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Investigations on the tensile strength of high-performance fiber reinforced concrete using statistical methods

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Abstract. This paper presents the investigations towards developing a better understanding on the contribution of steel fibers on the tensile strength of high-performance fiber reinforced concrete (HPFRC). An extensive experimentation was carried out with w/cm ratios ranging from 0.25 to 0.40 and fiber content ranging from zero to 1.5 percent with an aspect ratio of 80. For 32 concrete mixes, flexural and splitting tensile strengths were determined at 28 days. The influence of fiber content in terms of fiber reinforcing index on the flexural and splitting tensile strengths of HPFRC is presented. Based on the test results, mathematical models were developed using statistical methods to predict 28-day flexural and splitting tensile strengths of HPFRC for a wide range of w/cm ratios. The expressions, being developed with strength ratios and not with absolute values of strengths and are applicable to wide range of w/cm ratio and different sizes/shapes of specimens. Relationship between flexural and splitting tensile strengths has been developed using regression analysis and absolute variation of strength values obtained was within 3.85 percent. To examine the validity of the proposed model, the experimental results of previous researchers were compared with the values predicted by the model.

Keywords: silica fume; crimped steel fibers; fiber reinforcing index; high-performance fiber reinforced concrete; flexural strength; splitting tensile strength; modeling.

1. Introduction

The concept of using fibers in concrete is a routine one for introducing special properties to concrete. Balaguru and Shah (1992) have reported that the addition of steel fibers in concrete matrix improves all mechanical properties of concrete especially tensile strength and toughness. ACI Committee 544 (1982) states that SFRC is usually specified by strength and fiber content. Flexural strengths rather than compressive strength are generally specified for pavements, as reported in ACI Committee 544. It is well documented that use of silica fume in concrete results in significant improvement in mechanical properties and durability of concrete. The use of silica fume in concrete increases the C-S-H gel formation that is mainly responsible for the enhancement of strength, durability of concrete and reduction in pore structure in the transition zone and increased impermeability (Aïtcin 1998). Banja and Sengupta (1991, 2002) have observed that silica fume incorporation results in the improvement of the tensile strengths in the concrete and developed a

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statistical model of the compressive strength of SF concrete, involving non-dimensional variables, is independent of the specimen parameters. In spite of the wealth of information available in the existing literature on fiber reinforced concrete, the relationship between 1. flexural tensile strength and fiber reinforcing index; 2. splitting tensile strength and fiber reinforcing index; and 3. flexural and splitting tensile strengths, for predicting 28-day strength at any water-to-cementitious material ratio and fiber content in terms of fiber reinforcing index are quite limited. To combine the effect of both weight fraction and their aspect ratio, the reinforcing index, (RI = w_f*(l/d)) can be used as the fiber reinforcing parameter for a given type of fiber (Fanella and Naamam 1985). Wafa and Ashour (1992) have developed models for predicting the influence of fiber contents on strength properties (modulus of rupture, splitting tensile strength and compressive strength) of HSFRC and given different models for the same strength parameter but for different size of specimens. Relationship between modulus of ruptures considering size effect, regardless of fiber content, has also been developed. Ezheldin, *et al.* (1992) have developed a relationship between steel fiber reinforcing index (RI) and compressive strength of HSFRC with w/cm ratio of 0.35 and for fiber content ranging from 0 to 59 kg/m^3, and Nataraja, *et al.* (2001, 1999) have developed models for predicting 28-day strengths using regression analysis as a function of fiber reinforcing index (RI). In all the models only a particular w/cm ratio with varying fiber content was used. The absolute strength values have been dealt with in all the models and thus are valid for a particular w/cm ratio and type of specimen.

The aim of the present investigation is to overcome this inherent weakness and develop the statistical models involving non-dimensional parameters so that the effect of wide range of w/cm ratios and specimen size can be eliminated. Mansur, *et al.* (1999) have studied the effect of fiber volume fraction and size, shape and casting directions (both in vertical and horizontal) of specimens on compressive strength (ranging from 70 to 120 MPa), stiffness and ductility of HSFC. Based on the test results, mathematical models were developed to predict 28-day flexural and splitting tensile strengths of HPFRC for a wide range of w/cm ratios and at 5 and 10 percent silica fume replacement, which may serve as the useful tools to quantify the effects of fiber reinforcement in terms of fiber reinforcing index for assessing the tensile strengths of concrete. The relationship between 28-day flexural and splitting tensile strength has been derived by analyzing experimental data, using regression analysis. The information embodied in this paper is directed towards developing a better understanding on the contribution of crimped steel fiber reinforcement on the tensile strength of high-performance concrete (HPC).

2. Experimental program

Four basic mixes for plain concrete (SF concrete), designated FC1-0.0, FC2-0.0, FC3-0.0 and FC4-0.0 corresponding to the w/cm ratios of 0.4, 0.35, 0.30 and 0.25 were selected. Based on these basic mixes, fibrous concrete mixes were proportioned.

2.1. Materials and mixture proportions

Ordinary Portland cement-53 grade satisfying the requirements of IS: 12269-1987 and silica fume contained 88.7% of SiO_2, having fineness by specific surface area of 23000 m^2 /kg, a specific gravity of 2.25 were used in the ratio of 9.5:0.5 and 9:1 by weight of cementitious materials in all the mixes. The chemical Analysis of Silica fume (Grade 920-D) is given in Table 1(a). Fine aggregate of locally available river sand having fineness modulus of 2.55, a specific gravity of 2.63 and water absorption of 0.98%, and coarse aggregate of blue granite crushed stones with 12.5 mm maximum size having a specific gravity of 2.70 and water absorption of 0.65% were used. Fibers conforming to ASTM A820-01 have been used, are crimped steel fibers of diameter = 0.45 mm and length = 36 mm, giving an aspect ratio of 80. The properties of steel fibers used are given in Table 1(b). Mixtures were proportioned using ACI 544-1993 recommended guidelines and guidelines given in ACI 211.4R-1999 (part I), and IS: 10262-1992. Mixture proportions used in the test programme are summarized in Table 2. For each water to cemetitious materials ratio six fibrous concrete mixes were prepared with fiber volume fractions, V_f of 0.5, 1.0 and 1.5 percent. Due to the inclusion of the fibers some minor adjustments in terms different ingredients had to be made as shown in Table 2. Mixing water was adjusted to correct for aggregate absorption. Sulphonated naphthalene formaldehyde (SNF) as HRWR admixture (super-plasticizer) conforming to ASTM Type F with dosage range of 1.75% to 2.75% by weight of cementitious materials has been used to maintain the adequate workability of plain and fiber reinforced concrete.

2.2. Mixing and curing

Due to high cement and micro silica contents, the presence of small size coarse aggregate content and steel fiber content in fresh mix concrete, the efficient mixing of the HPFRC is more difficult than conventional concrete. For these reasons, super-plasticizer was used to produce uniform concrete without any segregation. Concrete was mixed using a tilting type mixer and specimens were cast using steel moulds, compacted with needle vibrator. For each mix, at least three $100 \times 100 \times 500$ mm prisms and three 150×300 mm cylinders were cast. Specimens were demolded 24 hours after casting and water cured at ~28°C until the age of testing at 28 days.

Table 1(a) Chemical analysis of silica fume (Grade 920-D) (Analyzed for mandatory parameters of ASTM C1240-1999)

Component	Result
Silicon dioxide, SiO_2	88.7%
Moisture content	0.7%
Loss of Ignition @ 975°C	1.8%
Carbon	0.9%

Table 1(b) Fiber characteristics (crimped steel fiber)

Geometry and properties	Value
Fiber diameter, d (mm)	0.45
Fiber length, l (mm)	36
Aspect ratio, l/d	80
Ultimate tensile strength, f_u (MPa)	910
Young's modulus, E_f (GPa)	200
Number of fibers per kg.	22410

Table 2 Mix proportioning of HPFRC (data for 1 m^3)

Mix Designation	w/cm	Cm kg (=C+SF)	FA kg	CA kg	SF kg	W kg	SP (%)	Steel fiber V_f(%)
FC1-0	0.4	438	691	1088	22	175	1.75	0
FC1-0.5	0.4	438	687	1079	22	175	1.75	0.5
FC1-1	0.4	438	682	1071	22	175	1.75	1
FC1-1.5	0.4	438	678	1062	22	175	1.75	1.5
FC1*-0	0.4	438	691	1088	43.8	175	1.75	0
FC1*-0.5	0.4	438	687	1079	43.8	175	1.75	0.5
FC1*-1	0.4	438	682	1071	43.8	175	1.75	1
FC1*-1.5	0.4	438	678	1062	43.8	175	1.75	1.5
FC2-0	0.35	486	664	1088	24.3	170	2	0
FC2-0.5	0.35	486	660	1079	24.3	170	2	0.5
FC2-1	0.35	486	655	1071	24.3	170	2	1
FC2-1.5	0.35	486	651	1062	24.3	170	2	1.5
FC2*-0	0.35	486	664	1088	48.6	170	2	0
FC2*-0.5	0.35	486	660	1079	48.6	170	2	0.5
FC2*-1	0.35	486	655	1071	48.6	170	2	1
FC2*-1.5	0.35	486	651	1062	48.6	170	2	1.5
FC3-0	0.3	550	624	1088	27.5	165	2.5	0
FC3-.5	0.3	550	620	1079	27.5	165	2.5	0.5
FC3-1	0.3	550	615	1071	27.5	165	2.5	1
FC3-.5	0.3	550	611	1062	27.5	165	2.5	1.5
FC3*-0	0.3	550	624	1088	55	165	2.5	0
RC3*-0.5	0.3	550	620	1079	55	165	2.5	0.5
FC3*-1	0.3	550	615	1071	55	165	2.5	1
FC3*-1.5	0.3	550	611	1062	55	165	2.5	1.5
FC4-0	0.25	640	562	1088	32	160	2.75	0
FC4-0.5	0.25	640	558	1079	32	160	2.75	0.5
FC4-1	0.25	640	553	1071	32	160	2.75	1
FC4-1.5	0.25	640	549	1062	32	160	2.75	1.5
FC4*-0	0.25	640	562	1088	64	160	2.75	0
FC4*-0.5	0.25	640	558	1079	64	160	2.75	0.5
FC4*-1	0.25	640	553	1071	64	160	2.75	1
FC4*-1.5	0.25	640	549	1062	64	160	2.75	1.5

In mix designation FC1 to FC4 and FC1* to FC4*, silica fume replacement is 5 and 10 percent respectively by weight of cementitious materials, after hyphen denotes fiber volume fraction, percent.
SP(%)-Super plasticizer in percent by weight of cementitious materials
Water present in Super plasticizer is excluded in calculating the water to cementitious materials ratio.
V_f(%) denotes Steel fiber volume fraction in percent in total volume of concrete.

2.3. Strength testing

2.3.1. Flexural strength

The flexural strength (Modulus of rupture) tests were conducted as per the specification of ASTM C 78-94 using $100 \times 100 \times 500$ mm prisms under third- point loading on a simply supported span of 400 mm The tests were conducted in a 100 kN closed loop hydraulically operated Universal testing machine. Samples were tested at a deformation rate of 0.1 mm/min. Three samples were used for computing the average strength and in few cases more samples (4 or 5) were considered.

2.3.2. Splitting tensile strength

The splitting tensile strength (indirect tensile strength) tests were conducted according to the specification of ASTM C 496-90 using 150×300 mm cylindrical specimens. The tests were conducted in a 1000 kN closed loop hydraulically operated Universal testing machine. Three samples were used for computing the average strength and in few cases more samples (4 or 5) were considered.

2.3.3. Compressive strength

Compressive strength test were performed according to IS: 516-1979 standards using 150 mm cubes loaded uniaxially. Each strength value was average of there specimens. The compressive strength ranges from 55.6 to 86.5 MPa is given in Table 3.

3. Analysis of results and statistical modeling

The results of this investigation are applicable to the material and the type of fibers used.

3.1. Flexural strength

Table 3 presents the variation of the flexural strength (modulus of rupture), f_{rf} on the effect of fiber content in terms of fiber reinforcing index and the flexural strength ratios of fiber reinforced and plain concrete (silica fume concrete). Fig. 1 shows flexural strength ratios, (f_{rf}/f_r) as a function of the fiber reinforcing index, RI of the concrete. The strength ratios can be utilized for the development of the generalized expressions which, being free from the influence of varying w/cm

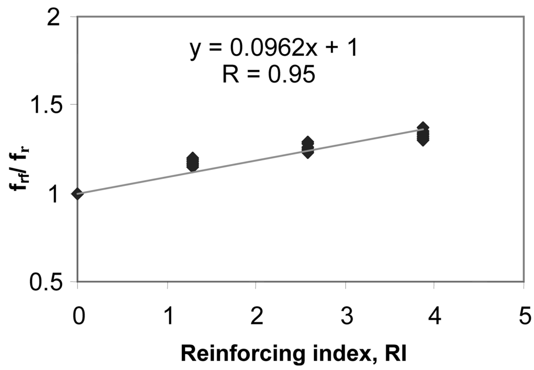

Fig. 1 Relationship between flexural tensile strength ratio and fiber reinforcing index

Table 3 28-day tensile strength of fiber reinforced concrete and plain concrete and their corresponding ratios

Mix Designation	w/cm ratio	Reinforcing Index, RI	f_c (MPa)	f_{rf} (MPa)	f_{rf}/f_r	f_{spf} (MPa)	f_{spf}/f_{spf}	Estimated by present model. Model Eq. (4)	Percent variation
FC1-0	0.4	0	55.62	5.61	1	3.88	1	5.678	-1.25
FC1-0.5		1.29	59.28	6.68	1.191	4.97	1.281	6.708	1.06
FC1-1		2.58	62.2	7.17	1.28	5.60	1.443	7.269	-1.38
FC1-1.5		3.88	62.85	7.46	1.33	6.04	1.556	7.648	-2.53
FC1*-0		0	61.03	6.21	1	4.38	1	6.161	0.79
FC1*-0.5		1.29	64.75	7.15	1.151	5.48	1.251	7.164	-0.19
FC1*-1		2.58	66.85	7.73	1.245	6.37	1.454	7.927	-2.55
FC1*-1.5		3.88	67.38	8.19	1.319	6.83	1.559	8.308	-1.44
FC2-0	0.35	0	62.32	6.28	1.000	4.41	1	6.344	-1.02
FC2-0.5		1.29	65.43	7.32	1.166	5.69	1.290	7.531	-2.88
FC2-1		2.58	67.72	7.88	1.255	6.31	1.431	8.074	-2.46
FC2-1.5		3.88	68.36	8.44	1.344	6.67	1.512	8.381	0.70
FC2*-0		0	66.87	6.75	1	4.75	1	6.669	1.17
FC2*-0.5		1.29	69.23	8.06	1.194	5.94	1.25	7.752	3.82
FC2*-1		2.58	71.4	8.54	1.265	6.65	1.4	8.364	2.06
FC2*-1.5		3.88	71.95	9.15	1.356	7.26	1.528	8.873	3.03
FC3-0	0.3	0	65.8	7.31	1	4.86	1	7.236	1.01
FC3-0.5		1.29	70.6	8.48	1.16	6.35	1.307	8.755	-3.24
FC3-1		2.58	72.41	9.05	1.238	6.73	1.385	9.009	0.45
FC3-1.5		3.88	72.94	9.58	1.311	7.15	1.471	9.384	2.05
FC3*-0		0	72.75	7.40	1	5.12	1	7.495	-1.28
FC3*-0.5		1.29	75.87	8.76	1.184	6.35	1.24	8.663	1.10
FC3*-1		2.58	76.96	9.32	1.259	7.18	1.402	9.410	-0.97
FC3*-1.5		3.88	77.29	10.13	1.369	7.71	1.506	9.872	2.55
FC4-0	0.25	0	75.21	7.80	1	5.15	1	7.687	1.45
FC4-0.5		1.29	80.74	9.11	1.166	6.58	1.278	9.065	0.49
FC4-1		2.58	82.97	9.62	1.232	7.51	1.458	9.909	-3.00
FC4-1.5		3.88	83.03	10.16	1.301	7.95	1.544	10.296	-1.34
FC4*-0		0	78.54	8.02	1	5.62	1.000	8.152	-1.65
FC4*-0.5		1.29	82.83	9.58	1.195	6.95	1.237	9.405	1.83
FC4*-1		2.58	85.91	10.36	1.292	8.05	1.432	10.383	- 0.22
FC4*-1.5		3.88	86.47	11.01	1.355	8.48	1.509	10.753	2.34

In mix designation FC1 to FC4 and FC1* to FC4*, silica fume replacement is 5 and 10 percent respectively, after hyphen denotes fiber volume fraction, percent

f_{rf} represents flexural strength of FRC, f_r refers to the strength of plain concrete

f_{spf} represents splitting tensile strength of FRC, f_{sp} refers to the strength of plain concrete

Fiber reinforcing index (RI) = w_f *(l/d) and average density of HPFRC = 2415 kg/m^3

Weight fraction (w_f) = (density of fiber/density of fibrous concrete)*V_f

Aspect ratio (l/d) = length of fiber/diameter of fiber.

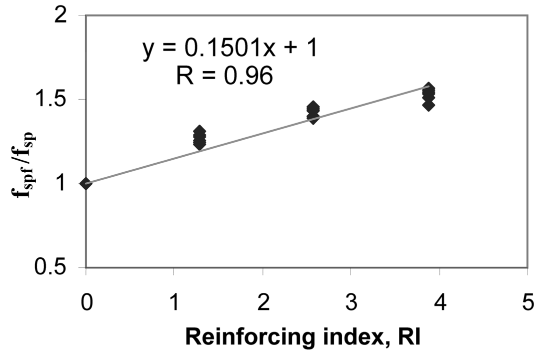

Fig. 2 Relationship between splitting tensile strength ratio and fiber reinforcing index

ratios and specimen parameters, can be used for the prediction of strength. The validity of model was investigated by examining relevant statistical coefficients (Bhattacharya 1977).

Based on the results of the present investigation, the equation for predicting the flexural strength ratio of fiber reinforced and plain concrete using linear regression analysis, has been obtained as:

$$f_{rf}/f_r = 1 + 0.096\ (\mathrm{RI}) \tag{1}$$

Where f_{rf} = flexural strength of HPFRC, MPa
f_r = flexural strength of plain concrete, MPa and
RI = steel fiber reinforcing index

The values of the correlation coefficient (R) and the standard error of estimate (s) have been obtained as 0.95 and 0.263 respectively. The percent variation in absolute has been obtained as 2.103.

3.2. Splitting tensile strength

Table 3 presents the variation of the Splitting tensile strength, f_{spf} on the effect of fiber content in terms of fiber reinforcing index and the Splitting tensile strength ratios of fiber reinforced and plain concrete (silica fume concrete). Fig. 2 shows splitting tensile strength ratios, (f_{spf}/f_{sp}) as a function of the fiber reinforcing index, RI of the concrete. These ratios can be utilized for the development of the generalized expressions which, being free from the influence of varying w/cm ratios and specimen parameters, can be used for the prediction of strength. The validity of model was investigated by examining relevant statistical coefficients (Bhattacharya 1977).

Based on the test results, using linear regression analysis, the splitting tensile strength ratio (f_{spf}/f_{sp}) of fiber reinforced and plain concrete may be predicted in terms of fiber reinforcing index, RI as follows:

$$f_{spf}/f_{sp} = 1 + 0.150\ (\mathrm{RI}) \tag{2}$$

Where f_{spf} = Splitting tensile strength of fiber reinforced concrete, MPa;
f_{sp} = Splitting tensile strength of plain concrete, MPa and

The values of the correlation coefficient (R) and the standard error of estimate (s) have been obtained as 0.96 and 0.274 respectively. The percent variation in absolute has been obtained as 3.092.

3.3. Relationship between flexural strength and splitting tensile strength

The situations, where there is no prior relationship known between variables, a scatter diagram is prepared and the information portrayed in this diagram is used in the search of the appropriate mathematical model. On analyzing the data of results from scatter diagram (Fig. 3), it was observed that 0.673 power might be appropriate for the test results. Accordingly, regression analysis was carried out for this model using method of least squares and the unknown parameters were determined. The validity of the model was investigated by examining relevant statistical coefficients.

The relationship between the 28-day flexural strength ratio of fiber reinforced and plain concrete (reference concrete) and 28-day splitting tensile strength ratio of fiber reinforced and plain concrete based on the results of the present investigation, for w/cm ratios ranging from 0.25 to 0.40, has been obtained as:

$$f_{rf} / f_r = 1.002\ (f_{spf} / f_{sp})^{0.673} \tag{3}$$

The values of the correlation coefficient (R) and the standard error of estimate (s) have been obtained as 0.98 and 0.163 respectively. The percent variation in absolute has been obtained as 1.645.

Eq. (3) can be written as:

$$f_{rf} = 1.002\, f_{spf}^{\ 0.673}\ (f_r / f_{sp}^{\ 0.673})$$

$$f_{rf} = 1.002\ K_1 f_{spf}^{0.673}$$

$$f_{rf} = K_2 f_{spf}^{0.673} \tag{4}$$

Where $K_1 = (f_r / f_{sp}^{0.673})$ and $K_2 = 1.002\ K_1$, is a constant which depends upon the w/cm ratio.

The values of constant, K_2 for different w/cm ratios is given in Table 4. Fig. 4 shows the variation of constant, $K2$ with respect to w/cm ratio as a non-linear curve, with correlation coefficient (R) as 0.93.

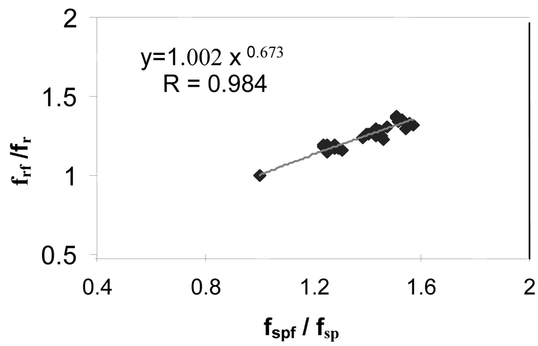

Fig. 3 Relationship between flexural tensile strength ratio and splitting tensile strength ratio

Table 4 Values for constant, K_2 for different water-cementitious material ratios

w/cm ratio	Average value of constant, K_2
0.40	2.280
0.35	2.337
0.30	2.499
0.25	2.551

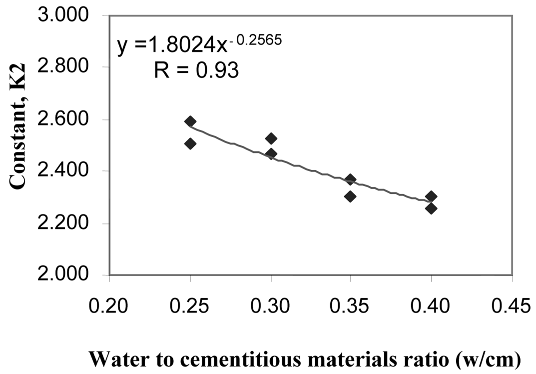

Fig. 4 Constant, $K2$ vs water to cementitious material ratio

3.4. Analysis of data

The coefficient of determination, R^2, for each linear equation was found to be close to unity, indicating that the linear model is a good description for the relationship between the two variables. Another statistical tool to evaluate the suitability of the linear model is by observing residuals plotted as a function of the strength(or) reinforcing index. If the relationship between X (independent variable) and Y (dependent variable) is linear and if the various assumptions made in regression analysis are true, then a plot of residuals against the values of X will show no apparent trend or pattern with changes in X. Fig. 5 shows the plot of standardized residuals against RI, for all the mixtures. Standardized residual is a raw residual divided by the standard error of estimate, which is a measure of the actual variability about the regression plane of the underlying population. If the residuals are normally distributed about the regression, 95 % of the standardized residuals should lie between -2 and +2 and 99% between -2.5 and +2.5. Fig. 5 shows almost 97% of the standardized residuals are between -2 and +2 and 100% between -2.5 and +2.5 indicating that there are no outlier or extreme residual values. The Fig. 5 also shows a reasonably well-scattered plot.

3.5. Validation of the model with results of previous researchers

In order to testify whether the proposed model is independent of the specimen parameters, strength

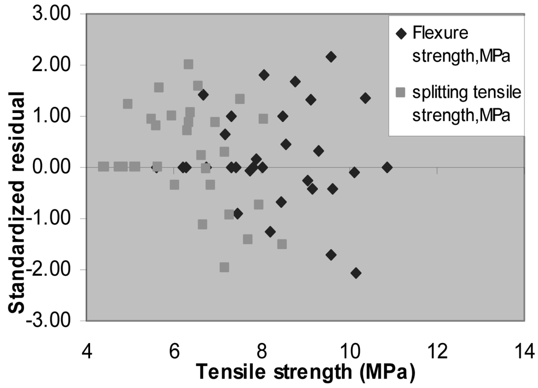

Fig. 5 Standardized residuals for tensile strengths

Table 5 Comparison between the results of previous researchers and the predicted values of the present model

Researchers and type of specimen used	w/cm ratio	Fiber volume fraction, percent	28-day flexural strength, MPa		Variation, Percent
			Experimental value	Predicted by the present model	
Wafa and Ashour	0.21	0	9.98	10.00	-0.22
Beam		0.5	11.42	11.93*	-4.43
Size:150×150×530 mm		0.75	12.98	12.33	5.01
		1.0	14.79	13.71	6.72
Size:100×100×350 mm	0.21	0	10.36	10.38	-0.20
Fiber-hooked end steel		0.5	13.27	12.38	6.70
		0.75	14.16	12.80	9.60
		1.0	15.55	14.23	8.47
Mohammadi and	0.35	1.0	5.35	5.36	-0.20
Kaushik		1.0	7.50	6.26	10.96
Beam:100×100×500 mm		1.0	7.16	6.91	3.55
Fiber-crimped steel					

*Refer Appendix-1

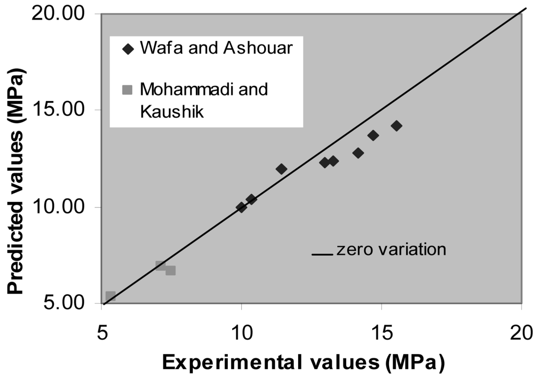

Fig. 6 Comparison between experimental and predicted values of flexural strength

results of steel fiber reinforced concrete at different w/cm ratios, obtained by different researchers on different specimens have been considered. A comparison between the experimental results obtained by Wafa and Ashour (1992), Mohammadi and Kaushisk (2003) and those predicted by the present model (Eq. 3) is presented in Table 5. The correlation of the experimental values of flexural strengths by various researchers and the corresponding values predicted by the model is presented in Fig. 6. The variation of strength calculated based on the prediction by the present model, indicates that, the performance of the model mainly depends upon the fiber types and aspect ratio of single or mixed type fibers. For FRC mixes containing hooked end steel fibers (Wafa and Ashour 1992), the maximum absolute variation obtained was 9.60 percent.

4. Conclusions

Extensive experimentation was carried out to determine the effect of steel fiber content in terms of fiber reinforcing index on tensile strength of HPC at water-cementitious material ratios ranging from 0.25 to 0.40

1. The addition of steel fibers by 1.50 percent volume fraction (RI = 3.88) results in an increase of 37.91 percent in the flexural tensile strength, and results in an increase of 55.94 percent in the splitting tensile strength compared with the unreinforced matrix.
2. On the basis of regression analysis of a large number of experimental results, the statistical models have been developed, which can serve as the useful tools for predicting and optimizing the tensile strength of high-performance fiber reinforced concrete over a wide range of w/cm ratios and fiber reinforcing index, RI varies from 0 to 3.88 (V_f ranges from 0 to 1.5 percent). These models involve the non- dimensional variables, are suitable for wide range of w/cm ratios and independent of specimen parameters.
3. The proposed models were found to have good accuracy in estimating the tensile strength of high-performance fiber reinforced concrete, where 92% of the estimated values are within ±5% of the actual values.
4. The validity of the model has been verified with results of different researchers and the variation obtained is within 11 percent.

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Notation

The following symbols are used in this paper

f_r = flexural strength of plain concrete (HPC), MPa
f_{rf} = flexural strength of HPFRC, MPa
f_{sp} = splitting tensile strength of plain concrete (HPC), MPa
f_{spf} = splitting tensile strength of HPFRC, MPa
f_c = compressive strength of HPFRC, MPa
HPC = high performance concrete
HPFRC = high performance fiber reinforced concrete
SFRC = steel fiber reinforced concrete
V_f = fiber volume fraction in percent
w_f = weight fraction
RI = fiber reinforcing index.
K_1, K_2 = constants

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Appendix-1

Typical calculation for finding Flexural strength, $\mathbf{f_{rf}}$ of HPFRC

$$(\mathbf{f_{rf}} / \mathbf{f_r}) = 1.002*(f_{spf}/f_{sp})^{0.673}$$
$$= 1.002*(8.38/6.45)^{0.673}$$
$$= 1.195$$
$$\mathbf{f_{rf}} = 1.195*9.98$$
$$= 11.93 \text{ MPa}$$

Experimental flexural strength, f_{rf} =11.42 MPa

Therefore variation = (11.42-11.93)/11.42*100
= -4.43 %

CC