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A methodology for remaining life prediction of concrete structural components accounting for tension softening effect

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Abstract. This paper presents methodologies for remaining life prediction of plain concrete structural components considering tension softening effect. Non-linear fracture mechanics principles (NLFM) have been used for crack growth analysis and remaining life prediction. Various tension softening models such as linear, bi-linear, tri-linear, exponential and power curve have been presented with appropriate expressions. A methodology to account for tension softening effects in the computation of SIF and remaining life prediction of concrete structural components has been presented. The tension softening effects has been represented by using any one of the models mentioned above. Numerical studies have been conducted on three point bending concrete structural component under constant amplitude loading. Remaining life has been predicted for different loading cases and for various tension softening models. The predicted values have been compared with the corresponding experimental observations. It is observed that the predicted life using bi-linear model and power curve model is in close agreement with the experimental values. Parametric studies on remaining life prediction have also been conducted by using modified bilinear model. A suitable value for constant '*k*' of modified bilinear model is suggested based on parametric studies.

Keywords: concrete fracture; fatigue; tension softening; stress intensity factor; crack growth; remaining life.

1. Introduction

Concrete is a widely used material that is required to withstand a large number of cycles of repeated loading in structures such as highways, airports, bridges and offshore structures. The present state-of-the-art of designing such structures against the fatigue mode of distress is largely empirical, gained by many years of experience. As long as the designer is dealing with structures made of similar to those for which the relationships were derived, the performance can be reasonably well predicted. However, as conditions change, a need exists for a rational approach. Concrete contains numerous flaws, such as holes or air pockets, precracked aggregates, lack of bond between aggregate and matrix, etc., from which cracks may originate. When the tensile strength of a material is reached in a structure, cracking will occur. During fatigue cyclic loading, the flaw is blunted and resharpened and it is reasonable to assume that the crack so formed will be the nucleus of crack propagation that may lead to failure, and that the crack will initiate after the first loading cycle. Cracks generally propagate in a direction, which is perpendicular to the maximum tensile stress. In heterogeneous materials, cracks tend to follow the weakest path in the material. While the

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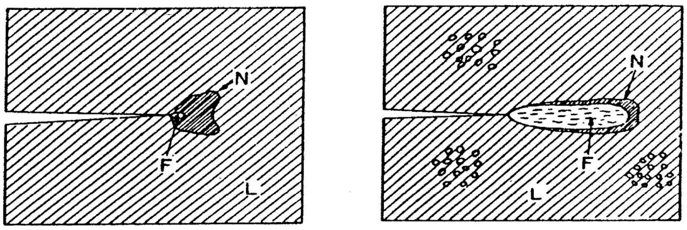


Fig. 1 FPZ in ductile and brittle materials

shape of the crack is likely to be highly irregular, it is expected that the irregularities will be smoothed out and the cracks will grow in a slow manner to a simple shape along which the stress intensity factor (SIF) is nearly uniform. Fracture mechanics is a rapidly developing field that has great potential for application to concrete structural design.

The fracture behavior of concrete is greatly influenced by the fracture process zone (FPZ). In concrete and rock fracture, the plastic flow is next to nonexistent and the nonlinear zone is almost entirely rolled by FPZ. Such materials are now commonly called quasi-brittle. The variation of FPZ along the structure thickness or width is usually neglected. The inelastic fracture response due to the presence of FPZ may then be taken into account by a cohesive pressure acting on the crack faces. Fig. 1 shows FPZ in brittle-ductile materials and quasi-brittle materials (Bazant 2002).

To model this behavior using discrete crack fracture mechanics, it is assumed that an initial crack begins to propagate at the proportional limit f_y and continues to propagate in a stable manner until the peak stress. When the crack extends in concrete, new crack surfaces are formed along the path of the initial crack tip. The newly formed crack surfaces may be in contact and this leads to toughening mechanisms in FPZ such as aggregate bridging. Further, they may continue to sustain some normal tensile stress that is characterized by a material tensile stress-separation relationship.

The first application of fracture mechanics to concrete was made by Kaplan (1961) using linear elastic fracture mechanics (LEFM) principles. Barenblatt (1959) and Dugdale (1960) made the first attempt to include the cohesive forces in the crack tip region within the limits of elasticity theory. Barenblatt (1959) assumed that cohesive forces act in a small zone near the crack ends such that the faces close smoothly. The distribution of these forces is generally unknown. For Dugdale (1960) model, the distribution of the closing forces is known and constant according to an elastic-perfectly plastic material. A major advance in concrete fracture was made by Hillerborg, *et al*. (1976), which includes the tension softening process zone through a fictitious crack ahead of the pre-existing crack whose lips are acted upon by closing forces such that there is no stress concentration at the tip of this extended crack. Bazant (1976) and Bazant and Cedolin (1979) used a smeared crack model to model cracking in concrete. In this model, the crack front is assumed to consist of a diffuse zone of microcracks and the stresses that close FPZ faces are represented through a stress-strain softening law.

Prasad and Krishnamoorthy (2002) developed a 2D computational model for investigation of crack formation and crack growth in plain and RC plane stress members. Attard and Tin-Loi (2005) conducted studies on numerical simulation of quasi-brittle fracture in concrete. Fracture was modeled through a constitutive softening-fracture law at the interface nodes, with the material within the triangular unit remaining linear elastic. Gasser and Holzapfel (2005), described methodologies for modeling 3D crack propagation in unreinforced concrete. It was mentioned that tensile failure involves progressive micro-cracking, debonding and other complex irreversible processes of internal damage. Wu *et al*. (2006), proposed an analytical model to predict the effective fracture toughness of concrete based on the fictitious crack model. The equilibrium equations of forces in the section were derived in combination with the plane section assumption. Slowik, *et al.* (2006) presented a method for determining tension softening curves of cementitious materials based on an evolutionary algorithm. Extensive research work was carried out towards numerical modelling of fracture and size effect in plain concrete using lattice model Raghu Prasad, *et al*. (2006). The concept of lattice model is discretization of the continuum by line elements such as bar and beam elements, which can transfer forces and moments.

When the structural components are subjected to repetitive live loads of high-stress amplitude, according to classical theory, applied loads result in in-plane tensile stresses at the bottom of the components. The stress-state in such structures is often simulated with three-point bending tests. Plain concrete subjected to flexural loading fails owing to crack propagation. Repeated loading results in a steady decrease in the stiffness of the structure, eventually leading to failure. It is of interest to characterize the material behavior subjected to such loading and study the crack propagation and remaining life resulting from such loading.

The current approaches used to evaluate fatigue performance are mainly empirical. Fatigue equations based on the well known S-N concept have been developed. Implementation of the conventional S-N approach requires time-consuming experimental data collection for a given design case followed by statistical analysis. The resulting information is not applicable to other design cases with different loading configurations or boundary conditions. A severe limitation of the S-N approach is the inherent empiricism as it does not use fundamental material parameters that can be determined for use in design or evaluation. Mechanistic approaches that utilize the concept of fracture mechanics to study crack propagation from fatigue loading have also been proposed. For example, Perdikaris, *et al.* (1987) showed that compliance measurements provide a convenient method for estimating the traction-free crack length of fatigued concrete specimens. Since then, few experimental investigations on fatigue crack propagation in concrete have been reported (Baluch, *et al.* 1987, Ramsamooj 1994, Stuart 1982, Subramanian, *et al.* 2000, Matsumoto 1999, Toumi and Turatsinze 1998, Slowik, *et al*. 1996, Bazant 1991, Ingraffea 1977, Mu, *et al.* 2004). The rate of fatigue crack growth in concrete exhibits an acceleration stage that follows an initial deceleration stage. In the deceleration stage the rate of crack growth decreases with increasing crack length, whereas in the acceleration stage there is a steady increase in crack growth rate up to failure. They have attempted to apply the fracture mechanics principles to describe the crack growth during the acceleration stage of fatigue crack growth in concrete. It has been observed that the Paris law coefficients are dependent on the material composition potentially explaining the large differences in the values of the Paris law coefficients. From literature, it has also been observed that the research work towards crack growth analysis and remaining life prediction of concrete structural components considering tension softening is limited (Baluch, *et al.* 1987, Ramsamooj 1994, Stuart 1982, Subramanian, *et al*. 2000, Matsumoto 1999, Toumi and Turatsinze 1998, Slowik, *et al*. 1996, Bazant 1991, Ingraffea 1977). There is a scope to conduct crack growth analysis and remaining life prediction of concrete structural components considering tension softening effect in to account. Further, it has been observed from the literature that there are numerous tension softening models to account for softening effect. There is scanty information in choosing the appropriate softening model for reliable remaining life prediction.

This paper presents methodologies for remaining life prediction of concrete structural components considering tension softening effect. Non-linear fracture mechanics principles have been used for crack growth analysis and remaining life prediction. Various tension softening models such as linear, bi-linear, tri-linear, exponential and power curve have been presented with appropriate expressions. A methodology to account for tension softening effects in the computation of SIF and remaining life prediction of concrete structural components has been presented. The tension softening effects has been represented by using any one of the models mentioned above. Numerical studies have been conducted on three point bending concrete structural component under constant amplitude loading. Remaining life has been predicted for different loading cases using various tension softening models and compared with the corresponding experimental observations. It is observed that the predicted life using bi-linear model and power curve model is in close agreement with the experimental values. Parametric studies on remaining life prediction have also been conducted by using modified bilinear model. A suitable value for constant '*k*' of modified bilinear model is suggested from parametric studies.

2. Concrete fracture models

Based on different energy dissipation mechanisms, NLFM models for quasi-brittle materials can be classified as a fictitious crack approach (cohesive crack model) and an equivalent-elastic crack approach. Fracture mechanics models using only the Dugdale-Barenblatt energy dissipation mechanism are usually referred to as the fictitious crack approach, whereas fracture mechanics models using only the Griffth-Irwin energy dissipation mechanism are usually referred to as the effective-elastic crack approach or equivalent-elastic crack approach.

The energy release rate for a mode I quasi-brittle crack, G_q, can be expressed as (Shah, *et al*. 1995).

$$G_q = G_{Ic} + G_\sigma \tag{1}$$

Brief description of fictitious crack model is presented below.

2.1. Fictitious crack approach (Cohesive crack model)

The fictitious crack approach assumes that energy to create the new surface is small compared to that required to separate them, and the energy rate term G_{IC} vanishes in Eq. (1). Fig. 2 shows the simulation of a newly formed crack structures and the corresponding fracture process zone (Shah, *et al*. 1995). As a result, the energy dissipation for crack propagation can be completely characterized by the cohesive stress-separation relationship $\sigma(\omega)$. Since all energy produced by the applied load is completely balanced by the cohesive pressure, Eq. (1) is reduced to (with G_{Ic} = 0)

$$G_q = \int_0^W t\sigma(w)dw \tag{2}$$

Eq. (2) is valid for structures with a constant thickness. The fictitious crack is assumed to initiate and propagate when the principal tensile stress reaches the tensile strength of material f_t.

Cohesive crack model requires a unique $\sigma(\omega)$ curve to quantify the value of energy dissipation.

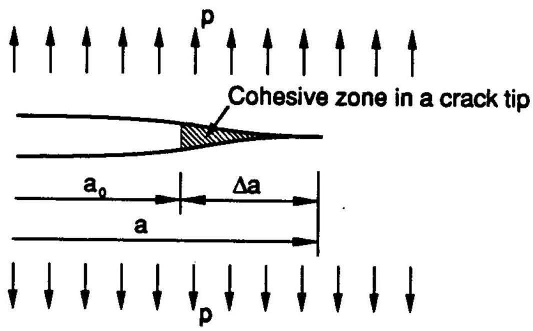


Fig. 2 Mode I crack for fictitious crack approach

Table 1 Different types of closing pressure for FPZ

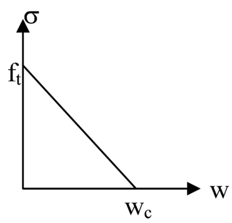
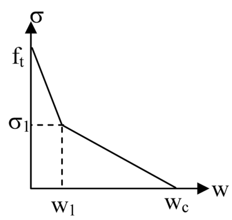
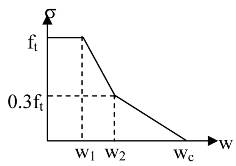
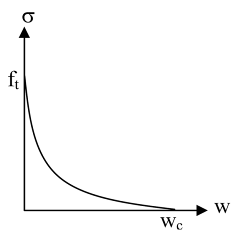
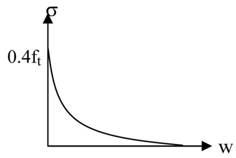
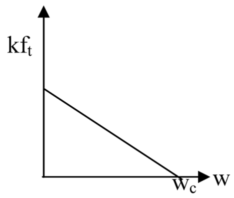
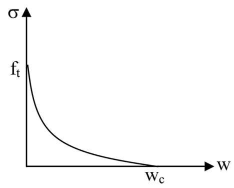
Type	Expression	Shape
Linear curve - Hillerborg, *et al.* (1976)	$\sigma = f_t(1 - w/w_c)$	
Bilinear curve - Roelfstra and Wittmann (1986)	$\sigma = \begin{cases} f_t - (f_t - \sigma_1)w/w_1 & \text{for} \quad w \le w_1 \\ \sigma_1 - \sigma_1(w - w_1)/(w_c - w_1) & \text{for} \quad w_1 > w \end{cases}$	
Trilinear curve - Liaw, *et al.* (1990)	$\sigma = \begin{cases} f_t & \text{for} \quad w \le w_1 \\ f_t - 0.7 f_t(w - w_1)(w_2 - w_1) & \text{for} \quad w_1 < w \le w_2 \\ 0.3 f_t(w_c - w)/(w_c - w_2) & \text{for} \quad w_2 < w \le w_c \end{cases}$	
Exponential curve - Footer, *et al.* (1986)	$\sigma = f_t\left(1 - \frac{w}{w_c}\right)^n$ where *n* is a fitting parameter	
Reinhardt (1985)	$\sigma = f_t\left\{1 - \left(\frac{w}{w_c}\right)^n\right\}$ where 0 < *n* < 1 is a fitting parameter	

Table 1 Different types of closing pressure for FPZ (Continued)

Type	Expression	Shape
Gopalaratnam and Shah (1985)	$\sigma = f_t \exp(kw^{\lambda})$ where k and λ are material parameters	
similar relationship was also suggested by Cedolin, *et al.* (1987)	$k = -0.06163$ and $\lambda = 1.01$ for concrete with f_c' values of 33-47 MPa.	
Power curve - Du, *et al.* (1990)	$\sigma = 0.4 f_t (1 - w/w_c)^{1.5}$	
Bilinear curve with $w_1 = 0$ - Figueiras and Owen (1984)	$\sigma = k f_t (1 - w/w_c)$ where, k=constant	
Power curve - Hordijk (1991)	$\sigma = f_c \left\{ \left[1 + \left(a_1 \frac{w}{w_c} \right)^3 \right]_{\exp} \left(-a_2 \frac{w}{w_c} \right) - \frac{w}{w_c} (1 + a_1^3) \exp(-a_2) \right\}$ where a_1 and a_2 are fitting parameters	

The choice of the $\sigma(\omega)$ function influences the prediction of the structural response significantly, and the local fracture behavior, for example the crack opening displacement, is particularly sensitive to the shape of $\sigma(\omega)$. Many different shapes $\sigma(\omega)$ curves, including linear, bilinear, trilinear, exponential, and power functions, have been used in the literature. Some of the widely used $\sigma(\omega)$ curves with appropriate expressions are listed in Table 1.

3. SIF accounting for tension softening and remaining life prediction

In this approach, one of the major assumptions is to use fracture mechanics principles to describe the crack growth phenomena during the acceleration stage of fatigue crack growth in concrete. The fatigue mechanism in plain concrete may be attributed to progressive bond deterioration between aggregates and matrix or by development of cracks existing in the concrete matrix. These two mechanisms may act together or separately, leading to complexity of the fatigue mechanism. It is well known fact that concrete typically exhibits nonlinear fracture processes because of the large FPZ, leading to LEFM based approach objectionable. Hence an analytical model for assessing the fatigue life of concrete accounting the tension softening effect is required. The following are the

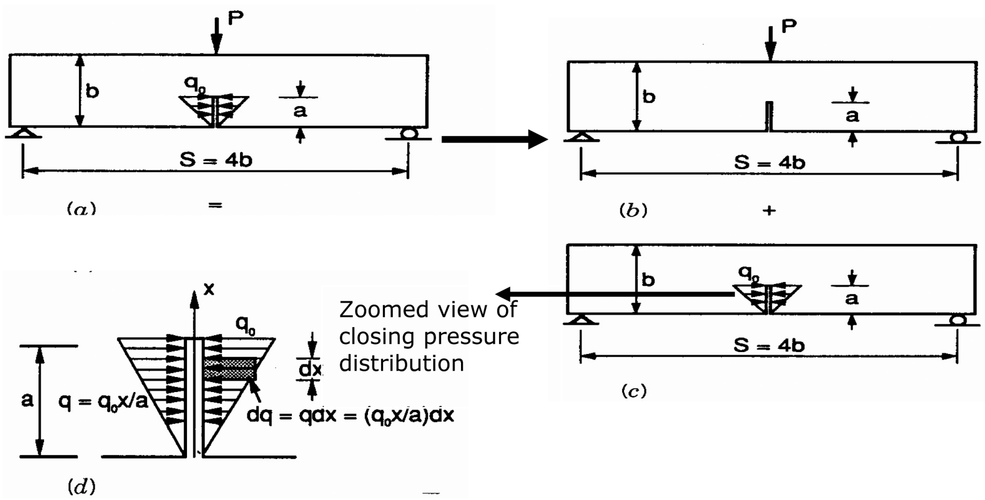


Fig. 3 Illustration of superposition principle

basic assumptions of tension softening.

Modelling assumptions

i. Plane sections of the beam remain plane after deformation

ii. Fictitious crack surface remains plane after deformation

iii. Normal closing fractions acting on the factious crack follow the linear stress crack opening displacement

iv. Fiber bending stress in the concrete along the bottom of the beam is equal to the fraction normal to the crack mouth at the bottom of the beam.

To incorporate the tension softening behavior, based on the principle of superposition the stress intensity factor (SIF) has to be modified as (Fig. 3),

$$K_I = K_I^P - K_I^q \tag{3}$$

where K_I^P is SIF for the concentrated load P in a three point bending beam geometry, and K_I^q is SIF due to the closing force applied on the effective crack face inside the process zone, which can be obtained through Green's function approach by knowing the appropriate softening relation. Superposition principle is used by accounting the non linearity in incremental form. SIF due to applied load and due to closing force will act in opposite directions. K_I will not become zero as the magnitude of K_I^q is around 10 to 20% of K_I^P .

3.1. Computation of K_I^P

SIF due to the concentrated load P can be calculated by using LEFM principles. A three-point bending beam is shown in Fig. 3(b). The SIF for the beam can be expressed as

$$K_I^P = \sigma\sqrt{\pi a} g_1\left(\frac{a}{b}\right) \quad \text{where, } \sigma = \frac{3PS}{2b^2 t} \tag{4}$$

where P = applied load, a = crack length, b = depth of the beam, t = thickness and $g_1(a/b)$ = geometry factor, depends on the ratio of span to depth of the beam and is given below for $S/b = 2.5$ (Tada, *et al*. 1985).

$$g_1\left(\frac{a}{b}\right) = \frac{1.0 - 2.5a/b + 4.49(a/b)^2 - 3.98(a/b)^3 + 1.33(a/b)^4}{(1-a/b)^{3/2}} \tag{5}$$

3.2. Computation of K_I^q

The incremental SIF due to the closing force dq can be written as, (Shah, *et al.* 1995)

$$dK_I^q = \frac{2}{\sqrt{\pi\Delta a}} aq\, g\left(\frac{a}{D}, \frac{x}{a}\right) \tag{6}$$

where dq can be expressed as function of softening stress distribution over the crack length Δa; the function 'g' represents the geometry factor.

Calculation of 'dq'

By using the above concept (Fig. 3(d)), cohesive crack can be modelled in the following manner (Fig. 4).

The crack opening displacement w at any point x is assumed to follow linear relationship (Fig. 4) and can be expressed as,

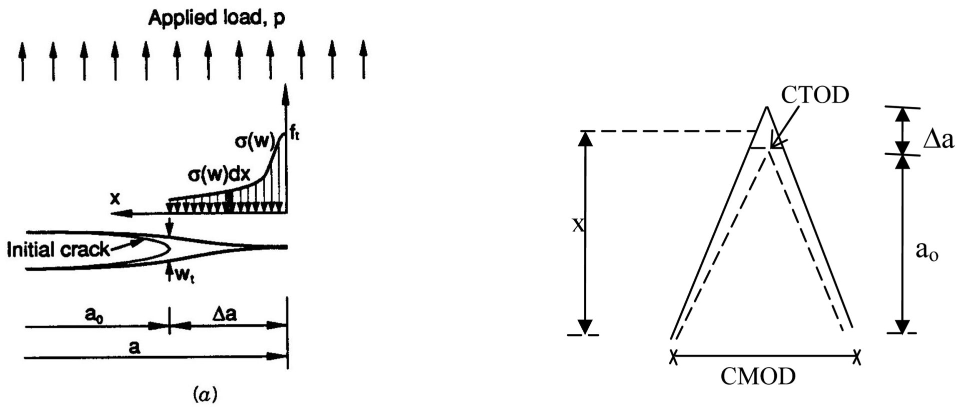
$$w = \delta\left(\frac{a_0 - x}{\Delta a} + 1\right) \qquad a_0 \le x \le a_{eff} \tag{7}$$

where δ is the crack tip opening displacement, a_0 is initial crack length and.

$$a_{eff} = a_0 + \Delta_a$$

As an example, let us consider linear softening law (refer Table 1)

$$\sigma = f_t(1 - w/w_c) \tag{8}$$



(a) Modelling of quasi-brittle Crack with crack surfaces in contact

(b) Schematic diagram of crack Opening displacement

Fig. 4 Cohesive crack modelling

Where f_t = tensile strength of concrete and w_c = critical crack opening displacement

Substituting for w from Eq. (7) in the linear softening law given by Eq. (8), one can obtain,

$$dq = \sigma = f_t\left\{1-\frac{\delta}{w_c}\left(\frac{a_0-x}{\Delta a}+1\right)\right\} \tag{9}$$

The crack opening displacement at any point $\delta(x)$ can be calculated using the following equation

$$\delta(x) = \mathrm{CMOD}\, g_3\left(\frac{a}{b},\frac{x}{a}\right) \text{where} \tag{10}$$

$$g_3\left(\frac{a}{b},\frac{x}{a}\right) = \left\{\left(1-\frac{x}{a}\right)^2+\left(1.081-1.149\frac{a}{b}\right)\left[\frac{x}{a}-\left(\frac{x}{a}\right)^2\right]\right\}^{1/2}$$

where CMOD is crack mouth opening displacement and is calculated using the following formula.

$$\mathrm{CMOD} = \frac{4\sigma a}{E} g_2\left(\frac{a}{b}\right) \tag{11}$$

where $g_2(a/b)$ is geometric factor, depends on the ratio of span to depth of the beam and is given below for S = 2.5b

$$g_2(a/b) = \frac{1.73-8.56a/b+31.2(a/b)^2-46.3(a/b)^3+25.1(a/b)^4}{(1-a/b)^{3/2}} \tag{12}$$

Hence, replacing dq in Eq. (6) and integrating over length Δa, K_I^q can be obtained as,

$$K_I^q = \int_{a_0}^{a_{eff}} \frac{2f_t}{\sqrt{\pi\Delta a}}\left\{1-\frac{\delta}{w_c}\left(\frac{a_0-x}{\Delta a}+1\right)\right\} g\left(\frac{a}{b},\frac{x}{a}\right) dx \tag{13}$$

where

$$g\left(\frac{a}{b},\frac{x}{a}\right) = \frac{3.52(1-x/a)}{(1-a/b)^{3/2}} - \frac{4.35-5.28x/a}{(1-a/b)^{1/2}} + \left[\frac{1.30-0.30(x/a)^{3/2}}{\sqrt{1-(x/a)^2}}+0.83-1.76\frac{x}{a}\right]\left[1-\left(1-\frac{x}{a}\right)\frac{a}{b}\right] \tag{14}$$

Similar expressions can be obtained for other models such as bilinear, trilinear, exponential, power law etc., Remaining life can be predicted by using any one of the standard crack growth equations (such as Paris, Erdogan-Ratwani, etc.,)

$$\frac{da}{dN} = f(\Delta K) \tag{15}$$

Here ΔK can be computed by using following expression

$$DK = K_{\max} - K_{\min}, \text{ where } K_{\max} = K^p - K^q \tag{16}$$

4. Numerical studies

Crack growth analysis and remaining life prediction studies have been carried out by using LEFM and NLFM principles for concrete three point bending specimens under constant amplitude loading. The details of the studies are presented below.

4.1. Problem1

This problem was studied by Toumi and Turatsinze (1998) for three point bending concrete specimen (refer Fig. 5).

Length (S) = 320 mm
Depth (b) = 80 mm
Thickness (t) = 50 mm
Initial crack length (a_o) = 2 to 4 mm
Compressive strength = 57 MPa
Tensile strength = 4.2 MPa
Fracture toughness = 0.63 MPa$\sqrt{m}$
Crack growth equation = Paris
Min. load =198.72 N

The bending tensile stress (f_b) can be calculated by using the formula given below

$$f_b = 3PS / 2tb^2 \tag{17}$$

Crack growth analysis and remaining life prediction has been carried out using LEFM and NLFM principles.

4.1.1. Using LEFM principles

Remaining life has been predicted for the different loading cases using LEFM Principles. Geometric factor is calculated by using the expression given below (Tada, *et al*. 1985).

$$g_1\left(\frac{a}{b}\right) = \frac{1.0-2.5a/b+4.49(a/b)^2-3.98(a/b)^3+1.33(a/b)^4}{(1-a/b)^{3/2}} \tag{18}$$

Table 2 shows the predicted remaining life values for various loading cases along with the experimental values presented by Toumi and Turatsinze (1998). From Table 2, it can be observed that there is maximum of about 12% difference between the predicted and experimental values.

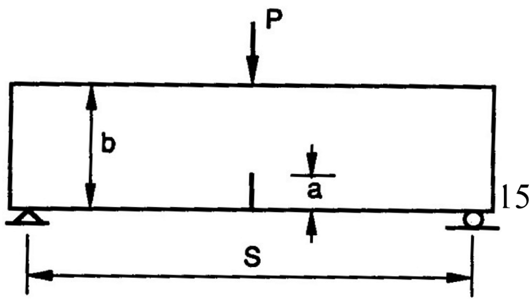


Fig. 5 Three point bending problem

Table 2 Predicted remaining life values using LEFM principles

S. No.	Max. Stress (MPa)	Crack growth constants		Remaining life (Cycles)		% diff.
		C (μm/cycle)	*m*	Present study	Literature (Exptl.) Toumi and Turatsinze (1998)	
1	1.125	6.45	4.18	28689	32222	10.96
2	1.05	0.33	2.31	57251	63611	9.98
3	0.975	0.26	2.25	62603	69444	9.82
4	0.9	2.04	2.6	16188	18333	11.7

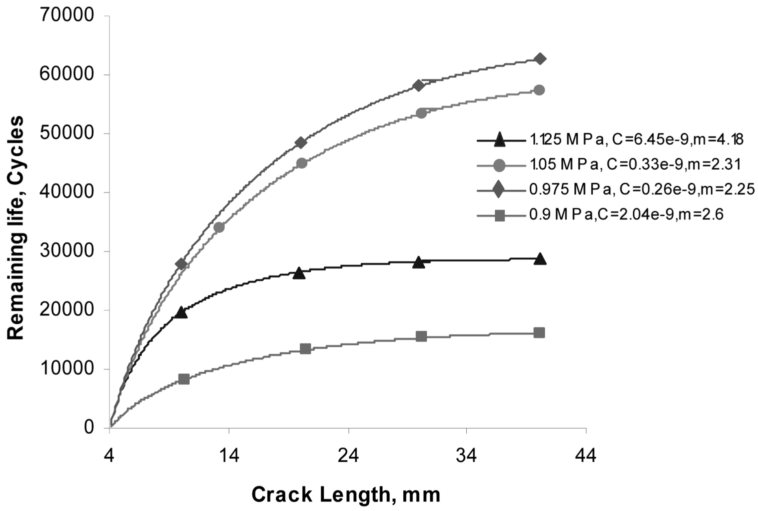


Fig. 6 Crack length vs remaining life - LEFM

Table 3 Remaining life prediction using various tension softening models

Max.stress MPa	Remaining life using						
	Linear	Bilinear	Trilinear	Expo. Model by Footer	Expo. Model by Reinhardt	Power model	Exptl Toumi and Turatsinze (1998)
1.125	33304	32251	32942	33308	33102	30887	32222
1.05	66747	63892	65032	66781	65348	61011	63611
0.975	74775	69692	70998	74791	71346	66592	69444
0.9	19102	18479	18801	19116	18892	17612	18333

The difference in the values is attributed to not considering the tension softening effect in the analysis. Fig. 6 shows the variation of predicted remaining life with crack length for differed loading cases.

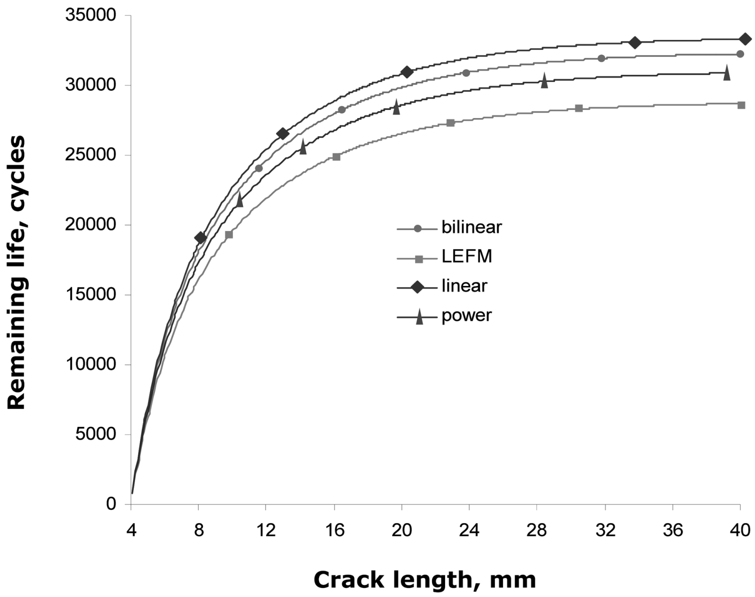


Fig. 7 Crack length Vs remaining life - NLFM

Table 4 Remaining life using modified bilinear model

Max.stress	Remaining life using modified Bilinear model, $\sigma = kf_t\left(1.0 - \frac{w}{wc}\right)$ with w_1=0					Exptl Toumi and Turatsinze (1998)
	k=0.9	k=0.8	k=0.7	k=0.6	k=0.5	
1.125	32809	32310	31826	31352	30982	32222
1.05	64842	63862	62740	61893	61063	63611
0.975	71102	69672	68412	67568	66510	69444
0.9	18678	18441	18096	17806	17417	18333

4.1.2. Using NLFM principles

Crack growth analysis and remaining life prediction has been carried out by accounting the tension softening effect. Various tension softening models are used to account for the tension softening effect in SIF computation. Table 3 shows the predicted remaining life by using various tension softening models along with the experimental values. From Table 3, it can be observed that the predicted remaining life using linear, tri-linear and exponential models is larger compared to corresponding experimental values. It can also be observed that the predicted remaining life using bi-linear and power curve models are in good agreement with the experimental values. Fig. 7 shows a plot between crack length and remaining life for various tension softening models. From the plot, it can be observed that the predicted remaining life is larger for linear model compared to other tension softening models.

Remaining life is also predicted by using modified bi-linear model. Table 4 shows the predicted remaining life for different values of k used in the modified bi-linear model. From the Table, it can be observed that the predicted remaining life with k = 0.7 and 0.6 is in very good agreement with the corresponding experimental values.

4.2. Problem 2

Crack growth studies and remaining life prediction has been carried out using LEFM and NLFM principles for concrete three point bending specimens under constant amplitude loading. This problem was experimentally studied by [Bazant and Schell 1993]. The details of the studies are presented below.

Beam depth (b) = 38.1, 107.8, 304.8 mm
Span (S) = 2.5 * D mm
Thickness (t) = 38.1 mm
Initial crack length = b/6 mm
Modulus of elasticity = 38,300 MPa
Tensile strength = 8.9 MPa
Other input details are given in Table 5

4.2.1. Using LEFM principles

Remaining life has been predicted for the different loading cases using LEFM Principles. Table 5 shows the predicted remaining life values for the various loading cases along with the experimental

Table 5 Predicted remaining life values using LEFM principles

Sl. No.	Details of beam, mm	Max. stress, MPa	Min. Stress, MPa	Crack growth constants		Remaining life		Diff(%)
				Log C	m	Present study	Exptl. Bazant and Schell (1993)	
1	b=38.1 S=95.25 t=38.1	0.291	0.0279	-65.264	8.525	29010	33409	13.17
2	b=107.8 S=268.75 t=38.1	0.07422	0.00675	-64.07	8.099	6779	7450	9.00
3	b=304.8 S=762 t=38.1	0.01915	0.00174	-66.235	8.248	36642	40867	10.338

Table 6 Remaining life prediction using various tension softening models

Max.stress MPa	Remaining life using						
	Linear	Bilinear	Trilinear	Expo. Model by Footer	Expo. Model by Reinhardt	Power curve	Exptl. Bazant and Schell (1993)
0.291	34862	33496	34129	34982	34672	31982	33409
0.07422	7789	7498	7662	7801	7754	7162	7450
0.01915	42812	41146	41970	42798	42486	39102	40867

values reported by Bazant and Schell (1993). From Table 5, it can be observed that there is maximum of about 13% difference between the predicted and experimental values. The difference in the values can be attributed to not considering the tension softening effect in the analysis.

4.2.2. Using NLFM principles

Crack growth analysis and remaining life prediction has been carried out by accounting for the tension softening effect. Various tension softening models are used to account for the tension softening effect in the SIF computation. Table 6 shows the predicted remaining life by using various tension softening models along with the experimental values. From Table 6, it can be observed that the predicted remaining life using linear, tri-linear and exponential models is higher compared to corresponding experimental values. It can also be observed that the predicted remaining life values using bi-linear and power curve models are in good agreement with the corresponding experimental values.

Remaining life is also predicted by using modified bi-linear model with different values of k. Table 7 shows the predicted remaining life obtained by using the modified bi-linear model. From the Table, it can be observed that the predicted remaining life with *k*=0.7 and 0.6 is in very good agreement with the corresponding experimental observations.

Table 7 Remaining life using modified bilinear model

Max.stress	Remaining life using modified Bilinear model, $\sigma = kf_t\left(1.0 - \frac{w}{wc}\right)$ With w_1=0					Exptl Bazant and Schell (1993)
	k=0.9	*k*=0.8	*k*=0.7	*k*=0.6	*k*=0.5	
0.291	34117	33546	32983	32484	32101	33409
0.07422	7692	7512	7368	7213	7146	7450
0.01915	41848	41098	40315	39684	39128	40867

Table 8 Predicted remaining life values using LEFM and NLFM principles

Sl. No	Max. Stress MPa	Stress ratio	Initial crack length, mm	Crack growth constants		Remaining life (Cycles) predicted by using		
						LEFM	NLFM (modified bi-linear)	
				C	*m*		(*k*=0.7)	(*k*=0.6)
1	0.5194	0.1	75	7.71e-25	3.12	38078*	43290	42612
2		0.2		5.78e-24	3.12	33176	37682	37103
3		0.3		1.72e-24	3.15	25436	28842	28367
4	0.692	0.1	75	7.71e-25	3.12	24536	27762	27412
5		0.2		5.78e-24	3.12	21987	24912	24492
6		0.3		1.72e-24	3.15	14789	16744	16482
7	0.4328	0.1	85	7.71e-25	3.12	25123	28382	28102
8		0.2		5.78e-24	3.12	22453	25391	25010
9		0.3		1.72e-24	3.15	17936	20112	19982

* – Experimental value 44000 [16]

4.3. Problem 3

Another example problem has been chosen for crack growth analysis and remaining life prediction. This problem was studied by Baluch, *et al*. (1987).

Length of supported span (s) = 1360 mm
Thickness (t) = 51 mm
Depth (b) = 152 mm
Fracture toughness = 1.16×10^6 N/m$^{3/2}$
Other input details are shown in the Table 8

Table 8 shows the predicted remaining life for different loading cases using LEFM and NLFM principles. From Table 8, it can be observed that there is about 11% diff. between the predicted value and the corresponding experimental value in the case of LEFM. Table 8 also shows the predicted remaining life using modified bi-linear model with k=0.7 and 0.6. It can be observed that the predicted values are in very good agreement with the experimental value. For other loading cases, the experimental values are not available in the literature for comparison. Since the methodologies for crack growth analysis and remaining life prediction accounting for tension softening effect have been well tested and verified for the previous problems, it can be assumed that the predicted remaining life is reliable.

5. Summary and concluding remarks

Methodologies for remaining life prediction of concrete structural components accounting for tension softening effects have been presented. Non-linear fracture mechanics principles have been used for crack growth analysis and remaining life prediction. Various tension softening models such as linear, bi-linear, tri-linear, exponential and power curve have been presented with appropriate expressions. A methodology to account for tension softening effects in the computation of SIF and remaining life prediction of concrete structural components has been presented. The tension softening effects has been represented by using any one of the models mentioned above. Numerical studies have been conducted on three point bending concrete structural component under constant amplitude loading. Remaining life has been predicted for different loading cases and for various tension softening models. The predicted values have been compared with the corresponding experimental observations. The main observations from the study are:

- The predicted life using linear, tri-linear and exponential models is higher compared to experimental values
- In general, the predicted life using bi-linear and power curve model is in close agreement with the experimental values

Parametric studies on remaining life prediction have also been carried out by using modified bilinear model. From the parametric studies, it is observed that the predicted remaining life with k=0.7 and 0.6 of modified bi-linear model is in very close agreement with the corresponding experimental values.

From the overall study, it can be concluded that the methodologies for crack growth analysis and remaining life prediction of concrete structural components can be effectively used for reliable remaining life prediction. A modified bi-linear model with k=0.7 and 0.6 can be used to account for tensioning softening effect in the analysis.

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