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Classical shell theory for instability analysis of concrete pipes conveying nanofluid

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Abstract. This paper deals with the instability analysis of concrete pipes conveying viscous fluid-nanoparticle mixture. The fluid is mixed by AL_2O_3 nanoparticles where the effective material properties of fluid are obtained by mixture rule. The applied force by the internal fluid is calculated by Navier-Stokes equation. The structure is simulated by classical cylindrical shell theory and using energy method and Hamilton's principle, the motion equations are derived. Based on Navier method, the critical fluid velocity of the structure is calculated and the effects of different parameters such as fluid velocity, volume percent of nanoparticle in fluid and geometrical parameters of the pipe are considered. The results present that with increasing the volume percent of nanoparticle in fluid, the critical fluid velocity increase.

Keywords: critical fluid velocity; concrete pipes classical shell theory; nanofluid; exact solution

1. Introduction

Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines, engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, in grinding, machining and in boiler flue gas temperature reduction. They exhibit enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid. Knowledge of the rheological behaviour of nanofluids is found to be critical in deciding their suitability for convective heat transfer applications. Nanofluids also have special acoustical properties and in ultrasonic fields display additional shear-wave reconversion of an incident compressional wave; the effect becomes more pronounced as concentration increases.

Pipes conveying fluid are of considerable interest in many fields such as oil and gas pipelines, pump discharge lines, propellant lines, reactor system components and so forth. However, the fluid-induced dynamics analysis of pipes is studied by researchers. Most of the studies in this field are reviewed by Paidoussis (1993) and Amabili (2008). Toorani and Lakis (2001) studied dynamic analysis of anisotropic cylindrical shells containing flowing fluid. Zhang *et al*. (2001) used a method for the dynamic analysis of initially tensioned orthotropic thin-walled cylindrical tubes conveying steady fluid flow, based on Sanders' non-linear theory of thin shells and the classical potential flow theory. A coupled formulation based on the semi-analytical finite element technique was developed by Jayaraj *et al*. (2002) for composite shells conveying fluid. A linear analysis of the vibratory behaviour of initially tensioned orthotropic circular cylindrical shells conveying a compressible inviscid fluid was presented by Zhang *et al*. (2002). A coupled fluid structure interaction problem was analyzed by Kadoli and Ganesan (2003) using semi-analytical finite element method involving composite cylindrical shells conveying hot fluid for free vibration and buckling behavior. The stability and parametric resonances of supported pipes conveying pulsating fluid were studied by Song and Zhang (2001) via numerical methods. The stability and chaotic motions of a standing pipe conveying fluid was studied by Wang and Ni (2006) based on numerical calculations, bifurcation diagram, time trace and phase portrait of the oscillations. The post-divergence behaviour of extensible fluid-conveying pipes supported at both ends was studied by Modarres-Sadeghi and Païdoussis (2009) using the weakly nonlinear equations of motion of Semler, Li and Païdoussis. The investigation of the three-dimensional nonlinear dynamics of a fluid-conveying pipe undergoing overall motions was carried out by Meng *et al*. (2011). The differential transformation method (DTM), was generalized by Ni *et al*. (2011) to analyze the free vibration problem of pipes conveying fluid with several typical boundary conditions. The application of transfer matrix method (TMM) to the vibration analysis of three-dimensional (3D) pipelines conveying fluid was performed by Dai *et al*. (2012). A fully three-dimensional, geometrically exact theory for flexible tubes conveying fluid was derived by Gay-Balmaz and Putkaradze (2015). Nonlinear equations of three-dimensional motion were established by Zhang *et al*. (2016) for fluid-conveying pipes with general boundary conditions. The effect of aspect ratio of length to diameter on the dynamic response of a fluid-conveying pipe was studied by Gu *et al*. (2016) using the Timoshenko beam model. Vibration and stability of

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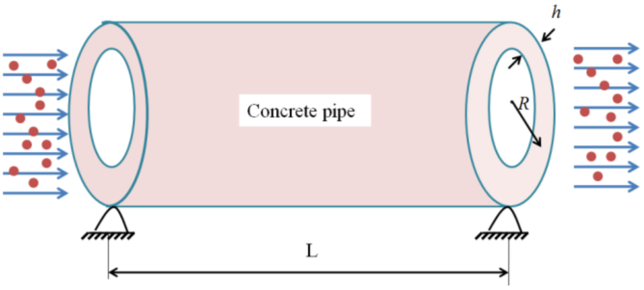

Fig. 1 A concrete pipe conveying fluid mixed by AL_2O_3 nanoparticles

concrete pipes reinforced with carbon nanotubes (CNTs) and Fe_2O_3 nanoparticles conveying fluid were presented by Zamani Nouri (2017, 2018).

In this paper, vibration and stability of concrete pipes conveying nanofluid is presented based on classical shell model. hTe force of the fluid is calculated by Navier-Stokes equation. Applying exact solution, the frequency and critical fluid velocity are obtained and the effects of the fluid velocity, volume percent of nanoparticle in fluid and geometrical parameters of the pipe are considered.

2. Mathematical modeling

As shown in Fig. 1, a concrete pipe with length of L, radius of R and thickness of h is considered. The pipe is conveying fluid which is mixed by AL_2O_3 nanoparticles.

Based on the classical shell theory, the displacement field can be written as (Reddy 1984)

$$\begin{aligned} u(x,\theta,z) &= u(x,\theta) - z\frac{\partial w(x,\theta)}{\partial x}, \\ v(x,\theta,z) &= v(x,\theta) - z\frac{\partial w(x,\theta)}{R\partial\theta}, \\ w(x,\theta,z) &= w(x,\theta). \end{aligned} \tag{1}$$

where (u,v,w) denote the displacement components at an arbitrary point (x,θ,z) in the concrete pipe, The strain-displacement relations can be expressed as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{x\theta} \end{Bmatrix} - z\begin{Bmatrix} k_{xx} \\ k_{\theta\theta} \\ k_{x\theta} \end{Bmatrix} = \left(\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{R\partial\theta} & \frac{1}{R} \\ \frac{\partial}{2R\partial\theta} & \frac{\partial}{2\partial x} & 0 \end{bmatrix} - z\begin{bmatrix} 0 & 0 & \frac{\partial^2}{\partial x^2} \\ 0 & 0 & \frac{\partial^2}{R^2\partial\theta^2} \\ 0 & 0 & 2\frac{\partial^2 w}{R\partial x\partial\theta} \end{bmatrix} \right) \times \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \tag{2}$$

The stress-strain relation of the structure is

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{x\theta} \end{Bmatrix}, \tag{3}$$

Noted that C_{ij} $(i,j=1,2,\ldots,6)$ are elastic constants. The strain energy of the structure can be written as

$$\begin{aligned} U = \int_A \Bigg(& N_x \frac{\partial u}{\partial x} - M_x \frac{\partial^2 w}{\partial x^2} + N_\theta\left(\frac{\partial v}{R\partial\theta} + \frac{w}{R}\right) \\ & - M_\theta \frac{\partial^2 w}{R^2\partial\theta^2} + N_{x\theta}\left(\frac{\partial u}{R\partial\theta} + \frac{\partial v}{\partial x}\right) - 2M_{x\theta}\frac{\partial^2 w}{R\partial\theta\partial x}\Bigg) dA, \end{aligned} \tag{4}$$

where the stress resultants can be defined as

$$\begin{Bmatrix} N_{xx} \\ N_{\theta\theta} \\ N_{x\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \end{bmatrix} dz, \tag{5}$$

$$\begin{Bmatrix} M_{xx} \\ M_{\theta\theta} \\ M_{x\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \end{bmatrix} zdz, \tag{6}$$

The kinetic energy of the concrete pipe may be written as

$$\begin{aligned} K = \int \Bigg(& \frac{\rho}{2}\left(\frac{h^3}{12}\left(\left(\frac{\partial^2 u}{\partial t\,\partial x}\right)^2 + \left(\frac{\partial^2 w}{\partial t\partial\theta}\right)^2 \right)\right) \\ & + h\left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \right)\Bigg) dA. \end{aligned} \tag{7}$$

The external work due to the internal fluid can be obtained from

$$W = \int_A (P_{Fluid}) w dA. \tag{8}$$

where P_{Fluid} can be obtained from the Navier-Stokes equation as (Zamani Nouri 2018)

$$\begin{aligned} P_{Fluid} = \Bigg[& -\rho_{eff} h_f \left(\frac{\partial^2 w}{\partial t^2} + 2v_x \frac{\partial^2 w}{\partial x\,\partial t} + v_x^2 \frac{\partial^2 w}{\partial x^2} \right) \\ & + \frac{h_f}{R^2}\frac{\partial}{\partial\theta}\left(\mu_{eff}\left(\frac{\partial^2 w}{\partial\theta\partial t} + v_x \frac{\partial^2 w}{\partial\theta\partial x} \right)\right) \\ & - \frac{2h_f}{R}\left(\mu_{eff}\left(\frac{\partial w}{\partial t} + v_x \frac{\partial w}{\partial x} \right)\right) + h_f \frac{\partial}{\partial x}\left(\mu_{eff}\left(\frac{\partial^2 w}{\partial x\,\partial t} + v_x \frac{\partial^2 w}{\partial x^2} \right)\right)\Bigg] \end{aligned} \tag{9}$$

where the effective viscosity $(\mu_{\mu_{eff}})$ and density $(\rho_{\mu_{eff}})$ of the fluid-nanoparticle may be calculated from mixture law as follows

$$\rho_{eff} = \phi\rho_n + (1-\phi)\rho_f, \tag{10}$$

$$\mu_{eff} = \phi\mu_n + (1-\phi)\mu_f, \tag{11}$$

where ρ_n, ρ_f, μ_n, μ_f and ϕ are nanoparticle density, fluid density, nanoparticle viscosity, fluid viscosity and volume fraction of nanoparticle in the fluid respectively.

The governing equations can be derived by Hamilton's principal as follows

$$\int_0^t (\delta U - \delta W - \delta K) dt = 0. \tag{12}$$

Substituting Eqs. (4), (7) and (8) into Eq. (12) yields the following motion equations

$$hC_{11}\frac{\partial^2 u}{\partial x^2} + \frac{hC_{12}}{R}\left(\frac{\partial^2 v}{\partial x\partial\theta} + \frac{\partial w}{\partial x}\right) +$$

$$\frac{hC_{66}}{R}\left(\frac{\partial^2 u}{R\partial\theta^2}+\frac{\partial^2 v}{\partial x\partial\theta}\right)=\rho h\frac{\partial^2 u}{\partial t^2}, \tag{13}$$

$$\frac{hC_{12}}{R}\frac{\partial^2 u}{\partial x\partial\theta}+\frac{hC_{22}}{R^2}\left(\frac{\partial^2 v}{\partial\theta^2}+\frac{\partial w}{\partial\theta}\right)+ hC_{66}\left(\frac{\partial^2 u}{R\partial x\partial\theta}+\frac{\partial^2 v}{\partial x^2}\right)=\rho h\frac{\partial^2 v}{\partial t^2}, \tag{14}$$

$$\begin{aligned}&\frac{h^3}{12}\left(-C_{11}\frac{\partial^4 w}{\partial x^4}-\frac{C_{12}}{R^2}\frac{\partial^4 w}{\partial x^2\partial\theta^2}\right)+\\ &\frac{h^3}{12}\left(-\frac{C_{12}}{R^2}\frac{\partial^4 w}{\partial x^2\partial\theta^2}-\frac{C_{22}}{R^4}\frac{\partial^4 w}{\partial\theta^4}\right)--\frac{hC_{12}}{R}\frac{\partial u}{\partial x}\\ &-\left(\frac{hC_{22}}{R}\right)\left(\frac{\partial v}{\partial\theta}+\frac{w}{R}\right)-\rho_f h\left[\frac{\partial^2 w}{\partial t^2}+2v_x\frac{\partial^2 w}{\partial x\partial t}+v_x^2\frac{\partial^2 w}{\partial x^2}\right]\\ &-\mu_0 h\left[\frac{\partial^3 w}{\partial x^2\partial t}+v_x\frac{\partial^3 w}{\partial x^3}+\frac{1}{R^2}\left(\frac{\partial^3 w}{\partial\theta^2\partial t}+v_x\frac{\partial^3 w}{\partial\theta^2\partial x}\right)\right]\\ &+\eta hH_x^2\left(\frac{\partial^2 w}{\partial x^2}\right)=\rho h\frac{\partial^2 w}{\partial t^2}\end{aligned} \tag{15}$$

3. Solution method

Based on Navier method, the dynamic amplitudes can be written for simply supported boundary conditions as

$$\mathrm{d}=\begin{Bmatrix}u\\ v\\ w\end{Bmatrix}=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\begin{Bmatrix}A_1\cos(\frac{m\pi x}{a})\cos(n\theta)\cos(\omega t)\\ A_2\sin(\frac{m\pi x}{a})\sin(n\theta)\cos(\omega t)\\ A_3\sin(\frac{m\pi x}{a})\cos(n\theta)\cos(\omega t)\end{Bmatrix}, \tag{16}$$

where ω represents vibration frequency of the pipe, m and n are half axial and circumferential wave numbers, respectively. Substituting Eq. (16) f into motion equations yields

$$[K][d]+[C]\omega+[M]\omega^2=0, \tag{17}$$

where [K], [C] and [M] are stiffness, damp and mass matrixes, respectively; $[d]=[A_1,A_2,A_3]$ is the dynamic vector. However, the critical fluid velocity and frequency can be obtained from Eq. (17).

4. Numerical results and discussion

A concrete pipe with length to radius ratio L/R=4 and thickness to radius ratio h/R=0.03 is considered with Young's modulus of E=20 GPa and Poisson's ratios of v=0.3.

To the best author's knowledge, no similar publications for vibration and instability of concrete pipes cannot found directly. However, the present work could be partially validated based on a simplified analysis suggested by Loy *et al*. (1997), Qu *et al*. (2013), Zhang *et al*. (2001b), Tang *et al*. (2016). However, vibration of simply supported classical cylindrical shells is investigated where the nonlinear terms in motion equations, fluid and nanoparticles are ignored. The structure parameters of the classical shell assumed as h/R=0.01, L/R=20, E=210 GPa, v=0.3, ρ=7850 Kg/m^3. A non-dimensional frequency is defined as. Table 3 illustrates the frequency of pipe for classical theory. As can be seen, the obtained results are close to those expressed in Loy *et al*. (1997), Qu *et al*. (2013), Zhang *et al*. (2001b), Tang *et al*. (2016), indicating validation of our work.

Table 1 Validation of present work

n	Loy *et al.* (1997)	Qu *et al.* (2013)	Zhang *et al.* (2001)	Tang *et al.* (2016)	Present
1	0.016101	0.016103	0.016101	0.016101	0.016234
2	0.009388	0.009382	0.009382	0.011225	0.011713
3	0.022108	0.022105	0.022105	0.022310	0.024902
4	0.042097	0.042095	0.042095	0.042139	0.044934
5	0.068008	0.068008	0.068008	0.068024	0.070855
6	0.099730	0.099730	0.099731	0.099738	0.102593
7	0.0137239	0.137239	0.0137240	0.137240	0.140108
8	0.180527	0.180528	0.180527	0.180530	0.183402
9	0.229594	0.229594	0.229596	0.229596	0.232472
10	0.284435	0.284436	0.284438	0.284439	0.287318

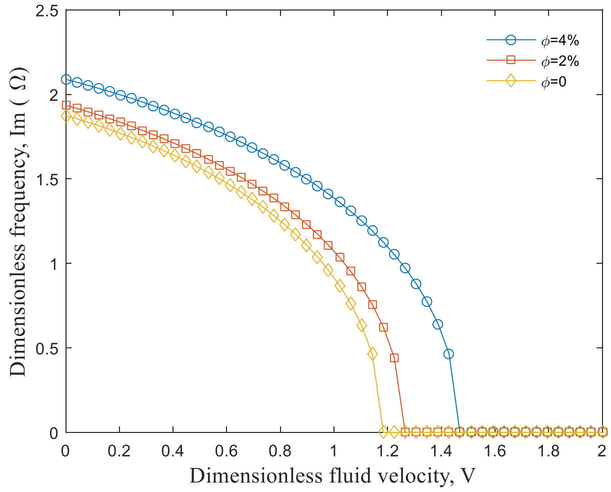

Fig. 2 The effect of nanoparticles volume percent on the frequency of structure

Figs. 2 and 3 show the variation of imaginary and real parts of dimensionless eigenvalues ($\Omega=\sqrt{E/\rho}\,\omega$) for the different values of nanoparticles volume percent versus dimensionless fluid velocity ($V=\sqrt{\rho/E}\,v_x$,).As can be seen, Im(Ω) decreases with increasing V, while the Re(Ω) remains zero. These imply that the system is stable. When the natural frequency becomes zero, critical velocity is reached, which the system loses its stability due to the divergence via a pitchfork bifurcation. Hence, the Eigen frequencies have the positive real parts, which the system becomes unstable. In this state, both real and imaginary parts of frequency become zero at the same point. Therefore, with increasing flow velocity, system stability decreases and became susceptible to buckling. As it can be seen, the non-dimensional frequency and critical fluid velocity increases with increasing the volume percent of nanoparticles.

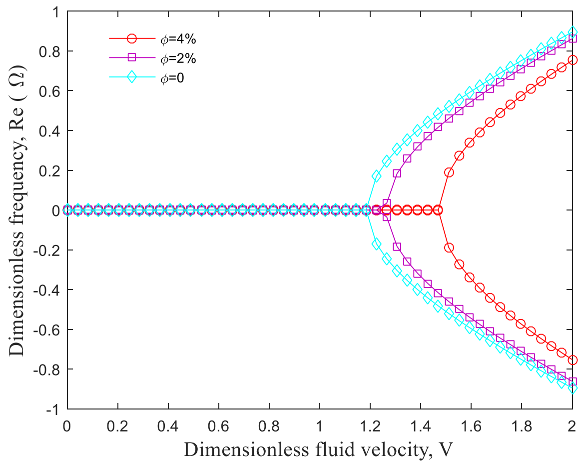

Fig. 3 The effect of nanoparticles volume percent on the damping of structure

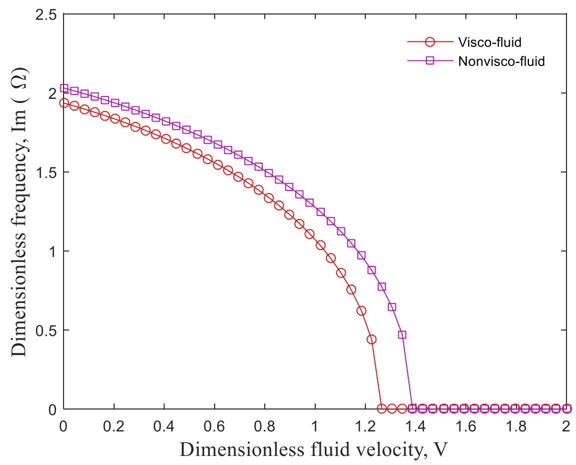

Fig. 4 The effect of fluid viscosity on the frequency of structure

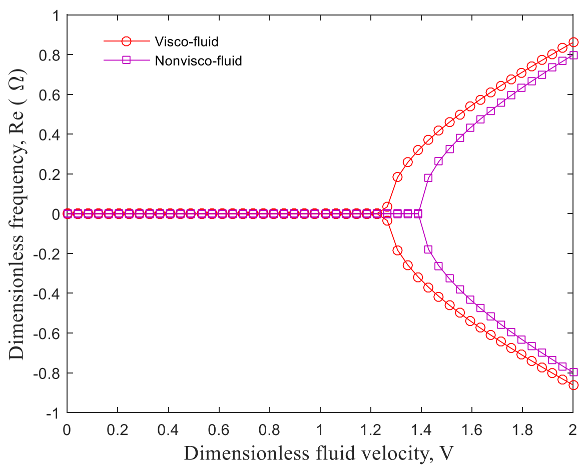

Fig. 5 The effect of fluid viscosity on the damping of structure

The effect of the fluid viscosity on the dimensionless frequency and damping of the concrete pipe with respect to dimensionless flow velocity is shown in Figs. 4 and 5. It is observed that considering fluid viscosity, the dimensionless frequency and critical fluid velocity decreases.

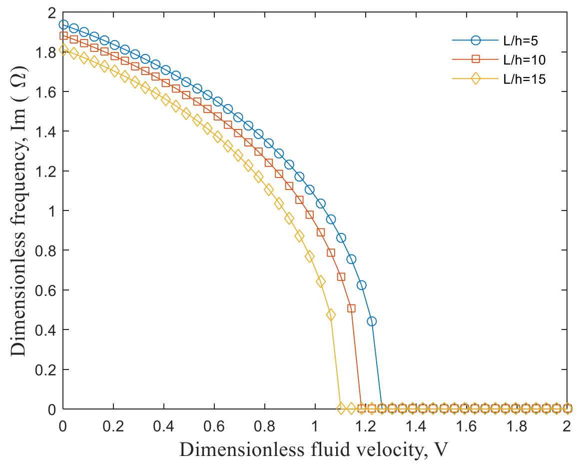

Fig. 6 The effect of length to thickness ratio on the frequency of structure

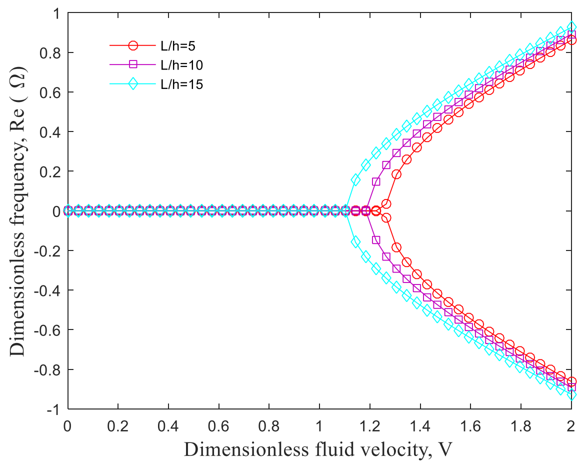

Fig. 7 The effect of length to thickness ratio on the damping of structure

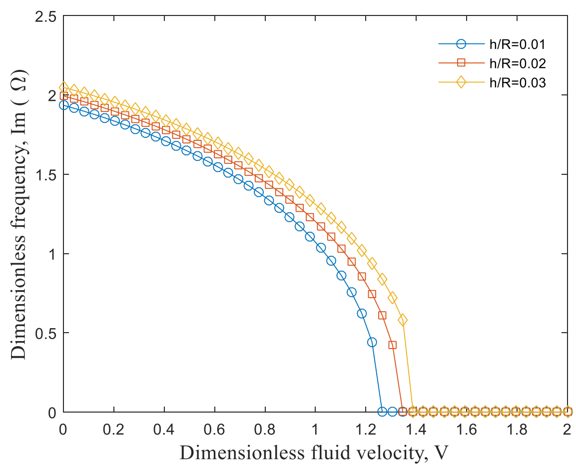

Fig. 8 The effect of thickness to radius ratio on the frequency of structure

Figs. 6 and 7 present the dimensionless frequency and damping of the structure versus dimensionless flow velocity for different length to thickness ratio of the pipe. It can be seen that the dimensionless frequency and critical fluid velocity of the pipe decreases with increasing the length to

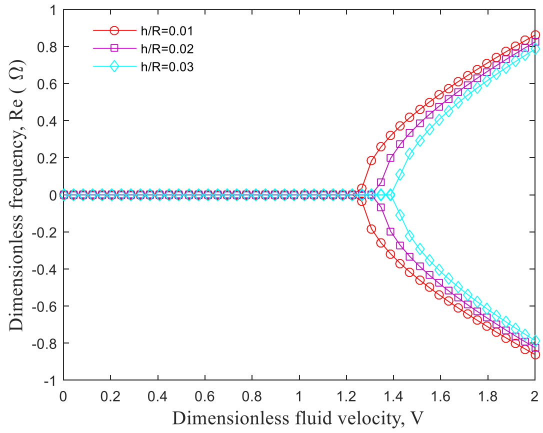

Fig. 9 The effect of thickness to radius ratio on the damping of structure

thickness ratio of the pipe. Figs. 8 and 9 present the dimensionless frequency and damping of the structure versus dimensionless flow velocity for different thickness to radius ratio of the pipe. As can be seen, with increasing the thickness to radius ratio of the pipe, the dimensionless frequency and critical fluid velocity increases. It is since with increasing the length to thickness ratio of the pipe and decreasing the thickness to radius ratio of the pipe, the stiffness decreases.

5. Conclusions

In this paper, vibration and stability of concrete pipe conveying nanofluid was presented. The force of the fluid was calculated by Navier-Stokes equation. Based on classical shell model, energy method and Hamilton's principle, the motion equations were derived. Applying exact solution, the frequency and critical fluid velocity of the structure were obtained and the effects of volume percent of nanoparticles, fluid velocity and viscosity as well as geometrical parameters of the pipes were considered. Results show that with increasing flow velocity, system stability decreases and became susceptible to buckling. The non-dimensional frequency and critical fluid velocity increases with increasing the volume percent of nanoparticles. It was observed that considering fluid viscosity, the dimensionless frequency and critical fluid velocity decreases. It can be seen that the dimensionless frequency and critical fluid velocity of the pipe decreases with increasing the length to thickness ratio of the pipe. In addition, with increasing the thickness to radius ratio of the pipe, the dimensionless frequency and critical fluid velocity increases.

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