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GMDH-based prediction of shear strength of FRP-RC beams with and without stirrups

Ali Kaveh[1a], Taha Bakhshpoori[*2] and Seyed Mahmood Hamze-Ziabari[1b]

[1]*Centre of Excellence for Fundamental Studies in Structural Engineering, Iran University of Science and Technology, Tehran-16, Iran*
[2]*Faculty of Technology and Engineering, Department of Civil Engineering, East of Guilan, University of Guilan, Rudsar-Vajargah, Iran*

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Abstract. In the present study, group method of data handling networks (GMDH) are adopted and evaluated for shear strength prediction of both FRP-reinforced concrete members with and without stirrups. Input parameters considered for the GMDH are altogether 12 influential geometrical and mechanical parameters. Two available and very recently collected comprehensive datasets containing 112 and 175 data samples are used to develop new models for two cases with and without shear reinforcement, respectively. The proposed GMDH models are compared with several codes of practice. An artificial neural network (ANN) model and an ANFIS based model are also developed using the same databases to further assessment of GMDH. The accuracy of the developed models is evaluated by statistical error parameters. The results show that the GMDH outperforms other models and successfully can be used as a practical and effective tool for shear strength prediction of members without stirrups (R^2=0.94) and with stirrups (R^2=0.95). Furthermore, the relative importance and influence of input parameters in the prediction of shear capacity of reinforced concrete members are evaluated through parametric and sensitivity analyses.

Keywords: shear strength prediction; FRP-RC beams; stirrup; GMDH; ANN; ANFIS

1. Introduction

Internal reinforcement by fiber reinforcement plastic or polymer (FRP) as an advanced polymer composite material has gained increasing popularity in the reinforced concrete building and civil/structural infrastructures during the last two decades. This popularity owes to the desired specific advantages such as light weight, high tensile strength, non-corrosive, and nonmagnetic properties despite disadvantages such as higher initial cost compared with steel reinforcement and as a result a number of types of FRP bars are now commercially available (GangaRao *et al.* 2006). FRP bars are a competitive option for internal reinforcement in beams more than other types of members and known as FRP-reinforced concrete (FRP-RC) beams. Because of the mechanical differences between FRP and steel bars, FRP-RC beams show different performances under flexure and shear failure. Over the last two decades several researchers have conducted experimental and theoretical studies toward accurate investigation of shear strength and more specifically flexural strength of FRP-RC beams (Smith and Teng 2002). In parallel many design codes developed equations for safe design of FRP-RC beams such as: ACI 440.1R-06 (2007), CSA S6-06 (2006), CSA S806-12 (2012), the Japan Society of Civil Engineering (JSCE) standard (1997), CNR DT-203/2006 (2007), Hoult *et al.* (2008).

Shear failure mode is as much as flexural failure importance because of its rapid progression (El-Chabib *et al.* 2005). Although shear resistance mechanisms of FRP-RC beams with and without stirrups are mainly similar to the steel reinforcement case, the material differences between FRP and steel result in different shear resistance performances (Jnaid and Aboutaha 2013). Available design codes are essentially developed by modifications on the shear design equations of steel reinforced beams with the aim of minimizing these differences. However, there are differences between these equations which result in variable performances of safety. Many studies have been conducted recently to better evaluate the current design codes based on the databases obtained from available experimental literature. For example, Razaqpur and Spadea (2014), Oller *et al.* (2015) recently studied comprehensively the shear design problem of FRP-RC beams with and without stirrups, among many others (Hoque 2006, Sas *et al.* 2009, Bulut *et al.* 2011, Machial *et al.* 2012, Zhang *et al.* 2014, Chowdhury and Islam 2015, Liang *et al.* 2017). Based on the literature it is generally accepted that current shear design equations of FRP-RC beams are conservative and in some cases un-conservative. It should be noted that as much as accurate performance prediction of members is desired by engineers and on the other hand this can be more curious in the case of FRP bars because of its high cost than steel.

Soft computing based techniques have emerged as more flexible, less assumption dependent and potentially self-adaptive approaches to generate predictive models for problems which by their nature are inherently complex,

∗Corresponding author, Professor
E-mail: tbakhshpoori@guilan.ac.ir
[a]Professor
E-mail: alikaveh@iust.ac.ir
[b]Ph.D. Student
E-mail: ziabari@civileng.iust.ac.ir

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nonlinear and dynamic (See and Openshaw 1999). The shear strength capacity of FRP-RC members with and without stirrups is supplied by several mechanisms. Therefore, it can be an inherently complex, nonlinear and dynamic function of many influential mechanical and geometrical parameters. In this regards the use of soft computing based approaches in the shear strength capacity assessment of FRP-RC members is attracted attention in the recent years but not as much as other prediction problems of RC members (Kaveh *et al.* 2016, Kaveh *et al.* 2017). Nehdi *et al.* (2007) applied genetic algorithm for this problem with 8 influential parameters for two cases of with and without stirrups using 100 and 68 data samples, respectively. Although a simple equation is developed but considering a semi-empirical equation was essential in their study. Kara (2011) addressed this problem for the case of without stirrups based on the gene expression programming (GEP) approach using 104 experimental data with 7 influential parameters. Mohammadi Golafshani *et al*. (2012) developed successfully artificial neural network and fuzzy logic based models using 179 different splice beam tests with six input variables. An explicit formulation is also available based on the developed model by GEP approach. Lee and Lee (2014) used artificial neural network (ANN) for this problem without stirrups using 110 test data with 6 influential parameters. ANN is also used by Bashir and Ashour (2012) for this problem. Nasrollahzadeh and Basiri (2014) solve this problem based on fuzzy inference system (FIS) using 128 and 69 data samples with 8 influential parameters for two cases of without and with stirrups, respectively, and present FIS based formulations for each case. Mohammadi Golafshani *et al.* (2015) used artificial neural network and genetic programming to develop predictive models for the bond strength of the GFRP bars in concrete, based on 159 experimental database with seven input variables. Shahnewaz *et al.* (2016) applied genetic algorithm in this problem with and without stirrups using 116 and 46 data samples, respectively, based on the different shear design equations proposed in FRP design guidelines. Very recently Mohammadi Golafshani and Ashour (2016) developed a new approach based on the biogeography-based programming (BBP) as an extension of Biogeography-Based Optimization (BBO) to address this problem for the case of without stirrups using 138 experimental specimens with 6 influential parameters and presented an optimum equation. used This study investigates efficiency of the GMDH network with the aim of improving the prediction capability of soft computing based approaches in this problem. GMDH is a relatively unexplored network. In GMDH the most important input variables, number of layers, neurons in hidden layers and optimal model structure are determined automatically. The network is thus composed of active neurons that organize themselves. The GMDH network learns in an inductive way and tries to build a function (called a polynomial model) which would result in the minimum error between the predicted value and expected output (Srinivasan 2008).

Recently gathered datasets: a set of 175 data samples by Razaqpur and Spadea (2014) and a set of 112 data samples by Oller *et al*. (2015) are used here which are more comprehensive than the data sets utilized in the previous studies as stated in the previous paragraph. The relative importance of significant parameters dealing with shear strength of FRP-reinforced members is also investigated through sensitivity analysis. The performance of proposed models evaluated by comparing against several codes of practice. Further evaluation also done based on comparison with an artificial neural network (ANN) model and an ANFIS based model developed in this study using the same databases. The accuracy of the developed models is evaluated by statistical error parameters. The results show that the GMDH outperforms other models and successfully can be used as a practical and effective tool for shear strength prediction of members without stirrups (R^2=0.94) and with stirrups (R^2=0.95). Furthermore, the relative importance and influence of input parameters in the prediction of shear capacity of reinforced concrete members is evaluated through parametric and sensitivity analyses.

The remaining sections of this paper are organized as follows. In Section 2, GMDH algorithm is described in brief. Third section states to the data description, involved input parameters and modeling process for predicting shear capacity of FRP-RC members with and without stirrups. The GMDH model performance is evaluated against empirical approaches and other soft computing based approaches (ANFIS and ANN) in Section 4 and related results and discussions are made. Sensitivity and parametric analyses are also conducted in this section to evaluate the robustness of GMDH model in capturing the underlying physical behaviors of FRP-RC members. At the end, the paper is concluded in Section 5.

2. Methodology

Recently, Group Method of Data Handling (GMDH) has been used in a great variety of areas such as data mining and knowledge discovery, forecasting and systems modeling, optimization and pattern recognition (Amanifard *et al.* 2008, Madandoust *et al.* 2012, Najafzadeh *et al.* 2015). It is originally designed and proposed by Ivakhnenko (1970) in the 1967s. The main advantage of GMDH method in comparison with ANN method as one of the most common data mining approaches is that the dependencies between input parameters and responses are represented in parametric form as an equation while these dependencies are hidden within neural network structures in ANN method. Besides that, ANN methods need an essential time of learning and therefore it is difficult to be applied for modeling and forecasting in real time system. The description of the GMDH network is outlined at the following.

2.1 Principle of the GMDH network

GMDH is a machine learning approach based on the polynomial theory of complex systems (Ivakhnenko and Ivakhnenko 2000). From this network, the most significant input parameters, number of layers, number of neurons used in middle layers, and optimal topology design of the network are defined automatically. Therefore, the GMDH network is included those of active neurons known as a selforganized model. The structure of GMDH network is configured thorough the training stage with polynomial model which produces the minimum error between the predicted value and observed output. The formal definition of the system identification problem is to find an approximate function $\hat{f}$ that can be used to predict the actual output y or a given input vector $X=(x_1, x_2, x_3, x_n)$ as close as possible to the actual output y. Therefore, n observations of multi-input-single-output data pairs are considered as

$$y_i = f\left(x_{i1}, x_{i2}, x_{i3}, \dots \mathrm{x}_{in}\right) \quad (i=1,2,\dots,M) \tag{1}$$

The general relationship between input and output variables can be expressed by a complicated discrete form of the Volterra function, a series in the form of

$$\begin{aligned} y = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij} x_i x_j + \\ \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} w_{ijk} x_i x_j x_k + \dots \end{aligned} \tag{2}$$

which is known as the Kolmogorov-Gabor polynomial (Najafzadeh *et al.* 2015). In the present study, a quadratic polynomial of the GMDH network is used that is written as

$$\begin{aligned} Quadratic: \ \hat{y} = w_0 + w_1 x_i + w_2 x_j + \\ w_3 x_i x_j + w_4 x_i^2 + w_5 x_j^2 \end{aligned} \tag{3}$$

The weighting coefficients of Eq. (3) are calculated using regression techniques such that the difference between actual output (y) and the calculated value ($\hat{y}$) for each pair of x_i and x_j as input variables is minimized. In this way, the weighting coefficients of the quadratic function $\hat{y}$ are obtained to optimally fit the output to the whole set of input-output data pairs, which is defined as

$$E = \frac{\sum_{i=1}^{M}\left[y_i - \hat{y}_i\right]^2}{M} \rightarrow \min \tag{4}$$

In this study, the GMDH network is improved using a back propagation algorithm. This method includes two main steps: (1) the weighting coefficients of the quadratic polynomial are determined using the least squares method from the input layer to output layer in the form of a forward path; and (2) the weighting coefficients were updated using a back-propagation algorithm in a backward path. This procedure may be continued until the error of the training network (E) is minimized.

3. Data set and modelling

To develop the GMDH model, shear strength capacity of FRP-RC members with and without stirrups was considered to be a function of the following parameters

$$V = f\left(\begin{aligned} f_c,\ \frac{a}{d},\ b,\ d,\ A_F,\ f_{Fu},\ E_F,\ \rho_F,\ A_{Fw}, \\ \rho_{Fw},\ E_{Fw},\ f_{Fuw} \end{aligned}\right) \tag{5}$$

where V is the ultimate shear capacity, including self-weight (kN); f_c is the average concrete cylinder compression strength (Mpa); a/d is the ratio of shear span to the effective depth; b is the cross section width (mm); d is the effective depth of the cross section (mm); A_F and A_{Fw} are the area of longitudinal and transversal reinforcement, respectively (mm^2); f_{Fu} and f_{Fuw} are the ultimate tensile strength of longitudinal and transversal reinforcement (Mpa), respectively; E_F and E_{Fw} are the Young's modulus of longitudinal and transversal reinforcement (Mpa), respectively; ρ_F and ρ_{Fw} are the longitudinal and transversal reinforcement ratio (%), respectively.

3.1 Data collection

Compiling a comprehensive database is requisite in generating new predictive models. Razaqpur and Spadea (2014), Oller *et al.* (2015) recently studied comprehensively the shear design problem of FRP-RC beams for the cases: with and without stirrups; and with stirrups, respectively. In the first study a set of 175 data samples is collected for the case of without stirrups and in the latter a set of 112 data samples is gathered. In order to generate GMDH models these data samples are benefited here which are more comprehensive than used data sets in previous studies as reviewed in the introduction section. The mentioned databases include different amounts and types of FRP materials, different sizes of members, a wide range of concrete compressive strength and also *a/d* values.

Table 1 Range of the employed parameters for model development

Parameter		Min	max	Mean	Standard deviation
b (mm)	With stirrups	135	450	222.46	73.12
	Without stirrups	89	1000	304.62	208.53
d (mm)	With stirrups	170	937	293.17	127.47
	Without stirrups	73	3000	351.22	526.38
a/d	With stirrups	1.19	7.53	2.56	0.97
	Without stirrups	1.12	16.22	4.12	2.16
f_c (Mpa)	With stirrups	20	50	33.30	7.15
	Without stirrups	20	93	43.44	17.74
A_F (mm^2)	With stirrups	157	8608	1347.10	1383.60
	Without stirrups	60	8608	950.73	1246.80
ρ_F (%)	With stirrups	0.51	3.98	1.79	0.72
	Without stirrups	0.22	3.98	1.15	0.72
E_F (Mpa)	With stirrups	29	137	65.18	24.69
	Without stirrups	29	148	70.97	41.30
f_{Fu} (Mpa)	With stirrups	397	2200	1119.7	345.53
	Without stirrups	397	2640	1084.40	560.16
V (KN)	With stirrups	20.5	599.30	184.25	110.06
	Without stirrups	9.8	291.5	67.78	55.62
E_{Fw} (Mpa)	With stirrups	30	144	73.27	33.97
ρ_{Fw}(%)	With stirrups	0.04	1.50	0.52	0.45
f_{Fuw} (Mpa)	With stirrups	322	2040	1056.1	368.91
b (mm)	With stirrups	135	450	222.46	73.12
d (mm)	Without stirrups	89	1000	304.62	208.53
	With stirrups	170	937	293.17	127.47

The statistical parameters of the geometrical and mechanical properties of the specimens are presented in Table 1. From this table, it can be seen that the range of input parameters, for example, concrete compressive strength and *a/d* values is remarkably wide. Therefore, the derived model can be reliably used for different cases in these ranges.

3.2 Data collection

To develop new models for predicting shear strength capacity of both FRP-reinforced members with and without stirrups using GMDH algorithm, the available datasets are randomly divided into training and testing subsets. The training data are taken for the learning of the algorithm. Furthermore, in the training process, 10-fold cross validation technique is employed to avoid the overfitting problem. The testing datasets are used to specify the generalization capability of the models to a set of new data they did not train with. In the other words, the testing data are employed to measure the performance of the models obtained by GMDH algorithm when applied to dataset which played no role in building the models. Out of the 112 data for FRP-reinforced members with stirrups, 90 data vectors (80%) are taken for the training process. The remaining 22 data (20%) were used for the testing of the models. For FRP-reinforced members without stirrups, 140 data vectors are used to train the algorithm and the remaining 35 data are used to validate the developed model.

Following data division, they are presented to the GMDH for model training. The GMDH returns the following selective polynomials for prediction of shear strength of members with stirrups as follows:

Layer 1:

$$\begin{aligned} L_1 = &-48.47 + 28.34\rho_t - 22.95\rho + 0.3775d + 79.35\rho_t\rho + \\ &0.3114d\rho_t + 0.246\rho d - 69.95\rho_t^2 - 17.44\rho^2 - 0.0002389d^2 \end{aligned} \tag{6a}$$

Layer 2:

$$\begin{aligned} L_2 = &177.4 - 0.2786L_1 - 148.6(a/d) + 1.218b \\ &+ 0.04975(a/d)L_1 + 0.00254bL_1 + 0.1112b(a/d) \\ &+ 0.0001706L_1^2 + 16.12(a/d)^2 - 0.003011b^2 \end{aligned} \tag{6b}$$

Layer 3:

$$\begin{aligned} L_3 = &65.87 + 0.3055L_2 - 0.5387E_t - 1.956f_c + \\ &0.002405E_t + 0.01221L_2 f_c + 0.009724f_c E_t + \\ &0.0003764L_2^2 + 0.0005084E_t^2 + 0.01084f_c^2 \end{aligned} \tag{6c}$$

Layer 4:

$$\begin{aligned} V(KN) = &-35.77 + 1.326L_3 - 113.8\rho_t + 0.3711E_r \\ &+ 0.06931\rho_t L_3 - 0.002938E_r L_3 + 1.989E_r\rho_t \\ &- 0.0004145L_3^2 - 10.83\rho_t^2 - 0.001098E_r^2 \end{aligned} \tag{6d}$$

The developed GMDH models for prediction of the shear strength of members without stirrups are as follows:

Layer 1:

$$\begin{aligned} L_1 = &52.39 + 0.05163A_F - 19.62(a/d) + 0.1712b \\ &- 0.002753(a/d)A_F - 1.619\times10^{-6}bA_F - 0.02226b(a/d) \\ &- 2.352\times10^{-6}A_F^2 + 1.613(a/d)^2 + 7.584\times10^{-5}b^2 \end{aligned} \tag{7a}$$

Layer 2:

$$\begin{aligned} L_2 = &14.51 - 0.06667L_1 + 0.01931A_F + 0.02435b \\ &- 0.001208A_F L_1 - 0.002139bL_1 + 7.231\times10^{-5}bA_F \\ &+ 0.02188L_1^2 + 1.673\times10^{-5}A_F^2 + 5.242\times10^{-5}b^2 \end{aligned} \tag{7b}$$

Layer 3:

$$\begin{aligned} L_3 = &39.54 + 0.1569L_2 - 0.08429f_{FU} + 0.8584E_F \\ &+ 0.0003209f_{FU}L_2 + 0.000393E_F L_2 - 0.0003528E_F f_{FU} \\ &+ 0.002545L_2^2 + 2.528\times10^{-5}f_{FU}^2 + 0.0006065E_F^2 \end{aligned} \tag{7c}$$

Layer 4:

$$\begin{aligned} L_4 = &8.036 + 0.4288L_3 + 0.01035A_F + 0.02742b \\ &- 0.0004398A_F L_3 - 0.0008504bL_3 + 3.693\times10^{-5}bA_F \\ &+ 0.007928L_3^2 + 5.776\times10^{-6}A_F^2 + 3.757\times10^{-6}b^2 \end{aligned} \tag{7d}$$

Layer 5:

$$\begin{aligned} V(KN) = &-18 + 0.9005L_4 + 0.008982L + 0.02835d \\ &- 1.856\times10^{-5}LL_4 - 0.0003146dL_4 + 8.801\times10^{-6}dL \\ &+ 0.0008526L_4^2 - 7.976\times10^{-7}L^2 - 1.322\times10^{-5}d^2 \end{aligned} \tag{7e}$$

4. Results and discussion

Correlation coefficient (R), root mean square error ($RMSE$), coefficient of determination (R^2), and $BIAS$ as the prevalent prediction error indicators are used here. Their governing formulae are as follows (Kaveh *et al*. 2017)

$$R = \frac{\sum_{i=1}^{N}(P_i - P_m)(O_i - O_m)}{\sqrt{(P_i - P_m)^2}\sqrt{(O_i - O_m)^2}} \tag{8}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(P_i - O_i)^2} \tag{9}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(O_i - P_i)^2}{\sum_{i=1}^{N}(O_i - O_m)^2} \tag{10}$$

$$BIAS = \frac{\sum_{i=1}^{N}(P_i - O_i)}{N} \tag{11}$$

where O_i is the measured value, P_i stands for prediction values; N is the number of data points, O_m is the mean value for the observation and P_m is the mean value of prediction. In fact, the R parameter was chosen to show correlation between predicted and measured values. If the R value is more than 0.8, it shows that there is strong correlation between measured and predicted values (Smith 1986). However, R sometimes may not necessarily indicate better model performance due to the tendency of the model to deviate toward higher or lower values, particularly when the data range is very wide and most of the data are distributed about their mean. Consequently, the coefficient of determination, R^2, was used because it can give unbiased

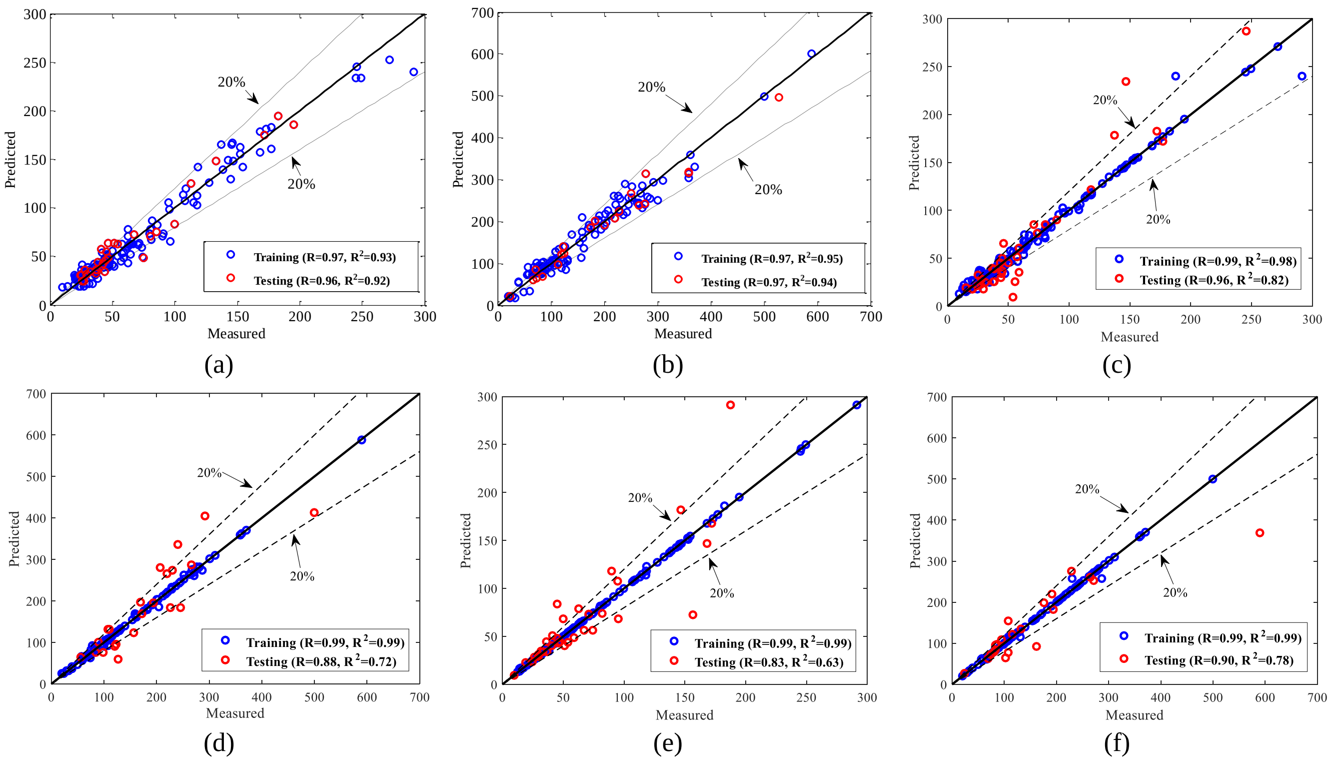


Fig. 1 Comparison between measured and predicted shear capacity *V* (KN) for training and testing dataset: (a) GMDH with stirrups; (b) GMDH without stirrups; (c) ANN with stirrups; (d) ANN without stirrups; (e) ANFIS with stirrups; and (f) ANFIS without stirrups

estimate and may be a better measure for model performance. In addition, *BIAS* and *RMSE* parameters should be at the minimum for having precise results (Kaveh *et al.* 2016).

To further confirmation of the predictive performances of developed GMDH model, two well-known soft computing approaches including ANN and ANFIS are also developed in this study. To achieve this, two multi-layer perceptron (MLP) neural networks based on Levenberg-Marquardt algorithm are developed to obtain the optimum structures for predicting shear strength of members with stirrups and without stirrups. Different number of hidden layers and different neurons in each hidden layer are applied to reach the best performance. The models with the best performances for both training and testing datasets are chosen as the predictive models. All simulations are done in the MATLAB programming language version R2012a. In this regard, the optimum topological structure obtained by MLP for members without stirrups is 10 input variables in an input layer, 5 neurons in hidden layer 1, and 3 neurons in hidden layer 2. The output layer is composed of *V* values predicted by ANN algorithm. For members with stirrups, 9 input variables in input layer, 6 neurons in hidden layer 1, and 7 neurons in hidden layer 2 are obtained for the best model.

To develop a predictive model for shear strength of FRP-RC beams with and without stirrups based on the ANFIS algorithm, MATLAB programming language version R2012a is also employed. The Genfis2 function based on the subtractive clustering method is used to generate the FIS structure. The strategy for finding the best structure of

Table 2 Performance of GMDH model in predicting shear strength for training and testing datasets

Model	Type	subsets	*BIAS*	*RMSE*	*R*	R^2
GMDH	With stirrups	Training	1.28×10^{-4}	21.23	0.9770	0.9546
		Testing	-0.0236	24.43	0.9705	0.9405
	Without stirrups	Training	-1.72×10^{-4}	12.62	0.9738	0.9300
		Testing	0.0126	13.05	0.9645	0.9293
ANN	With stirrups	Training	0.0144	4.7263	0.9989	0.9978
		Testing	4.9088	50.0794	0.8815	0.7205
	Without stirrups	Training	0.0385	7.4676	0.9912	0.9825
		Testing	1.3888	21.6965	0.9609	0.8228
ANFIS	With stirrups	Training	1.32×10^{-4}	4.7936	0.9987	0.9975
		Testing	-8.7570	53.5282	0.9052	0.7849
	Without stirrups	Training	-0.0049	0.8144	0.9989	0.9971
		Testing	-2.4513	37.5175	0.8307	0.6328

ANFIS is similar to the previous methods. At the first step, the member functions of the inputs are generated using subtractive clustering. Then a recursive least square method is used to tune the member function parameters. The number of member functions is gradually decreased by reducing the range of influence of cluster centers in a trial and error manner.

The performance of GMDH model besides ANN and ANFIS based models for estimating shear strength capacity of FRP-reinforced members with stirrups and without stirrups in the training and testing sets are illustrated in Fig. 1, which present the scatter between measured and predicted shear strength around the optimal line of equality. As shown, there is a little scatter around the optimal line

Table 3 Performance of various methods for prediction of shear capacity

Model		BIAS	RMSE	R	R^2
Deitz *et al*. (1999)	Without stirrups	9.6053	53.4496	0.6655	0.0714
Tureyen and Frosch (2003)	Without stirrups	21.2499	48.5423	0.8923	0.2341
Nehdi *et al*. (2007)	With stirrups	-14.5129	49.9430	0.8858	0.7489
Hoult *et al*. (2008)	With stirrups	-104.7465	125.3379	0.8757	-0.3120
	Without stirrups	-16.5428	30.5144	0.8964	0.6973
CSA S6 (2006)	With stirrups	-114.6302	135.2815	0.9053	-0.5285
	Without stirrups	-24.4146	37.6559	0.8759	0.5390
ACI (2007)	With stirrups	-34.7581	84.1493	0.7574	0.4086
	Without stirrups	-30.5228	42.9928	0.8640	0.3992
JSCE (1997)	With stirrups	-115.3977	138.0292	0.8682	-0.5912
	Without stirrups	-23.2551	36.0670	0.8837	0.5772
CNR (2007)	With stirrups	59.3698	97.3723	0.8660	0.2081
	Without stirrups	9.5247	38.1315	0.8209	0.5274
CSA S806 (2012)	With stirrups	-19.6729	41.7546	0.9414	0.8544
	Without stirrups	-4.0633	32.31	0.8198	0.6606
GMDH model (present study)	With stirrups	1.25×10-10	21.23	0.9770	0.9546
	Without stirrups	-9.14×10-10	12.62	0.9738	0.9482

between predicted and measured values of shear strength in both training and testing sets. The similar performances of the GMDH models on the training and testing data indicate that they have both good predictive ability and generalization performances.

The GMDH model performances are further confirmed analytically in Table 2, which contains four different performance measures including: *R*, R^2, *RMSE*, and *BIAS*. It is demonstrated that the models perform well in both training and testing sets and the performance of GMDH model for training set is consistent with testing sets.

To further evaluation of the GMDH models the results obtained for prediction of shear strength are compared with the traditional design equations and with other soft computing based techniques in the following first two subsections, respectively. Parametric analysis and sensitivity analysis are also conducted in the next two subsections to assess the influences of important parameters on shear strength and determining the most important predictive parameters.

4.1 Performance comparison with design equations

A comparative study was conducted to evaluate the performance of GMDH models against several codes of practice including ACI 440.1R-06 (440 2007), CSA S6-06 (2006), CSA S806-12 (2012), the Japan Society of Civil Engineering (JSCE) standard (1997), CNR DT-203/2006 (2007) and models by Deitz *et al.* (1999), Tureyen and Frosch (2003), Nehdi *et al.* (2007), Hoult *et al.* (2008). In order to measure the capability of the models, the mentioned four statistical error indicators are used. The prediction performance of different models for entire database is summarized in Table 3. Furthermore, Fig. 2 visualizes the histogram plots of the ratio of the experimental to predicted shear strength values for the entire database for models predict the both cases with and without stirrups. For FRP-RC members with stirrup, predictions of ACI, CSA S806, CSA S6, JSCE, and Hoult *et al*. (Hoult *et al.* 2008) underestimate the shear capacity while predictions of CNR model overestimate the shear capacity. However, the performance of CSA S806 is more accurate than others in prediction of shear capacity of members with stirrups according to Fig. 2 and Table 3. According to Table 3, the developed GMDH for FRP-RC members with stirrups improved the *RMSE* and R^2 values by 46% and 11.7%, respectively, in respect to CSA S806 models, which is the most accurate model among other design equations.

For FRP-RC members without stirrup, all models underestimate the shear strength except CNR model. The results of Hoult *et al*. (Hoult *et al*. 2008) and CSA S806 models are more accurate than other design models. However, the developed GMDH model performances are remarkably more accurate than other models.

According to the Table 3, the GMDH improved the *RMSE* and R^2 values by 58% and 35%, respectively, in respect to the best models among the design codes (Hoult *et al.* 2008). As shown in the Fig. 2, the ratio between predicted shear by GMDH model and measured shear are mostly concentrated on value 1. It can be interpreted that the developed GMDH model has less uncertainty and higher level of accuracy than the mentioned design methods.

4.2 Parametric analysis

Parametric analysis of the developed models is carried out with the aim to deeper understand of the shear strength of reinforced concrete with and without stirrups. The parametric analysis investigates the response of the predicted shear strength based on the GMDH model to a set of hypothetical input data generated over the ranges of the minimum and maximum data used for the model training. The methodology is based on the change of only one input variable at a time while the other variables are kept constant

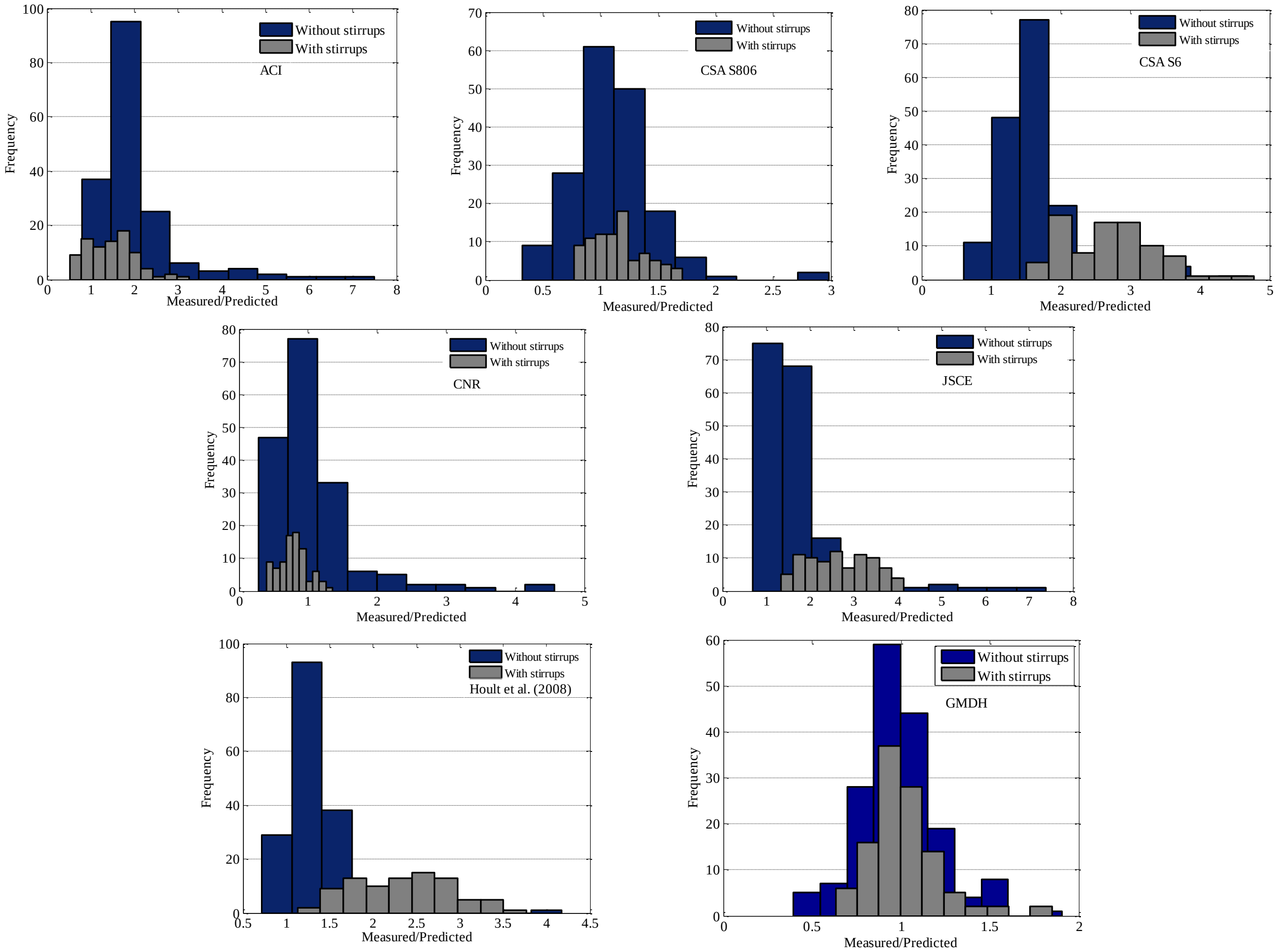
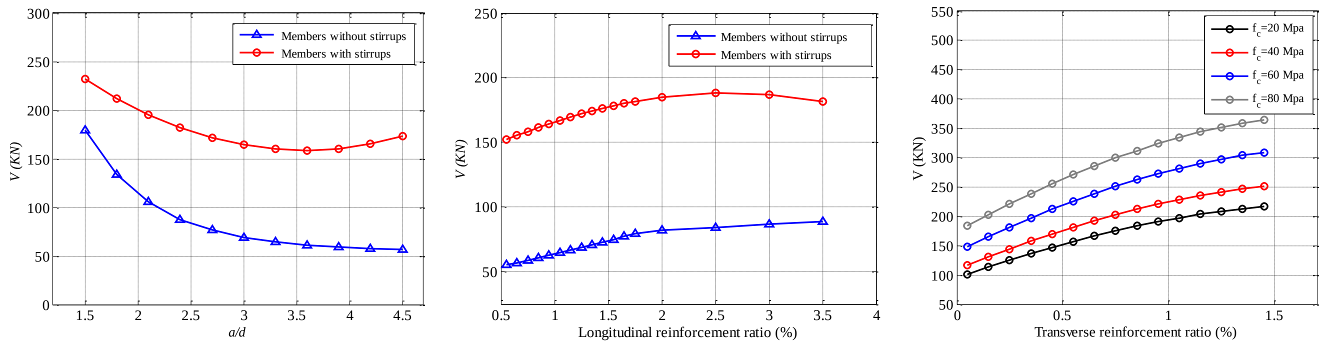


Fig. 2 Histogram of the measured/predicted shear strength values using different models



(a) (b) (c)

Fig. 3 Shear strength parametric analysis in the GMDH-based model for different ranges of: (a) shear span to depth ratio; (b) longitudinal reinforcement ratio; and (c) transversal reinforcement ratio

at the average values of their entire data sets. A set of synthetic data for the single varied parameter is generated by increasing the value in increments. These inputs are presented to the prediction equation and the shear strength is calculated. This procedure is repeated using another variable until the model response is tested for all input variables. The robustness of the GMDH models is determined by examining how well the predicted shear strength values agree with the underlying physical behavior of the shear capacity of RC members. Fig. 3 illustrates the tendency of the shear strength prediction of reinforced members with stirrups and without stirrups to the variations of the main design parameters including reinforcement ratio, shear span to depth ratio and concrete compressive strength.

From Fig. 3(a), it can be seen that shear strength decreases due to increasing shear span to depth ratio for both reinforced members with stirrups and without stirrups. It is also well-known that the failure mode is changed from shear comparison to diagonal tension for a/d=2.5.

According to Fig. 3(a), this mechanism has been captured by the GMDH model for both members with stirrups and without stirrups. In general, the shear strength of members with stirrups are more than the members without stirrups and the discrepancy between the amounts of shear strength for both members increases due to increasing the shear span to depth ratio. These results from parametric analysis are also in good agreement with structural engineering senses.

In Fig. 3(b), the variation of shear strength capacity with longitudinal reinforcement ratio (ρ_F) for both FRP-reinforced members with stirrups and without stirrups is shown. It can be observed that shear strength capacity increases due to increasing the longitudinal reinforcement. In general, shear strength in members without stirrups withstands with the shear of concrete and shear strength of longitudinal FRP. It can be expected that increasing of longitudinal reinforcement ratio leads to increase of shear strength capacity. The GMDH model correctly captured this effect of longitudinal reinforcement ratio on shear strength. For FRP-reinforced members with stirrups, the shear strength withstands with three components: shear of concrete, axial strength of stirrup and shear strength of longitudinal FRP. According to Fig. 3(b), the shear strength capacity of these members increases due to increasing longitudinal reinforcement ratio which is consistent with previous studies.

To consider the effect of transverse reinforcement ratio on shear strength in GMDH model, the tendency of the shear strength predictions to variations of transverse reinforcement ratio (ρ_{FW}) is illustrated in Fig. 3(c). It can be seen that shear strength is notably more sensitive to change of ρ_{FW} than changes of *a/d* and ρ_F. It has also been concluded in the literature that the shear strength in these members is mainly controlled by axial strength of stirrups (Ramirez *et al*. 1998, Campana *et al*. 2013). In Fig. 3(c), the effect of compressive strength (f_c) on shear strength is also depicted. As shown, the shear strength of FRP-reinforced members increases due to increasing f_c. These extracted results of GMDH model are also consistent with previous experiments and studies in the literature (Ramirez *et al*. 1998, Campana *et al*. 2013).

4.3 Sensitivity analysis

To determine the importance of each input variable on the shear strength of reinforced members with and without stirrups, the sensitivity analysis is performed. The analysis is conducted such that, one parameter of predictive variables is eliminated one by one to observe how the statistical error parameters of the developed GMDH network predictions is changed. Tables 4 and 5 present the sensitivity analysis of GMDH network for reinforced members with and without stirrups, respectively. Parameters *L* and *h* in these tables are span length and overall depth of cross section, respectively. In case of members with stirrups, results of sensitivity analysis show that the ρ_{FW} (R^2=0.86, *RMSE*=36.90, and *MSE*=13.64) is the most effective parameter on the shear strength whereas the f_{Fuw} (R^2=0.95, *RMSE*=21.33, *MSE*=450.77) has the least influence on shear strength modeled by the GMDH. The other effective parameters on shear strength are *a/d*, *d*, *b*, f_c,

Table 4 Sensitivity analysis of the governing parameters for reinforced members with stirrups

Model with stirrup	*MSE*	*RMSE*	*RRMSE*	R^2
Model in absence of *b*	878.1830	29.6452	0.2974	0.9116
Model in absence of ρ_{FW}	1362.2	36.9078	0.3703	0.8629
Model in absence of f_c	712.5966	26.6945	0.2679	0.9283
Model in absence of ρ_F	670.8496	25.9008	0.2599	0.9325
Model in absence of E_F	626.1115	25.0222	0.2511	0.9370
Model in absence of *a/d*	1240.8	35.2250	0.3535	0.8751
Model in absence of E_{FW}	638.1424	25.2615	0.2535	0.9358
Model in absence of f_{Fuw}	450.7797	21.2316	0.2130	0.9546
Model in absence of *d*	1043.7	32.3068	0.3242	0.8949

Table 5 Sensitivity analysis of the governing parameters for reinforced members without stirrups

Model without stirrups	*MSE*	*RMSE*	RRMSE	R^2
Model in absence of ρ_F	329.4017	18.1494	0.3272	0.8929
Model in absence of *h*	159.3090	12.6218	0.2276	0.9482
Model in absence of *d*	162.4776	12.7467	0.2298	0.9472
Model in absence of *L*	162.1723	12.7347	0.2296	0.9473
Model in absence of *a/d*	443.6473	21.0629	0.3797	0.8558
Model in absence of f_c	162.4776	12.7467	0.2298	0.9472
Model in absence of *b*	289.0096	17.0003	0.3065	0.9061
Model in absence of A_F	168.0521	12.9635	0.2337	0.9454
Model in absence of E_F	234.7373	15.3211	0.2762	0.9237
Model in absence of f_{Fu}	162.3883	12.7432	0.2297	0.9472

ρ_F, E_F, and E_{FW} ranked from higher to lower values, respectively. It should be mentioned that the contributions of *d*, ρ_{FW} and *a/d* are remarkably more than other parameters. Based on the Table 5, for members without stirrups, results of sensitivity analysis indicated that the *a/d* (R^2=0.85, *RMSE*=21.06, *MSE*=443.64) is the most effective parameter on the shear strength whereas the *h* (R^2=159.30, *RMSE*=12.62, *MSE*=443.64) has the least influence on shear strength for the GMDH model. The other effective parameters on shear strength were ρ_F, *b*, E_F, A_F, *d*, f_{Fu}, f_c, and *L* ranked from higher to lower values, respectively. It should be noted that results show that the *a/d* and ρ_F were the most important parameters whereas the other parameters had marginally effects on the prediction of shear strength.

To obtain new contribution of this study, effects of the GMDH output model and the output of the most common empirical equations on the variations of *a/d*, f_c, ρ_F, and ρ_{FW} are investigated. In this way, the discrepancy ratio (DR), known as the ratio between predicted and measured values, is employed to quantify the sensitivity of the proposed models to input parameters. DR value of 1 show a perfect agreement, while values greater (or smaller) than 1 indicate overestimation (or underestimation) of the shear strength.

Fig. 4 depicts variations of DR values versus the *a/d*, f_c, ρ_F, ρ_{FW} parameters for GMDH and three empirical models (which had the more accuracy than other models according to the Table 3) for FRP-reinforced members with stirrups. As it is shown, errors of CSA S6 and Hoult *et al*. (Hoult, Sherwood *et al.* (2008)) models are remarkably sensitive to change of mentioned parameters. It can be interpreted that

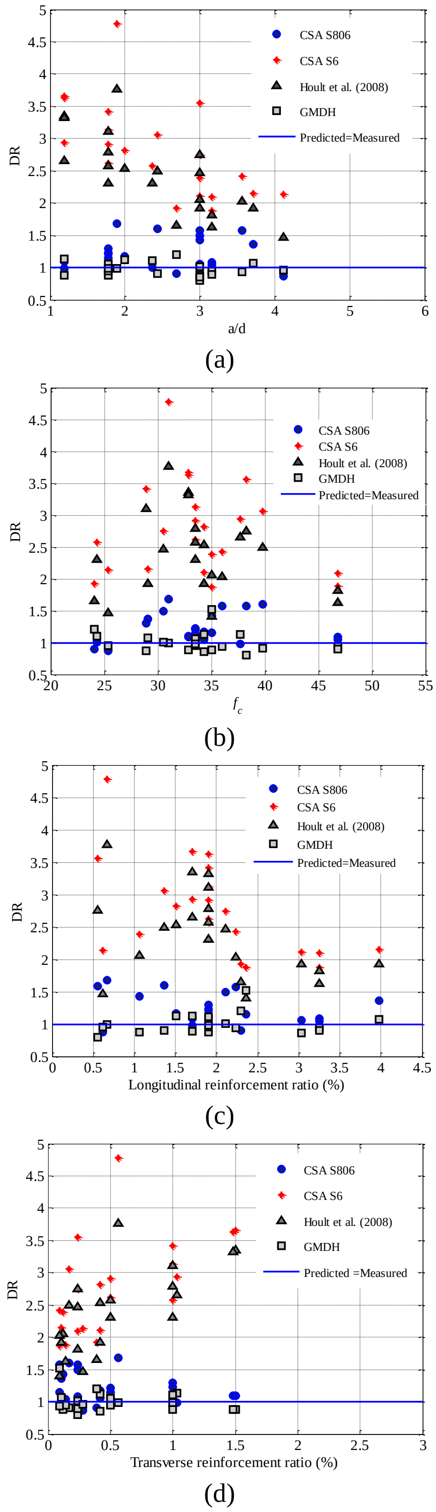


Fig. 4 Variation of discrepancy ratio (DR) between predicted and measured shear strength of reinforced members with stirrups with (a) *a/d*; (b) f_c; (c) ρ_f; and (d) ρ_t

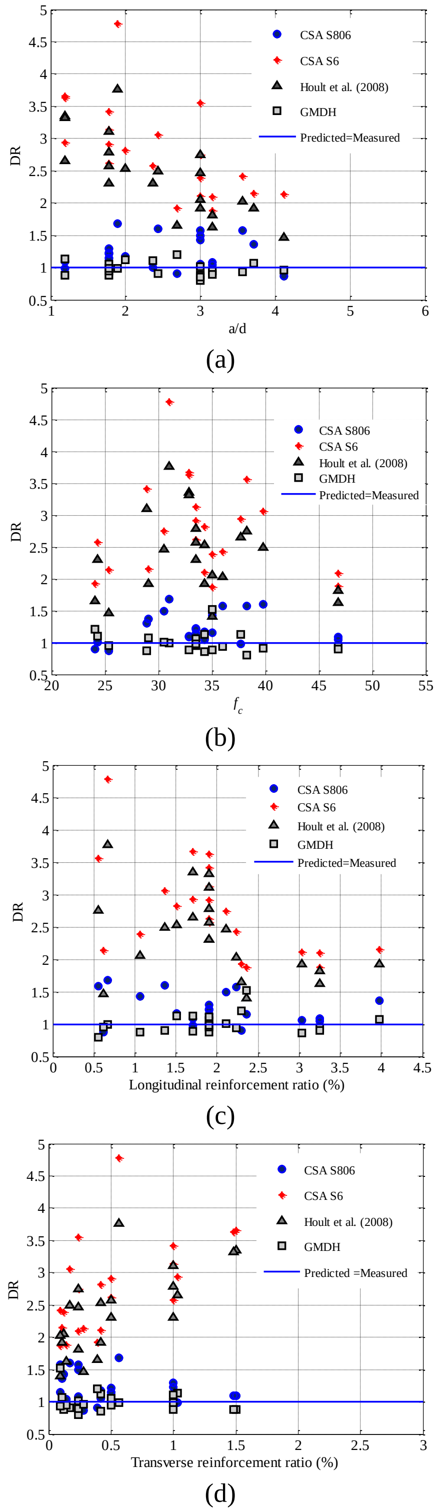


Fig. 5 Variation of discrepancy ratio (DR) between predicted and measured shear strength of reinforced members without stirrups with (a) *a/d*; (b) f_c; (c) ρ_f and (d) f_{Fu}

the mentioned models did not correctly incorporate predictive parameters in their models. In general, for having a good performance, model errors should be independent of the input parameters or less sensitive to them (Sahay and Dutta (2009)). The CSA S806 and GMDH errors are less sensitive to change of input parameters than other models. However, the CSA S806 errors also slightly show tendency to change of predictive parameters. As shown in Fig. 4, the GMDH errors in prediction of shear strength are completely independent of predictive parameters.

In Fig. 5, the variation of DR values with predictive parameters such as *a/d*, f_c, ρ_F, and f_{FU} for reinforced members without stirrups for different approaches are presented. From this figure, it can be seen that the scattering of JSCE and Hoult *et al*. (Hoult *et al*. 2008) are notable. These scattering are increased especially with variation of longitudinal reinforcement ratio (ρ_F). It can be interpreted that this predictive parameter is not incorporated appropriately in the mentioned models. As shown, the CSA S806 and GMDH model show less sensitivity to variation of predictive parameters. However, the GMDH model shows the better performance than CSA S806 in this aspect and its errors are completely independent of predictive parameters, especially longitudinal reinforcement ratio.

5. Conclusions

The GMDH network is utilized as a new alternative approach to formulate shear strength capacity of FRP-RC members with and without stirrups. Comprehensive datasets from literature were employed to develop the models for both reinforced members with stirrups and without stirrups. Effective parameters on shear strength including the effective depth, shear span-to-depth ratio, modulus of elasticity and ratio of the FRP longitudinal and transversal reinforcement, and compressive concrete strength were considered in modeling process. The proposed GMDH models give reliable estimation of the shear capacity of FRP-reinforced members. The GMDH models also produced better outcome than several codes of practice, i.e., ACI, CSA S806, CSA S6, CNR, JSCE, and Hoult *et al*. model. The developed GMDH for FRP-RC members without stirrups improved the RMSE and R2 values by 58% and 35%, respectively, in respect to the best models among the mentioned design codes. In case of FRP-RC members with stirrups, the developed GMDH improved the RMSE and R^2 values by 46% and 11.7%, respectively, in respect to CSA S806 models, which was the most accurate model among the other design equations. To further confirmation of the predictive performances of developed GMDH model, two well-known soft computing approaches including ANN and ANFIS are also developed in this study. The results show comparative performance of the GMDH network.

The relative importance of input parameters in the prediction of shear capacity is evaluated through sensitivity analysis. It is found that the shear span to depth ratio and longitudinal reinforcement ratio was the most important predictive parameters in prediction of shear strength of reinforced members without stirrups. In case of members with stirrups, the effective depth, the transverse reinforcement ratio, and shear span to depth ratio were the most effective parameters in prediction of shear capacity. In general, the results of sensitivity and parametric analyses indicate that the GMDH model is capable of capturing the underlying physical behavior of the shear strength of FRP-RC members.

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