i.e., all anhydrous cement is hydrated, hence $\xi=1$, or all free water is consumed. Upscaling towards the macroscale is performed in three steps, employing continuum micromechanics (Mori-Tanaka scheme) giving the creep parameter $J_{\text{eff}}^{\nu, \text{dev}}$, describing the long-term compliance rate under shear loading, as a function of the respective creep parameter $J_{\text{CSH}}^{\nu, \text{dev}}$ of CSH:

$$\begin{aligned}
 J_{\text{eff, Ib.2}}^{\nu, \text{dev}} &= J_{\text{CSH}}^{\nu, \text{dev}} \frac{f_{\text{CSH}} + \frac{5}{3}(f_l + f_g)}{f_{\text{CSH}}} \quad (\text{at Scale Ib.2}), \\
 J_{\text{eff, II}}^{\nu, \text{dev}} &= J_{\text{eff, Ib.2}}^{\nu, \text{dev}} \frac{f_m}{f_m + \frac{5}{2}(1 - f_m)} \quad (\text{at Scale II}), \\
 J_{\text{eff, III}}^{\nu, \text{dev}} &= J_{\text{eff, II}}^{\nu, \text{dev}} \frac{f_m}{f_m + \frac{5}{2}(1 - f_m)} \quad (\text{at Scale III}),
 \end{aligned} \tag{58}$$

where the indices “ l ”, “ g ”, and “ m ” refer to the liquid, gaseous, and matrix phase. Fig. 19 illustrates the error introduced by the aforementioned simplifications, showing both results from the multiscale model proposed in this paper (gray curves) and the long-term creep-compliance rate of the

¹³Specializing Eq. (25) for an incompressible matrix material gives $\beta=2/5$.

engineering model [Eq. (58), solid black curves]. The dashed curve in Fig. 19 represents the long-term response of the intrinsic compliance rate of CSH used as input, given by $J_{\text{CSH}}^{\nu, \text{dev}}$ as

$$\dot{J}_{\text{CSH}}^{\text{dev}}(t-\tau) \Big|_{(t-\tau) \gg \tau_{\text{CSH}}^{\nu, \text{dev}}} = \frac{J_{\text{CSH}}^{\nu, \text{dev}}}{t-\tau}. \quad (59)$$

The long-term compliance rate obtained from Eq. (58) underestimates the result from the multiscale model by approximately 20%, which is explained by the assumption of a simplified microstructure (pores and rigid inclusions) corresponding to a state of complete hydration. However, the reduction of the creep capacity in the second and third homogenization step towards the macroscale, with respect to the compliance rate of porous CSH, is similar to that obtained from the multiscale model presented in the previous section.

The presented multiscale model may be improved by considering the influence of the curing temperature (other than $T_0=20^\circ\text{C}$) on the creep compliance parameter $J_{\text{CSH}}^{\nu, \text{dev}}$, accounting for the thermal activation of the creep process through an Arrhenius term, i.e., $J_{\text{CSH}}^{\nu, \text{dev}}(T_0)\exp[E_a/R(1/T-1/T_0)]$, with E_a denoting the activation energy of the long-term creep process (in Bažant 1995, the latter was identified as $E_a/R=2700$ K). Moreover, since magnitude and duration of the creep process was found to depend on the relative humidity h , the latter should also enter the evolution law for the degree of hydration ξ , particularly relevant for larger values of ξ . Both modes of improvement are topics of ongoing research.

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Fruitful discussions on multiscale representation and modeling of concrete with Franz-Josef Ulm, Georgios Constantinides, and Matthieu Vandamme during research stays of the first two authors at the MIT (Cambridge, MA) are gratefully acknowledged. Financial support granted by the Austrian Science Fund (FWF) via project P15912-N07 is appreciated.

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A. Input parameter sets for multiscale model

This appendix summarizes input-parameter sets, characterizing various cement-based materials used to verify the multiscale model. The input parameters include the basic mix properties (water/cement-ratio w/c and cement content c), the elastic properties of the aggregate, as well as parameters characterizing the employed cement (Blaine fineness ϕ , medium initial radius of the clinker grains R , and the mass fractions of the clinker phases). Mineralogical analyses of OPC clinker give access to the mass fractions of lime CaO , silica SiO_2 , alumina Al_2O_3 , ferrite Fe_2O_3 , sulphate SO_3 , The mass fractions of the clinker phases are determined by the so-called *Bogue* calculation (Taylor 1997). For this calculation, the lime content m_{CaO} is reduced by the content bound in gypsum $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ (written in abbreviated form $\text{C}\bar{\text{S}}\text{H}_2$), with 1 mol SO_3 combining with 1 mol CaO to form gypsum or, equivalently, 80 g SO_3 binding 56 g CaO . Hence, the mass fraction of sulphate m_{SO_3} combines with $56/80m_{\text{SO}_3}$ of CaO . Other parameters, when required for verification of the model, e.g., the compressive strength and the aggregate/cement-ratio a/c for comparison of the modeled creep compliance with the B3 model (Bažant and Baweja 1997) are also listed in the following tables. Table 1 summarizes intrinsic elastic parameters of anhydrous cement and hydration products taken from Bernard, *et al.* (2003).

Table 1 Intrinsic elastic parameters of an hydrous cement and hydration products taken from Bernard, *et al.* (2003). In lack of experimental data the elastic properties of gypsum $\text{C}\bar{\text{S}}\text{H}_2$ and of reaction products from C_3A and C_4AF hydration were assumed to be equal to the elastic properties of the homogenized material of Scale Ib-1 (CSH)

Constituent	Young's modulus [GPa]	Poisson's ratio [–]
C_3S	135	0.3
C_2S	140	0.3
C_3A	145	0.3
C_4AF	125	0.3
CSH-LD	21.7	0.24
CSH-HD	29.4	0.24
CH	38	0.305

B. Determination of capillary depression $p_c(\xi)$ according to Pichler, *et al.* (2007)

Fig. 20(a) shows typical data obtained from mercury intrusion porosimetry (MIP) and image analysis of vacuum mixed cement pastes, characterized by no initial porosity, i.e., $f_g(\xi=0)=0$. Taking into account that MIP measurements provide reliable results for small pore sizes and image analysis is limited to greater pore-size ranges, a line approximating the respective ranges of the pore

size is introduced [see Fig. 20(b)]. Based on experimental data of cement pastes characterized by various values of the w/c -ratio and ξ (Cook and Hover 1999, Diamond and Leeman 1995), the slope of the line was identified as $k=0.2/\log_{10}(3 \times 10^7/1 \times 10^8)=0.135$, starting from a pore diameter of 2.5×10^{-9} m. According to Bye (1999), the latter corresponds to the smallest pore size at which menisci can form. The linear approximation of the cumulative pore-size distribution (similar to the proposed pore size distribution in Koenders and van Breugel (1997)) allows determination of the pore radius at the liquid-gas interface for a given volume fraction of the gaseous phase $f_g(\xi, w/c)$ [see Fig. 20(b)]. For the so-obtained value of $2R_w(f_g)$, the capillary pressure of the pore liquid can be computed as

$$p_c = \frac{2\gamma_{lg}}{R_w}, \quad (60)$$

Table 2 Input-parameter set for multiscale model for the shotcrete employed at the Lainzer tunnel

<i>basic mix properties:</i>		
water/cement-ratio w/c	[-]	0.48
cement content c	[kg/m ³]	380
<i>elastic properties of aggregate:</i>		
Young's modulus of aggregate E_a	[GPa]	50
Poisson's ratio of aggregate ν_a	[-]	0.3
<i>cement characteristics:</i>		
Blaine fineness ϕ	[cm ² /g]	4895
medium initial radius of clinker grains R	[μ m]	5
mass fractions of clinker phases		
m_{C_3S}	[%]	48.7
m_{C_2S}	[%]	22.8
m_{C_3A}	[%]	11.4
m_{C_4AF}	[%]	9.4
$m_{C\bar{S}H_2}$	[%]	7.7
<i>additional parameters for model verification:</i>		
curing temperature T (isothermal conditions)	[°C]	11
experiments conducted on specimens of age ...	[h]	2;4;8;12
	[d]	1;2;7;28
aggregate/cement-ratio a/c^1	[-]	5.3
compressive strength $f_{c\infty}^1$	[MPa]	40.0

¹parameters for comparison with B3 model (Bažant and Baweja 1997)

where $\gamma_{lg}=0.073$ N/m is the interfacial tension between the liquid and the gaseous phase.

This approach for determining the capillary depression was assessed by four different data sets reported in the open literature (Acker 2001, Hua, *et al.* 1995, Cook and Hover 1999, Jehng, *et al.* 1996). Fig. 21 shows the comparison of these test data and the model prediction based on $k=0.135$ and Eq. (60).

The model result for p_c as a function of the w/c -ratio and ξ were obtained from the calculation

Table 3 Input-parameter set for multiscale model (upscaling of creep properties) for concrete investigated in Laplante (1993)

Creep experiments published in ... type of concrete (notation in reference)		Laplante (1993)	
		BO	BTHP
<i>basic mix properties:</i>			
water/cement-ratio w/c	[-]	0.50	0.33
cement content c	[kg/m ³]	342	398
<i>elastic properties of aggregate:</i>			
Young's modulus of aggregate E_a	[GPa]	65	65
Poisson's ratio of aggregate ν_a	[-]	0.23	0.23
<i>cement characteristics:</i>			
Blaine fineness ϕ	[cm ² /g]	3466	3466
medium initial radius of clinker grains R	[μ m]	(8)	(8)
mass fractions of clinker phases			
m_{C_3S}	[%]	60.7	60.7
m_{C_2S}	[%]	18.7	18.7
m_{C_3A}	[%]	2.8	2.8
m_{C_4AF}	[%]	12.2	12.2
$m_{C\bar{S}H_2}$	[%]	2.5	2.5
others + inert	[%]	3.1	3.1
<i>additional parameters for model verification:</i>			
curing temperature T (isothermal conditions)	[°C]	20	20
loading times t_0	[h]	20; 27	20; 24; 28
	[d]	3; 7; 28	3; 7; 28
aggregate/cement-ratio a/c^1	[-]	5.46	4.84
compressive strength f_{c28}^1	[MPa]	40	80

¹parameters for comparison with B3 model (Bazant and Baweja 1997)Table 4 Input-parameter set for multiscale model (upscaling of creep properties) for concrete investigated in Hummel, *et al.* (1962)

Creep experiments published in ... type of cement (notation in reference)		Hummel, <i>et al.</i> (1962)				
		PZ 225	PZ 225	PZ 225	PZ 225	PZ 425
<i>basic mix properties:</i>						
water/cement-ratio w/c	[-]	0.38	0.45	0.55	0.65	0.55
cement content c	[kg/m ³]	350	345	334	328	334
<i>elastic properties of aggregate:</i>						
Young's modulus of aggregate ¹ E_a	[GPa]	(40)	(40)	(40)	(40)	(40)
Poisson's ratio of aggregate ν_a	[-]	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)
<i>cement characteristics:</i>						
Blaine fineness ϕ	[cm ² /g]	3900	3900	3900	3900	4575
medium initial radius of clinker grains R	[μ m]	(7)	(7)	(7)	(7)	(5)

Table 4 Continued

Creep experiments published in ... type of cement (notation in reference)		Hummel, <i>et al.</i> (1962)				
		PZ 225	PZ 225	PZ 225	PZ 225	PZ 425
mass fractions of clinker phases ²						
m_{C_3S}	[%]	49.1	49.1	49.1	49.1	53.6
m_{C_2S}	[%]	23.0	23.0	23.0	23.0	13.2
m_{C_3A}	[%]	8.0	8.0	8.0	8.0	10.7
m_{C_4AF}	[%]	7.6	7.6	7.6	7.6	9.6
$m_{C\bar{S}H_2}$	[%]	3.7	3.7	3.7	3.7	4.7
others + inert	[%]	8.6	8.6	8.6	8.6	8.2
<i>additional parameters for model verication:</i>						
curing temperature (isothermal conditions)	[°C]	20	20	20	20	20
loading times t_0	[d]	28	28	3; 28; 90	28	3; 28; 90
aggregate/cement-ratio a/c^3	[–]	5.4	5.4	5.4	5.4	5.4
compressive strength f_{c28}^3	[MPa]	41.4	34.9	28.6	20.3	43.5

¹Rhine gravel, ²4% ignition loss was taken into account ³parameters for comparison with B3 model (Bažant and Baweja 1997) (cylinder 15/30 cm)

Table 5 Input-parameter sets for multiscale model (upscaling of creep properties) for concrete investigated in Hummel, *et al.* (1962)

creep experiments published in ... type of aggregate (notation in reference)		Hummel, <i>et al.</i> (1962)			
		Rhine gravel	sandstone (green)	quartz (rounded)	quartz (crushed)
<i>basic mix properties:</i>					
water/cement-ratio w/c	[–]	0.55	0.73	0.55	0.55
cement content c	[kg/m ³]	337	370	337	337
<i>elastic properties of aggregate:</i>					
Young’s modulus of aggregate E_a	[GPa]	40.0	19.6	86.3	58.9
Poisson’s ratio of aggregate ν_a	[–]	(0.3)	(0.3)	(0.3)	(0.3)
<i>cement characteristics:</i>					
Blaine fineness ϕ	[cm ² /g]	4000	4000	4000	4000
medium initial radius of clinker grains R	[μm]	(7)	(7)	(7)	(7)
mass fractions of clinker phases ¹					
m_{C_3S}	[%]	41.8	41.8	41.8	41.8
m_{C_2S}	[%]	28.0	28.0	28.0	28.0
m_{C_3A}	[%]	8.2	8.2	8.2	8.2
m_{C_4AF}	[%]	9.5	9.5	9.5	9.5
$m_{C\overline{S}H_2}$	[%]	4.0	4.0	4.0	4.0
others + inert	[%]	8.5	8.5	8.5	8.5
<i>additional parameters for model verication:</i>					
curing temperature (isothermal conditions)	[°C]	20	20	20	20
loading times t_0	[d]	28	28	28	28
aggregate/cement-ratio a/c^2	[–]	5.39	4.09	5.46	5.40
compressive strength f_{c28}^2	[MPa]	22.3	24.0	23.9	29.7

¹1% free CaO was taken into account

²parameters for comparison with B3 model (Bažant and Baweja 1997) (cylinder 15/30 cm)

Table 6 Input-parameter sets for multiscale model (upscaling of creep properties) for concrete investigated in Hummel, *et al.* (1962)

Creep experiments published in ... type of aggregate (notation in reference)		Hummel, <i>et al.</i> (1962)			
		Carara marble	granite	basalt	sandstone (red)
<i>basic mix properties:</i>					
water/cement-ratio w/c	[-]	0.55	0.55	0.55	0.55
cement content c	[kg/m ³]	337	337	337	337
<i>elastic properties of aggregate:</i>					
Young's modulus of aggregate E_a	[GPa]	46.5	43.5	96.0	9.6
Poisson's ratio of aggregate ν_a	[-]	(0.15)	(0.2)	(0.2)	(0.3)
<i>cement characteristics:</i>					
Blaine fineness ϕ	[cm ² /g]	4000	4000	4000	4000
medium initial radius of clinker grains R	[μ m]	(7)	(7)	(7)	(7)
mass fractions of clinker phases ¹					
m_{C_3S}	[%]	41.8	41.8	41.8	41.8
m_{C_2S}	[%]	28.0	28.0	28.0	28.0
m_{C_3A}	[%]	8.2	8.2	8.2	8.2
m_{C_4AF}	[%]	9.5	9.5	9.5	9.5
$m_{C\bar{S}H_2}$	[%]	4.0	4.0	4.0	4.0
others + inert	[%]	8.5	8.5	8.5	8.5
<i>additional parameters for model verication:</i>					
curing temperature (isothermal conditions) [°C]		20	20	20	20
loading times t_0	[d]	28	28	28	28
aggregate/cement-ratio a/c^2	[-]	5.59	5.50	6.26	5.02
compressive strength f_{c28}^2	[MPa]	24.2	29.6	27.1	20.2

¹1% free CaO was taken into account²parameters for comparison with B3 model (Bažant and Baweja 1997) (cylinder 15/30 cm)

Table 7 Input-parameter sets for multiscale model (upscaling of creep properties) for concrete investigated in Athrushi (2003)

Creep experiments published in ... type of concrete (notation in reference)		Athrushi (2003)	
		BASE5	Maridal
<i>basic mix properties:</i>			
water/cement-ratio w/c	[-]	0.42	0.44
cement content c	[kg/m ³]	368.1	350
<i>elastic properties of aggregate:</i>			
Young's modulus of aggregate E_a	[GPa]	(40)1	(50)1
Poisson's ratio of aggregate ν_a	[-]	(0.3)	(0.3)
<i>cement characteristics²:</i>			
Blaine fineness ϕ	[cm ² /g]	3600	3600
medium initial radius of clinker grains R	[μ m]	(8)	(8)

Table 7 Continued.

Creep experiments published in ...		Athrusshi (2003)	
type of concrete (notation in reference)		BASE5	Maridal
mass fractions of clinker phases			
m_{C_3S}	[%]	(55)	(55)
m_{C_2S}	[%]	(23)	(23)
m_{C_3A}	[%]	6,0	6,0
m_{C_4AF}	[%]	(6,2)	(6,2)
$m_{C\bar{S}H_2}$	[%]	4.8	4.8
others	[%]	(5)	(5)
<i>additional parameters for model verification:</i>			
curing temperature (isothermal conditions)	[°C]	20	20
loading times t_0	[d]	1; 2; 3; 4; 6; 8	2; 3; 6
aggregate/cement-ratio a/c^3	[-]	5.12	5.24
compressive strength f_{c28}^3	[MPa]	81.0	69.0
Young's modulus E_{28}	[GPa]	34.3	42.2

¹chosen to match macroscopic stiffness evolution (E_{28})²Norcem Anleggsement CEM I 52.5 N-LA, data taken from www.norcem.org³parameters for comparison with B3 model (Bažant and Baweja 1997)Table 8 Determination of p_c for specific values of the w/c -ratio and ξ

1. Determination of f_i and f_g from the given values for the w/c -ratio and ξ on the basis of the relations given in Fig. 2.
2. Use of the linear relationship in Fig. 20(b) for determination of $R_{w,max}$ and, subsequently of $R_w(f_g)$ from $k = (f_g + f_i) / [\log_{10}(2R_{w,max}) \log_{10}(2.5 \times 10^9)]$, giving $-R_{w,max}(f_g + f_i)$, and $k = f_g / [\log_{10}(2R_{w,max}) - \log_{10}(2R_w(f_g))]$, giving $R_w(f_g)$, where $k = 0.135 = \text{const.}$ and f_i and f_g are obtained from Step 1.
3. Determination of p_c from $R_w(f_g)$ using Eq. (60).

scheme outlined in Table 8.

C. Creep model B3 according to Bažant and Baweja (1997)

Within the creep model B3, the (basic) creep compliance function $J(t, t_0)$ [psi⁻¹] reads

$$J(t, t_0) = q_1 + C_0(t, t_0) \quad (61)$$

where q_1 and $C_0(t, t_0)$ are given as

$$q_1 = \frac{6 \cdot 10^5}{E_{28}} \quad \text{with } E_{28} = 57000 f_c \quad (62)$$

and

$$C_0(t, t_0) = q_2 Q(t, t_0) + q_3 \log[1 + (t - t_0)^{0.1}] + q_4 \log(t/t_0), \quad (63)$$

respectively. Hereby, f_c [psi] denotes the 28 days standard cylinder compressive strength (1 psi =

6.895 MPa) and t_0 is the age of loading in days. The remaining parameters are calculated as

$$Q(t, t_0) = \left[0.086(t_0)^{\frac{2}{9}} + 1.21(t_0)^{\frac{4}{9}} \right]^{-1} \left[1 + \frac{\left[0.086(t_0)^{\frac{2}{9}} + 1.21(t_0)^{\frac{4}{9}} \right]^{-r(t_0)}}{[(t_0)^{-0.5} \log[1 + (t - t_0)^{0.1}]]^{r(t_0)}} \right], \quad (64)$$

$$r(t_0) = 1.7(t_0)^{0.12} + 8, \quad (65)$$

$$q_2 = 451.1 c^{0.5} f_c^{-0.9}, \quad (66)$$

$$q_3 = 0.29(w/c)^4 q_2, \quad (67)$$

and

$$q_4 = 0.29(a/c)^{-0.7}, \quad (68)$$

where the cement content c is given in [lf/ft³] (1 lf/ft³ = 16.03 kg/m³) and a/c denotes the aggregate/cement-ratio.