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Dynamic behaviour of stiffened and damaged coupled shear walls

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Abstract. The free vibration of stiffened and damaged coupled shear walls is investigated using the mixed finite element method. The anisotropic damage model is adopted to describe the damage extent of the reinforced concrete shear wall element. The internal energy of a locally damaged shear wall element is derived. Polynomial shape functions established by Kwan are used to present the component of displacements vector on each point within the wall element. The principle of virtual work is employed to deduce the stiffness matrix of a damaged shear wall element. The stiffened system is reinforced by an additional stiffening beam at some level of the structure. This induces additional axial forces, and thus reduces the bending moments in the walls and the lateral deflection, and increases the natural frequencies. The effects of the damage extent and the stiffening beam on the free vibration characteristics of the structure are studied. The optimal location of the stiffening beam for increasing as far as possible the first natural frequency of vibration is presented.

Keywords : free vibration; damaged reinforced concrete structures; coupled shear wall; finite element method.

1. Introduction

As the height of buildings increases, it becomes more and more important to provide the structures of buildings with sufficient stiffness against lateral loads arising from wind or earthquakes. Reinforced concrete shear walls are recognized as one of the more efficient structural systems for such purposes. However, such walls are very often weakened by vertical bands of openings which are required for doors, windows and corridors. Recently, in north Algeria, many high-rise buildings are constructed, using the box system which consists only of reinforced concrete walls with openings and slabs. In the earlier papers (Choo and Coull 1984, Li and Choo 1996, Aksogan, *et al.* 2003, Kuang and Chau 1999). it has been shown that the efficiency of coupled structural walls could under certain circumstance be increased significantly by addition of a stiffer beam or a rigid truss at the top or some level of the structure. Therefore, the earlier investigations (Choo, and Coull 1984, Li and Choo 1996, Aksogan, *et al.* 2003, Kuang and Chau 19991, Michael 1967, Kwan 1993) on the dynamic behavior of coupled shear wall structures are focused on the effect of flexible foundation and local deformations at beam-wall joints, carried out using a discrete-

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continuous approach and finite element method, whereas the vibration characteristics of damaged RC stiffened coupled shear walls has not been studied yet.

The structures of buildings such as coupled shear walls, are now decaying because of age, deterioration, misuse, lack of repair and in some cases, they were not designed for current demand. However, the damage may be defined as any deviation in the structures original geometric or material properties that may cause undesirable stresses, displacements, or vibration on the structure. These weakening and deviations may be due to cracks, loose bolts, broken welds, corrosion, fatigue, etc.

As the damage increases within the reinforced concrete structure, the alteration of the mechanical characteristics yields modal characteristic changes. In this way, Chen, *et al.* (1996) investigated the structural damage by means of the identification method of modal changes. At a critical damage level, they indicated that a decrease of the fundamental frequency up to 10% can be expected for steel beams. For reinforced concrete structures, the fundamental frequency reduction, related to the structural damage can be significantly larger. Pseudodynamic tests carried out at the European Laboratory for structural assessment (JRC-Ispra) in fact showed fundamental frequency reduction more than 60% Pegon, *et al.* (1998).

The main objective of the present study is to analyze the global behavior of the damaged stiffened RC coupled shear walls. The anisotropic damage model is incorporated to describe the damage extent of the coupled shear walls. The mixed finite element method (Kwan 1993) is employed to derive the stiffness matrix of equivalent damaged shear wall element. The FEM is employed to determine the dynamic characteristics in free vibration analysis problem. Numerical results are presented that relate the effect of damage state on the vibrations characteristics of the RC stiffened coupled shear walls.

2. Finite element for analysis of shear walls

Application of the finite element method to shear walls analysis can be dated back to 1960s. Theoretically, the finite element method, being the most powerful tool of analysis available, can be applied to any type of building structures. However, due to relatively low efficiency and high computing cost, full finite element analysis of shear wall have never been popular. The

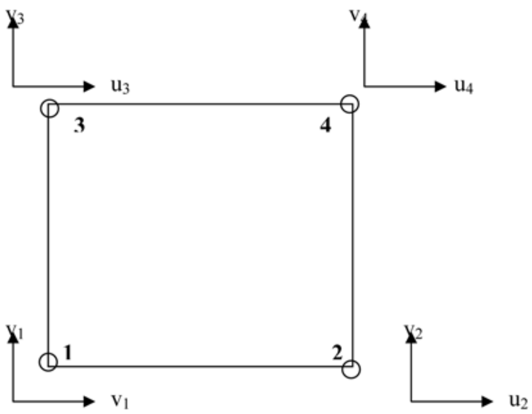

Fig. 1 Q4 element

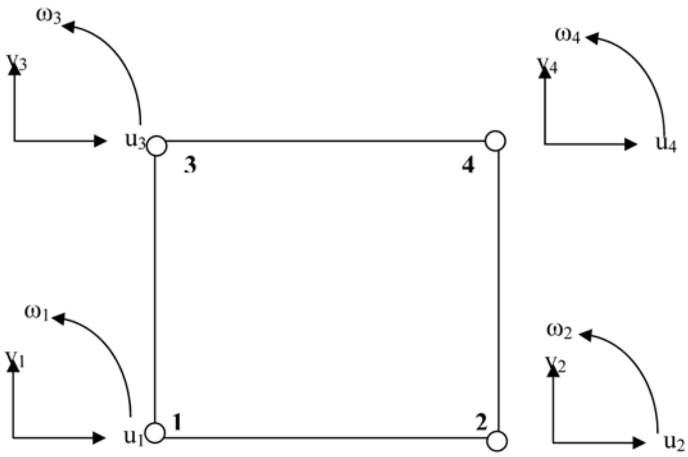

Fig. 2 Cheng's beam type element

principal causes for low efficiency of the method is the presence of parasitic shear in many of the lower order elements such as Q4 element which has two translation DOFs per node as shown in Fig. 1. Because displacement shape functions of this element are expressed in linear functions, deformation of element edges can be expressed by straight lines and the shear stress in an element are constant and cannot represent the actual stress distribution accurately if the finite element mesh is not fine. However, it is felt that the best method of dealing with parasitic shear is to avoid them by using elements that can exactly represents the strain state of pure bending.

To improve the computational efficiency of the finite element method, finite strip element (Cheung and Swaddiwudhipong 1978), and higher order element (Chan and Cheung 1979) and Lee element (1987) were developed to model the shear wall with the rotational DOF for represent the

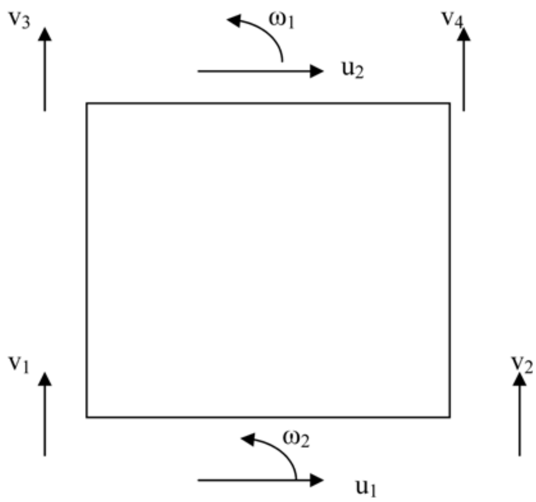

Fig. 3 Kwan's strain based element

strain state of pure bending, so to avoid parasitic shear problem.

Therefore, the 12 DOFs plane stress element as Cheung element (1983) and Lee element (1987) with drilling DOFs Fig. 2. was used in many research works as Kim and Lee (2003).

As suggested by Kwan (1992, 1993), by neglecting the lateral strain in the wall, which are generally of little significance. The DOF can be reduced from 12 to eight as shown in Fig. 3. Use of this simplified Cheung's element, which is computationally more efficient, is recommended rather the original Cheung's element.

Using the mixed finite element method, Kwan (1993) developed a wall element with the eight DOFs. This element included two existing elements, namely the simplified Cheung's element (1979) and Kwan's strain based element (1992).

3. Theoretical background

3.1. Material properties of damaged shear walls

Voyiadjis and Kattan (1992) proposed an anisotropic damage model, in which the elastic energy configuration of deformed and damaged state is equivalent to the elastic energy configuration of deformed but undamaged state. Based on this assumption, the relations of elastic constants of damaged state and undamaged state can be expressed as :

$$E_{11}^{d} = E_{11}(1-\Phi_{11})^2$$
$$G_{12}^{d} = 4\left(\frac{G_{12}(1-\Phi_{11})(1-\Phi_{22})}{(1-\Phi_{11})+(1-\Phi_{22})}\right)^2 \tag{1}$$

where E_{11}^{d}, G_{12}^{d} and E_{11}, G_{12} are the elastic constants of damaged and undamaged state, respectively, and Φ_{11} and Φ_{22} are damage variables. Hence, the material properties of the damaged shear wall can be represented by replacing the above elastic constants with the effective ones defined in Eq. (1). A convenient way to determining Φ_{11} and Φ_{22} is to utilize the damage law postulated by Yu, *et al*. (1989), Shen, *et al.* (2004) for concrete. It is given as:

$$\Phi_{22} = \frac{1}{2N_C+1}\left[\frac{\varepsilon_2}{\varepsilon_f^c}\right]^{NC} \tag{2}$$

$$\Phi_{11} = H\Phi_{22} \quad (H>1) \tag{3}$$

and

$$N_C = \frac{\sqrt{E_f^f}}{2(\sqrt{E_c}-\sqrt{E_f^c})} \tag{4}$$

where E_f^c is the tangential elastic modulus when the stress reaches its peak, E_c is the initial elastic modulus and ε_f^c is the failure strain and ε_2 is the current state of strain. H is a constant determined by experiments, for example, when E_c=49.49 GPa, N_C=3.65, H=3 and ν=0.2, then $\Phi_{22}=0.12048(\varepsilon_2 / \varepsilon_f^c)^{3.65}$.

3.2. Damaged shear wall stiffness matrix

Consider a shear walls with a damaged region (the shaded area) $S_d = b_d x\, h_d$ and O_d is its centroid. The shear wall element which has the total area $S_t = bx\ h,$ is subjected to lateral load. Let u to be the lateral displacement, v the vertical displacement and ω the rotation of the vertical fibres. The mixed finite element method established by Kwan (1993), is adopted to deduce the stiffness matrix of a proposed strengthened shear wall element**.** Hence, the displacement components at any point within the wall element may be expressed in the terms of the nodal DOF of the element as follows:

$$v = v_1\left(\frac{1}{2}-\frac{x}{b}\right)\left(\frac{1}{2}-\frac{y}{h}\right)+v_2\left(\frac{1}{2}+\frac{x}{b}\right)\left(\frac{1}{2}-\frac{y}{h}\right)+v_3\left(\frac{1}{2}-\frac{x}{b}\right)\left(\frac{1}{2}+\frac{y}{h}\right)+v_4\left(\frac{1}{2}+\frac{x}{b}\right)\left(\frac{1}{2}+\frac{y}{h}\right)$$
$$+\left(\left(\frac{6}{h}u_1-u_2\right)-3(\omega_1+\omega_2)\right)\left(\frac{1}{4}-\left(\frac{y}{h}\right)^2\left(\frac{x}{2}+\frac{b}{4}\right)\right) \tag{5}$$

$$u = u_1\left(\frac{1}{2}-\frac{3}{2}\left(\frac{y}{h}\right)+2\left(\frac{y}{h}\right)^3\right)+\omega_1 h\left(-\frac{1}{8}+\frac{1}{4}\left(\frac{y}{h}\right)+\frac{1}{2}\left(\frac{y}{h}\right)^2-\left(\frac{y}{h}\right)^3\right)+u_2\left(\frac{1}{2}+\frac{3}{2}\left(\frac{y}{h}\right)-2\left(\frac{y}{h}\right)^3\right)$$
$$+\omega_2 h\left(\frac{1}{8}+\frac{1}{4}\left(\frac{y}{h}\right)-\frac{1}{2}\left(\frac{y}{h}\right)^2-\left(\frac{y}{h}\right)^3\right) \tag{6}$$

The strain energy for each wall element can be written as:

$$U^e = U^e_B + U^e_S \tag{7}$$

Where:

U^e_B and U^e_S are the strain energy due to the bending and shear effects respectively.

The strain energy considering only the bending effect U^e_B is done as :

$$U^e_B = \frac{1}{2}\left[E_{11}t_1\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\varepsilon_y^2\, dxdy - E_{11}t_1 g_1\int_{yd-\frac{hd}{2}}^{yd+\frac{hd}{2}}\int_{xd-\frac{bd}{2}}^{xd+\frac{bd}{2}}\varepsilon_y^2 dxdy\right] \tag{8}$$

in which:

$$\varepsilon_y = \frac{dv}{dy} \tag{9}$$

and
$$g_1 = 1-(1-\Phi_{11})^2 \tag{10}$$

The strain energy which related to the shear effect may be written as:

$$U^e_S = \frac{1}{2}\left[G_{12}t_1\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\gamma_{xy}^2 dxdy - G_{12}t_1 g_2\int_{yd-\frac{hd}{2}}^{yd+\frac{hd}{2}}\int_{xd-\frac{bd}{2}}^{xd+\frac{bd}{2}}\gamma_{xy}^2 dxdy\right] \tag{11}$$

in which

$$\gamma_{xy} = \frac{dv}{dx}+\frac{du}{dy} \tag{12}$$

and
$$g_2 = 1-4\left(\frac{(1-\Phi_{11})(1-\Phi_{22})}{(1-\Phi_{11})(1-\Phi_{22})}\right)^2 \tag{13}$$

where E_{11} and G_{12} are the Young's modulus in the y direction and the shear modulus of the RC shear wall, the coordinate $(x_{d,}\ y_d)$ represent the position of the geometric centre of the damaged

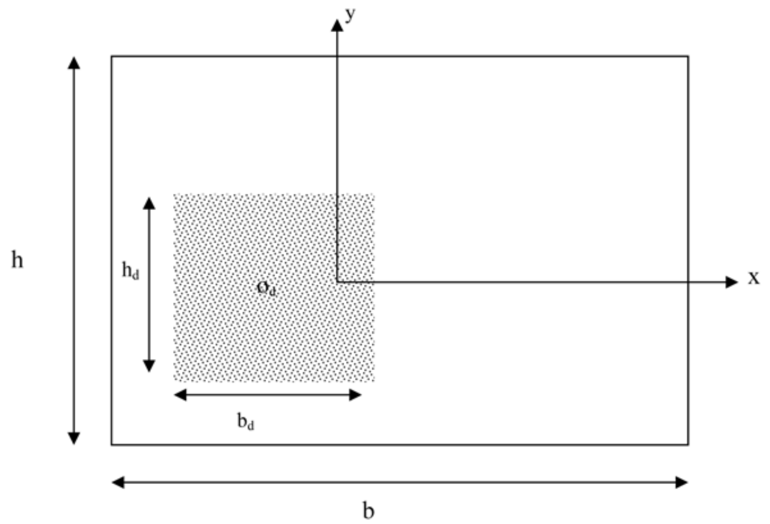

Fig. 4 Damage shear wall element

region O_d Fig. 4.

The following form of the strain energy for each wall element can be obtained :

$$U^e = \frac{1}{2} d_e^t K_w d_e \tag{14}$$

where the nodal displacement vector $d_e^t = \{u1, w1, v1, v2, u2, w2, v3, v4\}$

Since the finite element formulation is well established, no explicit procedure to determine the stiffness matrix K_w of a damaged and strengthened shear wall is given here.

3.3. Stiffness matrix of undamaged coupling beam

The coupling beam may be modelling by a standard two-nodes beam element with shear deformation taken into account Fig. 5. The stiffness matrix equation of the coupling beam is given by:

$$\begin{bmatrix} (M)_{1'} \\ (V)_{1'} \\ (M)_{2'} \\ (V)_{2'} \end{bmatrix} = K_b \begin{bmatrix} (\omega)_{1'} \\ (v)_{1'} \\ (\omega)_{2'} \\ (v)_{2'} \end{bmatrix} \tag{15}$$

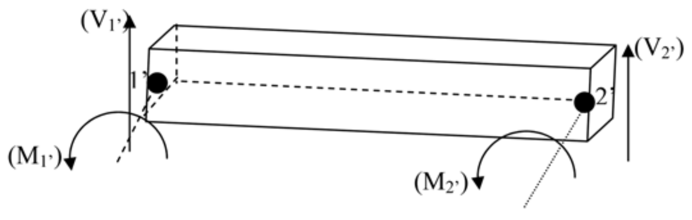

Fig. 5 Forces acting on beam element in local coordinate system

The stiffness matrix $\mathbf{K_b}$ is as given below:

$$K_b = \frac{E_{11}I_c}{(1+\mu)}\begin{bmatrix} \frac{(4+\mu)}{L} & \frac{6}{L^2} & \frac{(2-\mu)}{L} & \frac{-6}{L^2} \\ \frac{6}{L^2} & \frac{12}{L^3} & \frac{6}{L^2} & \frac{-12}{L^3} \\ \frac{(2-\mu)}{L} & \frac{6}{L^2} & \frac{(4+\mu)}{L} & \frac{-6}{L^2} \\ \frac{-6}{L^2} & \frac{-12}{L^3} & \frac{-6}{L^2} & \frac{12}{L^3} \end{bmatrix} \tag{16}$$

Where L and I_c are the span and the moment of inertia of the connecting beam respectively. μ is the shear deformation factor defined by:

$$\mu = \frac{12E_{11}I_c}{G_{(12)}A_sL^2} \tag{17}$$

in the above equation, A_s is the effective shear area of the beam section. For the case of rectangular beam, A_s may be taken as $\pi^2/12$ of the sectional area.

To allow for local deformation of the beam-wall joints, the flexible portions of the connecting beams are extended by half the beam depth at each end into the wall.

3.4. Free vibration analysis

For free vibration analysis, the inertia effects of the building is simulated by lumping the mass of each storey at the corresponding floor level. Rotatory inertia is neglected in comparison to the lateral and vertical inertias effects. Since at each floor level there is a floor slab which acts as rigid diaphragm, the whole floor is assumed to move horizontally and vertically as a rigid body. The mass matrix of the structure is taken as a diagonal matrix, employing the lumped mass assumption.

The circular frequencies are determined from the following standard frequency equation:

$$|K-\alpha^2 M| = 0 \tag{18}$$

where α is the circular frequency, $\mathbf{M}$ is the mass matrix and $\mathbf{K}$ is the stiffness matrix of the structure. The respective modal vectors D_i, are found by substituting each and every circular frequency, α_i, in the following equation at a time:

$$(K-\alpha_i M)\, D_i=0 \qquad i=1,2,\ldots\ldots m \tag{19}$$

m is the total number of obtained mode shape of a structure.

4. Numerical results

Examples structures were chosen both for verification and for application purposes.

For the problem under consideration, there are no suitable comparison results of damaged coupled shear walls in the open literature. To demonstrate the validity and accuracy of the proposed method,

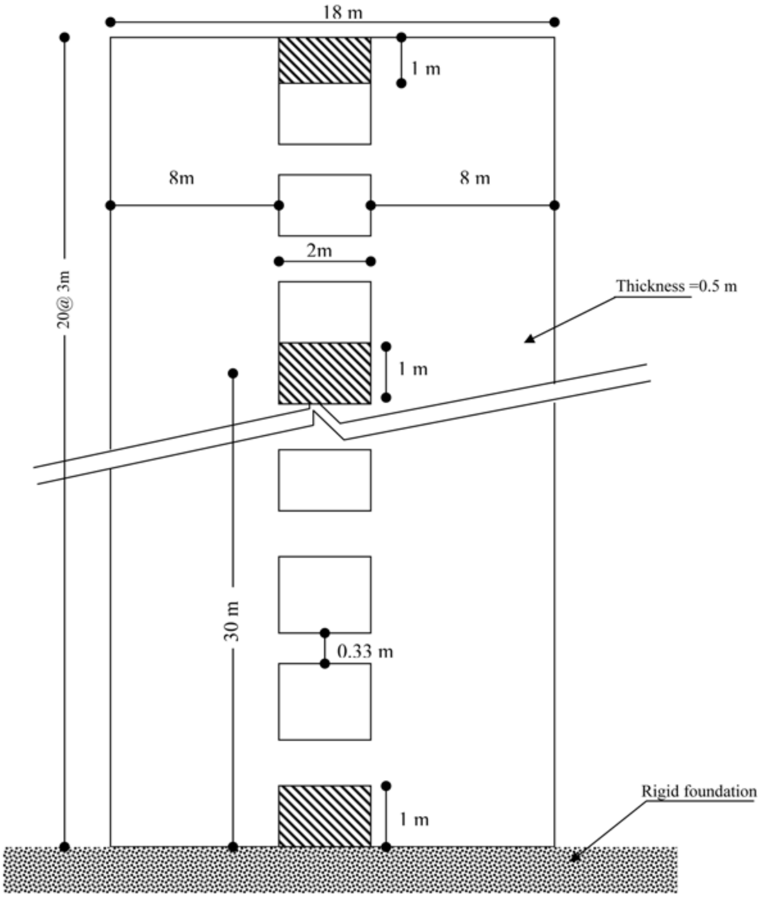

Fig. 6 20-storey stiffened coupled shear wall E_{11}= 15×10^6 KN m^{-2}; G_{12}= 6×10^6 KN m^{-2} and density of walls ρ = 2400 kg m^{-3}

we consider the free vibration of the undamaged coupled shear walls studied in references (Li and Choo 1996, Kuang and Chau 1999).

4.1. Verification

In order to verify the accuracy of the mechanical concept of the proposed method, a typical 20-storey and 25-storey coupled shear walls structures are analysed. The coupled shear walls shown in Fig. 6. (20-storey) is reinforced by a stiffening beams positioned at the top and the bottom of the walls and the middle of the structural height .Whereas, the coupled shear walls shown in Fig. 7 (25-storey), the stiffening beam is located at mid-level of the structure height.

The numerical results of the present analysis and those obtained by the discrete-continuous approach are compared in Table 1-2. The results indicate, by allowing more degrees of freedom and a more accurate representation of the inertia terms, the present analysis yields, in general, lower values for natural frequencies.

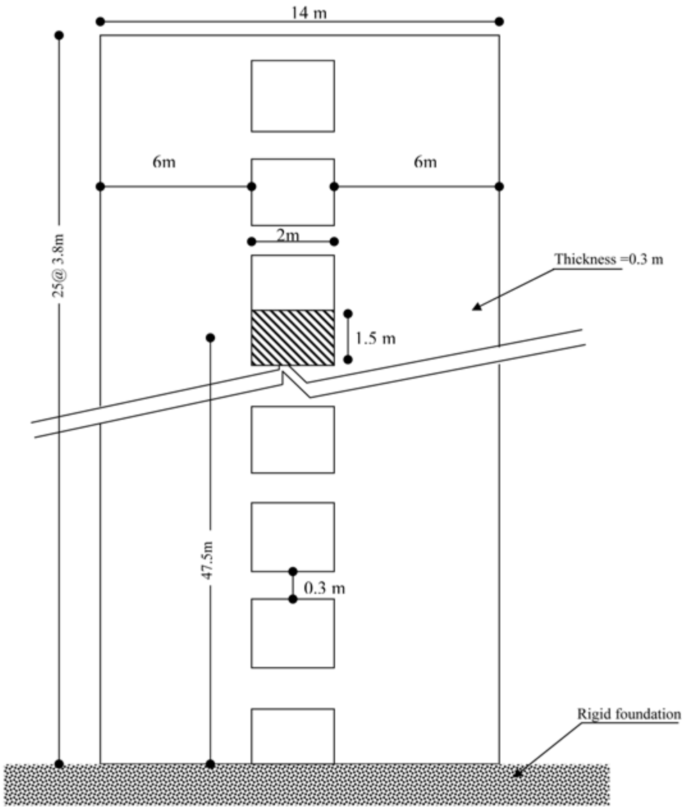

Fig 7 25-storey stiffened coupled shear wall E_{11}= 27.6×10^6 KN m^{-2}; G_{12}= 11.5×10^6 KN m^{-2} and density of walls ρ = 2400 kg m^{-3}

Table 1 Comparison studies of five firsts frequencies for coupled shear walls Fig. 5 (20-strorey)

Mode N°	Present (vertical Inertia neglected)	Present	Reference (Li and Choo 1996) the discrete-continuous approach
1	1.515	1.499	1.553
2	6.352	5.943	6.335
3	15.899	10.334	15.40
4	26.503	14.167	24.12
5	40.098	14.834	36.60

Table 2 Comparison studies of five firsts frequencies for coupled shear walls Fig. 6 (25-strorey)

Mode N°	Present (vertical Inertia neglected)	Present	Reference (Kuang and Chau 1999) the discrete-continuous approach
1	0.75	0.75	0.76
2	3.09	3.00	2.90
3	7.18	5.85	8.07
4	11.29	7.53	13.2
5	16.92	7.87	22.3

4.2. Numerical investigation

In order to illustrate the effects of the ratio S_D/S_T of the damaged area to the total area of the two walls (wall I and wall II) and its location H_D measured from the base Fig. 7 on the frequencies modes, a 20-storey reinforced concrete coupled shear wall with and without stiffening beam is analyzed as an example. The material properties are as follows: the modulus of elasticity is E_{11} =

Table 3 Variation of natural frequencies of damaged unstiffened coupled shear wall (the damage area considered from the base) (Hertz)

Mode N°	Undamaged	S_D/S_T =0.2	S_D/S_T =0.4	S_D/S_T =0.8
1	1.458	1.171	1.097	1.068
2	5.998	5.505	5.06	4.379
3	10.152	8.060	7.101	6.506
4	13.268	12.166	11.399	9.741
5	14.501	13.104	12.742	12.011

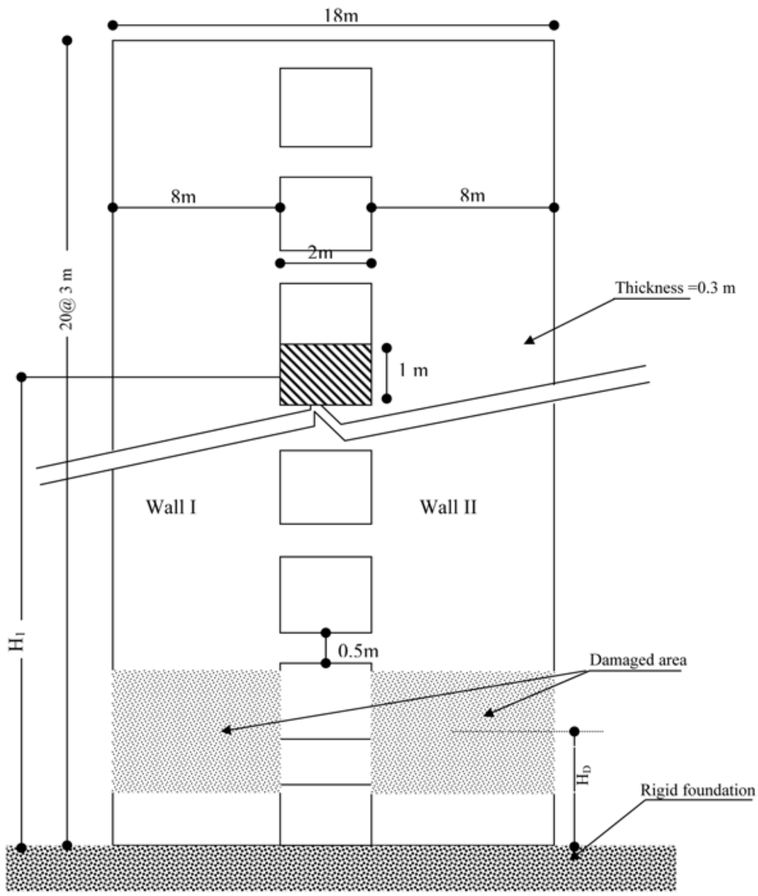

Fig. 8 Damaged coupled shear wall used in the numerical investigation

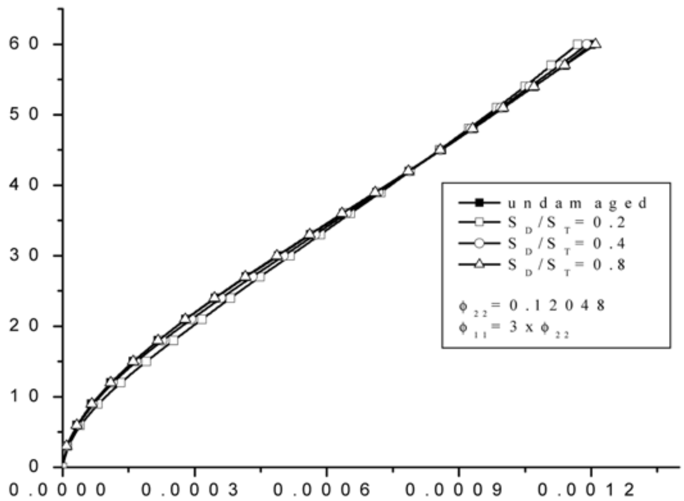

Fig. 9 Comparison of the first mode shapes (coupled shear wall without stiffening beam, the damaged area located at the bottom)

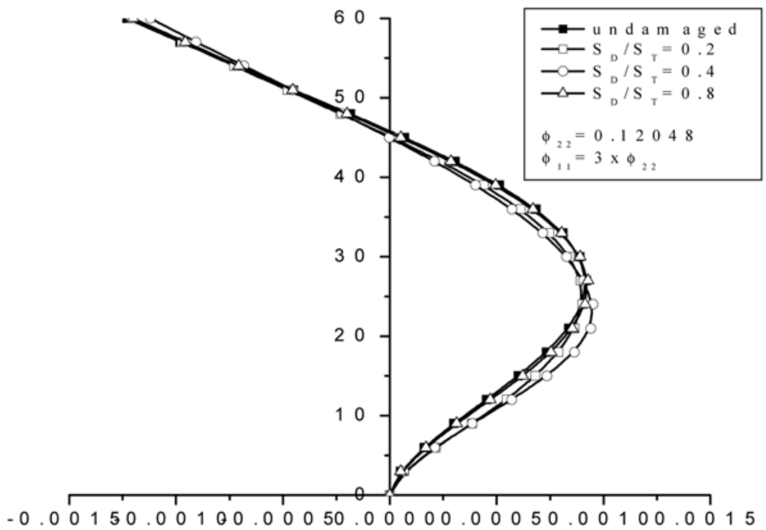

Fig. 10 Comparison of the second mode shapes (coupled shear wall without stiffening beam, the damaged area located at the bottom)

49490 MN/m^2, the modulus of elasticity in shear is G = 20620 MN/m^2.

In this study, the masses are lumped on each storey level and they are equal to 120t per storey. A damaged state is described through the incorporation of damaged variables Φ_{11} and Φ_{22}.

The first 5 natural frequencies of vibration are listed in Table 3. it can be seen that the damaged area affect significantly the free vibration characteristics of the structures, not only on the fundamental frequency but also on the higher modes.

Variation of the first natural frequency of the example coupled shear wall are plotted in Fig. 12 for different damages states. It can be seen that the natural frequency characteristics of the structure

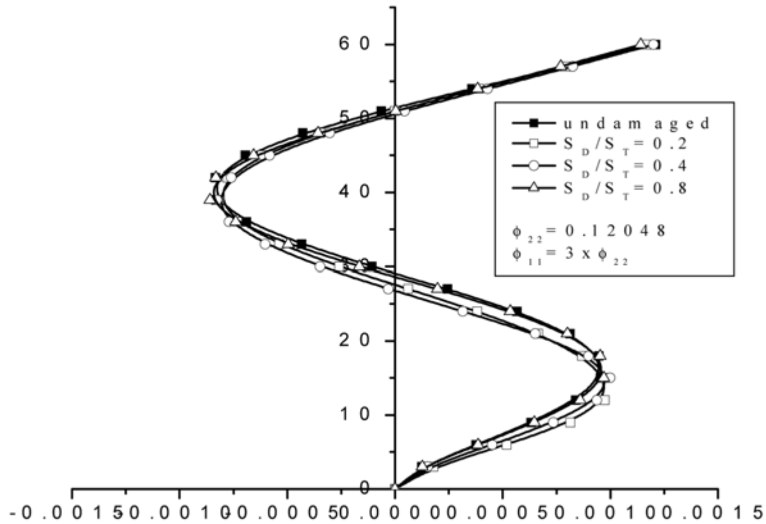

Fig. 11 Comparison of the third mode shapes (coupled shear wall without stiffening beam, the damaged area located at the bottom)

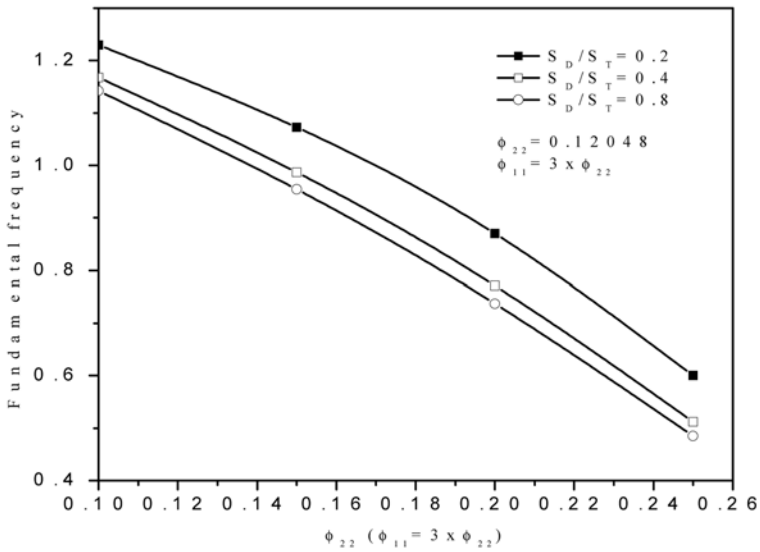

Fig. 12 The effect of the damage parameters on the fundamental frequency (coupled shear wall without stiffening beam, the damaged area located at the bottom)

is sensitive to the damage extent. The effect of the damage location on the free vibration characteristics of the structure is shown in Fig. 13, this effect become less significant when the damaged area is located at the top of the structure.

Figs. 14-15 present the optimal locations of the stiffening beam for increasing as far as possible the first natural frequency of vibration. It is obvious that the optimal location is at level between 0.4 and 0.5 of the structural height. This location is essentially the same as that required to maximize

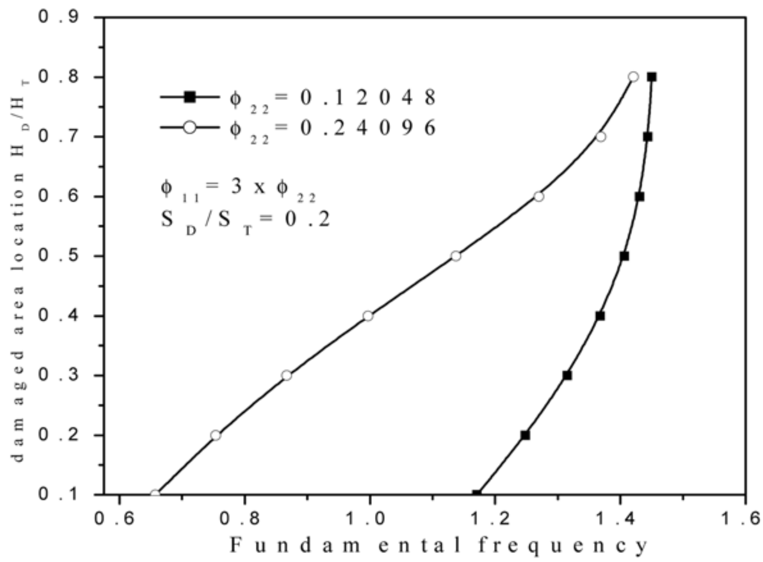

Fig. 13 The effect of the damage location on the fundamental frequency (coupled shear wall without stiffening beam)

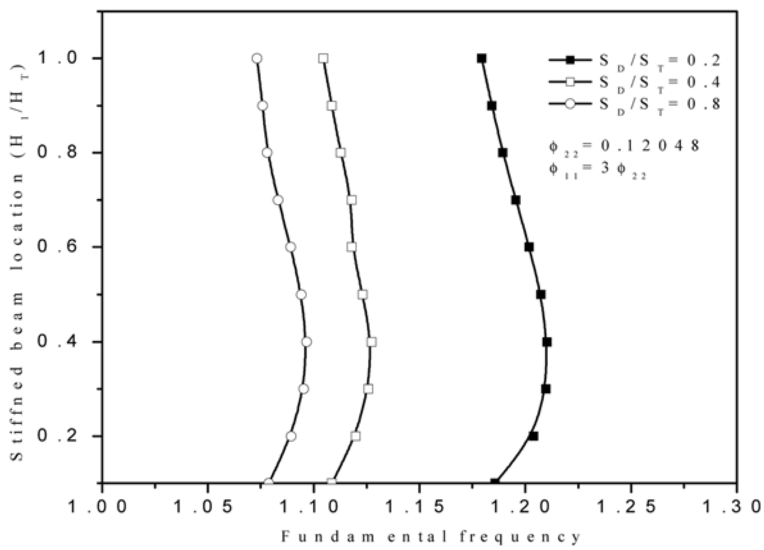

Fig. 14 The effect of the stiffened beam location on the fundamental frequency (the damaged area located at the bottom)

the first natural frequency of undamaged stiffened coupled shear wall, and also to minimise the lateral deflection of the structure, since the first mode of vibration corresponds closely to static deflected form of coupled shear walls with a stiffening beam.

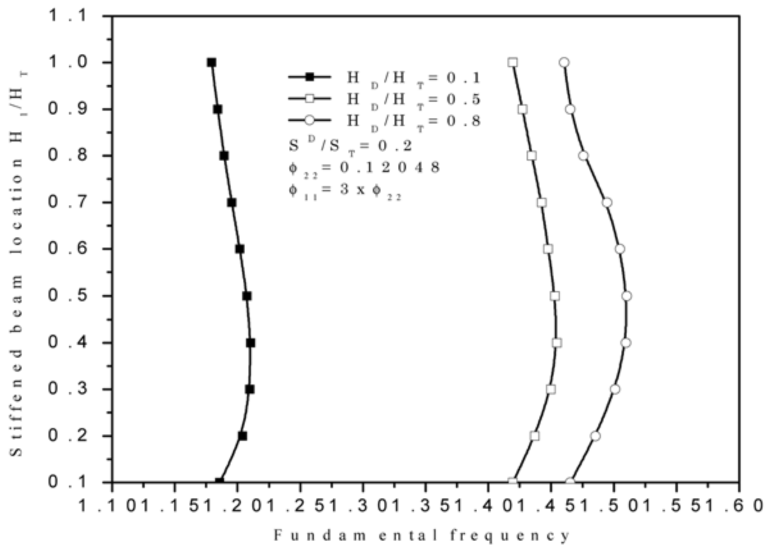

Fig. 15 The effect of the stiffened beam location on the fundamental frequencies

5. Conclusions

Free vibration characteristics of stiffened and damaged coupled shear wall has been studied using the mixed finite element method. Polynomial shape functions established by Kwan are used to express the displacements vector on each point of the wall element. The numerical investigation on the representative stiffened coupled shear wall show that the dynamic efficiency of the structure is enhanced by the addition of a stiffening beam to coupled shear walls, and that the first natural frequency can be increased significantly. The effect of the damaged extent and location has been also studied. From the results, it is observed that the damage extent has a significant effect on the fundamental frequency of vibration, when it is located at the bottom of the structure. This effect may be gradually decreasing, as the damage location be distant from the base.

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