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Reliabllity analysis of containment building subjected to earthquake load using response surface method

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Abstract. The seismic safety of reinforced concrete containment building can be evaluated by probabilistic analysis considering randomness of earthquake, which is more rational than deterministic analysis. In the safety assessment of earthquake-resistant structures by the deterministic theory, it is not easy to consider the effects of random variables but the reliability theory and random vibration theory are useful to assess the seismic safety with considering random effects. The reliability assessment of reinforced concrete containment building subjected to earthquake load includes the structural analysis considering random variables such as load, resistance and analysis method, the definition of limit states and the reliability analysis. The reliability analysis procedure requires much time and labor and also needs to get the high confidence in results. In this study, random vibration analysis of containment building is performed with random variables as earthquake load, concrete compressive strength, modal damping ratio. The seismic responses of critical elements of structure are approximated at the most probable failure point by the response surface method. The response surface method helps to figure out the quantitative characteristics of structural response variability. And the limit state is defined as the failure surface of concrete under multi-axial stress, finally the limit state probability of failure can be obtained simply by first-order second moment method. The reliability analysis for the multiaxial strength limit state and the uniaxial strength limit state is performed and the results are compared with each other. This study concludes that the multiaxial failure criterion is a likely limit state to predict concrete failure strength under combined state of stresses and the reliability analysis results are compatible with the fact that the maximum compressive strength of concrete under biaxial compression state increases

Keywords: seismic safety; containment building; response surface method; limit state function.

1. Introduction

The safety of nuclear power plant structures is of concern to regulatory agencies, the nuclear industry and the general public because of the serious socioeconomic consequences that could result from structural failure. In 1998, the US NRC introduced its new Reactor Oversight Process, with the concept of seven cornerstones as a basis for defining the safety scope in its new safety oversight model to be consistent with its mission of protecting the public health and safety with respect to civilian nuclear power plant operation (William 2005). Among the seven cornerstones barrier integrity derived from the probabilistic risk assessment approach to plant safety is included. The effects of aging infrastructure, treatment of uncertainties and containment modeling under a variety of accident conditions are related with containment building and facilities.

To ensure structural safety, nuclear power plant structures must be able to withstand all kinds of

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loads and load combinations that may be expected to occur during the lifetime of the plant. These loads include various static and dynamic loads, which are caused by operational, environmental and accidental conditions. It is recognized that the loads involve randomness and other uncertainties in nature. For example, we not only cannot predict the occurrence of an earthquake in advance, but also cannot precisely its intensity and duration. Similarly, the structural resistance cannot be determined precisely since the basic parameters such as material strength always exhibit statistical variation. In addition, the failure mechanism of a structure, which is needed to define the structural resistance, usually is very complicated and cannot be defined with certainty. Furthermore, structural behavior is always idealized to simplify the analysis. In view of randomness and uncertainty in loads, structural resistance and structural behavior, etc., a probabilistic approach for assessment of structural safety is a rational choice, since the theory of probability provides a framework for the formal treatment of uncertainties (Shinozuka 1981).

Structural reliability is generally defined as the probability that a structure will achieve a specified life without failure under a given loading. The life of structure is thus recognized as being a random variable. The prediction of structure life should be based on the probabilistic approach. Reliability theory is simply a probabilistic design approach to the problem of the design characteristic life.

A probability-based reliability analysis method for structures, particularly for containment structures, has been developed at the Brookhaven National Laboratory. This method, which incorporates the finite element analysis and random vibration theory, makes it possible to evaluate the reliability of structures under various static and dynamic loads in terms of limit state probabilities (Hwang 1983). Let the load and the resistance be random variables. Then, the estimation of the response involves solutions of propagation of uncertainty. Basic methods for uncertainty-propagation analysis are the integration techniques and the procedures based on the evaluation of the moments of the probability distribution. They cannot be easily used for large structural systems. Complexity here arises from the large number of variables, from the time dependence of the random excitations and spatial variation of the system parameters. The Monte Carlo approach can be utilized under these circumstances, but it may not be practical for complex structural systems primarily because of the high computational costs and times. A stochastic finite element technique, especially response surface method, makes use of a polynomial expansion of the structural response in terms of the spatial averages of the design variables. The method can provide information on the contribution to the response uncertainty from different sources of uncertainty. This procedure was studied by Bucher (1987), Faravelli (1989), Haldar (2000), and Veneziano (1983).

In this paper, the reliability analysis for a reinforced concrete containment building is performed using stochastic finite element technique and random vibration analysis. Especially to calculate the limit state probability of structure, failure criteria for concrete under combined state of stresses is defined as limit state (Lee 2004). The reliability analysis for the multiaxial strength limit state and the uniaxial strength limit state is performed and the results are compared with each other.

2. Earthquake load

Accelerograms of strong-motion earthquakes recorded on firm ground and at moderate epicentral distances are generally extremely irregular, and have the appearance of random-time functions. Fig. 1 shows the different structural responses, which are obtained from the dynamic analysis of containment building subjected to the same magnitude of earthquake load. Displacement at the apex

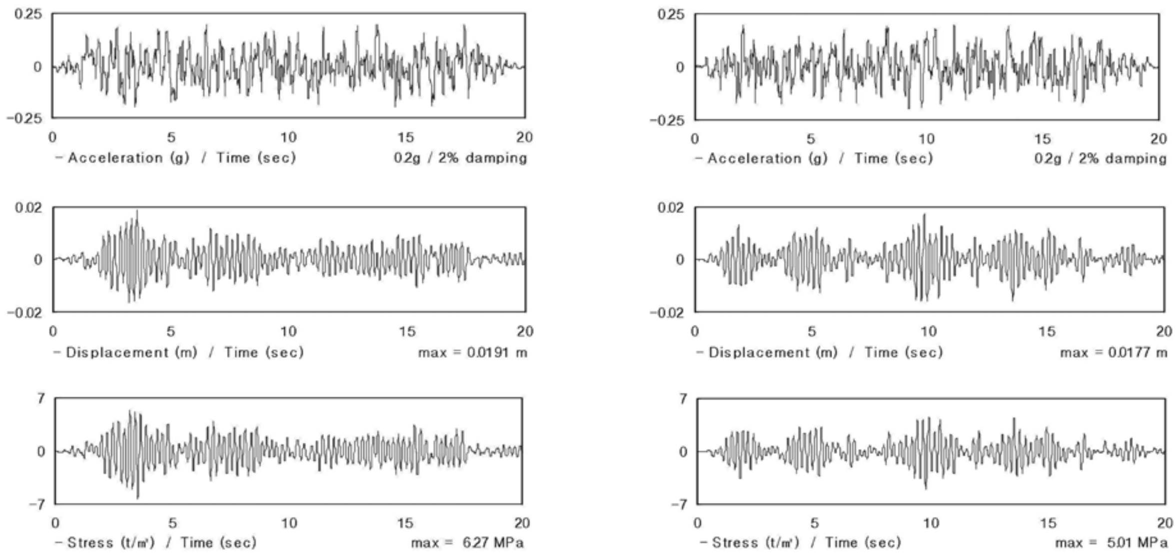

Fig. 1 The seismic responses of containment building for the same earthquake load

of containment building and meridional stress at the bottom of containment building from the numerical example performed in this paper are shown in the figure respectively. From the numerical example, the differences of responses at the same input loading are up to 8% in displacement and 25% in stress.

So the structural response may be obtained in probabilistic terms using random vibration analysis. This has led to a number of studies in which earthquakes are modeled by white noise and filtered white-noise processes.

The structural response to dynamic loads is a function of the amount and type of damping and therefore it is essential for obtaining realistic results in dynamic analysis to get the realistic damping values. The selection of an appropriate damping value then depends on the type of structure, the support conditions, and the magnitude of excitation. And the damping of structures under dynamic loading influences significantly the structural responses.

In a probabilistic approach to seismic-resistant design, a quantity of major interest is the probability that the random response to an earthquake of specified intensity will remain within specified barriers during the excitation. With few exceptions, the estimation of this probability of structural safety requires simulation of a large number of artificial earthquake motions. The mathematical modeling of earthquake motions, therefore, provides descriptions of the physical process for the purpose of predicting structural safety against earthquakes.

The ground acceleration is assumed to be idealized as a segment of finite duration of a stationary Gaussian process with mean zero and a Kanai-Tajimi spectrum.

The ground acceleration ahas the spectral density $S_a(\omega)$ as

$$S_a(\omega) = S_o \frac{1 + 4\xi_g^2\left(\frac{\omega}{\omega_g}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right]^2 + 4\xi_g^2\left(\frac{\omega}{\omega_g}\right)^2} \tag{1}$$

where the parameter S_o represents the intensity of white noise acceleration at bedrock, and ω_g and ξ_g are the dominant frequency and the damping ratio of the overlaying layers of soil, respectively. The values of ω_g and ξ_g depend on the soil conditions of the site.

The statistics of ω_g and ξ_g for different soil conditions have been reported in the previous studies (Ellingwood 1982, Hwang 1983). For soft soil sites, ω_g is estimated in the range of 2.4π to 3.5π, whereas for rock sites, ω_g ranges from 8π to 10π. As for the stiff soil sites, ω_g is between the values estimated for rock and soft soils. And the mean value of ξ_g is estimated as 0.6 and coefficient of variation (COV) is about 0.4.

The values of S_o in Eq. (1) can be determined based on the peak ground acceleration of earthquake, A_1, as follows:

$$A_1 = p_g\ \sigma_g \tag{2}$$

where p_g = peak factor (assumed to be 3.0 in this study).

Standard deviation of the ground acceleration, σ_g is given by

$$\sigma_g = \left[\int_{-\infty}^{\infty} S_a(\omega)d\omega\right]^{0.5} = \left[\pi\omega_g\left(\frac{1}{2\xi_g} + 2\xi_g\right)\right]^{0.5} S_o^{0.5} \tag{3}$$

Fig. 2 shows K-T spectrum for pga=1.0g, ω_g = 8.5π, ξ_g= 0.6 for rock site.

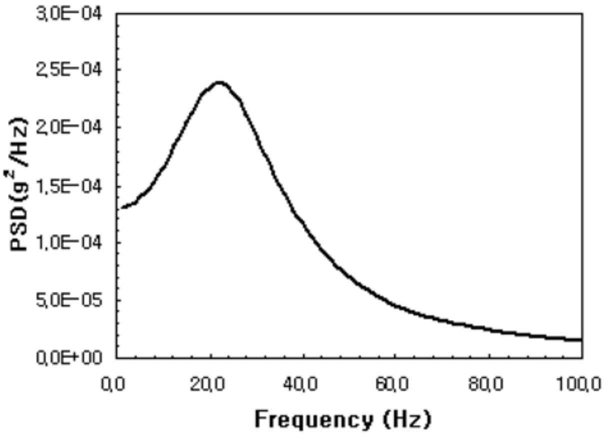

Fig. 2 Single-sided Kanai-Tajimi spectrum

3. Stochastic finite element analysis for the containment

3.1. Containment modeling

The concrete containment building considered in this study is shown in Fig. 3 and the three-dimensional finite element model is shown in Fig. 4. The containment consists of a circular cylindrical wall with a hemispherical dome on the top. The containment building is reinforced with hoop and meridional rebars in two internal and external layers with each 7.5 cm concrete cover.

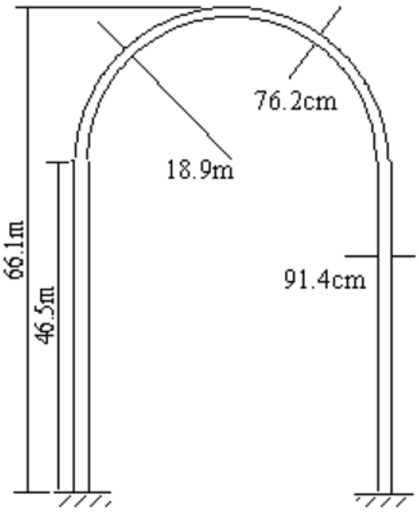

Fig. 3 Cross-section of containment building

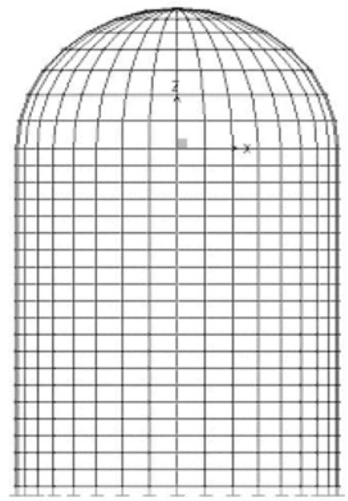

Fig. 4 FEM model for the containment

Also diagonal rebars is reinforced to resist the shear forces and steel liner is used to ensure against leakage. However, diagonal rebars and steel liner are not included in this study. Only hoop and meridional rebars are included in the modeling of reinforced concrete containment building. For the cylindrical portion of the containment, both the hoop and meridional reinforcing rebars are D57 with 30.5 cm spacing. The hemispherical dome is also reinforced with D44 by 30.5 cm spacing.

The finite element used in modeling is three dimensional shell element for concrete and one dimensional rebar element for rebars as described in the ABAQUS (2003) computer code. The boundary condition is modeled as fixed base. As can be seen from Fig. 4., the containment is divided into twenty-seven layers and each layer has thirty-two elements.

3.2. Random vibration analysis

The random vibration analysis is performed to get the information on the random structural responses of the concrete containment building subjected to random excitation such as earthquake generated ground acceleration. The dynamic characteristics of the structures are represented by the natural frequencies and associated mode shapes. The set of eigenmodes extracted in the eigenvalue analysis of structure are used to calculate the corresponding power spectral densities (PSD) of response variables(stresses, strains, displacements, etc.) and the variance and root mean square (RMS) values of these same variables. The first twenty natural frequencies shown in tabulated form

Table 1 Natural frequencies of the containment

Mode	1	2	3	4	5	6	7	8	9	10
Hertz	4.29	4.29	5.78	5.78	6.72	6.72	7.69	7.69	9.10	11.78
Mode	11	12	13	14	15	16	17	18	19	20
Hertz	11.78	11.95	11.95	12.04	12.04	12.59	12.59	13.10	14.56	14.56

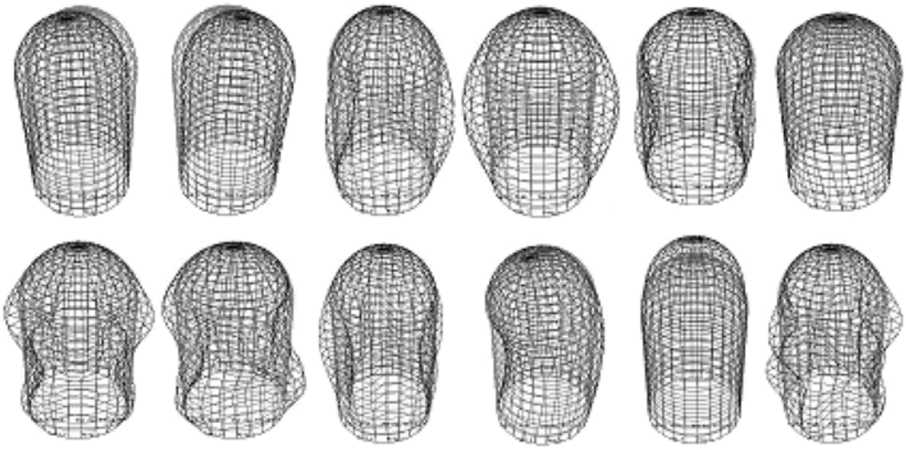

Fig. 5 Mode shapes (from the upper left ; No.1, 2, 3, 5, 7, 9, 10, 12, 14, 16, 18, 19)

in Table 1 are considered to obtain the random structural responses. Fig. 5 shows mode shapes and most of them have the pairs in the axi-symmetric shell structures. The RMS of response variables is computed by integrating the single-sided power spectral density of the variable over the frequency range. This integration is performed numerically by using the trapezoidal rule over the range of frequencies. The RMS of random responses is utilized to calculate the probabilistic properties (mean and variation) of maximum responses.

3.3. Random variables

To perform a reliability analysis, it is necessary to determine the probabilistic properties of loads and resistances. For concrete containments, several loads such as dead loads, live loads, prestress loads, thermal loads, pressure loads, wind loads, earthquake loads and impact loads, etc., are included in the design specifications. These are characterized by their sources, increasing magnitude or severity, and decreasing probability of occurrence, or increasing recurrence period. The present study disregards the randomness of loads except the earthquake load, which is modeled as a random process shown in chapter 2.

The structural resistance is the function of basic variables, which would include material properties (e.g. yield or ultimate stress of steel, crushing strength of concrete), and structural dimensions or section properties. The basic variables are random and contribute to the uncertainty in the structural resistance. Additional uncertainties in the structural resistance would arise from the analytical modeling of the structural system.

The weight density of the concrete is 23.5 kN/m^3, Young's modulus and Poisson's ratio are 2.45

Table 2 Statistical description of random variables

Random variables		Mean value	C.O.V	Distribution type
Compressive strength, f_c'	x_1	37.7 Mpa	0.14	Normal
Modal damping, ξ	x_2	0.05	0.15	Normal
K-T spectrum, ξ_g	x_3	0.6	0.4	Normal

$\times\ 10^4$ Mpa and 0.17, respectively. The mean value of concrete compressive strength is 37.7Mpa and the coefficient of variation is 0.14 with the Gaussian distribution. As for the steel, the Young's modulus and Poisson's ratio are 1.96×10^5 Mpa and 0.3, respectively. The mean value of yield strength is 480.2 Mpa and the coefficient of variation is 0.093 with the Gaussian distribution.

The three dominant random variables considered in the analysis are shown in tabulated form in Table 2 and the other random variables are not included in the study.

3.4. Response surface analysis

The seismic responses of containment are obtained from the response surface analysis. Three variables as shown in Table 2 are chosen to incorporate the randomness of the design variables in the response uncertainty. The response approximation used in this study is a second-order polynomial type presented by Bucher, *et al.* (1987). It can be represented as

$$g(x) = a_o + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} a_{ii} x_i^2 \tag{4}$$

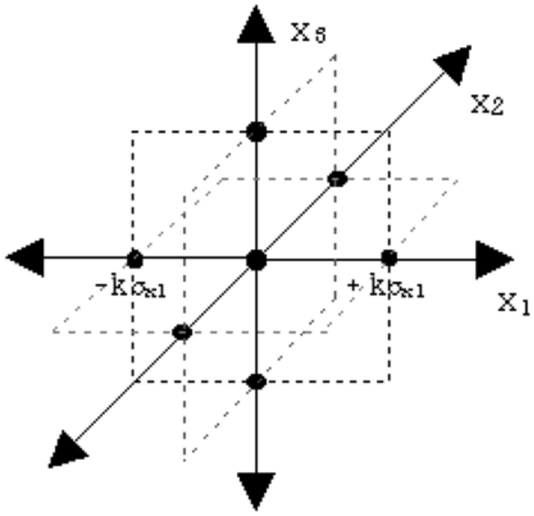

Fig. 6 Selection of sampling points

Table 3 Sampling points for response surface method

Random variable		f_c' (Mpa)	ξ	ξ_g
Center point	S1	37.7	0.05	0.6
Axial points	S2	48.3	0.05	0.6
	S3	27.1	0.05	0.6
	S4	37.7	0.065	0.6
	S5	37.7	0.035	0.6
	S6	37.7	0.05	0.84
	S7	37.7	0.05	0.36

Table 4 Results of random vibration analysis for each sampling point

Sampling points	RMS of concrete stress (Mpa)			RMS of steel stress (Mpa)	RMS of max. disp. (mm)
	f_{11}	f_{22}	f_{12}	f_{11}	
S1	0.16	0.98	0.044	7.25	3.80
S2	0.17	1.03	0.046	6.78	3.54
S3	0.15	0.91	0.042	8.01	4.25
S4	0.13	0.76	0.035	5.61	2.95
S5	0.23	1.39	0.063	10.31	5.40
S6	0.15	0.92	0.042	6.80	3.57
S7	0.16	0.98	0.045	7.23	3.79

where x_i = ith random variables as shown in Table 2; and a_o, a_i, and a_{ii} = unknown coefficients to be determined by solving a set of simultaneous equations.

The minimum number of simultaneous equations to get the unknown coefficients is ($2n$+1) for n random variables. Seven sampling points are evaluated at the center point of random variables and at the axial points which are $\pm k\sigma$ apart from center point. The center point can be the best estimated value (mean value) of random variable and s is the standard deviation of random variable. The determination of k value to evaluate axial points has no restriction and 2 or 3 are recommended for the good results.

The sampling points for response surface analysis are selected on the axis of random variables(see Fig. 6) and shown in tabulated form in Table 3.

The random vibration analysis at each sampling point is performed for the peak ground acceleration 0.2 g and the RMS of maximum seismic responses of the critical element, at which the failure is expected to occur first in the containment are shown in tabulated form in Table 4. The other load effects are assumed to be deterministic in nature and they don't be included in the random vibration analysis and in the response surface analysis. The results shown in Table 4 are only due to an earthquake load. The critical elements (elements 1,16,17,32 in this study) are assumed to be elements at the bottom layer in cylindrical wall, because the flexural mode is considered to be dominant for the containment subjected to the lateral excitations as an earthquake load. The RMS of maximum displacement in Table 4 is at the apex of the containment.

3.5. Maximum responses

The maximum structural response is one of the most important quantities in the analysis and design of structures under earthquake excitations. Total displacements of the structure under cyclic loading are divided into the elastic components and inelastic components. Generally, when the input motion is stationary, Gaussian, narrow-band random process, the peak response (displacement) distribution of linear system is Rayleigh. But it is reported that the peak response is Weibull-like behavior, when the input is nonstionary, Gaussian random process (Shinozuka and Yang 1969).

It has often been assumed that the strong shaking portion of typical earthquake accelerograms is stationary and that the corresponding structural response is, as well. In this case, standard deviations of responses are assumed to be constant with varying time.

Table 5 Mean and standard deviation of maximum responses for each sampling point

Sampling points	Concrete stress (Mpa)			Steel stress (Mpa)	Max. of disp. (mm)
	f_{11}	f_{22}	f_{12}	f_{11}	
S1	0.42(0.082)	2.50(0.49)	0.11(0.023)	18.49(3.64)	9.69
S2	0.44(0.085)	2.62(0.52)	0.11(0.024)	17.30(3.40)	9.03
S3	0.39(0.076)	2.32(0.46)	0.11(0.022)	20.44(4.02)	10.84
S4	0.32(0.064)	1.93(0.38)	0.09(0.018)	14.31(2.81)	7.53
S5	0.59(0.117)	3.56(0.70)	0.16(0.031)	26.30(5.17)	13.78
S6	0.39(0.077)	2.35(0.46)	0.11(0.022)	17.35(3.41)	9.11
S7	0.41(0.081)	2.50(0.49)	0.11(0.023)	18.44(3.63)	9.67

*standard deviation in parentheses

Let Y_m be the absolute global maximum of $Y(t)$ in the time interval (T_1, T_2), then the mean value, μ_{Ym}, and the standard deviation, σ_{Ym}, of the absolute global maximum response follow as (Yang 1981).

$$\mu_{Ym} = (K + 0.5772\ K^{1-\alpha})\sigma \tag{5}$$

$$\sigma_{Ym} = 1.28\sigma/K^{\alpha-1} \tag{6}$$

where $\alpha = 2.0$ (Rayleigh distribution), $K = [\alpha \ln[\omega_a(T_2 - T_1)/\pi]]^{1/\alpha}$, ω_a = the predominant frequency of the power spectral density of $Y(t)$.

In this study, $K \cong 2.3$ with $(T_2 - T_1) = 10$ sec, $\omega_a = 4.29$ Hz.

The statistical measures of the maximum responses at the sampling points are calculated by virtue of Eq. (5) and Eq.(6) and are shown in Table 5.

4. Reliability analysis of the containment

4.1. Limit state

For the reinforced concrete containment structures, the failure will occur when the reinforcing steels begin to yield or the concretes at the extreme fiber of the containment wall cross-section begin to crush. Thus the limit state condition can be expressed in terms of uniaxial strength as follows

$$f_s \geq f_y \tag{7}$$

$$f_c \geq 0.85 f_c' \tag{8}$$

where f_s is the stress in the rebars, f_y the yield stress of steel, f_c the compressive concrete stress at the extreme fibers, and f_c' the compressive concrete strength.

But the strength of concrete under multiaxial stresses is generally a function of the state of stress and cannot be predicted by limitations of simple tensile, compressive, and shearing stresses independently of each other (Chen 1982). Therefore, the strength of concrete elements can be properly determined only by considering the interaction of the various components of the state of stress. A failure criterion of isotropic materials based upon state of stress must be an invariant function of the state of stress, i.e., independent of the choice of the coordinate system by which stress is defined. The Drucker-Prager failure criterion as a smooth approximation to the Mohr-Coulomb failure surface or an extension of the von Mises failure surface is utilized to define the limit state for concrete under combined states of stress.

The Drucker-Prager failure surface, F_c is given by

$$F_c(I_1,J_2) = \delta I_1 + \sqrt{J_2} - \lambda = 0 \tag{9}$$

where $I_1 = f_x + f_y + f_z$, $J_2 = [(f_x - f_y)^2 + (f_y - f_z)^2 + (f_z - f_x)^2]/6 + f_{xy}^2 + f_{yz}^2 + f_{zx}^2$, and f_x, f_y, f_z are axial normal stresses and f_{xy}, f_{yz}, f_{zx} are tangential stresses in the cartesian coordinate syetem, and δ is a constant, which is chosen from the ratio of the ultimate stress reached in biaxial compression to the ultimate stress reached in uniaxial compression, and λ is a hardening parameter.

In uniaxial compression, the first stress invariant $I_1 = f_c'$, the second stress invariant $J_2 = f_c'^2/3$, where f_c' is the uniaxial compressive strength and in biaxial compression, $I_1 = 2f_{bc}'$, $J_2 = f_{bc}'^2/3$, where f_{bc}' is the biaxial compressive strength. The failure ratio, $\gamma_{bc} = f_{bc}'/f_c'$ is the ratio of the biaxial compressive strength to the uniaxial compressive strength and is assumed to be 1.15 in this study. Therefore, failure surface F_c can be obtained as

$$F_c(I_1, J_2) = F_L - F_R = 0 \tag{10}$$

where $F_L = \sqrt{J_2} - 0.07\, I_1$, $F_R = 0.51\, f_c'$.

The limit state can be look upon as the simplest application where the failure criterion is considered to contain just two variables: a load effect F_L and a resistance F_R, and the failure event is specified by $F_L - F_R \geq 0$. In the reliability analysis, the load effect F_L, dimensionally consistent with F_R can be approximated by the response surface method, which the method provides information on the contribution to the response uncertainty from different sources of uncertainty.

4.2. Load effects

Loads acting on nuclear structures can generally be classified into load process models such as permanent loads, sustained loads, and transient loads. For concrete containments, the ASME code section III, division 2 defines explicit and precise load categories. For example, load generated by the safe shutdown earthquake (SSE) is included in the extreme environmental load category. The load combination (LC) of abnormal/extreme environmental category is as follows;

$$LC = 1.0D + 1.0L + 1.0F + 1.0P_a + 1.0T_a + 1.0E_{ss} + 1.0R_a + 1.0R_r \tag{11}$$

where, D=dead load, L=live load, F=prestress load, P_a=accidental pressure load, T_a=accidental thermal load, E_{ss}=safe shutdown earthquake load, R_a=pipe reactions from thermal conditions, and R_r=local effects on the containment due to design basis accident.

Dealing only with D, L, F, P_a, T_a, and E_{ss} for the time being, the meridional and hoop stresses of concrete at the extreme fibers are shown in Table 6, where the plus sign means tension. The concrete stresses in the critical element are assumed to govern the limit state in this study and the steel stresses are neglected in performing the reliability analysis.

Table 6 Maximum concrete stresses in the cylindrical wall

Loads	Magnitude	Stress(Mpa)		
dead load	23.5 kN/m^3	meridional	f_D	–1.45
live load	0.005 Mpa	meridional	f_L	–0.052
prestress load	0.41 Mpa	meridional hoop	f_F	–5.11 –10.21
accidental internal pressure	0.41 Mpa	meridional hoop	f_P	+ 4.36 + 8.81
accidental temperature load	outside temp.: –18.9°C accident temp. : 132.2°C	meridional hoop	f_T	–12.26 –12.26
earthquake load	0.2 g, 5% non-exceedance	meridional hoop	f_E	–3.31 –0.55

Table 7 Coefficients of response surface for random variables

Approximation of response		Hoop direction	Meridional direction	F_L	
		f_{ch}	f_{cm}	$f_{ch} + f_{cm}$	f_{cm}
Sampling points	S1	14.21	17.82	7.18	9.04
	S2	14.33	17.98	7.24	9.12
	S3	14.27	17.59	7.12	8.92
	S4	14.18	17.07	6.95	8.66
	S5	14.54	19.22	7.66	9.75
	S6	14.28	17.62	7.13	8.94
	S7	14.31	17.82	7.19	9.04
Coefficients of response surface	a_0	18.09	23.50	9.49	11.89
	a_1	-5.81E-02	4.22E-02	5.68E-03	2.30E-02
	a_2	-78.67	-216.11	-79.22	-109.67
	a_3	-1.83	1.67	0.29	0.83
	a_{11}	8.08E-04	-3.14E-05	0.0	-1.79E-04
	a_{22}	666.67	1444.44	555.56	733.33
	a_{33}	1.48	-1.74	-0.35	-0.87

The hoop and meridional maximum concrete stress in the cylindrical wall of containment including D, L, F, P_a, T_a, and E_{ss}, and the load effect F_L are calculated at each sampling point. The load effect F_L can be expressed as Eq. (4) by virtue of response surface method and thus the coefficients of response surface shown in Table 7 are used to calculate the first-order approximate mean and variance of load effect, F_L expressed as Taylor series.

4.3. Limit state probabilities of failure

The probability of failure is computed by performing the first order second moment (FOSM) reliability analysis. The reliability index, b of the limit state function by mean value FOSM method is as follows

$$\beta = \frac{E[F_R] - E[F_L]}{\sqrt{Var[F_R] + Var[F_L]}} \quad (12)$$

where $E[F_R]$ = mean value of F_R, $E[F_L]$ = mean value of F_L, $Var[F_R]$ = variance of F_R, $Var[F_L]$ = variance of F_L.

Also, advanced FOSM method is utilized to search the most probable failure point (MPFP) and to calculate the reliability index, β. Introduce the set of uncorrelated reduced variates, $u_i = (x_i - \mu_i)/\sigma_i$ with the basic variables $(x_1, x_2, \ldots, x_n)$, then the point on the failure surface, $(u_1^*, u_2^*, \ldots, u_n^*)$, having the minimum distance to the origin may be determined by minimizing the function b, subjected to the contstraint $F_c(u)$=0; that is,

$$\textbf{\textit{Minimize}}\ \beta = \sqrt{u^T u} \quad \text{subjected to } F_c(u) = 0 \quad (13)$$

For this purpose, the iterative procedure is used as;

$$u^{k+1} = \frac{F_u^{kT} u^k - F_c(u^k)}{F_u^{kT} F_u^k} F_u^k \quad (14)$$

where, $F_u^k = \left\{ \frac{\partial F_c}{\partial u_1}, \frac{\partial F_c}{\partial u_2}, \ldots, \frac{\partial F_c}{\partial u_c} \right\}^T$ at $u = u^k$

Searching the most probable failure point, $(u_1^*, u_2^*, \ldots, u_n^*)$, the minimum distance β is

$$\beta = -\frac{F_u^{*T} u^*}{\sqrt{F_u^{*T} F_u^*}} \quad (15)$$

The state of stress considered in this study is the biaxial stress, that is, the meridional stress and the hoop stress at the bottom layer in the cylindrical wall.

The load effect F_L is expressed as the polynomial type of random variable function,

$$F_L = a_0 + \sum_{i=1}^{3} a_i x_{ii} + \sum a_{ii} x_i^2$$

$$= 9.49 + 5.68e{-3} \times x_1 - 79.22 \times x_2 + 0.29 \times x_3 + 555.56 \times x_2^2 - 0.35 \times x_3^2 \quad (16)$$

Expanding F_L in a Taylor series about the mean value μ_{x_i}, if the series is truncated at the linear terms, the first-order approximate mean and variance of load effect are

$$E[F_L] \cong a_0 + \sum_{i=1}^{3} a_i u_{x_i} + \sum_{i=1}^{3} a_{ii} u_{x_i}^2 = 7.18 \text{ Mpa} \quad (17)$$

$$Var[F_L] \cong \sum_{i=1}^{3} Var[x_i] \left(\frac{\partial F_L}{\partial x_i} \right)^2 = 0.54 \text{ Mpa} \quad (18)$$

The first-order approximate mean and variance of resistance are

$$E[F_R] = 19.23 \text{ Mpa} \quad (19)$$

$$Var[F_R] = 7.23 \text{ Mpa} \tag{20}$$

The reliability index, β of the limit state function by mean value FOSM method is

$$\beta = 4.317 \ (P_f = 3.58\text{e-}5) \tag{21}$$

And the first-order approximations are evaluated at MPFP and the reliability index, β is calculated by advanced FOSM as follows

$$\beta = 4.464 \ (P_f = 1.88\text{e-}5) \tag{22}$$

When the only one axial stress is considered in the limit state, the load effect F_L is expressed as

$$F_L = 11.89 + 2.30e{-}2\times x_1 - 109.67\times x_2 + 0.83\times x_2 - 1.79e{-}4\times x_1^2 + 733.33\times x_2^2 - 0.87\times x_3^2 \tag{23}$$

In the similar manner the reliability index can be calculated by mean value FOSM and advanced FOSM.

The MPFP on the limit state function, reliability index, and the limit state probabilities of failure are summarized in table 8. And when the only one axial stress such as the meridional stress, which is larger than the hoop stress in the critical elements, is considered to calculate the limit state probability of failure, the results are shown in Table 8.

It is seen that the reliability index for the case which the biaxial stress is included, is different from that for the case which only the meridional stress is included, and the former is larger than the latter when the state of stress is biaxial compression. This is consistent with the fact that the maximum compressive strength increases for the biaxial compression state. It is expected that the reliability index under biaxial compression-tension state will be smaller than under uniaxial compression state, even though the hoop concrete stress rarely happens to be in tension because the prestress force is introduced to resist the internal pressure load, which produces the tension stresses in the meridional and hoop direction. In addition, the reliability index by AFOSM is obtained on the MPFP of limit state and is larger than by mean value FOSM, and the MPFP moves from the mean value point like as the resistance term decreases and the load effect term increases.

4.4. Uniaxial strength limit state

Even though the strength of concrete under multiaxial stresses cannot be properly predicted by simple compressive stress, similar works on the simple limit state function are done to compare with reliability analysis results from the Drucker-Prager failure criterion. The probability of failure

Table 8 Results of reliability analysis for the containment building

Considering stress		Bi-axial stress included		One axial stress included	
Reliability method		MV FOSM Mean	AFOSM MPFP	MV FOSM Mean	AFOSM MPFP
Limit state	F_R	19.23	14.20	19.23	17.98
	F_L	7.18	7.24	9.04	9.17
Reliability index, β		4.317	4.464	3.526	3.762
Probability of failure		3.58E-05	1.88E-05	7.97E-04	3.38E-04

in the uniaxial strength limit state is computed by use of Eq. (8). Meridional stress (f_{cm}) larger than hoop stress at the bottom layer of cylindrical wall is used to calculate the reliability index of limit state function, that is $f_{cm} - 0.85f'_c = 0$.

The meridional concrete stress at the bottom of the cylindrical wall, f_{cm} is also expressed as the polynomial type of which coefficients are shown in Table 7,

$$f_{cm} = 23.50 + 4.22e{-}2 \times x_1 - 216.11 \times x_2 + 1.67 \times x_3 - 3.14e{-}5 \times x_1^2 - 1444.44 \times x_2^2 - 1.74 \times x_3^2 \quad (24)$$

The first-order approximate mean and variance of meridional concrete stress, f_{cm} are

$$E[f_{cm}] \cong a_0 + \sum_{i=1}^{3} a_i u_{x_i} + \sum_{i=1}^{3} a_{ii} u_x^2 = 17.82 \text{ Mpa} \quad (25)$$

$$Var[f_{cm}] \cong \sum_{i=1}^{3} Var[x_i] \left(\frac{\partial f_{cm}}{\partial x_i}\right)^2 = 4.28 \text{ Mpa} \quad (26)$$

The first-order approximate mean and variance of resistance are

$$E[0.85f'_c] = 32.05 \text{ Mpa} \quad (27)$$

$$Var[0.85f'_c] = 20.13 \text{ Mpa} \quad (28)$$

The reliability index, β of the limit state function by mean value FOSM method is

$$\beta = 2.880 \ (P_f = 6.31\text{e-}3) \quad (29)$$

And the reliability index, b by advanced FOSM is

$$\beta = 3.144 \ (P_f = 2.85\text{e-}3) \quad (30)$$

MPFPs of resistance and load effect are 21.26 Mpa and 18.07 Mpa respectively.

As seen from the above, the reliabilities based on the simple compressive strength limit state differ greatly from results of the multiaxial strength limit state, especially from the case that biaxial stress are included to calculate the limit state probability of failure. It should be emphasized that the multiaxial failure criterion is a more likely limit state than the simple failure criterion to predict concrete failure strength under combined state of stresses.

5. Conclusions

A reliability analysis for the containment building subjected to earthquake load is performed under various assumptions and idealizations. The numerical example is worked out by virtue of the structural analysis considering random variables such as load, resistance and analysis method, the limit state function of biaxial stress states, and the level II reliability methods. The structural analysis includes the random vibration analysis to deal with the randomness of earthquake load and the stochastic finite element analysis to get the information on the contribution to the response uncertainty from different sources of uncertainty. The Drucker-Prager failure criterion is adopted to define the limit state for concrete under combined states of stress, and the load effects in the limit state are approximated on the most probable failure point by the response surface method. The response surface approximation like as a polynomial type makes it easy to apply the reliability methods such as level II methods to the complicated limit state of structure. The reliability analysis for the multiaxial strength limit state and the uniaxial strength limit state is performed and the results are compared with each other. This study concludes that the multiaxial failure criterion is a likely limit state to predict concrete failure strength under combined state of stresses and the reliability analysis results are compatible with the fact that the maximum compressive strength of concrete under biaxial compression state increases. But further research efforts investigating more complex failure criterion are needed to overcome the shortcoming of the Drucker-Prager failure criterion.

References

ABAQUS(2003), *Standard User's Manual,* Hibbitt, Karlsson & Sorensen, Inc.(HKS).

ASME(1980), Boiler and pressure vessel code, Section III, Division 2.

Bucher, C. G., Bourgund, U. (1987), “Efficient use of response surface methods”, Report No.9-87, Institute of Engineering Mechanics, University of Innsbruck, Austria.

Chen, W. F. (1982), *Plasticity in Reinforced Concrete*, McGraw-Hill.

Ellingwood, B. R., and Batts, M. E. (1982), “Characterization of earthquake forces for probability-based design of nuclear structures”, NUREG/CR-2945, September.

Faravelli, L.(1989), “Response-surface approach for reliability analysis”, *J. Eng. Mech.*, ASCE, **115**(12), 2763-2781.

Haldar, A., and Huh, J. (1999), “Reliability analysis of structures subjected to dynamic loadings using nonlinear SFEM” *Internatinal Union of Theoretical and Applied Mechanics, Nonlinearity and Stochastic Structural Dynamics.*

Haldar, A., and Mahadevan, S. (2000), *Reliability Assessment Using Stochastic Finite Element Analysis,* John Wiley & Sons, Inc..

Hwang, H., Kao, S., and Reich, M. (1982), “Probabilistic models for materials used in a reinforced concrete containment”, BNL Technical Report, BNL/NUREG-51618.

Hwang, H., Wang, P. C. and Reich, M. (1983), “Probabilistic models for operational and accidental loads on seismic category I structures”, U.S. Nuclear Regulatory Commission Report, NUREG/CR-3342, BNL, Washington, D.C., June.

Park, H.K., and *et al.* (1983), “Analysis and design of containment building”, Korea Power Engineering Company, Inc..

Lee, S. and Oh, B. (2004), “Reliability analysis of reinforced concrete containment building subjected to earthquake load”, *Proceedings of the Fourth International Conference on Concrete under Severe Conditions*, CONSEC‘04, Seoul, Korea, June 27-30, **1**, 1007-1014.

Shinozuka, M., *et al.* (1981), “Probability based load criteria for the design of nuclear structures: a critical review of the state of the art”, U.S. Nuclear Regulatory Commission Report, NUREG/CR-1979, BNL, Washington, D.C., April.

Shinozuka, M., Hwang, H. and Reich, M. (1983), “Reliability assessment of reinforced concrete containment structures”, BNL/NUREG-51661, Feb.

Shinozuka, M., and Yang, J-N. (1969), “On the bound of first excursion probability”, *J. Eng. Mech. Div.*, ASCE, **95**(EM2), April, 363-377.

Veneziano, D., Casciati, F., and Faravelli, L. (1983), “Method of seismic fragility for complicated systems”, *Proceedings of the 2nd Committee on the Safety of Nuclear Installations(CSNI) Specialistic Meeting on Probabilistic Methods in Seismic Risk Assessment for NPP,* Livermore, California.

Yang, J. N., and Liu, S. C. (1981), “Distribution of maximum and statistical response spectra”, *J. Eng. Mech. Div.*, ASCE, **107**(EM6), 1089-1102.

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