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Soil foundation effect on the vibration response of concrete foundations using mathematical model

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Abstract. In this paper, vibration analysis of concrete foundations resting on soil medium is studied. The soil medium is simulated by Winkler model considering spring element. The concrete foundation is modeled by thick plate elements based on classical plate theory (CPT). Utilizing energy method consists of potential energy, kinetic energy and external works in conjunction with Hamilton's principle, the motion equations are derived. Assuming the simply supported boundary condition for the concrete foundation, the Navier method is used for calculating the frequency of the structure. The effect of different parameters such as soil medium, mode numbers, length to width ratio and length to thickness ratio of the concrete foundation are shown on the frequency of the structure. At the first, the results are validated with other published works in order to show the accuracy of the obtained results. The results show that considering the soil medium, the frequency of the structure increases significantly.

Keywords: vibration analysis; concrete foundation; soil medium; analytical method; classical plate theory

1. Introduction

Dynamic loads such as wind, earthquake, impact and etc are applied to the concrete foundation in the buildings always which leads to vibration of the structure. However, presenting a mathematical model for these structures for vibration analysis is essential (Boadu *et al*. 2017).

Vibration analysis of concrete foundations has been studied by different researchers. Arslan and Durmuş (2013) described effects of infill walls on behavior of RC frame with low strength, including numerical modeling, modal testing and finite-element model updating. A procedure for fatigue reliability prediction of prestressed reinforced concrete (PSC) highway bridges was proposed by Zhu *et al*. (2014). Modal parameters such as natural frequencies, mode shapes and damping ratios of RC frames with low strength were determined by Arslan and Durmuş (2014) for different construction stages using ambient vibration test. Real-time tracking of concrete vibration effort for intelligent concrete consolidation was presented by Gong *et al*. (2015). Behavior of fresh concrete that is under vibration using mass-spring model (MSM) was presented by Aktas (2016). A method to improve the damping characteristics of polymer concretes was investigated by Ahn *et al*. (2016). Aktas and Ozerdem (2016) developed models to accurately predict the behavior of fresh concrete exposed to vibration using artificial neural networks (ANNs) model and regression model (RM). Kimindiri Kimani and Kaewunruen (2017) highlighted a study undertaken on the free vibration of a precast steel-concrete composite slab panel for track support. Free vibrations of steel-concrete composite beams were analyzed by Li *et al*. (2017) using the dynamic stiffness approach. Nonlinear vibration of embedded nanocomposite concrete was investigated by Shokravi (2017) based on Timoshenko beam model. Vibration and stability of concrete pipes reinforced with carbon nanotubes (CNTs) conveying fluid were presented by Zamani Nouri (2017). An extensive research was undertaken by Cao *et al*. (2018) to study the vibration serviceability of a long-span and light-weight floor subjected to human loading experimentally and numerically. Rezaiee-Pajand *et al*. (2018) devoted to two new techniques for free vibration analysis of concrete arch dam-reservoir systems. In order to study the natural vibration characteristics of steel-concrete composite truss beam (SCCTB), the influence of multiple factors such as interface slip, shear deformation and moment of inertia were considered by Jiang *et al*. (2018). A sensitivity study was conducted by Wodzinowski *et al*. (2018) to examine the effect of various design parameters on the free-vibration response of curved composite concrete-steel I-girder bridges.

None of the above works has analyzed vibration of concrete foundations based on mathematical models. In the present study, vibration response of concrete foundations resting on soli medium is presented. The soil medium is simulated by Winkler model. Utilizing CPT, energy method and Hamilton's principle, the motion equations are derived. Navier method is used for calculating the frequency of the concrete foundation. The effect of different parameters such as soil medium, mode numbers, length to width ratio and length to thickness ratio of the concrete foundation are shown on the frequency of the structure.

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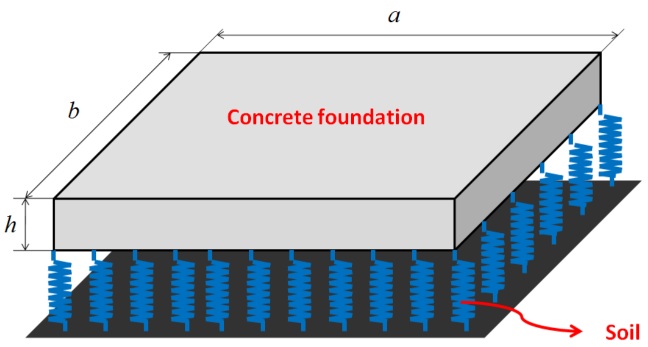

Fig. 1 Concrete foundation resting on soil medium

2. Mathematical modeling

A concrete foundation resting on soil medium is shown in Fig. 1 with length, width and thickness of a, b and h, respectively. The soil medium is simulated by spring elements.

2.1 Strain relations

Based on CPT, the displacements of the structure can be expressed as (Reddy 2002)

$$u(x,y,z,t)=u(x,y,t)-z\frac{\partial w(x,y,t)}{\partial x}, \tag{1}$$

$$v(x,y,z,t)=v(x,y,t)-z\frac{\partial w(x,y,t)}{\partial y}, \tag{2}$$

$$w(x,y,z,t)=w(x,y,t), \tag{3}$$

where u, v and w are the mid-plane displacement components in the x-, y- and z- directions, respectively. Based on Eqs. (1)-(3), the strain-displacement equations can be derived as

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}-z\frac{\partial^2 w}{\partial x^2}, \tag{4}$$

$$\varepsilon_{yy}=\frac{\partial v}{\partial y}-z\frac{\partial^2 w}{\partial y^2}, \tag{5}$$

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2z\frac{\partial^2 w}{\partial x\,\partial y}, \tag{6}$$

2.2 Stress relations

Based on Hook's law, the stress-strain relations can be written as

$$\sigma_{xx}=C_{11}\,\varepsilon_{xx}+C_{12}\,\varepsilon_{yy}, \tag{7}$$

$$\sigma_{yy}=C_{12}\,\varepsilon_{xx}+C_{22}\,\varepsilon_{yy}, \tag{8}$$

$$\tau_{xy}=C_{66}\,\gamma_{xy}, \tag{9}$$

where C_{ij} are elastic constants.

2.3 Potential energy

The potential energy of the concrete foundation based on CPT can be given by

$$U=\frac{1}{2}\int_A\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\sigma_{xx}\varepsilon_{xx}+\sigma_{yy}\varepsilon_{yy}+\tau_{xy}\gamma_{xy}\right)dzdA \tag{10}$$

Substituting Eqs. (4)-(6) into Eq. (10) yields

$$U=\frac{1}{2}\int_A\left\{N_{xx}\frac{\partial u}{\partial x}+N_{yy}\frac{\partial v}{\partial y}+N_{xy}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right.$$
$$\left.-M_{xx}\frac{\partial^2 w}{\partial x^2}-M_{yy}\frac{\partial^2 w}{\partial y^2}-2M_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\right\}\mathrm{dxdy}, \tag{11}$$

where the stress resultant relations can be defined as

$$\begin{bmatrix}N_{xx}\\N_{yy}\\N_{xy}\end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}\sigma_{xx}\\\sigma_{yy}\\\tau_{xy}\end{bmatrix}dz, \tag{12}$$

$$\begin{bmatrix}M_{xx}\\M_{yy}\\M_{xy}\end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}\sigma_{xx}\\\sigma_{yy}\\\tau_{xy}\end{bmatrix}z\,dz, \tag{13}$$

2.4 Kinetic energy

The kinetic energy of the concrete foundation is

$$K=\frac{\rho}{2}\int\left(I_0\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)\right.$$
$$\left.+I_2\left(\left(\frac{\partial^2 w}{\partial x\,\partial t}\right)^2+\left(\frac{\partial^2 w}{\partial y\,\partial t}\right)^2\right)\right)dxdy, \tag{14}$$

where ρ is the density of the structure and

$$\begin{Bmatrix}I_0\\I_2\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}\rho\\\rho z^2\end{bmatrix}dz. \tag{15}$$

2.5 Soil medium work

The external work due to the soil medium can be written as

$$W_s=\iint(-K_s w)w\,dxdy, \tag{16}$$

where K_s is the soil spring constant.

2.6 Hamilton's principal

The Hamilton's principal can be written as follows

$$\int_0^t(\delta U-\delta K-\delta W_s)dt=0. \tag{17}$$

Substituting Eqs. (11), (14) and (16) into Eq. (17) yields the following motion equations

$$\delta u: C_{11}\frac{\partial^2 u}{\partial x^2}+C_{12}\frac{\partial^2 v}{\partial y\,\partial x}+C_{66}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\,\partial y}\right)=\rho\frac{\partial^2 u}{\partial t^2}, \tag{18}$$

$$\delta v: C_{66}\left(\frac{\partial^2 u}{\partial y\,\partial x}+\frac{\partial^2 v}{\partial x^2}\right)+C_{12}\frac{\partial^2 u}{\partial y\,\partial x}+C_{222}\frac{\partial^2 v}{\partial y^2}=\rho\frac{\partial^2 v}{\partial t^2}, \quad (19)$$

$$\delta w: -\frac{C_{11}h^3}{12}\frac{\partial^4 w}{\partial x^4}-\frac{C_{12}h^3}{12}\frac{\partial^4 w}{\partial y^4}-\frac{C_{66}h^3}{3}\frac{\partial^4 w}{\partial x^2\partial y^2} - K_s w=\rho h\frac{\partial^2 w}{\partial t^2}, \quad (20)$$

3. Analytical approach

Considering simply supported boundary condition for the concrete foundation, the displacements can be assumed as

$$u(x,y,t)=u_0\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{i\omega t}, \quad (21)$$

$$v(x,y,t)=v_0\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{i\omega t}, \quad (22)$$

$$w(x,y,t)=w_0\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{i\omega t}, \quad (23)$$

where m and n, denotes the axial and lateral mode numbers, respectively and ω is the frequency of the structure. Substituting Eqs. (21)-(23) into Eqs. (18)-(20) yields

$$[K][d]=[0], \quad (24)$$

where $[d]=[u_0,v_0,w_0]$. Setting the determinate of Eqs. (24) to zero, the frequency of the structure can be derived.

4. Numerical results and discussion

A concrete foundation with Yong's modulus of E_m=20 GPa and Poisson's ratio of v_m=0.2 is considered which has length to width ratio of a/b=2 and length to thickness ratio of a/b=15. The spring constants of soil medium are reported in Table 1 (Bowles 1988). It should be noted that the spring constant of soil medium is calculated by the following relations (Bowles 1988)

$$k_s=\frac{2E_s}{\left(1-v_s^2\right)+2H/B} \quad (25)$$

where E_s and v_s are soil Young's modulus and soil Poisson's ratio, respectively; B and H are width and height of the soil, respectively.

At the first, the results are validated with Whitney (1987) neglecting soil medium (K_s=0). The non-dimensional frequency of a plate with material properties the same as Whitney (1987) is shown in Table 2 for first four vibrational modes. As it is observed in Table 2 the results of present work are math with the work of Whitney (1987).

Table 1 Spring constants of soil mediums

Soil	(N/m^2) K_s
Loose sand	4800
Dense sand	64000
Clayey medium dense sand	32000
Clayey soil	12000

Table 2 Validation of this work

Method	Mode number			
	1	2	3	4
Whitney (1987)	15.171	33.248	44.387	66.820
Present	15.182	33.233	44.376	66.812

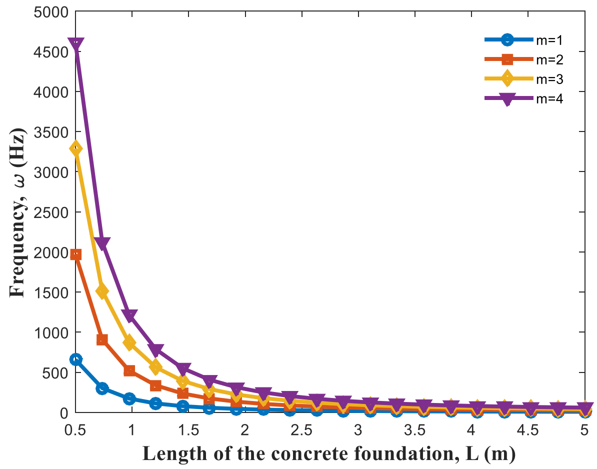

Fig. 2 The effect of the axial mode number on the frequency of the structure

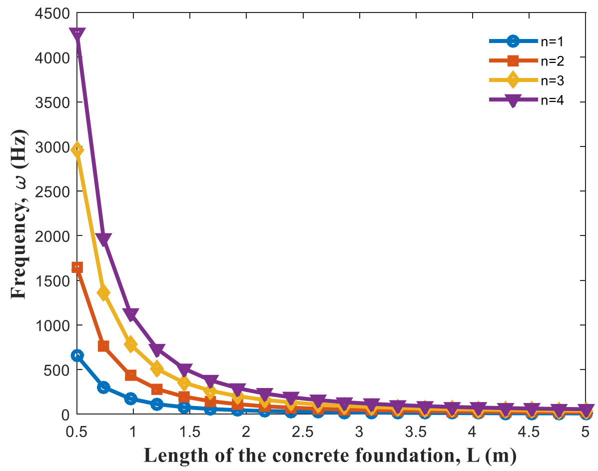

Fig. 3 The effect of the lateral mode number on the frequency of the structure

Figs. 2 and 3 illustrate the effect of the axial and lateral mode numbers on the frequency of the structure versus concrete foundation length, respectively. It can be seen that with increasing the axial and lateral mode numbers, the frequency increases. In addition, with increasing the length of the concrete foundation, the frequency is decreased since with increasing the length of the concrete foundation, the stiffness of the structure decreases.

Fig. 4 presents the effect of different soil mediums on the frequency of the concrete foundation against the concrete foundation length. Five cases of without soil medium, with loose sand, clayey soil, clayey medium dense sand and dense sand are considered. It is shown that the

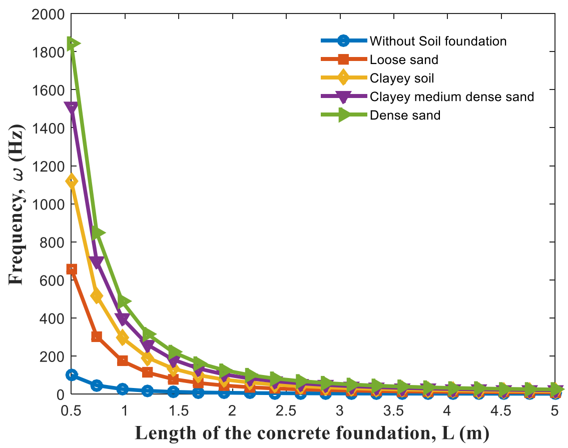

Fig. 4 The effect of the different soil mediums on the frequency of the structure

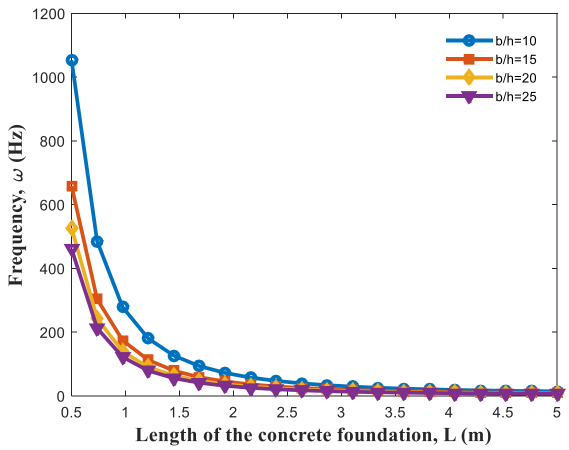

Fig. 6 The effect of the concrete foundation width to thickness ration on the frequency of the structure

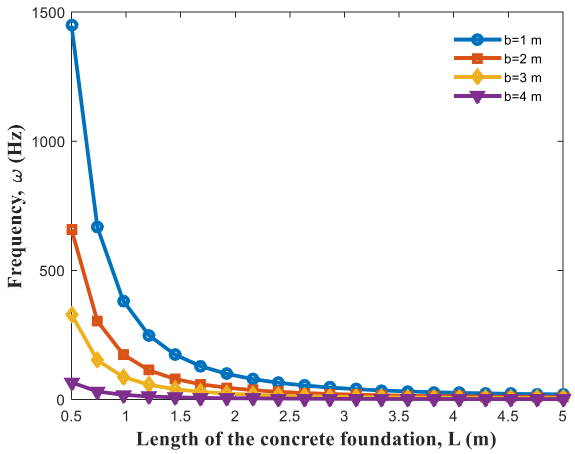

Fig. 5 The effect of the concrete foundation width on the frequency of the structure

concrete foundation with soil medium has the lowest frequency with respect to the structure resting on soil medium. In addition, the frequency of the harder soil is higher than softer ones. For example, the frequency of the concrete foundation resting on dense sand is higher than the frequency of the concrete foundation resting on loose sand.

The frequency of the concrete foundation versus length is demonstrated in Figs. 5 and 6 for different width and width to thickness ratio of the structure. As can be seen, increasing the width and width to thickness ratio of the structure leads to lower frequency. It is due to the fact that increasing the width and width to thickness ratio of the structure reduces the stiffness of the structure.

5. Conclusions

In this paper, vibration analysis of the concrete foundations resting on soil medium was presented mathematically. The soil medium was simulated by Winker model. Using CPT, energy method and Hamilton's principle, the motion equations were derived. The Navier method was used for obtaining the frequency of the structure. The effects of different parameters such as mode numbers, soil medium and geometrical parameters of the concrete foundation were shown in the frequency of the structure. The results show that with increasing the axial and lateral mode numbers, the frequency increases. It was shown that the concrete foundation with soil medium has the lowest frequency with respect to the structure resting on soil medium. In addition, the frequency of the harder soil was higher than softer ones. Furthermore, increasing the width and width to thickness ratio of the structure leads to lower frequency.

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