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The coupling effect of drying shrinkage and moisture diffusion in concrete

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Abstract. Drying shrinkage of concrete occurs due to the loss of moisture and thus, it is controlled by moisture diffusion process. On the other hand, the shrinkage causes cracking of concrete and affects its moisture diffusion properties. Therefore, moisture diffusion and drying shrinkage are two coupled processes and their interactive effect is important for the durability of concrete structures. In this paper, the two material parameters in the moisture diffusion equation, i.e., the moisture capacity and humidity diffusivity, are modified by two different methods to include the effect of drying shrinkage on the moisture diffusion. The effect of drying shrinkage on the humidity diffusivity is introduced by the scalar damage parameter. The effect of drying shrinkage on the moisture capacity is evaluated by an analytical model based on non-equilibrium thermodynamics and minimum potential energy principle for a two-phase composite. The mechanical part of drying shrinkage is modeled as an elastoplastic damage problem. The coupled problem of moisture diffusion and drying shrinkage is solved using a finite element method. The present model can predict that the drying shrinkage accelerates the moisture diffusion in concrete, and in turn, the accelerated drying process increases the shrinkage strain. The coupling effects are demonstrated by a numerical example.

Keywords: drying shrinkage; moisture diffusion; damage; composite mechanics; concrete.

1. Introduction

Drying shrinkage of concrete is one of the major problems that affect the durability of concrete structures. The shrinkage of concrete due to moisture loss results in large local stress which may

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reach the tensile strength of concrete and thus generates microcracks. In recent years, many researchers have investigated the interactive effect of moisture diffusion and the cracking resulted from moisture gradient, temperature gradient and mechanical loading. Much research work indicated that the microcraking produced during hardening, drying and heating of concrete and the cracking resulted from the mechanical loading could increase diffusivity of concrete and thus, accelerate the mass diffusion process in concrete.

Bazant, *et al.* (1987) performed both experimental and theoretical studies on the effect of cracking resulted from drying on the permeability and diffusivity of concrete. They observed that the cracked specimens had more weight loss than the uncracked specimens, which means that the drying process is faster in the cracked specimens. Samaha and Hover (1992) investigated the effects of air-drying and oven-drying on the transport properties of concrete. They observed that in air-dried concrete the shrinkage cracks are limited to a small depth from the exposed surface. From this observation, they concluded that those cracks due to air-dry do not affect greatly the transport properties of concrete. On the other hand, they noticed severe cracking in the oven-dried concrete samples. Those cracks are uniformly distributed through the sample and resulted in severe internal damage. Thus, the cracks induced by the oven dry contribute significantly to the degradation of transport properties of concrete. Gerard, *et al*. (1996) investigated the effect of mechanically induced cracking on the permeability of normal and high strength concrete. From the observations of the cracking patterns, it was concluded that the damage is controlled by the length and aperture of the cracks. A micro-macro model was developed to predict the effect of damage evolution on the diffusion properties of concrete. Gerard and Marchand (2000) performed a theoretical study on the effect of both isotropic and anisotropic crack networks on the steady state diffusion properties of severely damaged concrete. The diffusion equation was simplified by assuming that the cracks are uniform in size and distributed evenly on one or two-dimensional grid. The study concluded that the cracking could accelerate the diffusion process in concrete. They also proposed a model for the effect of cracking on the diffusivity of concrete based on the crack density and mean crack aperture.

This paper focuses mainly on the effect of isotropic damage induced by drying shrinkage on the moisture diffusion. The outline of the coupling effect was reported briefly in a conference proceeding (Ababneh, *et al*. 2001). The diffusion model and the shrinkage model used in the present study are multiphase and multiscale models, in which the influential parameters at macro-, meso-, and micro-scales are taken into account; and concrete design parameters are included as inputs for the material models. The shrinkage-induced damage will be characterized by a coupled damage-plasticity model. In the following sections, the diffusion model and the shrinkage model will be briefly described first, and then the coupling between the diffusion parameters and the shrinkage-induced damage will be discussed in detail, which is the main focus of this paper. Finally, numerical examples will be provided and the effects of several important material parameters will be discussed.

2. Modeling moisture diffusion in concrete (without considering shrinkage-induced damage)

2.1. Basic formulation of the governing equations

The isothermal moisture flux in concrete can be formulated using Fick's first law by two different methods. In the first method, the moisture flux (J) is proportional to the gradient of the water content and can be written as

$$J = -D_w \ grad(w_e) \tag{1}$$

In the second method, the moisture flux (J) is proportional to the gradient of the pore relative humidity, and can be written as

$$J = -D_H \, grad(H), \tag{2}$$

where w_e and H are the evaporable water content and the pore relative humidity, respectively. D_w and D_H has two different physical meanings in above two formulations. In Eq. (6), D_w is the moisture diffusivity and in Eq. (2), D_H is the humidity diffusivity.

The isothermal rate of change of total water content, w, in concrete can be expressed using the mass balance equation or Fick's second law:

$$\frac{\partial w}{\partial t} = -div(J) \tag{3}$$

in which $w = w_e + w_{ne}$, where w_{ne} is the nonevaporable or chemically combined water content in the concrete. The substitution of the moisture flux (J), described in Eq. (1) and (2) into the mass balance equation, Eq. (3), will result in two formulations of the nonlinear moisture diffusion equation:

$$\frac{\partial(w_e + w_{ne})}{\partial t} = div(D_w grad\ w_e) \tag{4}$$

$$\frac{\partial w}{\partial H}\frac{\partial H}{\partial t} = div(D_H grad\ H) \tag{5}$$

where $\partial w/\partial H$ is the moisture capacity and t is the time. The nonlinearity of the moisture diffusion equation, Eq. (4), is resulted from the dependence of the moisture diffusivity, D_w, on the moisture content, w. The nonlinearity of Eq. (5), is resulted from the dependence of $\partial w/\partial H$ and D_H on the relative humidity, H.

The formulation of the moisture diffusion equation based on the water content, i.e., Eq. (4), yields two state variables w_e and w_{ne}. Both of them vary with the time because of the hydration of Portland cement, even if there is no moisture exchange with the surrounding environment. On the other hand, the formulation based on relative humidity, i.e., Eq. (5), yields only one variable H. In the present work the second formulation in terms of relative humidity H, described by Eq. (2) and (5) will be used. In order to find a numerical solution for the governing partial differential equation of moisture diffusion in concrete, Eq. (5), the material parameters involved should be determined.

2.2. Moisture capacity

Concrete is a two-phase material, in which the aggregates are considered as the inclusions and represent one phase, and the cement paste is considered as the matrix and represents the other phase. Then, without considering the effect of shrinkage of concrete, the moisture capacity of concrete can be calculated as a certain average of the moisture capacities of the two constituent phases:

$$\left(\frac{\partial w}{\partial H}\right)_{avg} = f_{agg}\left(\frac{\partial w}{\partial H}\right)_{agg} + f_{cp}\left(\frac{\partial w}{\partial H}\right)_{cp} \tag{6}$$

where f_{agg} and f_{cp} are the weight percentages of the aggregates and cement paste. The moisture capacities of the aggregate and cement paste in Eq. (6) can be evaluated by using the model developed by Xi, *et al.* (1994a, 1994b) and Xi (1995a, 1995b).

2.3. Humidity diffusivity

The moisture diffusivity of concrete can be predicted by the composite model developed by Christensen (1979):

$$D_H = D_{Hcp}\left(1+\frac{g_i}{[1-g_i]/3+1/[(D_{Hagg}/D_{Hcp})-1]}\right) \tag{7}$$

in which g_i is the aggregate volume fraction, D_{Hcp} is the humidity diffusivity of the cement paste and D_{Hagg} is the humidity diffusivity of the aggregates. The composite model shown in Eq. (7) is suitable for composite materials made of particles (inclusions) with a broad spectrum of size distribution and without privileged direction. This model is apparently suitable for concrete in which aggregates, gravels and sands, of various sizes are embedded in an interconnected matrix.

Eq. (7) can be considered as the model at meso-scale level, in which D_{Hcp} and D_{Hagg} must be determined from a lower scale level. The humidity diffusivity of aggregates, D_{Hagg}, in concrete is very small comparing with the diffusivity of concrete and can be neglected in Eq. (7). The humidity diffusivity of cement paste, D_{Hcp}, can be predicted by the empirical model developed by Xi, *et al.* (1995b), which will not be described here.

3. Drying shrinkage of concrete

Drying shrinkage of concrete has been evaluated by many methods. One of the methods is developed for structural members, and it represents averaged shrinkage strain over a large scale. In this case, many influential parameters, such as size and shape of the structure member, properties of the concrete, and effect of the environment have to be taken into account. As a result, most of the prediction models at the structural level have been based are expressed in the form of empirical equations. Another method is for a representative volume element of concrete in a smaller scale, the so-called mesoscale including mortar and gravel as two constituent phases. In this case, the predicting model is a constitutive relationship among the shrinkage, the internal humidity, and related material parameters such as water-cement ratio of concrete. The third method is multiscale method. It considers the gravel and cement paste as two constituent phases at mesoscale level and calculates the effect of the two phases on the shrinkage of concrete, and then it further considers the cement paste as a multiphase composite at microscale levels and calculates the effect of the constituent phases on the shrinkage of cement paste. In this study, we used the multiscale model to characterize the shrinkage of concrete, which will be explained in next two sections.

3.1. Shrinkage of concrete by the mesoscale modeling method

Concrete is considered to be a composite spherical system (Xi and Jennings 1997) at the mesoscale level as shown in Fig. 1. Each spherical element in the model is composed of aggregates as the center cores, and the cement paste as the matrix. The shrinkage strain and bulk modulus of

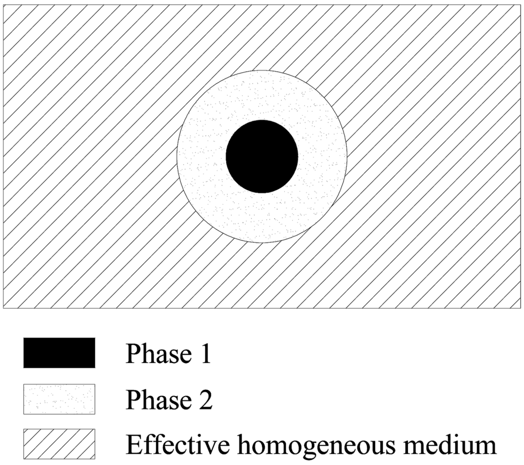

Fig. 1 Three-phase effective media model

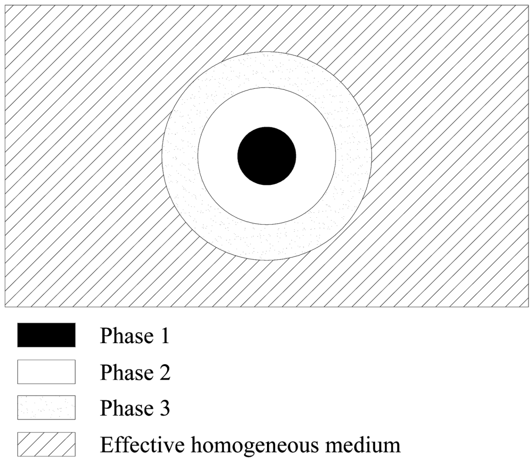

Fig. 2 Four-phase composite model

concrete, ε_{conc}^{sh} and K_{conc}, can be evaluated for the composite system:

$$\varepsilon_{conc}^{sh} = \frac{K_1 \cdot \varepsilon_1^{sh} g_i(3K_2+4G_2)+K_2\varepsilon_2^{sh}(1-g_i)(3K_1+4G_2)}{K_2(3K_1+4G_2)-4g_iG_2(K_2-K_1)} \tag{8}$$

$$K_{conc} = \frac{K_2(3K_1+4G_2)-4g_iG_2(K_2-K_1)}{(3K_1+4G_2)+3g_i(K_2-K_1)} \tag{9}$$

where ε_1^{sh} and ε_2^{sh}, K_1 and K_2, and G_1 and G_2 are the shrinkage, bulk modulus and shear modulus of phase 1 (aggregates) and phase 2 (cement paste), respectively; g_i is the volume fraction of aggregate.

The shrinkage of concrete is mainly due to the shrinkage of cement paste (ε_2^{sh}), while the shrinkage of the aggregates can be neglected (ε_1^{sh}=0) and so, Eq. (8) becomes:

$$\varepsilon_{conc}^{sh} = \varepsilon_2^{sh}\left\{1-\frac{g_i(K_1/K_2)}{1+(K_1/K_2-1)\dfrac{3+4g_i(G_2/K_2)}{3+4(G_2/K_2)}}\right\} \tag{10}$$

3.2. Shrinkage of cement paste by multiscale modeling method

Following the same multiscale idea for concrete, the cement paste can also be considered as a composite spherical system. But, the scale level of the system is at microscale as shown in Fig 2. In this microscale composite model, the unhydrated cement particles and the crystalline phases including calcium hydroxide (*CH*) crystals and ettringite crystals form the center cores (phase 1), inner product of calcium silicate hydrate (*C-S-H*) gel forms the phase 2, and the outer hydration product forms the phase 3 (Jennings and Tennis 1994). Both the inner product and the outer product are *C-S-H* but the density of the inner product is higher than that of the outer product (Diamond and Bonen 1993), and the shrinkage of *C-S-H* is mainly from the outer product. The bulk modulus and shrinkage strain of cement paste (i.e. ε_2^{sh} and K_2 in Eqs. 8, 9, and 10) can be evaluated as the bulk modulus and shrinkage strain of the effective homogeneous composite at the microscale (Xi and Jennings 1997):

$$\varepsilon_{cp}^{sh} = \frac{K_{eff}^{12}(\varepsilon_{eff}^{sh})^{12}(f_1+f_2)(3K_3+4G_3)+K_3\varepsilon_3^{sh}(1-f_1-f_2)(3K_{eff}^{12}+4G_3)}{K_3(3K_{eff}^{12}+4G_3)-4(f_1+f_2)(K_3-K_{eff}^{12})G_3} \tag{11}$$

$$K_{cp} = K_3 + \frac{(f_1+f_2)(K_{eff}^{12}-K_3)}{1+(1-f_1-f_2)((K_{eff}^{12}-K_3)/(K_3+4G_3/3))} \tag{12}$$

where,

$$(\varepsilon_{eff}^{sh})^{12} = \frac{K_1\varepsilon_1^{sh}(f_1/(f_1+f_2))(3K_2+4G_2)+K_2\varepsilon_2^{sh}(1-f_1/(f_1+f_2))(3K_1+4G_2)}{K_2(3K_1+4G_2)-4(f_1/(f_1+f_2))(K_2-G_1)G_2} \tag{13}$$

$$K_{eff}^{12} = K_2 + \frac{(f_1/(f_1+f_2))(K_1-K_2)}{1+(1-f_1/(f_1+f_2))((K_1-K_2)/((K_2+4G_2)/3))} \tag{14}$$

in which the subscript represents phase 1, 2, and 3; f, K, G, and ε^{sh} are volume fraction, bulk modulus, shear modulus and shrinkage of phases 1, 2 and 3 of the cement paste.

From test results using the environmental scanning electron microscopy (Neubauer, *et al.* 1997), it was found that the shrinkage of the phase 1 (unhydrated cement particles, calcium hydroxide crystals and other crystalline phases) is very small and can be neglected ($\varepsilon_1^{sh} = 0$). Also, It was found that the shrinkage of inner products (phase 2) in the range of relative humidity between 30% and 100% is very small and can be neglected ($\varepsilon_2^{sh} = 0$). As a results Eq. (13) vanishes, and Eq. (11) becomes:

$$\varepsilon_{cp}^{sh} = \frac{K_3\varepsilon_3^{sh}(1-f_1-f_2)(3K_{eff}^{12}+4G_3)}{K_3(3K_{eff}^{12}+4G_3)-4(f_1+f_2)(K_3-K_{eff}^{12})G_3} \tag{15}$$

The volume fraction of the three phases can be calculated as follows:

$$f_1 = f_{core}+f_{CH}+f_{AFm} \tag{16}$$

$$f_3 = [0.706(w/c)+0.131](1-f_1-f_{pores})+f_{pores} \tag{17}$$

$$f_2 = 1-f_1-f_3 \tag{18}$$

in which f_{core}, f_{CH} and f_{AFm} are the volume fraction of anhydrous core of the cement particles, calcium hydroxide (*CH*) crystals and *AFm* phase (ettringite), *w/c* is the water-cement ratio and f_{pores} is volume fraction of capillary pores. The volume fractions are functions of age of concrete, and they can be evaluated using proper models for cement hydration and for microstructure of cement paste. In this study, a comprehensive model developed by Jennings and Tennis (1994) is used.

4. The interaction of drying shrinkage and moisture diffusion

The interactive effect of drying shrinkage and moisture diffusion can be studied using one of the two alternatives. The first alternative is to consider that the stress or strain induced by drying shrinkage is one of the driving forces for moisture diffusion, and thus, there will be an additional term in the moisture diffusion equation, which corresponds to the effect of the stress or the strain

(Majorana and Mazars 1997). The second alternative is to consider the effect of the drying shrinkage on the two material parameters of the diffusion equation (Eq. 5), i.e., moisture capacity and moisture diffusivity. In this case, the stress or strain is not appeared explicitly in the diffusion equation, but as an internal parameter. We applied the second alternative in the present study. The effects of drying shrinkage on the two material parameters are incorporated into the diffusion equation by two different methods. For the humidity diffusivity, the effect of the shrinkage-induced damage in concrete is considered by using composite damage mechanics.

For the moisture capacity, the effect of differential shrinkage between cement paste and aggregate is considered. These two methods will be described in the following two sections.

4.1. The effect of shrinkage-induced damage on humidity diffusivity

The scalar damage model for stiffness of distressed materials, first introduced by Kachanov (1958), was based on the assumption that the area (or the volume) with damage cannot hold any load. Therefore, there is a reduction in the effective cross section area as the level of damage increases, which can be generally expressed by a reduction in the modulus of elasticity.

$$\sigma = (1-\omega)\,\boldsymbol{E}_0 : \varepsilon \tag{19}$$

where σ and ε are the stress and strain tensors, $\boldsymbol{E}_0$ is the initial elastic stiffness tensor, and ω is the scalar damage parameter.

Following the same concept, we may assume that, for the moisture diffusion in concrete, the damaged area (or volume) cannot effectively resist the penetration of moisture as the intact concrete does. With the increase of damage, the concrete diffusivity increases. There is a reduction in the effective cross section area. Therefore, the effect of damage on the moisture diffusivity can be expressed in terms of the same scalar damage parameter in Eq. (19). The diffusivity of distressed concrete, D_H, may be simply expressed in terms of the scalar damage parameter, ω (Ababneh, *et al.* 2001).

$$D_H(H,\omega) = \frac{D_H(H)}{1-\omega} \tag{20}$$

in which $D_H(H)$ is the moisture diffusivity of intact concrete, and $D_H(H, \omega)$ is the moisture diffusivity with the effect of damage. The problem with Eq. (20) is that when $\omega = 1$ (100% damage), $D_H(H, \omega)$ goes to infinity. This is due to the fact that the damaged volume is considered as void in concrete. Thus, the stiffness of void is zero in the stress analysis Eq. (19), and the diffusivity of void is infinity in the transport analysis Eq. (20).

Another method to incorporate the effect of damage on the humidity diffusivity is to use the concept of composite damage mechanics for transport properties of distressed concrete (Xi 2002, Xi and Nakhi 2005). In this method, the distressed concrete is considered as Phase 1, and the intact concrete Phase 2. The major difference is that the distressed concrete is not considered as void but as a damaged material with increased diffusivity, so the diffusivity of Phase 1 is higher than Phase 2, but not infinity. This is a more realistic model and it also solves the numerical problem associated with Eq. (20). The volume fraction of distressed concrete is equal to the damage parameter, ω. Then, the two-phase composite model for effective conductivity of composites developed by Christensen (1979) can be used

$$D_H(H,\omega) = D_2\left[1 + \frac{\omega}{(1-\omega)/3 + 1/[(D_1/D_2) - 1]}\right] \tag{21}$$

in which, $D_2 = D_H(H, \omega = 0)$ is the humidity diffusivity of intact concrete as calculated by Eq. (7) and $D_1 = D_H(H, \omega = 1)$ is the humidity diffusivity of the damaged concrete. The methods that can be used to evaluate D_1 has been described by Xi and Nakhi (2005). In this study, the diffusivity of distressed concrete will be determined by Eq. (21). The remaining question is how to determine the scalar damage parameter ω with a given shrinkage strain, which will be discussed later.

4.2. The effect of drying shrinkage on moisture capacity

The coupling effect between drying shrinkage strain and moisture capacity is similar to the effect of thermal strain on heat capacity, which was studied by Rosen and Hashin (1970) using a generalized method based on extreme energy principle. Similarly, Xi (1995a and 1995b), based on non-equilibrium thermodynamics and minimum potential energy principle, derived an analytical model for the effect of shrinkage strain on moisture capacity for two-phase composite materials as follows:

$$\frac{\partial w}{\partial H} = \left(\frac{\partial w}{\partial H}\right)_{avg} + \psi\left\{9\frac{H^0}{RT^0}\left(\frac{\beta_i - \beta_m}{\frac{1}{K_m} - \frac{1}{K_i}}\right)^2\left[\overline{\left(\frac{1}{K}\right)} - \frac{1}{K_{conc}}\right]\right\} \tag{22}$$

with,

$$\overline{\left(\frac{1}{K}\right)} = \frac{g_m}{K_m} + \frac{g_i}{K_i} \quad \text{and} \quad K_{conc} = K_m + \frac{g_i(K_i - K_m)}{1 + g_m[(K_i - K_m)/(K_m + (4/3)G_m)]} \tag{23}$$

in which $(\partial w/\partial H)_{avg}$ is the moisture capacity of concrete without considering the effect of shrinkage as shown in Eq. (6), ψ is a factor used to convert the units of the second term in the right hand side of Eq. (9) from mole/m^3 to gram/gram, R = gas constant in (Joules/mole/K), H^o and T^o are the reference humidity and temperature (in Kelvin), β_i and β_m are the shrinkage coefficients for the inclusions (aggregate) and the matrix (cement paste), K_i , K_m and K_{conc} are the bulk modulus of the inclusions, the matrix, and concrete, G_m is the shear modulus of the matrix, and g_i, g_m are the volume fraction of the inclusions and the matrix.

In the case that the shrinkages of the inclusion and the matrix are the same, the second term in Eq. (22) vanishes, which means that there is no coupling effect between the shrinkage and moisture capacity. Otherwise, the coupling effect must be taken into account. For concrete, $\beta_m \geq \beta_i$ and $K_i \geq K_m$ (the matrix shrinks more than the inclusion and the inclusion is stiffer than the matrix), the second term on the right-hand side of Eq. (22) depends strongly on shrinkage values of the constituent phases. With increasing β_m/β_i the value of the second term increases drastically. Numerical analyses by Xi (1995a and 1995b) showed that the influence of the shrinkage ratio is more significant than that of the ratio of bulk modulus, and furthermore, the second term is always positive when $\beta_m \geq \beta_i$ and $K_i \geq K_m$, which means that the differential shrinkage between the two constituent phases increases the moisture capacity of the concrete.

5. Framework for coupling between elastoplastic and damage with the presence of shrinkage

When shrinkage strain is evaluated using Eq. (10), the scalar damage parameter ω is needed in order to determine the effect of the damage on the diffusivity, Eq. (21). ω will be determined based on a combined plasticity and damage theory.

In small strain theory of plasticity, the strain tensor can be decomposed into the elastic part ε^e and the plastic part ε^p. Considering the presence of shrinkage strain tensor ε^{sh}, the total strain tensor can be expressed as:

$$\varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^{sh} \tag{24}$$

Recalling shrinkage of concrete ε^{sh}_{conc} from Eq. (10), the shrinkage strain tensor is evaluated by:

$$\varepsilon^{sh} = \varepsilon^{sh}_{conc}\boldsymbol{I} \tag{25}$$

where $\boldsymbol{I}$ is second order of identity tensor. The constitutive relation can then be written as:

$$\sigma = \boldsymbol{E}:(\varepsilon - \varepsilon^p - \varepsilon^{sh}) \tag{26}$$

in which $\boldsymbol{E}$ is the elastic stiffness tensor.

The elastoplastic and damage models are coupled by exploiting the effective stress concept known from continuum damage mechanics. The effective stress represents the redistributed stress over undamaged or effective area after the damage has taken place. Based on the scalar damage model described earlier in Eq. (19), the relation between the effective stress $\bar{\sigma}$ and the nominal stress σ can be expressed as:

$$\bar{\sigma} = \frac{\sigma}{1-\omega} \tag{27}$$

where ω is the damage parameter. It is assumed that $0 \le \omega \le \omega_c$, where ω_c is the critical damage in which a complete local rupture takes place. In practice, a $\omega_c = 1$ is usually employed.

Considering the initial elastic stiffness $\boldsymbol{E_o}$, we have

$$\bar{\sigma} = \boldsymbol{E_o}:(\varepsilon - \varepsilon^{\boldsymbol{p}} - \varepsilon^{\boldsymbol{sh}}) \tag{28}$$

and its corresponding time derivative takes the form:

$$\dot{\bar{\sigma}} = \boldsymbol{E_o}:(\dot{\varepsilon} - \dot{\varepsilon}^{\boldsymbol{p}} - \dot{\varepsilon}^{\boldsymbol{sh}}) \tag{29}$$

From Eqs. (26), (27) and (28), the constitutive relation becomes:

$$\sigma = (1-\omega)\boldsymbol{E}_o:(\varepsilon - \varepsilon^p - \varepsilon^{sh}) = \boldsymbol{E}:(\varepsilon - \varepsilon^p - \varepsilon^{sh}); \quad \boldsymbol{E} = (1-\omega)\boldsymbol{E}_o \tag{30}$$

Following the effective stress concept, it is logical to assume that the plastic flow takes place only in undamaged area. Thus, the formulae from elastoplastic theory that are dependent on stress must be modified by substituting the effective stress in place of the nominal stress. The problem can then be solved by using standard elastoplastic theory.

For our study, elastoplastic behavior of concrete is assumed to follow the yield function based on the pressure-sensitive Drucker-Prager criterion and damage evolution of concrete to follow the damage model for concrete developed by Mazars (1984). In Mazars's model, equivalent strain $\tilde{\varepsilon}$ is introduced, which is defined as $\tilde{\varepsilon}=\sqrt{\langle\varepsilon\rangle:\langle\varepsilon\rangle}$, where ε are the principal strains. The equivalent strain $\tilde{\varepsilon}$ controls the evolution of the damage in that when the equivalent strain is higher than a threshold value, the damage start to evolve, and when the equivalent strain is smaller than a threshold value, there is no damage evolution. By coupling these theories and considering the presence of shrinkage strains, we can establish the framework for evaluating the effect of drying shrinkage and moisture diffusion on concrete structures.

6. Numerical implementation

The mechanical and transport theories and models described in the previous sections were implemented in a finite element code for a nonlinear transient analysis of diffusion problem and a

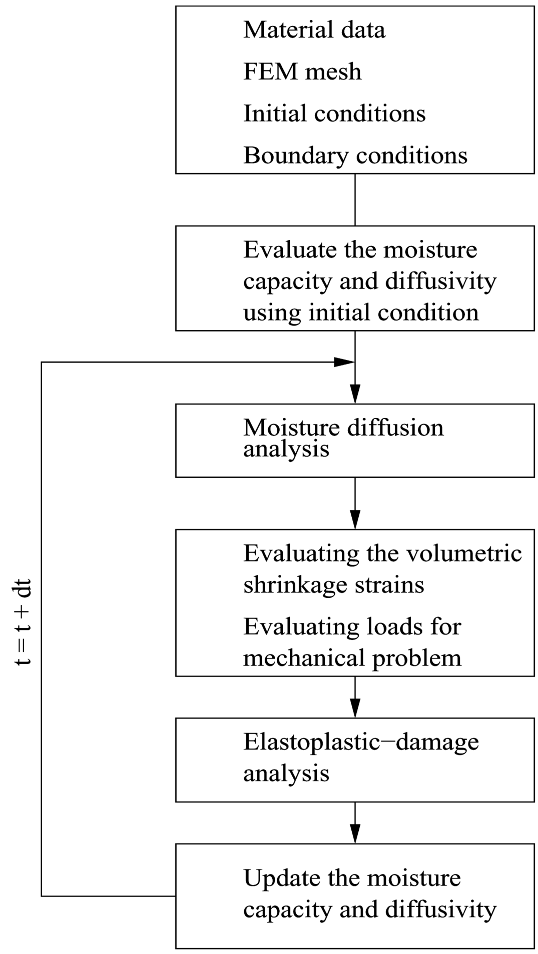

Fig. 3 Outline of the numerical simulation method

nonlinear analysis of solid mechanics. The coupled mechanical-diffusion problem is then solved using a staggered scheme. The outline of the scheme is shown in Fig. 3. This scheme can be categorized as a semi-coupled scheme in which the diffusion and the mechanical problem are solved one after another. In another word, during the diffusion analysis, the variables affected by mechanical problem are assumed as constants and vice versa.

The detail procedures can be described as follows. Given a time increment, the moisture diffusion problem is solved first. The moisture capacity and the humidity diffusivity are updated by considering the effects of shrinkage-induced damage obtained from previous time step. Since both moisture capacity and the diffusivity depend on the relative humidity, the diffusion problem is nonlinear and solved iteratively using Newton-Raphson algorithm. Once the moisture distribution is solved, the shrinkage strain is then determined using the multiscale shrinkage model presented earlier. These shrinkage strains act as loads in the mechanical problem. The combined elastoplastic-damage are then used to obtain the damage parameter in each element.

It should be pointed out that the solution of elastoplastic response is separated from the damage problem. First, the elastoplastic problem is solved and then the damage problem is solved for evaluating the damage parameter using the strains and stresses resulted from the elastoplastic analysis. The damage parameter is then used to update humidity diffusivity of the concrete. The same procedure will repeat for all time steps.

To solve the elastoplastic problem, the Generalized Cutting Plane Algorithm is employed for stress return algorithm (Simo and Ortiz 1985). The cutting plane algorithm is a fully explicit method that involves only functional evaluation. The solution strategy of the algorithm needs iteration, which is handled using Newton-Rapshon algorithm.

The other issue that needs to be addressed is mesh objectivity in the finite element analysis. It is well documented that the local isotropic damage model suffers from mesh dependency. To overcome this problem, we used the notion of equivalent length h, corresponds to representative dimension of mesh size. The idea is to have the fracture energy dissipated uniformly over the equivalent length. In this study, the equivalent length is assumed to be related to the area of element mesh A_e and taken as:

$$h = \sqrt{2A_e} \tag{50}$$

The material parameters for Mazars' damage model depend on the characteristic length l_c, which defines the heterogeneity scale of material. Thus, in the numerical analysis, the scalar damage parameters obtained from Mazars' damage model need to be calibrated numerically so that the fracture energy is dissipated uniformly over the equivalent length h.

Comparison with experimental data for the elastoplastic damage model The present elastoplastic

Table 1 Representative values of parameters used in analysis

Elastic parameters	Plastic parameters	Damage parameters	Diffusion parameters
E = 27580 MPa	$\alpha = 0.82$	$k_0 = 0.0001$	Water to cement ratio, $w/c = 0.55$
$\nu = 0.18$	$\alpha_\psi = 0.2$	$A_t = 0.81$	Aggregate volume fraction, $g_i = 0.65$
	$\tau_0 = 6.18$ Mpa	$B_t = 10450$	Cement type Type I
	$\tau_u = 8.15$ Mpa	$A_c = 1.34$	Curing time 28 days
	$\lambda = 3000$	$B_c = 2537$	

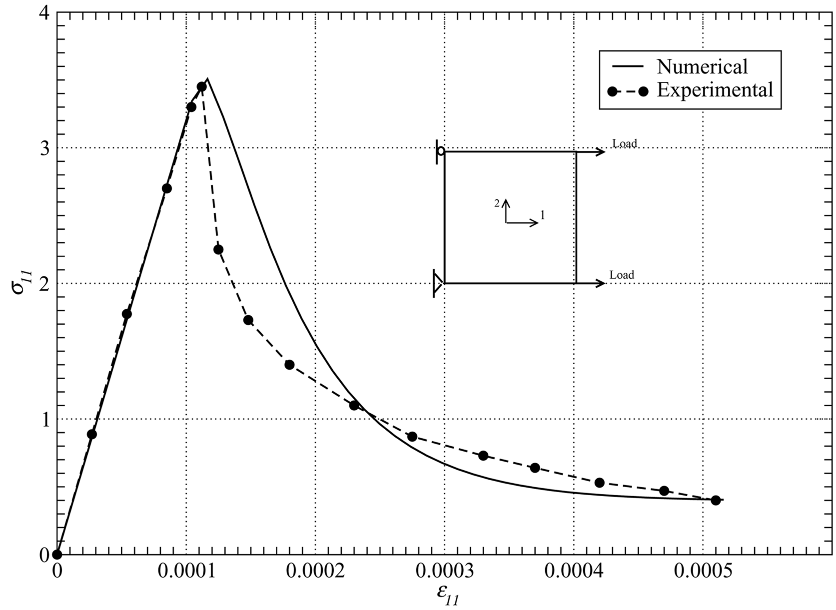

Fig. 4 Comparison with experimental data of tensile stress-strain diagram Stress vs strain

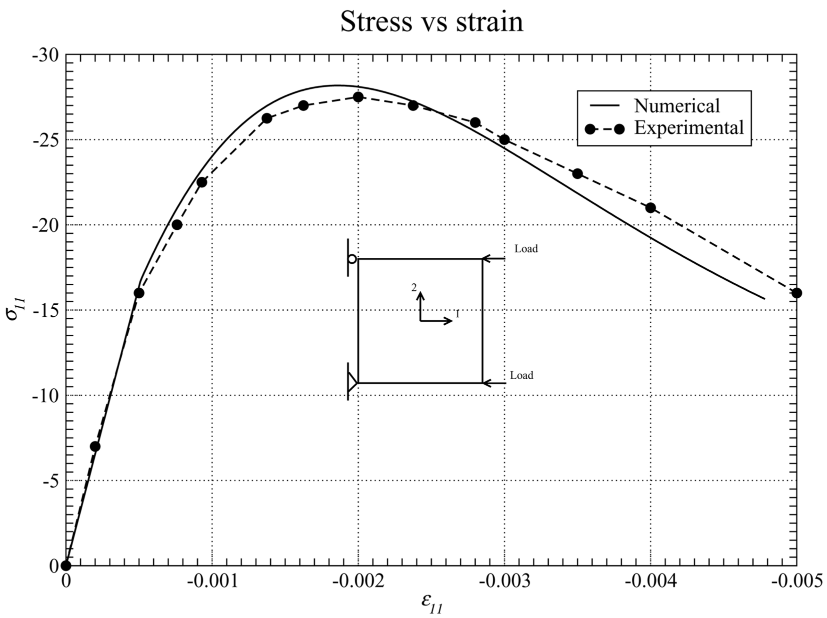

Fig. 5 Comparison with experimental data of compressive stress-strain diagram

damage model was compared with available experimental data to make sure that the model is capable of handling the problem at hand. The test data of tensile loading experiments by Gopalarantnam and Shah (1985) and the compressive loading experiments by Karsan and Jirsa (1969) were used for the comparisons. The material properties used in the numerical analyses are:

(1) for tensile case: $\boldsymbol{E_o} = 3.1 \times 10^4$ MPa, $k_o = 0.000112$, $A_t = 0.89$, $B_t = 12000$; and (2) for compressive case: $E_o = 3.17 \times 10^4$ MPa, $k_o = 0.000112$, $A_c = 0.65$, $B_c = 2000$. For both cases, $l_c = 82.6$ were used. Other parameters are listed in Table 1.

Figs. 4 and 5 show the numerical results of stress-strain diagrams in tensile and compressive cases, respectively. The numerical analyses were carried out using a single patch element of size 82.6×82.6 mm. As one can see in the figures, the numerical results agree well with the experimental data.

6.1. A complete numerical example for the drying process and the shrinkage-induced damage

As a numerical example, a 20 cm depth concrete slab similar to the slab shown in Fig. 6 was analyzed. The concrete has a water-cement ratio (w/c) of 0.55, aggregate volume fraction (g_i) of 0.65, average compressive strength (28 days) of 34 MPa and moist cured for 28 days before drying. The rest of parameters used in the analysis are listed on Table 1. The slab, which is initially saturated ($H_{ini} = 100\%$), is exposed to drying on the top surface ($H_{env} = 50\%$) after curing. The effect of micro-cracking due to drying shrinkage is taken into account through the scalar damage parameter. The mechanical analysis is carried out based on the assumption of plane strain condition. The finite element mesh used for both mechanical and diffusion analysis is shown in Fig. 7. The finite element mesh consists of 3000 quadrilateral elements.

The variation of pore relative humidity, the shrinkage strain, the equivalent strain, the scalar damage parameter and the total strains in the horizontal direction at the middle section are shown in Figs. 8 through 12. The time of exposure is started after the curing period of concrete. Fig. 8 shows the drying processes at different depths in the concrete. At early stage, the rate of drying is higher near the exposed surface. After the pore relative humidity near the surface approaching the

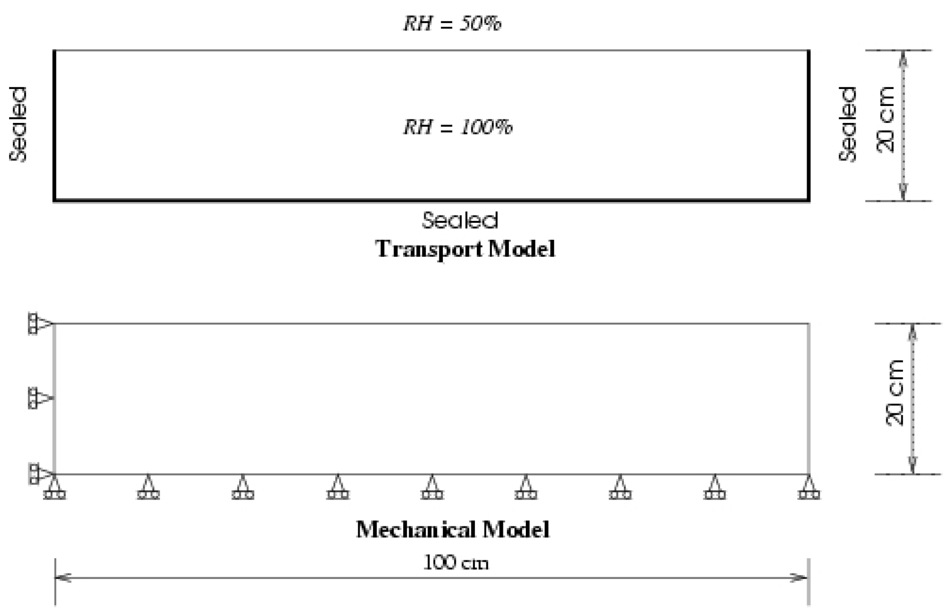

Fig. 6 Boundary conditions for the transport and mechanical analyses of a concrete slab

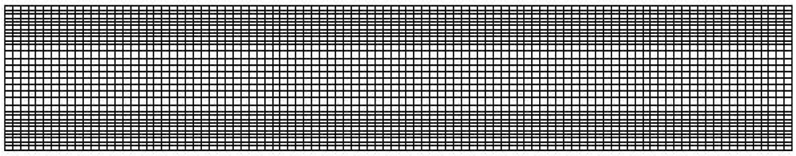

Fig. 7 Finite element mesh used in the numerical analyses

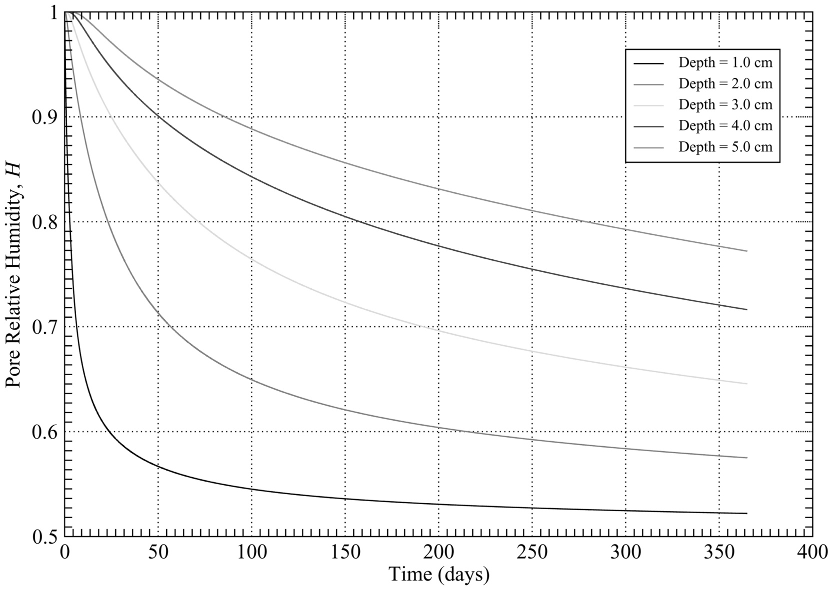

Fig. 8 Variation of the internal relative humidity with time of exposure

environmental condition (the boundary condition), the rate of drying becomes lower than those at deeper location.

Fig. 9 shows that the variations of shrinkage strain in the concrete specimen at different depths with increasing time of exposure. It is apparent that, the rate of the strain accumulation increases with an increase of exposure time. Since the shrinkage strain of concrete is directly related to the drying process, the rate of shrinkage strain behaves similarly to that of drying. Fig. 10 and Fig. 11 show the increase of the equivalent strain and the scalar damage parameter due to the drying process, respectively. The threshold value of equivalent strain in the damage model is taken at 0.0001. In the present example, since the equivalent strain at the depth deeper than 3 cm does not reach the threshold value of 0.0001, the damage takes place only within 3 cm of depth. From Fig. 11, one can see that the damage starts approximately within 1 day at 1 cm depth, 7 days at 2 cm depth and 70 days at 3 cm depth. The maximum values showed in both equivalent strain and damage parameter can be explained by considering the total strain in horizontal direction (see Fig. 12). It shows that at a certain time, the strains, especially those near the surface, start to go to the opposite direction (i.e. tensile strain decreases). This happens due the fact that the rate of shrinkage strain near the concrete surface becomes lower than those at deeper locations.

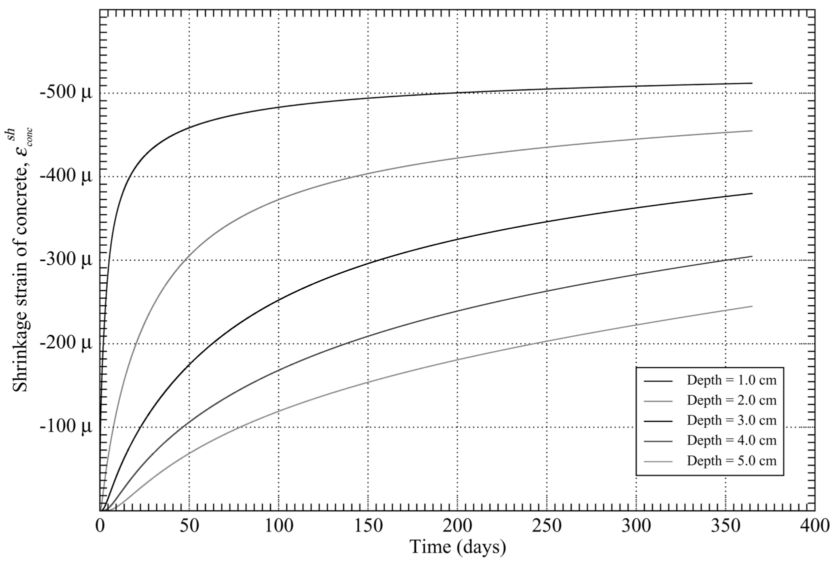

Fig. 9 Variation of concrete shrinkage strain with time of exposure

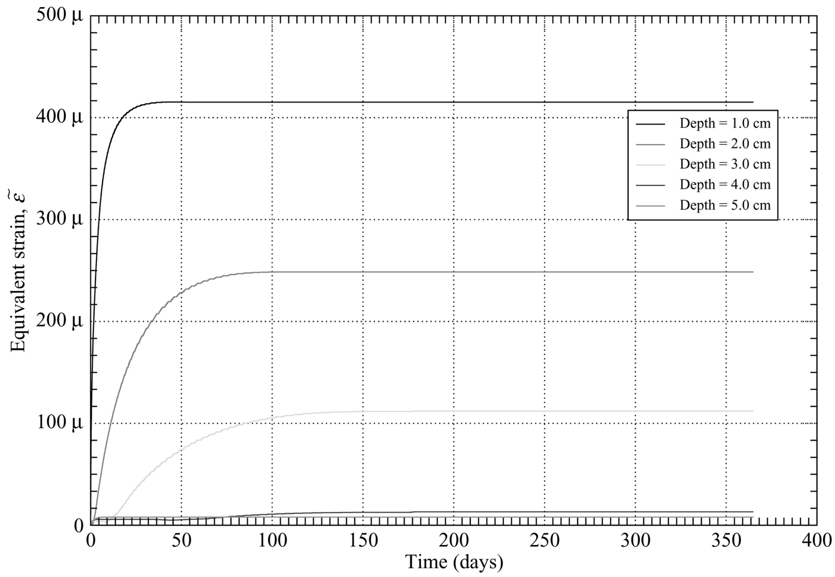

Fig. 10 Variation of equivalent strain with time of exposure

6.2. The effect of shrinkage-induced damage on moisture diffusion

The drying shrinkage creates damage in the concrete. The effect of the damage on the diffusion of moisture is taken into account by the two models developed in the present study for the humidity diffusivity and the moisture capacity. Figs. 13 and 14 show the coupling effect: profiles of the relative humidity and shrinkage strain in the concrete at different exposure times with and without considering the effect of the damage. The starting point of the profiles is from the exposed surface.

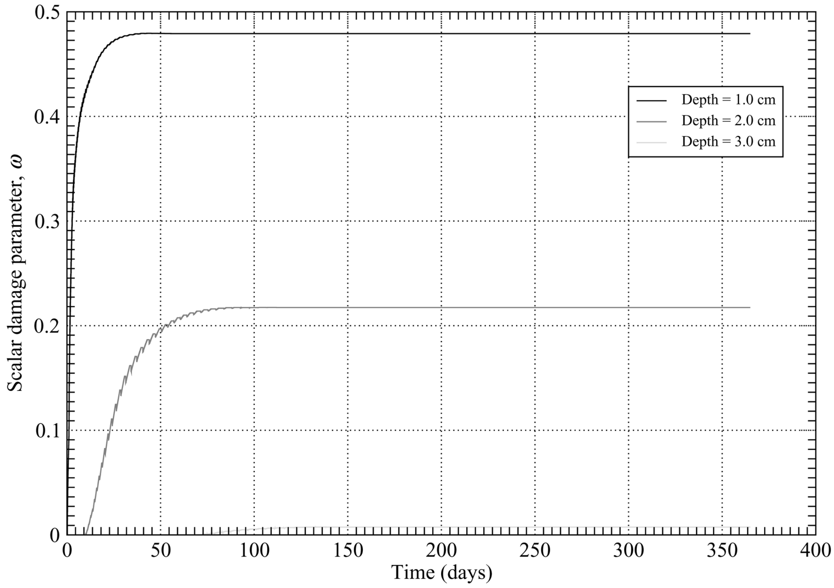

Fig. 11 Variation of scalar damage parameter with time of exposure

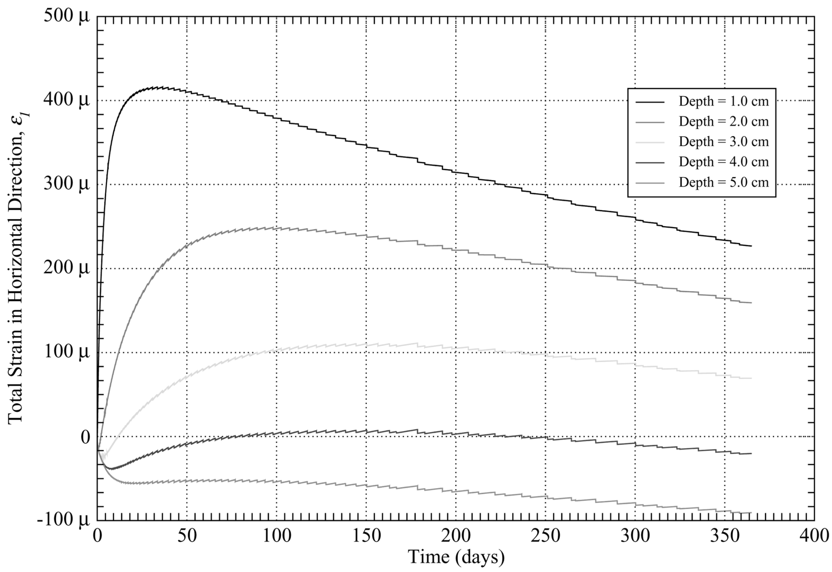

Fig. 12 Variation of total strain in horizontal direction with time of exposure

In Fig. 13, all curves eventually reach the equilibrium distribution of 50% (the steady state), which is the environmental relative humidity. One can see that the humidity profiles including the shrinkage-induced damage approach the equilibrium distribution in a faster rate, which indicates that the shrinkage-induced damage accelerates the diffusion process and increases the rate of drying. On the other hand, as shown in Fig. 14, the accelerated drying process increases the shrinkage strain of concrete. For example, comparing the two curves exposed after thirty 30 days with and without the

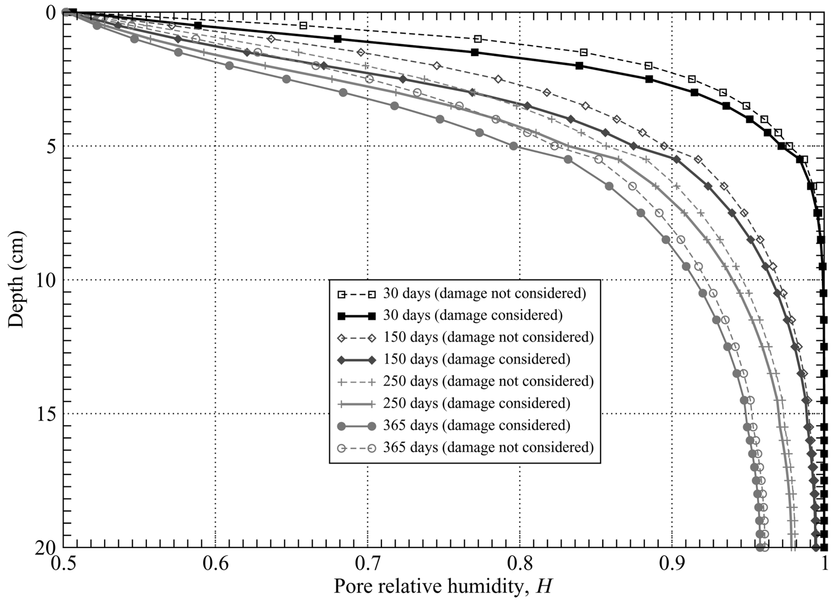

Fig. 13 The effect of damage on the drying process of the concrete

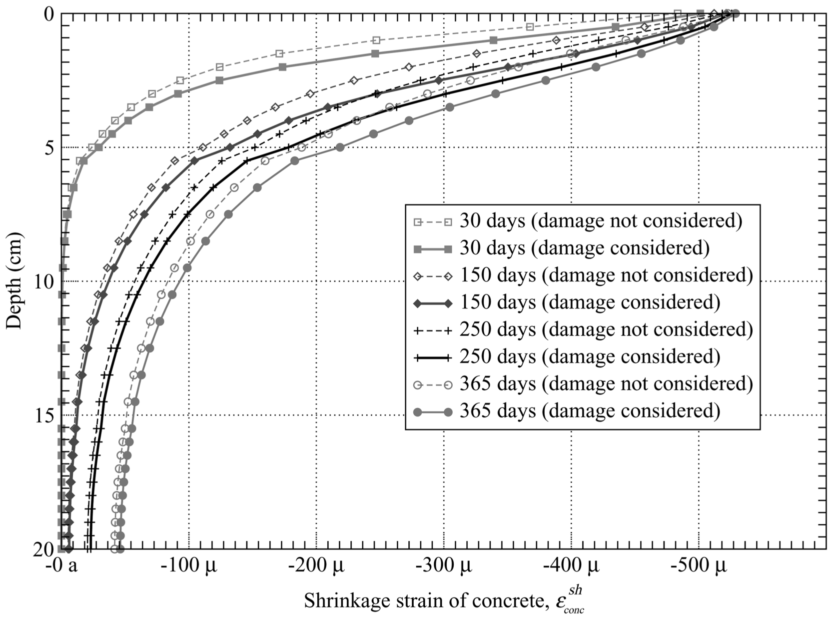

Fig. 14 The effect of damage on the shrinkage of concrete

damage, the curve with the effect of the damage is higher.

Figs. 13 and 14 show that there is an active interplay between the drying process and the shrinkage-induced damage. Basically the moisture loss causes drying shrinkage, which generates the damage. In turn, the damage increases diffusivity of concrete, and thus accelerates the moisture transfer. It is worth noting that Figs. 8-14 illustrate the numerical results of one group of material parameters for a specific concrete slab. The significance of the present new model considering the effect of drying shrinkage depends on many material parameters, such as water-cement ratio. With higher water-cement ratio, the drying shrinkage is higher. As a result, the shrinkage-induced damage is higher, and so does the rate of acceleration of the moisture diffusion process.

7. Conclusions

1. The multiscale models (at micro-, meso- and macro-scale) are used for modeling the moisture diffusion and drying shrinkage processes of concrete. The diffusion equation and the model for drying shrinkage of concrete are established for a representative volume of element at the macro-scale. At the meso-scale, concrete is considered as a two-phase composite with the aggregate as inclusion and the cement paste as matrix. At the micro-scale, the influential parameters on microstructure of cement paste, such as water-cement ratio and curing time, are included.

2. The effect of shrinkage-induced damage is characterized by modifying the two material parameters in the moisture diffusion equation, i.e., the moisture capacity and the humidity diffusivity. In this manner, the shrinkage-induced stress or strain is not shown in the diffusion equation as an explicit driving force, but as an implicit parameter.

3. The effect of shrinkage-induced damage on humidity diffusivity is characterized by a similar manner as the effect of isotropic damage on the secant modulus proposed by Kachanov. Furthermore, the effect of the damage can also be characterized by recently developed composite damage mechanics. The general trend is that with increasing damage, the humidity diffusivity increases.

4. The effect of drying shrinkage on the moisture capacity is included by an analytical model based on non-equilibrium thermodynamics and minimum potential energy principle for a two-phase composite. The influence of the drying shrinkage depends mainly on the difference between the shrinkage coefficients of the two constituent phases: the aggregate and the cement paste. When the two shrinkage coefficients are the same, the moisture capacity of the concrete is simply the weighted average of the moisture capacities of the two components; when the two shrinkage coefficients are different, the coupling effect must be taken into account. With increasing differential shrinkage, the moisture capacity of concrete increases.

5. An elastoplastic damage model is used to characterize the shrinkage-induced damage. The total strain is decomposed into the elastic strain, the plastic strain, and the shrinkage strain. The coupling between elastoplastic and damage is formed by considering the effective stress concept. The nominal stress is transformed into effective stress using the scalar damage parameter. The plastic strain may be evaluated using conventional plasticity methods, and the total strain is used to evaluate the scalar damage parameter based on the scalar damage model. The elastoplastic damage model is validated by available test data for different loading scenarios.

6. The present method can predict the basic trends for the coupling effect between the drying shrinkage of concrete and the moisture diffusion in concrete. The mechanical part of drying shrinkage problem is handled within the framework of elastoplastic damage model. The shrinkage-induced damage increases the humidity diffusion coefficient; the differential shrinkage between the aggregate and the cement paste increases the moisture capacity of concrete. As a result, the moisture diffusion process in concrete is accelerated by the drying shrinkage. In turn, the accelerated drying process increases the shrinkage of concrete.

Acknowledgements

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